Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.5-Secant/118-4.5.1.2-d-sec-^n-a+b-sec-^m

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [879]. This is test number [118].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (879)	0.00 (0)
Mathematica	98.86 (869)	1.14 (10)
Maple	83.62 (735)	16.38 (144)
Fricas	69.28 (609)	30.72 (270)
Mupad	36.75 (323)	63.25 (556)
Maxima	36.52 (321)	63.48 (558)
Giac	30.60 (269)	69.40 (610)
Sympy	5.57 (49)	94.43 (830)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

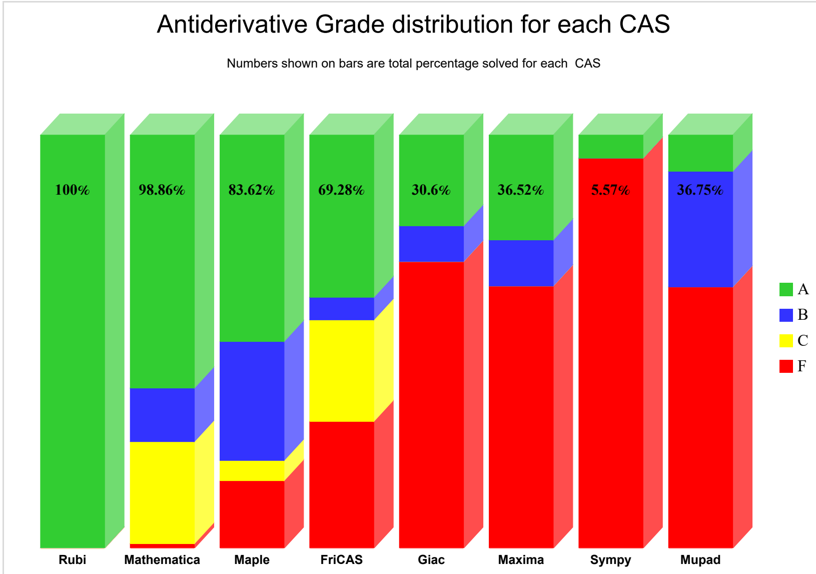
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

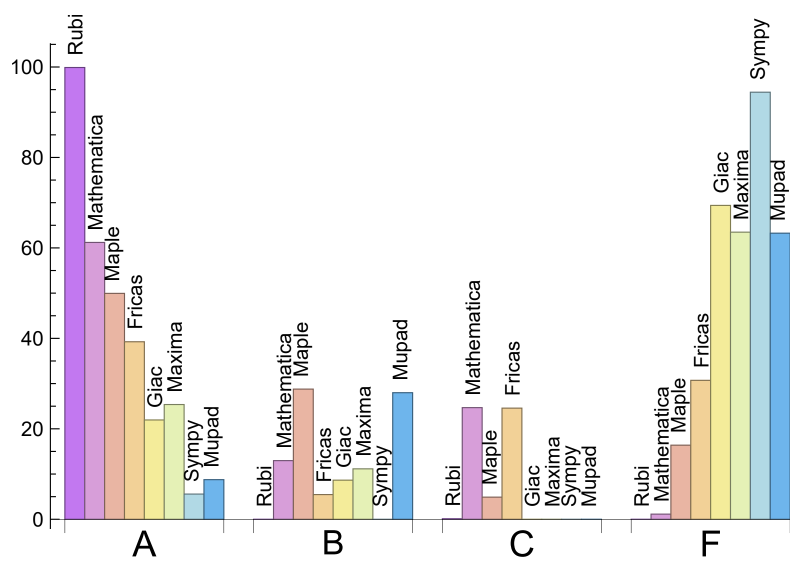
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.89	0.00	0.11	0.00
Mathematica	61.21	12.97	24.69	1.14
Maple	49.94	28.78	4.89	16.38
Fricas	39.25	5.46	24.57	30.72
Maxima	25.37	11.15	0.00	63.48
Giac	21.96	8.65	0.00	69.40
Mupad	N/A	27.99	0.00	63.25
Sympy	5.57	0.00	0.00	94.43

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	10	100.00 %	0.00 %	0.00 %
Maple	144	100.00 %	0.00 %	0.00 %
Fricas	270	64.81 %	35.19 %	0.00 %
Giac	610	93.93 %	5.08 %	0.98 %
Maxima	558	84.77 %	7.71 %	7.53 %
Sympy	830	67.83 %	16.63 %	15.54 %
Mupad	556	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

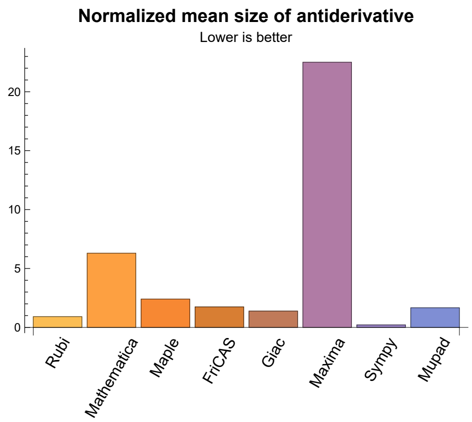
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

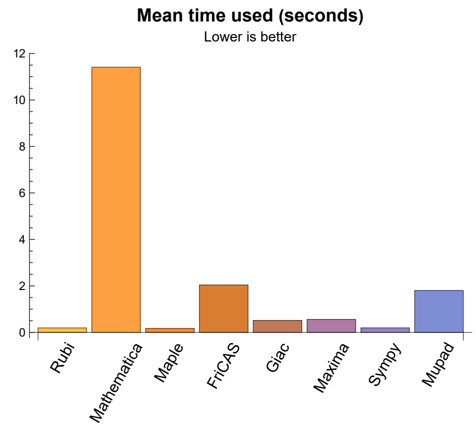
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.19	152.93	0.91	128.00	1.00
Mathematica	11.41	1206.94	6.30	154.00	1.06
Maple	0.17	501.73	2.40	214.00	1.72
Maxima	0.56	3928.25	22.51	104.00	1.21
Fricas	2.04	242.61	1.74	175.00	1.44
Sympy	0.19	3.88	0.21	0.00	0.00
Giac	0.52	155.65	1.39	110.00	1.30
Mupad	1.80	242.43	1.67	88.00	0.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
5
10
15
20
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
12
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {159, 160, 161, 162}

Mathematica {111, 112, 123, 131, 139, 146, 147, 151, 152, 157, 158, 163, 164, 264, 267, 270, 271, 280, 281, 282, 283, 284, 285, 287, 289, 294, 297, 298, 301, 302, 305, 306, 307, 310, 311, 314, 315, 318, 319, 320, 329, 330, 331, 333, 334, 335, 337, 338, 339, 345, 346, 347, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 384, 392, 395, 408, 415, 416, 417, 426, 427, 428, 433, 434, 440, 441, 442, 531, 535, 537, 538, 539, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 560, 562, 563, 564, 568, 570, 571, 572, 573, 576, 618, 626, 627, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 837, 840, 841, 842, 843, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 859, 860, 861, 862, 863, 864, 866, 867, 868, 869, 870, 871, 872, 873, 874, 878, 879}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

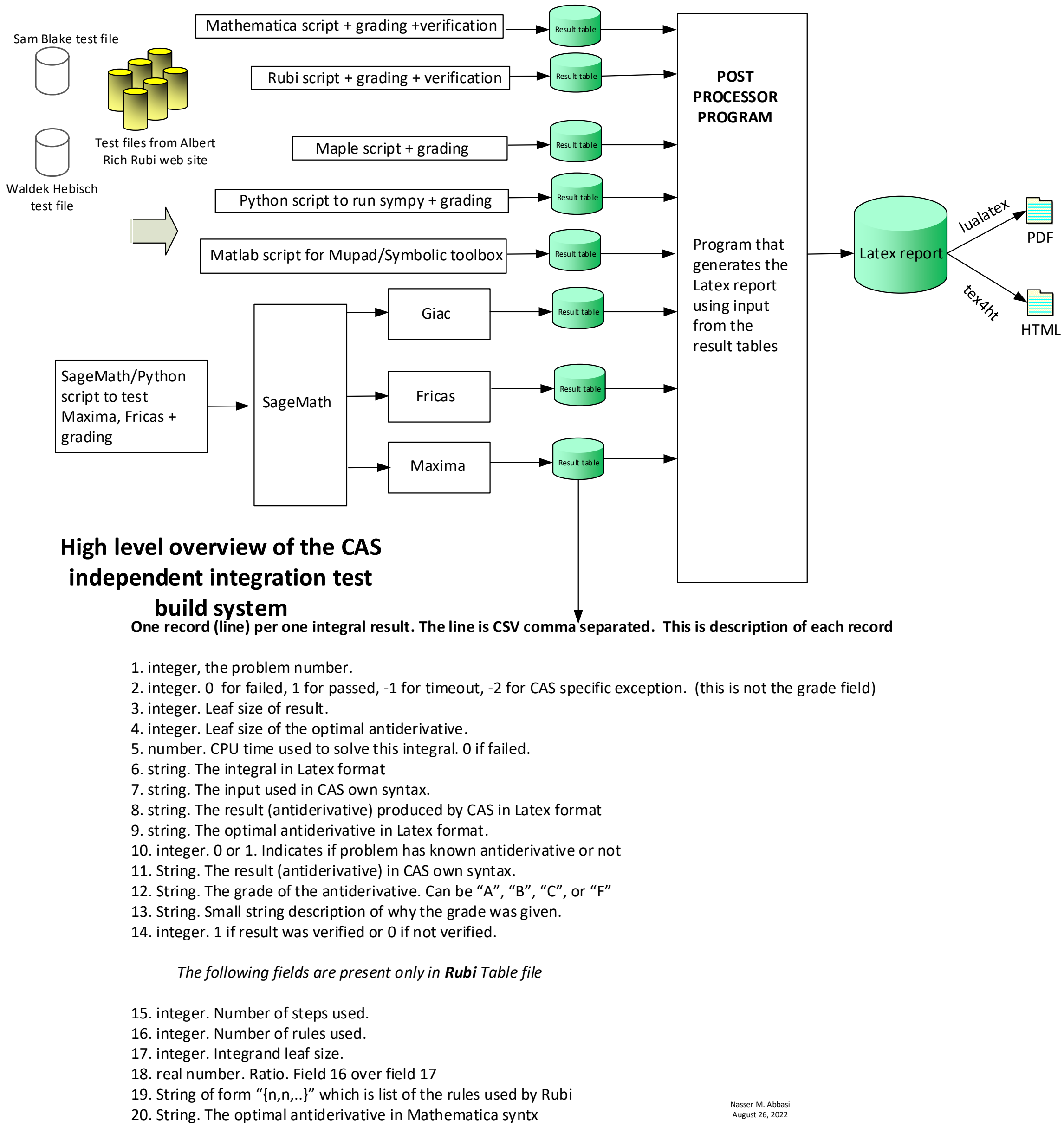
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

B grade: { }

C grade: { 286 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 36, 37, 38, 39, 40, 41, 46, 47, 49, 50, 51, 55, 56, 57, 59, 60, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 137, 140, 165, 166, 167, 168, 169, 170, 171, 175, 189, 204, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 286, 291, 295, 296, 299, 300, 303, 304, 308, 309, 312, 313, 316, 317, 341, 342, 343, 344, 363, 385, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 439, 440, 443, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 531, 532, 533, 534, 537, 538, 539, 540, 544, 545, 546, 547, 549, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 570, 571, 572, 573, 574, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 627, 629, 630, 631, 632, 633, 636, 637, 638, 639, 644, 645, 646, 649, 650, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 782, 783, 784, 785, 786, 787, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 875, 876, 877 }

B grade: { 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 42, 43, 44, 45, 48, 52, 53, 54, 58, 61, 62, 70, 71, 77, 78, 80, 88, 146, 147, 151, 152, 157, 158, 163, 164, 253, 260, 280, 281, 282, 283, 284, 297, 298, 301, 302, 305, 306, 310, 311, 314, 315, 318, 319, 329, 330, 331, 333, 334, 335, 337, 338, 339, 345, 346, 347, 348, 431, 438, 480, 529, 530, 551, 568, 576, 616, 626, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 822, 878, 879 }

C grade: { 96, 97, 98, 111, 112, 113, 114, 115, 117, 118, 123, 131, 133, 138, 139, 141, 142, 143, 144, 145, 148, 149, 150, 153, 154, 155, 156, 159, 160, 161, 162, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 233, 234, 244, 272, 273, 274, 275, 276, 277, 278, 279, 285, 287, 289, 290, 294, 307, 320, 321, 322, 325, 326, 327, 328, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 418, 419, 441, 442, 505, 535, 536, 541, 542, 543, 548, 550, 552, 553, 560, 561, 567, 569, 575, 577, 578, 628, 634, 635, 640, 641, 642, 643, 647, 648, 654, 655, 661, 662, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874 }

F grade: { 288, 292, 293, 323, 324, 332, 336, 340, 444, 445 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 99, 100, 101, 102, 107, 108, 109, 110, 112, 116, 118, 121, 123, 142, 167, 168, 169, 170, 171, 176, 177, 178, 182, 183, 184, 185, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 248, 249, 250, 251, 253, 254, 255, 256, 257, 260, 261, 262, 263, 264, 269, 270, 271, 351, 352, 354, 355, 359, 360, 361, 367, 368, 369, 370, 374, 375, 376, 377, 378, 379, 381, 382, 383, 384, 386, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 404, 405, 406, 407, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 534, 558, 559, 581, 582, 583, 584, 585, 589, 590, 592, 596, 598, 599, 604, 605, 606, 607, 608, 611, 612, 650, 678, 679, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793, 794, 795, 798, 799, 806, 813, 819, 820, 858 }

B grade: { 94, 95, 96, 97, 98, 103, 104, 105, 106, 111, 113, 114, 115, 117, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 165, 166, 172, 173, 174, 175, 179, 180, 181, 186, 187, 188, 189, 194, 204, 210, 218, 219, 220, 226, 227, 228, 235, 236, 243, 244, 245, 246, 247, 252, 258, 259, 265, 266, 267, 268, 353, 356, 357, 358, 362, 363, 364, 365, 366, 371, 372, 373, 380, 385, 387, 388, 396, 402, 403, 408, 409, 439, 440, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 586, 587, 588, 591, 593, 594, 595, 597, 600, 601, 602, 603, 609, 610, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 630, 631, 632, 633, 637, 638, 639, 644, 645, 646, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 670, 671, 796, 797, 800, 801, 802, 803, 804, 805, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 848, 849, 850, 855, 856, 857, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873 }

C grade: { 628, 629, 634, 635, 636, 640, 641, 642, 643, 647, 648, 649, 654, 655, 661, 662, 669, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 684, 840, 841, 845, 846, 847, 851, 852, 853, 854, 859, 860, 861, 867, 868, 874 }

F grade: { 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 242, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 441, 442, 443, 444, 445, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 875, 876, 877, 878, 879 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 221, 222, 229, 238, 247, 248, 269, 400, 401, 407, 414, 422, 423, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 523, 524, 525, 526, 527, 528, 529, 530, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { 30, 41, 42, 44, 48, 94, 95, 96, 97, 98, 103, 104, 111, 112, 117, 118, 218, 219, 220, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 398, 399, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 419, 420, 421, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade: { }

F grade: { 90, 91, 92, 93, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 257, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 427, 441, 442, 443, 444, 445, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 132, 133, 134, 135, 137, 140, 141, 142, 218, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 270, 271, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 446, 447, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 493, 494, 495, 496, 500, 501, 503, 504, 505, 508, 513, 523, 524, 525, 526, 527, 528, 529, 530, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { 4, 5, 14, 117, 118, 129, 130, 131, 136, 138, 139, 219, 220, 227, 228, 243, 244, 252, 259, 265, 266, 267, 268, 269, 449, 450, 460, 488, 489, 497, 498, 499, 502, 506, 507, 509, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 521, 522 }

C grade: { 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 630, 631, 632, 633, 637, 638, 639, 644, 645, 646, 650, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 837, 838, 839, 842, 843, 844, 848, 849, 850, 855, 856, 857, 858, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873 }

F grade: { 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 634, 635, 636, 640, 641, 642, 643, 647, 648, 649, 654, 655, 661, 662, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 840, 841, 845, 846, 847, 851, 852, 853, 854, 859, 860, 861, 867, 868, 874, 875, 876, 877, 878, 879 }

2.1.6 Sympy

A grade: { 4, 5, 6, 449, 450, 451, 686, 691, 693, 698, 703, 705, 707, 712, 720, 721, 722, 723, 724, 725, 732, 733, 734, 735, 749, 750, 751, 752, 753, 754, 755, 759, 760, 761, 762, 763, 764, 770, 771, 772, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 717, 718, 719, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 756, 757, 758, 765, 766, 767, 768, 769, 773, 774, 775, 776, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.1.7 Giac

A grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 99, 100, 101, 102, 107, 108, 109, 110, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 141, 142, 455, 470, 471, 472, 480, 481, 482, 488, 489, 491, 493, 494, 495, 497, 499, 500, 501, 502, 504, 505, 506, 507, 508, 512, 514, 516, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 745, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { 4, 5, 6, 14, 15, 93, 94, 95, 96, 97, 98, 103, 104, 106, 111, 112, 113, 114, 116, 117, 118, 124, 125, 132, 133, 140, 244, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 476, 477, 478, 479, 483, 484, 485, 486, 487, 490, 492, 496, 498, 503, 509, 510, 511, 513, 515, 517, 518, 519, 520 }

C grade: { }

F grade: { 105, 115, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 742, 743, 744, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.1.8 Mupad

A grade: { 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 99, 100, 101, 102, 107, 108, 109, 110, 116, 221, 222, 223, 224, 229, 230, 231, 232, 238, 239, 240, 241, 242, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816 }

C grade: { }

F grade: { 94, 95, 96, 97, 98, 103, 104, 105, 106, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 226, 227, 228, 233, 234,

235, 236, 237, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260,
261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281,
282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302,
303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323,
324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344,
345, 346, 347, 348, 349, 350, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388,
389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409,
410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430,
431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536,
537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557,
558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578,
579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599,
600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620,
621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641,
642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662,
663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683,
684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711,
713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 817, 818, 819, 820, 821, 822, 823, 824, 825,
826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846,
847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867,
868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	73	95	99	0	110	130
N.S.	1	1.00	0.89	0.86	1.12	1.16	0.00	1.29	1.53
time (sec)	N/A	0.045	0.180	0.149	0.287	2.272	0.000	0.459	3.941

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	60	70	88	0	96	102
N.S.	1	1.00	0.95	0.95	1.11	1.40	0.00	1.52	1.62
time (sec)	N/A	0.035	0.156	0.099	0.293	2.363	0.000	0.447	2.511

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	47	58	74	0	80	75
N.S.	1	1.00	1.00	1.00	1.23	1.57	0.00	1.70	1.60
time (sec)	N/A	0.034	0.024	0.087	0.282	2.944	0.000	0.475	1.057

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	29	60	37	63	47
N.S.	1	1.00	1.00	1.25	1.21	2.50	1.54	2.62	1.96
time (sec)	N/A	0.017	0.015	0.039	0.291	2.515	2.550	0.438	0.685

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	36	41	49	20
N.S.	1	1.00	1.00	1.50	1.44	2.25	2.56	3.06	1.25
time (sec)	N/A	0.005	0.004	0.026	0.292	2.807	1.087	0.450	0.610

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	21	20	17	17	39	15
N.S.	1	1.00	1.73	1.40	1.33	1.13	1.13	2.60	1.00
time (sec)	N/A	0.014	0.010	0.059	0.284	2.613	1.133	0.433	0.600

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	38	34	29	0	56	50
N.S.	1	1.00	0.84	1.00	0.89	0.76	0.00	1.47	1.32
time (sec)	N/A	0.025	0.060	0.079	0.279	3.085	0.000	0.409	1.063

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	46	42	0	72	55
N.S.	1	1.00	1.06	0.91	0.85	0.78	0.00	1.33	1.02
time (sec)	N/A	0.031	0.075	0.093	0.282	3.649	0.000	0.422	0.654

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	57	53	0	86	79
N.S.	1	1.00	0.96	0.79	0.75	0.70	0.00	1.13	1.04
time (sec)	N/A	0.038	0.112	0.104	0.282	3.052	0.000	0.443	4.170

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	487	111	133	124	0	138	170
N.S.	1	1.00	3.99	0.91	1.09	1.02	0.00	1.13	1.39
time (sec)	N/A	0.070	1.623	0.115	0.278	2.100	0.000	0.454	5.700

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	877	112	145	111	0	122	141
N.S.	1	1.00	9.14	1.17	1.51	1.16	0.00	1.27	1.47
time (sec)	N/A	0.065	6.441	0.072	0.286	2.902	0.000	0.528	3.960

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	318	75	85	96	0	106	112
N.S.	1	1.00	4.30	1.01	1.15	1.30	0.00	1.43	1.51
time (sec)	N/A	0.060	0.707	0.069	0.291	2.745	0.000	0.471	2.466

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	219	70	81	83	0	90	83
N.S.	1	1.00	4.06	1.30	1.50	1.54	0.00	1.67	1.54
time (sec)	N/A	0.041	0.648	0.045	0.280	2.509	0.000	0.459	1.175

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	171	44	41	76	0	79	56
N.S.	1	1.00	5.03	1.29	1.21	2.24	0.00	2.32	1.65
time (sec)	N/A	0.021	0.523	0.052	0.284	2.911	0.000	0.422	0.710

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	47	44	52	53	0	79	33
N.S.	1	1.00	1.38	1.29	1.53	1.56	0.00	2.32	0.97
time (sec)	N/A	0.040	0.017	0.066	0.286	2.809	0.000	0.445	0.700

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	48	36	0	64	57
N.S.	1	1.00	0.76	1.16	1.07	0.80	0.00	1.42	1.27
time (sec)	N/A	0.046	0.044	0.072	0.291	3.186	0.000	0.443	1.074

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	41	64	61	49	0	80	61
N.S.	1	1.00	0.72	1.12	1.07	0.86	0.00	1.40	1.07
time (sec)	N/A	0.063	0.092	0.088	0.290	3.560	0.000	0.440	0.656

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	53	90	83	63	0	96	89
N.S.	1	1.00	0.61	1.03	0.95	0.72	0.00	1.10	1.02
time (sec)	N/A	0.059	0.148	0.098	0.286	2.907	0.000	0.450	4.193

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	61	96	95	76	0	112	105
N.S.	1	1.00	0.59	0.93	0.92	0.74	0.00	1.09	1.02
time (sec)	N/A	0.085	0.149	0.131	0.288	2.393	0.000	0.458	4.419

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	487	146	179	124	0	138	170
N.S.	1	1.00	4.27	1.28	1.57	1.09	0.00	1.21	1.49
time (sec)	N/A	0.096	1.550	0.095	0.286	2.760	0.000	0.473	5.482

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	877	123	156	111	0	122	141
N.S.	1	1.00	9.43	1.32	1.68	1.19	0.00	1.31	1.52
time (sec)	N/A	0.086	6.432	0.069	0.286	2.136	0.000	0.486	4.053

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	154	94	104	98	0	106	112
N.S.	1	1.00	2.14	1.31	1.44	1.36	0.00	1.47	1.56
time (sec)	N/A	0.056	5.572	0.059	0.286	4.009	0.000	0.515	2.574

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	235	80	91	98	0	100	88
N.S.	1	1.00	3.56	1.21	1.38	1.48	0.00	1.52	1.33
time (sec)	N/A	0.035	0.957	0.050	0.284	4.070	0.000	0.472	0.737

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	211	55	64	91	0	80	57
N.S.	1	1.00	4.40	1.15	1.33	1.90	0.00	1.67	1.19
time (sec)	N/A	0.048	0.901	0.074	0.278	3.170	0.000	0.478	0.700

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	71	74	65	0	100	88
N.S.	1	1.00	1.37	1.20	1.25	1.10	0.00	1.69	1.49
time (sec)	N/A	0.049	0.071	0.082	0.292	3.057	0.000	0.481	0.725

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	71	50	0	80	63
N.S.	1	1.00	0.70	1.17	1.13	0.79	0.00	1.27	1.00
time (sec)	N/A	0.055	0.065	0.095	0.290	3.422	0.000	0.462	0.670

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	51	100	94	63	0	96	89
N.S.	1	1.00	0.60	1.18	1.11	0.74	0.00	1.13	1.05
time (sec)	N/A	0.072	0.133	0.111	0.286	3.494	0.000	0.490	4.173

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	63	121	117	76	0	112	105
N.S.	1	1.00	0.60	1.15	1.11	0.72	0.00	1.07	1.00
time (sec)	N/A	0.080	0.160	0.128	0.284	3.322	0.000	0.439	4.377

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	73	143	143	89	0	128	121
N.S.	1	1.00	0.57	1.11	1.11	0.69	0.00	0.99	0.94
time (sec)	N/A	0.101	0.207	0.157	0.281	3.641	0.000	0.533	3.492

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	211	204	270	137	0	154	199
N.S.	1	1.00	1.55	1.50	1.99	1.01	0.00	1.13	1.46
time (sec)	N/A	0.136	0.840	0.131	0.283	3.293	0.000	0.467	4.649

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	498	157	190	124	0	138	170
N.S.	1	1.00	4.49	1.41	1.71	1.12	0.00	1.24	1.53
time (sec)	N/A	0.106	1.536	0.086	0.287	2.416	0.000	0.492	5.470

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	877	142	175	111	0	122	141
N.S.	1	1.00	9.14	1.48	1.82	1.16	0.00	1.27	1.47
time (sec)	N/A	0.083	6.426	0.075	0.293	2.782	0.000	0.499	3.989

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	773	104	116	110	0	116	117
N.S.	1	1.00	8.49	1.14	1.27	1.21	0.00	1.27	1.29
time (sec)	N/A	0.062	6.275	0.061	0.301	3.350	0.000	0.442	0.878

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	272	91	110	111	0	129	115
N.S.	1	1.00	3.73	1.25	1.51	1.52	0.00	1.77	1.58
time (sec)	N/A	0.057	1.385	0.092	0.287	3.119	0.000	0.488	0.905

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	241	82	85	105	0	129	117
N.S.	1	1.00	3.30	1.12	1.16	1.44	0.00	1.77	1.60
time (sec)	N/A	0.058	1.795	0.106	0.302	2.941	0.000	0.471	0.891

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	91	93	97	80	0	116	93
N.S.	1	1.00	1.25	1.27	1.33	1.10	0.00	1.59	1.27
time (sec)	N/A	0.062	0.108	0.113	0.281	3.391	0.000	0.467	0.687

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	56	111	104	63	0	96	89
N.S.	1	1.00	0.64	1.28	1.20	0.72	0.00	1.10	1.02
time (sec)	N/A	0.074	0.107	0.110	0.282	2.288	0.000	0.457	4.125

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	133	128	76	0	112	105
N.S.	1	1.00	0.62	1.30	1.25	0.75	0.00	1.10	1.03
time (sec)	N/A	0.085	0.156	0.151	0.287	2.738	0.000	0.474	4.428

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	73	169	165	89	0	128	121
N.S.	1	1.00	0.57	1.33	1.30	0.70	0.00	1.01	0.95
time (sec)	N/A	0.115	0.208	0.091	0.278	1.972	0.000	0.491	3.418

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	83	185	187	102	0	144	137
N.S.	1	1.00	0.56	1.26	1.27	0.69	0.00	0.98	0.93
time (sec)	N/A	0.120	0.277	0.118	0.298	3.499	0.000	0.513	3.636

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	229	248	314	150	0	170	228
N.S.	1	1.00	1.47	1.59	2.01	0.96	0.00	1.09	1.46
time (sec)	N/A	0.151	1.288	0.147	0.291	2.654	0.000	0.569	4.843

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	374	134	205	124	0	114	96
N.S.	1	1.00	3.63	1.30	1.99	1.20	0.00	1.11	0.93
time (sec)	N/A	0.071	3.352	0.094	0.282	4.572	0.000	0.445	0.933

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	250	108	162	112	0	101	95
N.S.	1	1.00	2.94	1.27	1.91	1.32	0.00	1.19	1.12
time (sec)	N/A	0.069	1.446	0.068	0.294	3.025	0.000	0.440	0.727

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	194	74	119	97	0	84	67
N.S.	1	1.00	3.80	1.45	2.33	1.90	0.00	1.65	1.31
time (sec)	N/A	0.075	0.828	0.049	0.300	2.724	0.000	0.456	0.679

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	109	46	75	65	0	54	31
N.S.	1	1.00	2.87	1.21	1.97	1.71	0.00	1.42	0.82
time (sec)	N/A	0.049	0.193	0.043	0.273	2.710	0.000	0.444	0.647

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	17	17	23	22	0	16	16
N.S.	1	1.00	0.77	0.77	1.05	1.00	0.00	0.73	0.73
time (sec)	N/A	0.017	0.028	0.036	0.293	2.312	0.000	0.436	0.589

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	58	32	49	37	0	28	23
N.S.	1	1.00	2.00	1.10	1.69	1.28	0.00	0.97	0.79
time (sec)	N/A	0.010	0.135	0.038	0.494	2.799	0.000	0.440	0.638

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	89	56	92	46	0	58	66
N.S.	1	1.00	2.02	1.27	2.09	1.05	0.00	1.32	1.50
time (sec)	N/A	0.042	0.242	0.060	0.485	2.877	0.000	0.462	0.662

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	117	74	133	57	0	73	89
N.S.	1	1.00	1.58	1.00	1.80	0.77	0.00	0.99	1.20
time (sec)	N/A	0.066	0.255	0.066	0.498	2.325	0.000	0.446	0.725

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	143	85	176	70	0	88	70
N.S.	1	1.00	1.52	0.90	1.87	0.74	0.00	0.94	0.74
time (sec)	N/A	0.065	0.330	0.072	0.498	3.102	0.000	0.444	0.907

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	173	100	217	79	0	101	98
N.S.	1	1.00	1.47	0.85	1.84	0.67	0.00	0.86	0.83
time (sec)	N/A	0.072	0.321	0.080	0.507	2.659	0.000	0.431	2.454

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	300	120	190	162	0	122	122
N.S.	1	1.00	2.44	0.98	1.54	1.32	0.00	0.99	0.99
time (sec)	N/A	0.127	1.998	0.087	0.281	3.747	0.000	0.510	0.735

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	247	92	145	146	0	106	92
N.S.	1	1.00	2.78	1.03	1.63	1.64	0.00	1.19	1.03
time (sec)	N/A	0.109	1.260	0.069	0.281	2.947	0.000	0.460	0.688

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	160	62	98	114	0	77	43
N.S.	1	1.00	2.42	0.94	1.48	1.73	0.00	1.17	0.65
time (sec)	N/A	0.083	0.375	0.073	0.283	3.117	0.000	0.469	0.643

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	45	32	46	49	0	31	30
N.S.	1	1.00	0.82	0.58	0.84	0.89	0.00	0.56	0.55
time (sec)	N/A	0.046	0.074	0.047	0.274	3.301	0.000	0.493	0.596

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	32	47	51	0	31	30
N.S.	1	1.00	1.09	0.58	0.85	0.93	0.00	0.56	0.55
time (sec)	N/A	0.036	0.143	0.048	0.284	2.408	0.000	0.457	0.597

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	112	46	72	80	0	50	35
N.S.	1	1.00	1.96	0.81	1.26	1.40	0.00	0.88	0.61
time (sec)	N/A	0.049	0.295	0.046	0.502	2.992	0.000	0.439	0.630

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	151	72	118	90	0	79	91
N.S.	1	1.00	2.10	1.00	1.64	1.25	0.00	1.10	1.26
time (sec)	N/A	0.095	0.510	0.069	0.500	2.807	0.000	0.455	0.698

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	177	88	164	99	0	95	113
N.S.	1	1.00	1.61	0.80	1.49	0.90	0.00	0.86	1.03
time (sec)	N/A	0.124	0.383	0.077	0.513	2.484	0.000	0.441	0.735

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	199	101	207	108	0	108	135
N.S.	1	1.00	1.60	0.81	1.67	0.87	0.00	0.87	1.09
time (sec)	N/A	0.133	0.357	0.083	0.492	2.482	0.000	0.460	0.782

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	351	135	211	206	0	139	141
N.S.	1	1.00	2.17	0.83	1.30	1.27	0.00	0.86	0.87
time (sec)	N/A	0.201	1.021	0.119	0.294	2.247	0.000	0.453	0.679

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	294	105	165	190	0	122	111
N.S.	1	1.00	2.30	0.82	1.29	1.48	0.00	0.95	0.87
time (sec)	N/A	0.178	1.334	0.089	0.282	3.957	0.000	0.518	0.682

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	209	75	119	158	0	94	58
N.S.	1	1.00	1.99	0.71	1.13	1.50	0.00	0.90	0.55
time (sec)	N/A	0.155	0.514	0.076	0.299	2.709	0.000	0.502	0.688

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	57	45	67	75	0	46	45
N.S.	1	1.00	0.69	0.54	0.81	0.90	0.00	0.55	0.54
time (sec)	N/A	0.085	0.116	0.065	0.281	2.876	0.000	0.488	0.616

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	32	47	73	0	31	30
N.S.	1	1.00	0.86	0.39	0.57	0.88	0.00	0.37	0.36
time (sec)	N/A	0.065	0.178	0.078	0.283	2.876	0.000	0.481	0.597

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	86	45	67	75	0	46	45
N.S.	1	1.00	1.04	0.54	0.81	0.90	0.00	0.55	0.54
time (sec)	N/A	0.056	0.235	0.055	0.288	2.504	0.000	0.476	0.619

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	162	59	92	116	0	68	81
N.S.	1	1.00	1.84	0.67	1.05	1.32	0.00	0.77	0.92
time (sec)	N/A	0.079	0.285	0.056	0.497	1.892	0.000	0.429	0.692

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	169	85	137	126	0	96	113
N.S.	1	1.00	1.64	0.83	1.33	1.22	0.00	0.93	1.10
time (sec)	N/A	0.149	0.574	0.077	0.487	2.025	0.000	0.494	0.728

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	181	101	184	135	0	113	137
N.S.	1	1.00	1.23	0.69	1.25	0.92	0.00	0.77	0.93
time (sec)	N/A	0.193	0.577	0.092	0.519	2.641	0.000	0.510	0.758

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	403	148	231	250	0	155	160
N.S.	1	1.00	2.09	0.77	1.20	1.30	0.00	0.80	0.83
time (sec)	N/A	0.274	1.497	0.069	0.285	2.985	0.000	0.485	0.754

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	349	118	186	234	0	139	130
N.S.	1	1.00	2.19	0.74	1.17	1.47	0.00	0.87	0.82
time (sec)	N/A	0.245	1.263	0.060	0.292	3.286	0.000	0.467	0.702

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	193	88	139	202	0	110	83
N.S.	1	1.00	1.42	0.65	1.02	1.49	0.00	0.81	0.61
time (sec)	N/A	0.217	0.915	0.095	0.298	4.541	0.000	0.516	0.668

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	69	56	87	99	0	59	58
N.S.	1	1.00	0.58	0.47	0.72	0.82	0.00	0.49	0.48
time (sec)	N/A	0.114	0.208	0.075	0.274	9.540	0.000	0.493	0.673

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	58	87	99	0	59	58
N.S.	1	1.00	0.78	0.52	0.78	0.88	0.00	0.53	0.52
time (sec)	N/A	0.107	0.245	0.086	0.284	4.039	0.000	0.476	0.662

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	58	87	99	0	59	58
N.S.	1	1.00	0.88	0.52	0.78	0.88	0.00	0.53	0.52
time (sec)	N/A	0.088	0.273	0.077	0.277	2.200	0.000	0.455	0.675

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	58	87	99	0	59	58
N.S.	1	1.00	1.00	0.52	0.78	0.88	0.00	0.53	0.52
time (sec)	N/A	0.081	0.257	0.059	0.293	5.492	0.000	0.502	0.669

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	224	72	112	152	0	83	102
N.S.	1	1.00	2.02	0.65	1.01	1.37	0.00	0.75	0.92
time (sec)	N/A	0.121	0.399	0.059	0.512	3.103	0.000	0.457	0.725

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	263	98	158	162	0	112	137
N.S.	1	1.00	2.09	0.78	1.25	1.29	0.00	0.89	1.09
time (sec)	N/A	0.208	0.461	0.102	0.540	6.683	0.000	0.461	0.755

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	289	114	204	171	0	128	159
N.S.	1	1.00	1.64	0.65	1.16	0.97	0.00	0.73	0.90
time (sec)	N/A	0.270	0.564	0.099	0.498	8.727	0.000	0.483	0.811

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	401	131	206	278	0	155	149
N.S.	1	1.00	2.00	0.66	1.03	1.39	0.00	0.78	0.74
time (sec)	N/A	0.325	1.847	0.066	0.279	6.533	0.000	0.532	0.715

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	219	101	159	246	0	126	99
N.S.	1	1.00	1.24	0.57	0.90	1.39	0.00	0.71	0.56
time (sec)	N/A	0.294	1.858	0.067	0.288	7.016	0.000	0.517	0.691

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	97	71	107	123	0	72	127
N.S.	1	1.00	0.61	0.45	0.67	0.77	0.00	0.45	0.80
time (sec)	N/A	0.150	0.178	0.054	0.292	4.098	0.000	0.497	0.764

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	97	58	87	123	0	59	58
N.S.	1	1.00	0.61	0.36	0.55	0.77	0.00	0.37	0.36
time (sec)	N/A	0.150	0.220	0.103	0.282	4.044	0.000	0.527	0.720

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	110	45	67	123	0	46	45
N.S.	1	1.00	0.79	0.32	0.48	0.88	0.00	0.33	0.32
time (sec)	N/A	0.131	0.251	0.096	0.283	3.066	0.000	0.491	0.656

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	125	58	87	123	0	59	58
N.S.	1	1.00	0.87	0.41	0.61	0.86	0.00	0.41	0.41
time (sec)	N/A	0.113	0.260	0.085	0.302	15.421	0.000	0.505	0.689

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	138	71	107	123	0	72	127
N.S.	1	1.00	0.97	0.50	0.75	0.86	0.00	0.50	0.89
time (sec)	N/A	0.102	0.290	0.066	0.281	4.586	0.000	0.508	0.763

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	280	85	132	188	0	100	125
N.S.	1	1.00	1.94	0.59	0.92	1.31	0.00	0.69	0.87
time (sec)	N/A	0.145	0.549	0.068	0.502	3.736	0.000	0.465	0.798

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	319	111	178	198	0	129	159
N.S.	1	1.00	2.01	0.70	1.12	1.25	0.00	0.81	1.00
time (sec)	N/A	0.275	0.684	0.096	0.518	2.541	0.000	0.485	0.832

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	345	127	224	207	0	145	181
N.S.	1	1.00	1.60	0.59	1.04	0.96	0.00	0.67	0.84
time (sec)	N/A	0.347	0.761	0.122	0.506	2.486	0.000	0.510	0.950

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	58	82	0	82	0	120	331
N.S.	1	1.00	0.48	0.67	0.00	0.67	0.00	0.98	2.71
time (sec)	N/A	0.138	0.145	0.324	0.000	2.674	0.000	0.826	5.925

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	48	72	0	72	0	101	115
N.S.	1	1.00	0.56	0.84	0.00	0.84	0.00	1.17	1.34
time (sec)	N/A	0.104	0.124	0.115	0.000	2.679	0.000	0.837	4.458

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	36	62	0	60	0	82	108
N.S.	1	1.00	0.64	1.11	0.00	1.07	0.00	1.46	1.93
time (sec)	N/A	0.056	0.108	0.098	0.000	4.016	0.000	0.794	1.389

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	42	0	41	0	62	41
N.S.	1	1.00	1.12	1.62	0.00	1.58	0.00	2.38	1.58
time (sec)	N/A	0.022	0.074	0.080	0.000	3.750	0.000	0.753	0.191

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	60	89	146	133	0	130	-1
N.S.	1	1.00	1.62	2.41	3.95	3.59	0.00	3.51	-0.03
time (sec)	N/A	0.014	0.097	0.133	0.567	3.547	0.000	0.958	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	123	791	242	0	282	-1
N.S.	1	1.00	1.00	1.98	12.76	3.90	0.00	4.55	-0.02
time (sec)	N/A	0.044	0.207	0.276	0.592	3.226	0.000	0.814	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	47	221	1059	270	0	378	-1
N.S.	1	1.00	0.46	2.17	10.38	2.65	0.00	3.71	-0.01
time (sec)	N/A	0.085	0.106	0.173	0.628	3.289	0.000	1.023	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	47	310	1921	290	0	475	-1
N.S.	1	1.00	0.34	2.25	13.92	2.10	0.00	3.44	-0.01
time (sec)	N/A	0.123	0.100	0.197	0.749	2.360	0.000	1.022	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	47	399	6638	310	0	571	-1
N.S.	1	1.00	0.27	2.29	38.15	1.78	0.00	3.28	-0.01
time (sec)	N/A	0.170	0.096	0.228	0.872	2.825	0.000	0.988	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	70	93	0	98	0	180	429
N.S.	1	1.00	0.43	0.57	0.00	0.60	0.00	1.11	2.65
time (sec)	N/A	0.193	0.503	0.121	0.000	2.639	0.000	1.090	6.701

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	60	83	0	87	0	151	346
N.S.	1	1.00	0.52	0.72	0.00	0.75	0.00	1.30	2.98
time (sec)	N/A	0.135	0.181	0.106	0.000	2.632	0.000	1.163	5.084

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	48	73	0	74	0	121	116
N.S.	1	1.00	0.56	0.85	0.00	0.86	0.00	1.41	1.35
time (sec)	N/A	0.092	0.136	0.093	0.000	3.335	0.000	0.976	4.364

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	63	0	61	0	93	111
N.S.	1	1.00	0.64	1.07	0.00	1.03	0.00	1.58	1.88
time (sec)	N/A	0.046	0.093	0.086	0.000	4.471	0.000	0.925	1.237

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	181	997	235	0	195	-1
N.S.	1	1.00	1.14	2.74	15.11	3.56	0.00	2.95	-0.02
time (sec)	N/A	0.027	0.218	0.114	0.606	3.755	0.000	1.146	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	89	125	803	248	0	278	-1
N.S.	1	1.00	1.37	1.92	12.35	3.82	0.00	4.28	-0.02
time (sec)	N/A	0.084	0.210	0.111	0.626	3.707	0.000	1.138	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	108	222	0	278	0	0	-1
N.S.	1	1.00	1.02	2.09	0.00	2.62	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.382	0.132	0.000	2.749	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	120	311	0	300	0	535	-1
N.S.	1	1.00	0.83	2.16	0.00	2.08	0.00	3.72	-0.01
time (sec)	N/A	0.138	0.547	0.164	0.000	2.272	0.000	1.732	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	80	105	0	121	0	209	542
N.S.	1	1.00	0.39	0.52	0.00	0.60	0.00	1.03	2.67
time (sec)	N/A	0.266	0.217	0.132	0.000	2.681	0.000	1.148	9.325

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	70	95	0	108	0	180	456
N.S.	1	1.00	0.48	0.65	0.00	0.74	0.00	1.23	3.12
time (sec)	N/A	0.167	0.518	0.112	0.000	3.164	0.000	1.209	8.176

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	60	85	0	95	0	151	349
N.S.	1	1.00	0.52	0.73	0.00	0.82	0.00	1.30	3.01
time (sec)	N/A	0.118	0.185	0.102	0.000	2.952	0.000	1.123	4.580

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	50	75	0	82	0	122	146
N.S.	1	1.00	0.56	0.84	0.00	0.92	0.00	1.37	1.64
time (sec)	N/A	0.069	0.101	0.104	0.000	3.525	0.000	1.061	4.506

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	360	214	1395	310	0	225	-1
N.S.	1	1.00	3.67	2.18	14.23	3.16	0.00	2.30	-0.01
time (sec)	N/A	0.081	6.375	0.132	0.614	2.622	0.000	1.273	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	189	128	1383	276	0	368	-1
N.S.	1	1.00	2.01	1.36	14.71	2.94	0.00	3.91	-0.01
time (sec)	N/A	0.111	2.432	0.122	0.630	2.953	0.000	1.676	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	150	224	0	294	0	364	-1
N.S.	1	1.00	1.42	2.11	0.00	2.77	0.00	3.43	-0.01
time (sec)	N/A	0.119	0.595	0.135	0.000	4.162	0.000	1.273	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	151	313	0	320	0	539	-1
N.S.	1	1.00	1.05	2.17	0.00	2.22	0.00	3.74	-0.01
time (sec)	N/A	0.178	0.824	0.158	0.000	3.813	0.000	1.870	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	161	402	0	346	0	0	-1
N.S.	1	1.00	0.88	2.21	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.809	0.185	0.000	3.707	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	42	0	44	0	57	36
N.S.	1	1.00	1.11	1.56	0.00	1.63	0.00	2.11	1.33
time (sec)	N/A	0.026	0.115	0.180	0.000	2.813	0.000	0.677	0.787

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	188	91	146	182	0	65	-1
N.S.	1	1.00	4.95	2.39	3.84	4.79	0.00	1.71	-0.03
time (sec)	N/A	0.015	0.608	0.116	0.545	2.673	0.000	0.677	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	260	103	791	294	0	134	-1
N.S.	1	1.00	4.00	1.58	12.17	4.52	0.00	2.06	-0.02
time (sec)	N/A	0.051	0.913	0.153	0.602	3.423	0.000	0.525	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	314	0	347	0	150	-1
N.S.	1	1.00	0.76	2.24	0.00	2.48	0.00	1.07	-0.01
time (sec)	N/A	0.207	0.234	0.135	0.000	2.568	0.000	1.171	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	86	221	0	316	0	113	-1
N.S.	1	1.00	0.83	2.12	0.00	3.04	0.00	1.09	-0.01
time (sec)	N/A	0.115	0.155	0.123	0.000	2.660	0.000	1.134	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	83	121	0	262	0	108	-1
N.S.	1	1.00	1.14	1.66	0.00	3.59	0.00	1.48	-0.01
time (sec)	N/A	0.060	0.086	0.093	0.000	2.697	0.000	1.123	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	64	95	0	158	0	59	-1
N.S.	1	1.00	1.39	2.07	0.00	3.43	0.00	1.28	-0.02
time (sec)	N/A	0.027	0.056	0.086	0.000	3.873	0.000	0.783	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	5402	141	0	294	0	69	-1
N.S.	1	1.00	63.55	1.66	0.00	3.46	0.00	0.81	-0.01
time (sec)	N/A	0.053	24.032	0.087	0.000	3.759	0.000	0.873	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	105	201	0	417	0	326	-1
N.S.	1	1.00	0.97	1.86	0.00	3.86	0.00	3.02	-0.01
time (sec)	N/A	0.125	0.134	0.125	0.000	3.443	0.000	1.077	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	118	380	0	446	0	423	-1
N.S.	1	1.00	0.80	2.59	0.00	3.03	0.00	2.88	-0.01
time (sec)	N/A	0.180	0.255	0.148	0.000	3.606	0.000	1.147	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	124	417	0	414	0	210	-1
N.S.	1	1.00	0.68	2.28	0.00	2.26	0.00	1.15	-0.01
time (sec)	N/A	0.298	0.461	0.135	0.000	3.916	0.000	1.213	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	114	322	0	387	0	179	-1
N.S.	1	1.00	0.79	2.22	0.00	2.67	0.00	1.23	-0.01
time (sec)	N/A	0.203	0.307	0.125	0.000	4.308	0.000	1.264	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	104	225	0	336	0	157	-1
N.S.	1	1.00	0.99	2.14	0.00	3.20	0.00	1.50	-0.01
time (sec)	N/A	0.119	0.317	0.110	0.000	5.595	0.000	1.234	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	94	222	0	329	0	108	-1
N.S.	1	1.00	1.22	2.88	0.00	4.27	0.00	1.40	-0.01
time (sec)	N/A	0.069	0.196	0.096	0.000	3.508	0.000	1.175	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	93	220	0	327	0	108	-1
N.S.	1	1.00	1.21	2.86	0.00	4.25	0.00	1.40	-0.01
time (sec)	N/A	0.053	0.116	0.083	0.000	4.059	0.000	0.907	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	5524	370	0	491	0	47	-1
N.S.	1	1.00	48.46	3.25	0.00	4.31	0.00	0.41	-0.01
time (sec)	N/A	0.086	24.147	0.088	0.000	3.906	0.000	0.695	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	129	384	0	518	0	387	-1
N.S.	1	1.00	0.90	2.67	0.00	3.60	0.00	2.69	-0.01
time (sec)	N/A	0.184	0.863	0.124	0.000	2.624	0.000	1.238	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	197	560	0	536	0	473	-1
N.S.	1	1.00	1.06	3.03	0.00	2.90	0.00	2.56	-0.01
time (sec)	N/A	0.273	3.035	0.149	0.000	3.061	0.000	1.677	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	135	417	0	455	0	219	-1
N.S.	1	1.00	0.74	2.28	0.00	2.49	0.00	1.20	-0.01
time (sec)	N/A	0.296	1.316	0.138	0.000	3.066	0.000	1.218	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	125	316	0	404	0	188	-1
N.S.	1	1.00	0.86	2.18	0.00	2.79	0.00	1.30	-0.01
time (sec)	N/A	0.203	0.734	0.119	0.000	2.383	0.000	1.215	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	116	323	0	403	0	139	-1
N.S.	1	1.00	1.08	3.02	0.00	3.77	0.00	1.30	-0.01
time (sec)	N/A	0.121	0.658	0.113	0.000	2.549	0.000	1.276	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	115	315	0	399	0	139	-1
N.S.	1	1.00	1.07	2.94	0.00	3.73	0.00	1.30	-0.01
time (sec)	N/A	0.101	0.646	0.088	0.000	2.396	0.000	1.264	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	52	315	0	403	0	139	-1
N.S.	1	1.00	0.49	2.94	0.00	3.77	0.00	1.30	-0.01
time (sec)	N/A	0.080	0.066	0.083	0.000	2.474	0.000	1.006	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	5564	550	0	585	0	78	-1
N.S.	1	1.00	38.64	3.82	0.00	4.06	0.00	0.54	-0.01
time (sec)	N/A	0.124	24.190	0.104	0.000	2.506	0.000	0.801	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	169	552	0	606	0	424	-1
N.S.	1	1.00	0.97	3.17	0.00	3.48	0.00	2.44	-0.01
time (sec)	N/A	0.263	1.902	0.135	0.000	4.902	0.000	1.380	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	94	83	0	161	0	67	-1
N.S.	1	1.00	1.96	1.73	0.00	3.35	0.00	1.40	-0.02
time (sec)	N/A	0.031	0.407	0.108	0.000	2.659	0.000	0.530	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	127	119	0	301	0	69	-1
N.S.	1	1.00	1.46	1.37	0.00	3.46	0.00	0.79	-0.01
time (sec)	N/A	0.054	0.487	0.099	0.000	2.432	0.000	0.771	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	105	0	0	0	0	0	-1
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	0.290	0.079	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	85	0	0	0	0	0	-1
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.141	0.059	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	66	0	0	0	0	0	-1
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.061	0.053	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	691	0	0	0	0	0	-1
N.S.	1	1.00	8.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	5.073	0.059	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	2700	0	0	0	0	0	-1
N.S.	1	1.00	35.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	16.247	0.069	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	96	0	0	0	0	0	-1
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.337	0.073	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	106	0	0	0	0	0	-1
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	0.458	0.057	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	66	0	0	0	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.092	0.050	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	2694	0	0	0	0	0	-1
N.S.	1	1.00	31.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	16.000	0.059	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	2700	0	0	0	0	0	-1
N.S.	1	1.00	31.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	16.144	0.072	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	108	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.407	1.507	0.082	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	95	0	0	0	0	0	-1
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.211	0.073	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	85	0	0	0	0	0	-1
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.117	0.060	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	65	0	0	0	0	0	-1
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.077	0.053	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	718	0	0	0	0	0	-1
N.S.	1	1.00	9.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	4.807	0.053	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	240	0	0	0	0	0	-1
N.S.	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	2.097	0.072	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	766	766	111	0	0	0	0	0	-1
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.787	0.598	0.083	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	98	0	0	0	0	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.607	0.287	0.072	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	90	0	0	0	0	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.514	0.348	0.059	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	744	744	68	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	0.074	0.054	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	3007	0	0	0	0	0	-1
N.S.	1	1.00	33.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	16.424	0.053	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	3011	0	0	0	0	0	-1
N.S.	1	1.00	33.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	16.111	0.070	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	115	384	0	188	0	0	-1
N.S.	1	1.00	0.76	2.54	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.265	0.115	0.000	0.590	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	83	368	0	167	0	0	-1
N.S.	1	1.00	0.67	2.99	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.207	0.085	0.000	0.829	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	68	148	0	124	0	0	-1
N.S.	1	1.00	0.70	1.53	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.155	0.060	0.000	1.019	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	49	150	0	107	0	0	-1
N.S.	1	1.00	0.65	2.00	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.088	0.110	0.000	0.565	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	225	0	125	0	0	-1
N.S.	1	1.00	0.72	2.23	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.148	0.053	0.000	1.317	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	93	219	0	145	0	0	-1
N.S.	1	1.00	0.73	1.72	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.201	0.062	0.000	0.718	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	103	270	0	156	0	0	-1
N.S.	1	1.00	0.68	1.79	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.279	0.054	0.000	0.932	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	285	439	0	215	0	0	-1
N.S.	1	1.00	1.52	2.35	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.108	2.172	0.105	0.000	0.596	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	269	386	0	202	0	0	-1
N.S.	1	1.00	1.67	2.40	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.096	1.517	0.093	0.000	0.703	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	264	371	0	179	0	0	-1
N.S.	1	1.00	2.02	2.83	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.085	1.228	0.086	0.000	0.860	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	185	0	77	0	0	-1
N.S.	1	1.00	0.75	2.89	0.00	1.20	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.151	0.070	0.000	0.606	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	156	228	0	134	0	0	-1
N.S.	1	1.00	1.46	2.13	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.807	0.049	0.000	0.569	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	136	250	0	157	0	0	-1
N.S.	1	1.00	1.01	1.85	0.00	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.082	1.091	0.057	0.000	1.552	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	149	272	0	170	0	0	-1
N.S.	1	1.00	0.93	1.69	0.00	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.099	1.310	0.056	0.000	0.482	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	287	439	0	215	0	0	-1
N.S.	1	1.00	1.53	2.35	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.153	2.125	0.106	0.000	0.475	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	267	386	0	200	0	0	-1
N.S.	1	1.00	1.70	2.46	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.140	1.804	0.096	0.000	0.958	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	187	371	0	179	0	0	-1
N.S.	1	1.00	1.43	2.83	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.358	0.093	0.000	1.341	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	169	172	0	148	0	0	-1
N.S.	1	1.00	1.29	1.31	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.118	1.069	0.061	0.000	1.156	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	171	250	0	156	0	0	-1
N.S.	1	1.00	1.31	1.91	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.017	0.059	0.000	0.896	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	170	0	0	-1
N.S.	1	1.00	0.91	1.69	0.00	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.141	1.474	0.057	0.000	0.955	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	-1
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.171	1.873	0.064	0.000	0.785	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	287	492	0	228	0	0	-1
N.S.	1	1.00	1.35	2.31	0.00	1.07	0.00	0.00	-0.00
time (sec)	N/A	0.201	3.842	0.120	0.000	0.701	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	279	439	0	215	0	0	-1
N.S.	1	1.00	1.49	2.35	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.168	2.629	0.108	0.000	1.153	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	286	386	0	202	0	0	-1
N.S.	1	1.00	1.78	2.40	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.148	2.733	0.099	0.000	1.019	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	70	292	0	121	0	0	-1
N.S.	1	1.00	0.59	2.47	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.317	0.105	0.000	0.817	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	184	194	0	162	0	0	-1
N.S.	1	1.00	1.16	1.22	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.145	1.206	0.063	0.000	1.236	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	180	272	0	170	0	0	-1
N.S.	1	1.00	1.12	1.69	0.00	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.162	1.266	0.060	0.000	0.612	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	-1
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.178	1.797	0.062	0.000	0.703	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	293	273	0	196	0	0	-1
N.S.	1	1.00	1.38	1.28	0.00	0.92	0.00	0.00	-0.00
time (sec)	N/A	0.212	4.702	0.071	0.000	0.887	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	291	413	0	248	0	0	-1
N.S.	1	1.00	1.77	2.52	0.00	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.106	3.081	0.083	0.000	0.646	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	262	253	0	196	0	0	-1
N.S.	1	1.00	1.93	1.86	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.092	1.680	0.063	0.000	1.320	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	201	200	0	184	0	0	-1
N.S.	1	1.00	1.83	1.82	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.569	0.057	0.000	0.832	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	202	198	0	184	0	0	-1
N.S.	1	1.00	1.84	1.80	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.531	0.062	0.000	1.829	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	317	199	0	186	0	0	-1
N.S.	1	1.00	2.83	1.78	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.093	1.480	0.073	0.000	0.514	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	318	215	0	207	0	0	-1
N.S.	1	1.00	2.27	1.54	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.106	4.029	0.069	0.000	0.840	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	347	229	0	217	0	0	-1
N.S.	1	1.00	2.07	1.36	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.115	2.423	0.080	0.000	0.894	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	287	413	0	328	0	0	-1
N.S.	1	1.00	1.42	2.04	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.193	3.486	0.094	0.000	0.741	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	252	405	0	278	0	0	-1
N.S.	1	1.00	1.43	2.30	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.174	1.309	0.073	0.000	0.543	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	242	257	0	277	0	0	-1
N.S.	1	1.00	1.62	1.72	0.00	1.86	0.00	0.00	-0.01
time (sec)	N/A	0.150	1.179	0.060	0.000	0.484	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	188	0	150	0	0	-1
N.S.	1	1.00	1.27	2.44	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.359	0.063	0.000	0.553	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	239	257	0	277	0	0	-1
N.S.	1	1.00	1.60	1.72	0.00	1.86	0.00	0.00	-0.01
time (sec)	N/A	0.164	1.370	0.095	0.000	1.167	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	260	257	0	277	0	0	-1
N.S.	1	1.00	1.71	1.69	0.00	1.82	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.925	0.092	0.000	0.937	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	257	270	0	287	0	0	-1
N.S.	1	1.00	1.44	1.52	0.00	1.61	0.00	0.00	-0.01
time (sec)	N/A	0.171	1.688	0.093	0.000	0.804	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	271	283	0	297	0	0	-1
N.S.	1	1.00	1.36	1.42	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.186	1.769	0.096	0.000	0.873	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	378	453	0	404	0	0	-1
N.S.	1	1.00	1.53	1.83	0.00	1.64	0.00	0.00	-0.00
time (sec)	N/A	0.255	4.972	0.105	0.000	0.722	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	371	555	0	354	0	0	-1
N.S.	1	1.00	1.68	2.51	0.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.239	1.948	0.077	0.000	0.836	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	274	268	0	353	0	0	-1
N.S.	1	1.00	1.41	1.37	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.229	4.700	0.065	0.000	0.443	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	371	270	0	353	0	0	-1
N.S.	1	1.00	1.90	1.38	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.233	1.770	0.067	0.000	0.573	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	371	270	0	353	0	0	-1
N.S.	1	1.00	1.90	1.38	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.230	1.693	0.081	0.000	0.826	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	272	270	0	353	0	0	-1
N.S.	1	1.00	1.39	1.38	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.228	5.113	0.101	0.000	0.862	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	386	270	0	353	0	0	-1
N.S.	1	1.00	1.98	1.38	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.241	1.905	0.099	0.000	0.967	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	285	283	0	363	0	0	-1
N.S.	1	1.00	1.29	1.28	0.00	1.64	0.00	0.00	-0.00
time (sec)	N/A	0.242	2.327	0.094	0.000	0.850	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	297	296	0	373	0	0	-1
N.S.	1	1.00	1.20	1.20	0.00	1.51	0.00	0.00	-0.00
time (sec)	N/A	0.259	2.725	0.100	0.000	0.966	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	100	221	1264	356	0	0	-1
N.S.	1	1.00	0.86	1.91	10.90	3.07	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.544	0.364	0.605	2.863	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	75	186	662	302	0	0	-1
N.S.	1	1.00	1.04	2.58	9.19	4.19	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.245	0.151	0.579	2.183	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	54	150	241	189	0	0	-1
N.S.	1	1.00	1.46	4.05	6.51	5.11	0.00	0.00	-0.03
time (sec)	N/A	0.039	0.124	0.142	0.568	2.278	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	52	20	49	0	0	53
N.S.	1	1.00	1.08	1.44	0.56	1.36	0.00	0.00	1.47
time (sec)	N/A	0.034	0.097	0.132	0.532	2.764	0.000	0.000	0.537

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	49	68	113	66	0	0	69
N.S.	1	1.00	0.64	0.88	1.47	0.86	0.00	0.00	0.90
time (sec)	N/A	0.078	0.162	0.150	0.544	3.881	0.000	0.000	1.297

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	61	80	203	78	0	0	82
N.S.	1	1.00	0.53	0.70	1.77	0.68	0.00	0.00	0.71
time (sec)	N/A	0.120	0.196	0.145	0.550	3.186	0.000	0.000	1.668

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	71	90	293	88	0	0	93
N.S.	1	1.00	0.46	0.59	1.92	0.58	0.00	0.00	0.61
time (sec)	N/A	0.171	0.260	0.154	0.562	3.092	0.000	0.000	2.202

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	112	246	2361	400	0	0	-1
N.S.	1	1.00	0.70	1.54	14.76	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.577	0.187	0.661	2.595	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	99	214	2244	370	0	0	-1
N.S.	1	1.00	0.82	1.78	18.70	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.429	0.164	0.638	2.493	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	184	1143	310	0	0	-1
N.S.	1	1.00	1.00	2.45	15.24	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.288	0.145	0.574	2.406	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	86	174	274	307	0	0	-1
N.S.	1	1.00	1.13	2.29	3.61	4.04	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.376	0.129	0.561	3.708	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	50	71	38	69	0	0	70
N.S.	1	1.00	0.63	0.90	0.48	0.87	0.00	0.00	0.89
time (sec)	N/A	0.082	0.228	0.135	0.536	3.830	0.000	0.000	1.406

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	60	81	210	80	0	0	81
N.S.	1	1.00	0.52	0.70	1.81	0.69	0.00	0.00	0.70
time (sec)	N/A	0.127	0.279	0.138	0.563	3.352	0.000	0.000	1.773

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	72	93	303	92	0	0	94
N.S.	1	1.00	0.45	0.58	1.88	0.57	0.00	0.00	0.58
time (sec)	N/A	0.173	0.359	0.146	0.564	2.847	0.000	0.000	2.325

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	80	103	396	103	0	0	105
N.S.	1	1.00	0.40	0.51	1.97	0.51	0.00	0.00	0.52
time (sec)	N/A	0.212	0.546	0.148	0.578	2.272	0.000	0.000	3.022

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	582	286	3860	446	0	0	-1
N.S.	1	1.00	2.91	1.43	19.30	2.23	0.00	0.00	-0.00
time (sec)	N/A	0.241	8.333	0.149	0.848	2.628	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	458	254	3469	420	0	0	-1
N.S.	1	1.00	2.86	1.59	21.68	2.62	0.00	0.00	-0.01
time (sec)	N/A	0.196	7.856	0.170	0.792	3.019	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	226	2826	386	0	0	-1
N.S.	1	1.00	0.88	1.88	23.55	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.474	0.155	3.313	2.772	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	91	199	11494	346	0	0	-1
N.S.	1	1.00	0.81	1.78	102.62	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.708	0.154	0.654	1.953	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	103	195	593	364	0	0	-1
N.S.	1	1.00	0.87	1.65	5.03	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.453	0.151	0.592	2.636	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	64	85	60	87	0	0	85
N.S.	1	1.00	0.54	0.71	0.50	0.73	0.00	0.00	0.71
time (sec)	N/A	0.133	0.329	0.134	0.538	2.456	0.000	0.000	1.835

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	74	95	323	100	0	0	96
N.S.	1	1.00	0.47	0.61	2.07	0.64	0.00	0.00	0.62
time (sec)	N/A	0.175	0.350	0.150	0.571	4.132	0.000	0.000	2.316

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	80	105	422	113	0	0	107
N.S.	1	1.00	0.40	0.52	2.10	0.56	0.00	0.00	0.53
time (sec)	N/A	0.246	0.584	0.151	0.565	3.513	0.000	0.000	2.958

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	90	115	521	126	0	0	356
N.S.	1	1.00	0.37	0.48	2.16	0.52	0.00	0.00	1.48
time (sec)	N/A	0.286	0.374	0.157	0.572	3.280	0.000	0.000	6.925

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	45	0	121	50	0	0	55
N.S.	1	1.00	1.18	0.00	3.18	1.32	0.00	0.00	1.45
time (sec)	N/A	0.042	0.086	0.102	0.509	2.623	0.000	0.000	0.746

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	54	147	257	205	0	0	-1
N.S.	1	1.00	1.46	3.97	6.95	5.54	0.00	0.00	-0.03
time (sec)	N/A	0.044	0.148	0.576	0.559	2.914	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	299	124	385	233	0	98	-1
N.S.	1	1.00	7.87	3.26	10.13	6.13	0.00	2.58	-0.03
time (sec)	N/A	0.048	1.929	0.209	0.587	2.622	0.000	0.796	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	125	222	876	481	0	0	-1
N.S.	1	1.00	0.98	1.73	6.84	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.264	0.174	0.590	4.534	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	89	184	476	351	0	0	-1
N.S.	1	1.00	0.94	1.94	5.01	3.69	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.118	0.143	0.584	3.061	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	99	90	160	0	0	-1
N.S.	1	1.00	1.34	1.77	1.61	2.86	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.055	0.134	0.543	3.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	102	102	104	281	0	0	-1
N.S.	1	1.00	1.10	1.10	1.12	3.02	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.197	0.125	0.549	3.186	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	120	120	282	318	0	0	-1
N.S.	1	1.00	0.92	0.92	2.15	2.43	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.258	0.147	0.569	2.695	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	117	130	357	342	0	0	-1
N.S.	1	1.00	0.69	0.77	2.11	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.240	1.131	0.151	0.569	2.712	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	252	281	4934	579	0	0	-1
N.S.	1	1.00	1.45	1.61	28.36	3.33	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.601	0.151	0.895	2.575	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	220	240	2122	559	0	0	-1
N.S.	1	1.00	1.64	1.79	15.84	4.17	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.540	0.148	0.605	2.580	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	220	146	15721	338	0	0	-1
N.S.	1	1.00	2.27	1.51	162.07	3.48	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.530	0.136	1.189	2.636	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	120	146	1031	340	0	0	-1
N.S.	1	1.00	1.24	1.51	10.63	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.185	0.133	0.577	3.487	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	145	175	7176	378	0	0	-1
N.S.	1	1.00	1.06	1.28	52.38	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.469	0.140	0.624	2.467	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	150	193	33960	398	0	0	-1
N.S.	1	1.00	0.85	1.09	191.86	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.913	0.145	0.966	2.745	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	163	203	0	418	0	0	-1
N.S.	1	1.00	0.75	0.94	0.00	1.93	0.00	0.00	-0.00
time (sec)	N/A	0.359	1.346	0.158	0.000	2.728	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	340	454	9048	667	0	0	-1
N.S.	1	1.00	1.59	2.12	42.28	3.12	0.00	0.00	-0.00
time (sec)	N/A	0.386	1.247	0.164	4.553	3.022	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	308	406	4988	665	0	0	-1
N.S.	1	1.00	1.77	2.33	28.67	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.695	0.151	0.871	3.047	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	308	210	84332	426	0	0	-1
N.S.	1	1.00	2.25	1.53	615.56	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.681	0.154	22.973	3.021	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	266	210	2875	422	0	0	-1
N.S.	1	1.00	1.94	1.53	20.99	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.135	1.848	0.139	0.882	3.931	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	146	208	3049	426	0	0	-1
N.S.	1	1.00	1.07	1.52	22.26	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.939	0.145	1.014	2.891	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	186	236	258456	446	0	0	-1
N.S.	1	1.00	1.05	1.33	1460.20	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.267	1.310	0.155	3.027	2.822	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	165	254	148823	466	0	0	-1
N.S.	1	1.00	0.76	1.17	685.82	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.357	2.374	0.154	3.557	2.787	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	140	253	1643	337	0	0	-1
N.S.	1	1.00	1.11	2.01	13.04	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.431	0.200	0.596	2.376	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	111	218	873	297	0	0	-1
N.S.	1	1.00	1.31	2.56	10.27	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.307	0.142	0.576	3.369	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	76	178	473	223	0	0	-1
N.S.	1	1.00	1.41	3.30	8.76	4.13	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.103	0.132	0.567	2.461	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	40	95	87	88	0	0	-1
N.S.	1	1.00	1.48	3.52	3.22	3.26	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.030	0.123	0.536	3.199	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	90	96	101	144	0	0	-1
N.S.	1	1.00	1.45	1.55	1.63	2.32	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.177	0.117	0.539	2.448	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	118	116	279	163	0	0	-1
N.S.	1	1.00	1.20	1.18	2.85	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.258	0.136	0.553	2.951	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	122	126	354	174	0	0	-1
N.S.	1	1.00	0.91	0.94	2.64	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.314	0.142	0.560	2.865	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	71	0	0	0	0	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.266	0.102	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	71	0	0	0	0	0	-1
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.148	0.091	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	71	0	0	0	0	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.254	0.163	0.086	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	716	716	71	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	0.248	0.101	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	71	0	0	0	0	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	0.231	0.090	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	71	0	0	0	0	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	0.150	0.090	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	662	662	71	0	0	0	0	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.411	0.158	0.088	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	715	715	71	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	0.195	0.088	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	760	0	0	0	0	0	-1
N.S.	1	1.00	9.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	7.153	0.089	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	749	0	0	0	0	0	-1
N.S.	1	1.00	9.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	7.794	0.090	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	3346	0	0	0	0	0	-1
N.S.	1	1.00	44.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	20.439	0.089	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	585	0	0	0	0	0	-1
N.S.	1	1.00	7.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	7.350	0.090	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	1982	0	0	0	0	0	-1
N.S.	1	1.00	25.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	23.248	0.076	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	2618	0	0	0	0	0	-1
N.S.	1	1.00	33.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	25.469	0.073	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	79	274	0	0	0	0	0	-1
N.S.	1	0.24	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.087	9.733	0.072	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	2325	0	0	0	0	0	-1
N.S.	1	1.00	29.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	31.334	0.072	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	0.742	0.168	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	286	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	1.517	0.074	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	222	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	1.099	0.060	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	106	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.172	0.072	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.904	0.059	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	8.247	0.048	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	433	0	0	0	0	0	-1
N.S.	1	1.00	2.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	62.752	0.082	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.418	0.072	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.042	0.080	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	2938	0	0	0	0	0	-1
N.S.	1	1.00	49.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	15.949	0.076	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	2990	0	0	0	0	0	-1
N.S.	1	1.00	48.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	16.462	0.075	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.416	0.082	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.055	0.076	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	2951	0	0	0	0	0	-1
N.S.	1	1.00	40.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	6.234	0.073	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	3003	0	0	0	0	0	-1
N.S.	1	1.00	41.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	6.253	0.070	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.400	0.085	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.053	0.079	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	2951	0	0	0	0	0	-1
N.S.	1	1.00	40.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	6.207	0.073	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	3003	0	0	0	0	0	-1
N.S.	1	1.00	41.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	6.238	0.069	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	435	0	0	0	0	0	-1
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	11.152	0.087	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	86	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.437	0.078	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.100	0.087	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	2964	0	0	0	0	0	-1
N.S.	1	1.00	48.59	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.092	6.244	0.081	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	2992	0	0	0	0	0	-1
N.S.	1	1.00	44.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	6.266	0.076	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.384	0.078	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.133	0.082	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	2977	0	0	0	0	0	-1
N.S.	1	1.00	39.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	6.220	0.076	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	3005	0	0	0	0	0	-1
N.S.	1	1.00	38.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	6.217	0.072	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.368	0.079	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.132	0.080	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	2977	0	0	0	0	0	-1
N.S.	1	1.00	39.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	6.213	0.079	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	3005	0	0	0	0	0	-1
N.S.	1	1.00	38.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	6.208	0.075	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	458	0	0	0	0	0	-1
N.S.	1	1.00	2.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	26.256	0.103	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	377	0	0	0	0	0	-1
N.S.	1	1.00	3.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	15.700	0.088	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	236	0	0	0	0	0	-1
N.S.	1	1.00	5.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	74.199	0.095	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.106	1.363	0.089	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.115	74.986	0.082	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	363	0	0	0	0	0	-1
N.S.	1	1.00	2.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	5.021	0.079	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	222	0	0	0	0	0	-1
N.S.	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.515	0.081	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	377	0	0	0	0	0	-1
N.S.	1	1.00	2.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	2.389	0.085	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	236	0	0	0	0	0	-1
N.S.	1	1.00	3.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.592	0.084	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	2246	0	0	0	0	0	-1
N.S.	1	1.00	31.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	14.412	0.079	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	255	0	0	0	0	0	-1
N.S.	1	1.00	2.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	2.528	0.085	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	2248	0	0	0	0	0	-1
N.S.	1	1.00	25.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	6.266	0.083	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.103	0.087	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	2248	0	0	0	0	0	-1
N.S.	1	1.00	26.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	6.218	0.079	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	257	0	0	0	0	0	-1
N.S.	1	1.00	3.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.539	0.080	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	2250	0	0	0	0	0	-1
N.S.	1	1.00	25.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	6.266	0.080	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.321	0.102	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	2248	0	0	0	0	0	-1
N.S.	1	1.00	28.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	6.213	0.077	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	257	0	0	0	0	0	-1
N.S.	1	1.00	3.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.544	0.082	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	2250	0	0	0	0	0	-1
N.S.	1	1.00	23.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	6.246	0.081	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.328	0.092	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	154	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	1.388	0.083	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	123	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.669	0.066	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	95	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.224	0.057	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.109	0.076	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	711	0	0	0	0	0	-1
N.S.	1	1.00	8.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	6.791	0.073	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	3781	0	0	0	0	0	-1
N.S.	1	1.00	45.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	17.033	0.095	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	2529	0	0	0	0	0	-1
N.S.	1	1.00	25.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	15.583	0.099	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	2225	0	0	0	0	0	-1
N.S.	1	1.00	23.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	15.001	0.102	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	2424	0	0	0	0	0	-1
N.S.	1	1.00	25.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	16.370	0.102	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	3349	0	0	0	0	0	-1
N.S.	1	1.00	34.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	19.869	0.095	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	490	270	0	148	0	0	87
N.S.	1	1.00	4.41	2.43	0.00	1.33	0.00	0.00	0.78
time (sec)	N/A	0.071	6.096	0.095	0.000	1.037	0.000	0.000	1.178

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	232	219	0	137	0	0	80
N.S.	1	1.00	2.67	2.52	0.00	1.57	0.00	0.00	0.92
time (sec)	N/A	0.063	5.592	0.079	0.000	0.812	0.000	0.000	0.783

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	222	225	0	125	0	0	53
N.S.	1	1.00	3.64	3.69	0.00	2.05	0.00	0.00	0.87
time (sec)	N/A	0.052	5.153	0.077	0.000	0.721	0.000	0.000	0.763

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	155	150	0	107	0	0	27
N.S.	1	1.00	4.43	4.29	0.00	3.06	0.00	0.00	0.77
time (sec)	N/A	0.042	1.821	0.102	0.000	0.746	0.000	0.000	0.196

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	209	148	0	156	0	0	60
N.S.	1	1.00	3.67	2.60	0.00	2.74	0.00	0.00	1.05
time (sec)	N/A	0.053	5.035	0.078	0.000	0.971	0.000	0.000	1.032

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	444	368	0	175	0	0	87
N.S.	1	1.00	5.35	4.43	0.00	2.11	0.00	0.00	1.05
time (sec)	N/A	0.060	6.131	0.101	0.000	1.180	0.000	0.000	1.171

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	477	384	0	188	0	0	87
N.S.	1	1.00	4.30	3.46	0.00	1.69	0.00	0.00	0.78
time (sec)	N/A	0.070	6.159	0.114	0.000	0.596	0.000	0.000	1.282

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	294	437	0	199	0	0	87
N.S.	1	1.00	2.18	3.24	0.00	1.47	0.00	0.00	0.64
time (sec)	N/A	0.078	4.689	0.128	0.000	0.714	0.000	0.000	1.377

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	548	260	0	175	0	0	136
N.S.	1	1.00	3.73	1.77	0.00	1.19	0.00	0.00	0.93
time (sec)	N/A	0.136	6.128	0.098	0.000	0.964	0.000	0.000	1.184

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	516	272	0	162	0	0	129
N.S.	1	1.00	4.26	2.25	0.00	1.34	0.00	0.00	1.07
time (sec)	N/A	0.125	6.109	0.093	0.000	1.352	0.000	0.000	1.039

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	235	250	0	149	0	0	104
N.S.	1	1.00	2.47	2.63	0.00	1.57	0.00	0.00	1.09
time (sec)	N/A	0.114	5.856	0.106	0.000	0.819	0.000	0.000	0.919

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	224	228	0	134	0	0	59
N.S.	1	1.00	3.34	3.40	0.00	2.00	0.00	0.00	0.88
time (sec)	N/A	0.102	5.364	0.082	0.000	0.997	0.000	0.000	0.885

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	185	0	97	0	0	82
N.S.	1	1.00	0.89	4.20	0.00	2.20	0.00	0.00	1.86
time (sec)	N/A	0.084	0.169	0.092	0.000	0.895	0.000	0.000	1.214

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	470	371	0	187	0	0	109
N.S.	1	1.00	5.16	4.08	0.00	2.05	0.00	0.00	1.20
time (sec)	N/A	0.114	6.155	0.108	0.000	0.806	0.000	0.000	1.272

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	503	386	0	202	0	0	114
N.S.	1	1.00	4.16	3.19	0.00	1.67	0.00	0.00	0.94
time (sec)	N/A	0.126	6.191	0.128	0.000	0.796	0.000	0.000	1.378

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	531	439	0	215	0	0	114
N.S.	1	1.00	3.61	2.99	0.00	1.46	0.00	0.00	0.78
time (sec)	N/A	0.135	6.235	0.129	0.000	1.125	0.000	0.000	1.456

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	548	260	0	175	0	0	206
N.S.	1	1.00	3.73	1.77	0.00	1.19	0.00	0.00	1.40
time (sec)	N/A	0.193	6.124	0.098	0.000	1.020	0.000	0.000	1.150

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	516	272	0	162	0	0	143
N.S.	1	1.00	4.26	2.25	0.00	1.34	0.00	0.00	1.18
time (sec)	N/A	0.170	6.112	0.108	0.000	0.950	0.000	0.000	1.026

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	233	250	0	148	0	0	104
N.S.	1	1.00	2.56	2.75	0.00	1.63	0.00	0.00	1.14
time (sec)	N/A	0.146	5.960	0.099	0.000	0.724	0.000	0.000	0.975

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	240	172	0	180	0	0	104
N.S.	1	1.00	2.64	1.89	0.00	1.98	0.00	0.00	1.14
time (sec)	N/A	0.142	4.907	0.098	0.000	0.909	0.000	0.000	1.016

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	479	371	0	187	0	0	126
N.S.	1	1.00	5.26	4.08	0.00	2.05	0.00	0.00	1.38
time (sec)	N/A	0.143	6.192	0.118	0.000	0.832	0.000	0.000	1.640

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	501	386	0	200	0	0	154
N.S.	1	1.00	4.28	3.30	0.00	1.71	0.00	0.00	1.32
time (sec)	N/A	0.165	6.214	0.124	0.000	0.498	0.000	0.000	1.585

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	531	439	0	215	0	0	145
N.S.	1	1.00	3.61	2.99	0.00	1.46	0.00	0.00	0.99
time (sec)	N/A	0.181	6.242	0.138	0.000	0.756	0.000	0.000	1.642

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	314	229	0	208	0	0	-1
N.S.	1	1.00	2.45	1.79	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.143	1.692	0.088	0.000	0.995	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	292	215	0	198	0	0	-1
N.S.	1	1.00	2.92	2.15	0.00	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.127	2.118	0.085	0.000	0.781	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	270	199	0	186	0	0	-1
N.S.	1	1.00	3.75	2.76	0.00	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.112	1.697	0.088	0.000	0.672	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	262	198	0	184	0	0	-1
N.S.	1	1.00	3.74	2.83	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.057	0.086	0.000	1.297	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	263	200	0	184	0	0	-1
N.S.	1	1.00	3.76	2.86	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.112	1.064	0.089	0.000	0.813	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	303	253	0	236	0	0	-1
N.S.	1	1.00	3.16	2.64	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.126	1.733	0.101	0.000	0.525	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	338	413	0	258	0	0	-1
N.S.	1	1.00	2.73	3.33	0.00	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.135	3.676	0.129	0.000	1.030	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	366	283	0	288	0	0	-1
N.S.	1	1.00	2.29	1.77	0.00	1.80	0.00	0.00	-0.01
time (sec)	N/A	0.204	2.394	0.097	0.000	0.754	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	341	270	0	278	0	0	-1
N.S.	1	1.00	2.47	1.96	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.191	1.666	0.105	0.000	0.683	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	374	257	0	268	0	0	-1
N.S.	1	1.00	3.34	2.29	0.00	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.179	6.182	0.103	0.000	1.264	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	656	257	0	268	0	0	-1
N.S.	1	1.00	6.02	2.36	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.174	6.179	0.099	0.000	0.515	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	63	188	0	150	0	0	-1
N.S.	1	1.00	1.11	3.30	0.00	2.63	0.00	0.00	-0.02
time (sec)	N/A	0.079	0.227	0.102	0.000	1.152	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	312	257	0	268	0	0	-1
N.S.	1	1.00	2.86	2.36	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.180	4.983	0.103	0.000	0.499	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	393	405	0	318	0	0	-1
N.S.	1	1.00	2.89	2.98	0.00	2.34	0.00	0.00	-0.01
time (sec)	N/A	0.187	6.254	0.118	0.000	0.736	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	372	413	0	338	0	0	-1
N.S.	1	1.00	2.30	2.55	0.00	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.210	2.213	0.155	0.000	0.876	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	391	296	0	364	0	0	-1
N.S.	1	1.00	1.89	1.43	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.299	2.751	0.118	0.000	0.912	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	375	283	0	354	0	0	-1
N.S.	1	1.00	2.07	1.56	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.278	2.176	0.106	0.000	0.704	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	357	270	0	344	0	0	-1
N.S.	1	1.00	2.30	1.74	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.261	1.821	0.111	0.000	0.907	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	721	270	0	344	0	0	-1
N.S.	1	1.00	4.65	1.74	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.257	6.232	0.109	0.000	0.792	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	342	270	0	344	0	0	-1
N.S.	1	1.00	2.21	1.74	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.258	1.676	0.108	0.000	0.660	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	342	270	0	344	0	0	-1
N.S.	1	1.00	2.21	1.74	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.254	1.581	0.119	0.000	0.708	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	721	268	0	344	0	0	-1
N.S.	1	1.00	4.65	1.73	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.254	6.206	0.114	0.000	1.011	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	372	555	0	394	0	0	-1
N.S.	1	1.00	2.06	3.07	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.269	2.038	0.132	0.000	0.587	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	402	453	0	414	0	0	-1
N.S.	1	1.00	1.94	2.19	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	2.579	0.164	0.000	0.480	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	80	80	293	79	0	0	-1
N.S.	1	1.00	0.52	0.52	1.92	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.219	0.602	0.831	2.999	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	61	70	203	69	0	0	-1
N.S.	1	1.00	0.53	0.61	1.77	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.181	0.124	0.548	2.407	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	49	58	113	57	0	0	-1
N.S.	1	1.00	0.64	0.75	1.47	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.125	0.122	0.541	3.628	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	50	20	49	0	0	-1
N.S.	1	1.00	1.08	1.39	0.56	1.36	0.00	0.00	-0.03
time (sec)	N/A	0.070	0.111	0.118	0.524	1.919	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	74	142	241	180	0	0	-1
N.S.	1	1.00	1.30	2.49	4.23	3.16	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.173	0.125	0.552	2.546	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	90	178	662	325	0	0	-1
N.S.	1	1.00	0.98	1.93	7.20	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.258	0.156	0.566	2.999	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	100	213	1264	355	0	0	-1
N.S.	1	1.00	0.74	1.57	9.29	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.511	0.142	0.577	4.257	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	72	83	303	84	0	0	-1
N.S.	1	1.00	0.45	0.52	1.88	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.300	0.125	0.558	3.902	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	60	71	210	72	0	0	-1
N.S.	1	1.00	0.52	0.61	1.81	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.224	0.125	0.549	3.051	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	50	61	38	61	0	0	-1
N.S.	1	1.00	0.63	0.77	0.48	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.179	0.118	0.548	3.499	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	81	172	274	298	0	0	-1
N.S.	1	1.00	0.84	1.79	2.85	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.152	0.106	0.552	2.535	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	92	182	1143	337	0	0	-1
N.S.	1	1.00	0.97	1.92	12.03	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.349	0.129	0.567	4.417	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	99	212	2244	369	0	0	-1
N.S.	1	1.00	0.71	1.51	16.03	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.453	0.138	0.630	3.713	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	112	244	2361	391	0	0	-1
N.S.	1	1.00	0.62	1.36	13.12	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.545	0.154	0.667	4.262	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	90	95	422	105	0	0	-1
N.S.	1	1.00	0.45	0.47	2.10	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.250	0.125	0.565	3.429	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	74	85	323	92	0	0	-1
N.S.	1	1.00	0.47	0.54	2.07	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.271	0.121	0.556	2.137	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	64	75	60	79	0	0	-1
N.S.	1	1.00	0.54	0.63	0.50	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.258	0.118	0.529	3.382	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	93	185	593	339	0	0	-1
N.S.	1	1.00	0.67	1.34	4.30	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.255	0.135	0.574	3.718	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	90	197	11494	373	0	0	-1
N.S.	1	1.00	0.68	1.49	87.08	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.323	0.131	0.643	2.738	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	95	216	2826	385	0	0	-1
N.S.	1	1.00	0.68	1.54	20.19	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.708	0.141	3.223	2.873	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	180	244	3469	411	0	0	-1
N.S.	1	1.00	1.00	1.36	19.27	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.246	5.330	0.150	0.789	2.850	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	190	276	3860	437	0	0	-1
N.S.	1	1.00	0.86	1.25	17.55	1.99	0.00	0.00	-0.00
time (sec)	N/A	0.287	5.560	0.120	0.840	2.839	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	136	120	357	324	0	0	-1
N.S.	1	1.00	0.72	0.63	1.89	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.321	0.134	0.554	3.179	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	116	110	282	300	0	0	-1
N.S.	1	1.00	0.77	0.73	1.87	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.208	0.131	0.549	3.769	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	100	98	104	281	0	0	-1
N.S.	1	1.00	0.88	0.87	0.92	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.101	0.113	0.538	3.008	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	76	95	91	90	160	0	0	-1
N.S.	1	1.36	1.70	1.62	1.61	2.86	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.081	0.125	0.528	3.151	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	109	172	476	342	0	0	-1
N.S.	1	1.00	0.81	1.27	3.53	2.53	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.092	0.140	0.559	3.114	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	145	212	876	520	0	0	-1
N.S.	1	1.00	0.86	1.26	5.21	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.242	0.135	0.569	3.232	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	178	247	1646	550	0	0	-1
N.S.	1	1.00	0.84	1.17	7.80	2.61	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.426	0.154	0.594	3.652	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	152	193	0	400	0	0	-1
N.S.	1	1.00	0.64	0.81	0.00	1.69	0.00	0.00	-0.00
time (sec)	N/A	0.406	0.971	0.143	0.000	4.086	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	133	183	33960	380	0	0	-1
N.S.	1	1.00	0.68	0.93	172.39	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.303	0.665	0.129	0.945	4.607	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	138	173	7176	360	0	0	-1
N.S.	1	1.00	0.88	1.10	45.71	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.943	0.125	0.614	3.147	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	131	138	1031	340	0	0	-1
N.S.	1	1.00	1.12	1.18	8.81	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.466	0.141	0.557	2.440	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	248	136	15721	338	0	0	-1
N.S.	1	1.00	2.12	1.16	134.37	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.925	0.128	1.130	2.294	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	248	228	2122	550	0	0	-1
N.S.	1	1.00	1.43	1.31	12.20	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.805	0.132	0.592	2.513	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	242	271	4934	614	0	0	-1
N.S.	1	1.00	1.13	1.27	23.06	2.87	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.991	0.145	0.868	3.078	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	144	244	148823	448	0	0	-1
N.S.	1	1.00	0.61	1.03	627.95	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.412	1.185	0.135	3.539	3.202	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	141	234	258456	428	0	0	-1
N.S.	1	1.00	0.72	1.19	1311.96	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.314	0.962	0.138	2.976	3.037	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	168	200	3049	408	0	0	-1
N.S.	1	1.00	1.07	1.27	19.42	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.212	1.295	0.148	0.992	4.608	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	224	198	2875	408	0	0	-1
N.S.	1	1.00	1.43	1.26	18.31	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.177	3.886	0.129	0.857	3.721	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	341	200	84332	408	0	0	-1
N.S.	1	1.00	2.17	1.27	537.15	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.178	1.441	0.132	22.658	3.279	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	328	396	4988	638	0	0	-1
N.S.	1	1.00	1.53	1.85	23.31	2.98	0.00	0.00	-0.00
time (sec)	N/A	0.350	1.283	0.144	0.871	3.793	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	348	444	9048	702	0	0	-1
N.S.	1	1.00	1.37	1.75	35.62	2.76	0.00	0.00	-0.00
time (sec)	N/A	0.450	1.699	0.149	4.418	3.865	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	308	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	1.646	0.131	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	266	0	0	0	0	0	-1
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	1.380	0.110	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	105	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.167	0.102	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.919	0.065	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	7.809	0.090	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	73	95	99	0	164	152
N.S.	1	1.00	0.89	0.86	1.12	1.16	0.00	1.93	1.79
time (sec)	N/A	0.045	0.265	0.088	0.297	4.091	0.000	0.475	3.574

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	60	70	88	0	122	109
N.S.	1	1.00	0.95	0.95	1.11	1.40	0.00	1.94	1.73
time (sec)	N/A	0.038	0.174	0.065	0.274	3.125	0.000	0.468	2.809

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	47	58	74	0	107	85
N.S.	1	1.00	1.00	1.00	1.23	1.57	0.00	2.28	1.81
time (sec)	N/A	0.034	0.029	0.046	0.301	3.577	0.000	0.443	1.476

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	29	60	37	63	47
N.S.	1	1.00	1.00	1.25	1.21	2.50	1.54	2.62	1.96
time (sec)	N/A	0.019	0.017	0.035	0.256	2.767	2.621	0.460	0.793

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	36	41	49	57
N.S.	1	1.00	1.00	1.50	1.44	2.25	2.56	3.06	3.56
time (sec)	N/A	0.006	0.005	0.026	0.261	3.021	1.023	0.434	0.803

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	21	20	17	17	39	17
N.S.	1	1.00	1.73	1.40	1.33	1.13	1.13	2.60	1.13
time (sec)	N/A	0.015	0.011	0.054	0.255	2.072	1.120	0.438	0.744

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	38	34	29	0	82	31
N.S.	1	1.00	0.92	1.00	0.89	0.76	0.00	2.16	0.82
time (sec)	N/A	0.027	0.075	0.073	0.265	3.780	0.000	0.426	0.813

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	46	42	0	98	55
N.S.	1	1.00	1.06	0.91	0.85	0.78	0.00	1.81	1.02
time (sec)	N/A	0.032	0.079	0.076	0.256	4.426	0.000	0.436	0.831

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	57	53	0	140	75
N.S.	1	1.00	0.96	0.79	0.75	0.70	0.00	1.84	0.99
time (sec)	N/A	0.043	0.141	0.086	0.256	3.000	0.000	0.442	0.832

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	70	69	64	0	154	113
N.S.	1	1.00	1.00	0.76	0.75	0.70	0.00	1.67	1.23
time (sec)	N/A	0.044	0.095	0.100	0.254	2.625	0.000	0.442	4.748

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	118	110	132	136	0	272	221
N.S.	1	1.00	0.87	0.81	0.98	1.01	0.00	2.01	1.64
time (sec)	N/A	0.078	0.620	0.093	0.262	3.683	0.000	0.496	3.823

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	111	144	133	0	258	184
N.S.	1	1.00	0.75	1.01	1.31	1.21	0.00	2.35	1.67
time (sec)	N/A	0.070	0.301	0.099	0.258	2.933	0.000	0.480	3.664

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	84	100	0	178	141
N.S.	1	1.00	0.89	0.92	1.05	1.25	0.00	2.22	1.76
time (sec)	N/A	0.064	0.247	0.065	0.264	2.357	0.000	0.458	3.066

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	45	69	80	93	0	129	99
N.S.	1	1.00	0.76	1.17	1.36	1.58	0.00	2.19	1.68
time (sec)	N/A	0.041	0.113	0.055	0.266	2.934	0.000	0.425	1.529

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	43	40	74	0	77	181
N.S.	1	1.00	0.97	1.30	1.21	2.24	0.00	2.33	5.48
time (sec)	N/A	0.018	0.082	0.042	0.254	2.948	0.000	0.451	0.866

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	46	43	51	52	0	78	73
N.S.	1	1.00	1.39	1.30	1.55	1.58	0.00	2.36	2.21
time (sec)	N/A	0.039	0.022	0.073	0.259	2.289	0.000	0.461	0.846

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	47	40	0	96	42
N.S.	1	1.00	0.92	1.02	0.94	0.80	0.00	1.92	0.84
time (sec)	N/A	0.047	0.084	0.088	0.258	3.809	0.000	0.441	0.854

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	63	60	52	0	153	72
N.S.	1	1.00	1.02	1.09	1.03	0.90	0.00	2.64	1.24
time (sec)	N/A	0.066	0.163	0.100	0.260	3.348	0.000	0.487	0.825

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	86	89	82	77	0	224	93
N.S.	1	1.00	0.85	0.88	0.81	0.76	0.00	2.22	0.92
time (sec)	N/A	0.065	0.189	0.123	0.258	2.134	0.000	0.427	0.866

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	97	95	94	86	0	247	117
N.S.	1	1.00	0.87	0.86	0.85	0.77	0.00	2.23	1.05
time (sec)	N/A	0.092	0.288	0.117	0.256	3.930	0.000	0.478	0.894

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	120	148	181	170	0	367	258
N.S.	1	1.00	0.63	0.78	0.96	0.90	0.00	1.94	1.37
time (sec)	N/A	0.221	0.908	0.115	0.260	2.245	0.000	0.487	4.766

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	90	125	158	140	0	330	226
N.S.	1	1.00	0.69	0.96	1.22	1.08	0.00	2.54	1.74
time (sec)	N/A	0.144	0.467	0.092	0.263	2.903	0.000	0.499	4.798

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	96	106	126	0	205	157
N.S.	1	1.00	0.71	0.97	1.07	1.27	0.00	2.07	1.59
time (sec)	N/A	0.093	0.268	0.072	0.259	2.256	0.000	0.478	3.151

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	82	93	112	0	145	136
N.S.	1	1.00	0.75	1.12	1.27	1.53	0.00	1.99	1.86
time (sec)	N/A	0.036	0.174	0.060	0.268	2.751	0.000	0.443	0.955

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	88	57	66	94	0	131	97
N.S.	1	1.00	1.31	0.85	0.99	1.40	0.00	1.96	1.45
time (sec)	N/A	0.077	0.353	0.079	0.282	2.922	0.000	0.496	0.923

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	105	73	76	72	0	137	123
N.S.	1	1.00	1.33	0.92	0.96	0.91	0.00	1.73	1.56
time (sec)	N/A	0.083	0.160	0.085	0.256	2.900	0.000	0.484	1.025

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	80	76	73	66	0	170	77
N.S.	1	1.00	0.80	0.76	0.73	0.66	0.00	1.70	0.77
time (sec)	N/A	0.103	0.133	0.096	0.269	2.870	0.000	0.468	0.862

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	100	102	95	84	0	297	250
N.S.	1	1.00	0.81	0.83	0.77	0.68	0.00	2.41	2.03
time (sec)	N/A	0.133	0.298	0.119	0.256	3.081	0.000	0.468	3.834

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	130	123	119	110	0	332	287
N.S.	1	1.00	0.81	0.77	0.74	0.69	0.00	2.08	1.79
time (sec)	N/A	0.136	0.306	0.136	0.263	3.709	0.000	0.508	3.926

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	159	145	145	132	0	431	350
N.S.	1	1.00	0.86	0.78	0.78	0.71	0.00	2.33	1.89
time (sec)	N/A	0.169	0.344	0.141	0.256	2.644	0.000	0.462	3.298

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	154	209	275	217	0	592	370
N.S.	1	1.00	0.63	0.86	1.13	0.89	0.00	2.43	1.52
time (sec)	N/A	0.319	1.007	0.134	0.266	2.324	0.000	0.526	4.878

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	125	162	195	182	0	461	304
N.S.	1	1.00	0.70	0.91	1.09	1.02	0.00	2.58	1.70
time (sec)	N/A	0.209	0.796	0.110	0.262	2.381	0.000	0.509	4.970

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	101	147	180	163	0	360	245
N.S.	1	1.00	0.69	1.01	1.23	1.12	0.00	2.47	1.68
time (sec)	N/A	0.171	0.546	0.098	0.258	2.591	0.000	0.464	4.922

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	77	109	121	138	0	221	185
N.S.	1	1.00	0.72	1.02	1.13	1.29	0.00	2.07	1.73
time (sec)	N/A	0.087	0.319	0.077	0.264	2.466	0.000	0.463	1.029

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	280	96	115	130	0	179	152
N.S.	1	1.00	2.69	0.92	1.11	1.25	0.00	1.72	1.46
time (sec)	N/A	0.154	0.547	0.108	0.260	3.201	0.000	0.454	1.040

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	119	87	90	116	0	170	150
N.S.	1	1.00	1.10	0.81	0.83	1.07	0.00	1.57	1.39
time (sec)	N/A	0.154	0.710	0.104	0.260	3.487	0.000	0.468	1.034

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	128	98	102	98	0	212	158
N.S.	1	1.00	1.11	0.85	0.89	0.85	0.00	1.84	1.37
time (sec)	N/A	0.172	0.172	0.108	0.257	3.427	0.000	0.469	1.115

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	104	116	109	96	0	318	123
N.S.	1	1.00	0.72	0.80	0.75	0.66	0.00	2.19	0.85
time (sec)	N/A	0.215	0.243	0.119	0.273	3.290	0.000	0.473	0.884

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	133	138	133	121	0	425	330
N.S.	1	1.00	0.77	0.80	0.77	0.70	0.00	2.46	1.91
time (sec)	N/A	0.247	0.523	0.142	0.261	2.813	0.000	0.463	3.821

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	156	174	170	150	0	550	214
N.S.	1	1.00	0.73	0.82	0.80	0.70	0.00	2.58	1.00
time (sec)	N/A	0.265	0.458	0.095	0.263	2.574	0.000	0.504	1.094

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	114	160	198	183	0	380	274
N.S.	1	1.00	0.72	1.01	1.25	1.16	0.00	2.41	1.73
time (sec)	N/A	0.167	0.617	0.100	0.267	2.180	0.000	0.474	1.364

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	258	252	0	557	0	286	1021
N.S.	1	1.00	1.64	1.61	0.00	3.55	0.00	1.82	6.50
time (sec)	N/A	0.326	2.791	0.255	0.000	3.408	0.000	0.528	2.453

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	238	192	0	485	0	211	1002
N.S.	1	1.00	2.00	1.61	0.00	4.08	0.00	1.77	8.42
time (sec)	N/A	0.192	1.187	0.210	0.000	3.134	0.000	0.533	1.646

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	115	123	0	392	0	152	119
N.S.	1	1.00	1.35	1.45	0.00	4.61	0.00	1.79	1.40
time (sec)	N/A	0.117	0.406	0.145	0.000	3.766	0.000	0.487	1.213

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	102	83	0	290	0	120	186
N.S.	1	1.00	1.50	1.22	0.00	4.26	0.00	1.76	2.74
time (sec)	N/A	0.080	0.090	0.134	0.000	3.156	0.000	0.485	1.063

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	185	0	77	40
N.S.	1	1.00	0.98	0.90	0.00	3.78	0.00	1.57	0.82
time (sec)	N/A	0.044	0.044	0.071	0.000	2.806	0.000	0.464	0.894

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	65	0	230	0	218	186
N.S.	1	1.00	1.02	1.10	0.00	3.90	0.00	3.69	3.15
time (sec)	N/A	0.038	0.091	0.092	0.000	2.708	0.000	0.478	1.089

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	72	97	0	277	0	126	395
N.S.	1	1.00	0.95	1.28	0.00	3.64	0.00	1.66	5.20
time (sec)	N/A	0.076	0.148	0.106	0.000	3.067	0.000	0.476	1.310

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	138	0	334	0	178	592
N.S.	1	1.00	0.88	1.25	0.00	3.04	0.00	1.62	5.38
time (sec)	N/A	0.201	0.250	0.131	0.000	2.880	0.000	0.466	1.811

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	122	179	0	401	0	249	654
N.S.	1	1.00	0.82	1.21	0.00	2.71	0.00	1.68	4.42
time (sec)	N/A	0.320	0.354	0.155	0.000	2.975	0.000	0.447	2.064

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	153	256	0	482	0	393	2678
N.S.	1	1.00	0.79	1.33	0.00	2.50	0.00	2.04	13.88
time (sec)	N/A	0.481	0.606	0.164	0.000	3.579	0.000	0.487	3.425

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	357	277	0	909	0	299	2500
N.S.	1	1.00	1.61	1.25	0.00	4.09	0.00	1.35	11.26
time (sec)	N/A	0.433	6.135	0.372	0.000	5.971	0.000	0.517	8.010

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	162	209	0	760	0	331	3159
N.S.	1	1.00	0.99	1.27	0.00	4.63	0.00	2.02	19.26
time (sec)	N/A	0.241	1.120	0.280	0.000	3.984	0.000	0.494	6.811

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	146	166	0	596	0	203	2848
N.S.	1	1.00	1.25	1.42	0.00	5.09	0.00	1.74	24.34
time (sec)	N/A	0.156	0.412	0.253	0.000	3.985	0.000	0.495	6.727

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	118	0	329	0	150	92
N.S.	1	1.00	0.98	1.39	0.00	3.87	0.00	1.76	1.08
time (sec)	N/A	0.082	0.211	0.116	0.000	2.909	0.000	0.478	1.118

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	83	118	0	332	0	150	92
N.S.	1	1.00	0.97	1.37	0.00	3.86	0.00	1.74	1.07
time (sec)	N/A	0.073	0.250	0.122	0.000	2.910	0.000	0.472	1.043

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	138	151	0	484	0	179	2886
N.S.	1	1.00	1.27	1.39	0.00	4.44	0.00	1.64	26.48
time (sec)	N/A	0.112	0.501	0.127	0.000	3.546	0.000	0.463	6.672

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	172	184	0	565	0	837	3169
N.S.	1	1.00	1.18	1.26	0.00	3.87	0.00	5.73	21.71
time (sec)	N/A	0.233	0.787	0.152	0.000	3.294	0.000	0.547	7.011

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	144	224	0	660	0	264	2500
N.S.	1	1.00	0.69	1.08	0.00	3.17	0.00	1.27	12.02
time (sec)	N/A	0.404	0.822	0.193	0.000	3.486	0.000	0.494	8.323

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	176	263	0	757	0	335	2500
N.S.	1	1.00	0.67	1.01	0.00	2.90	0.00	1.28	9.58
time (sec)	N/A	0.581	1.139	0.211	0.000	2.731	0.000	0.461	9.085

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	205	296	0	1354	0	383	2500
N.S.	1	1.00	0.89	1.29	0.00	5.89	0.00	1.67	10.87
time (sec)	N/A	0.488	4.866	0.517	0.000	4.204	0.000	0.563	9.290

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	194	255	0	1153	0	347	2500
N.S.	1	1.00	1.03	1.36	0.00	6.13	0.00	1.85	13.30
time (sec)	N/A	0.291	1.395	0.439	0.000	3.750	0.000	0.506	9.497

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	113	184	0	594	0	253	204
N.S.	1	1.00	0.76	1.23	0.00	3.99	0.00	1.70	1.37
time (sec)	N/A	0.168	0.447	0.243	0.000	2.576	0.000	0.513	3.235

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	115	195	0	565	0	277	210
N.S.	1	1.00	0.86	1.46	0.00	4.22	0.00	2.07	1.57
time (sec)	N/A	0.140	0.412	0.202	0.000	3.220	0.000	0.534	3.374

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	115	186	0	595	0	254	204
N.S.	1	1.00	0.86	1.40	0.00	4.47	0.00	1.91	1.53
time (sec)	N/A	0.127	0.493	0.184	0.000	2.969	0.000	0.530	3.183

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	205	237	0	919	0	322	2500
N.S.	1	1.00	1.18	1.37	0.00	5.31	0.00	1.86	14.45
time (sec)	N/A	0.228	0.806	0.182	0.000	3.157	0.000	0.468	9.222

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	229	270	0	1037	0	357	2500
N.S.	1	1.00	1.03	1.21	0.00	4.65	0.00	1.60	11.21
time (sec)	N/A	0.445	0.912	0.230	0.000	2.781	0.000	0.552	8.999

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	199	310	0	1158	0	1723	2500
N.S.	1	1.00	0.67	1.05	0.00	3.91	0.00	5.82	8.45
time (sec)	N/A	0.670	2.127	0.254	0.000	3.468	0.000	0.736	9.285

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	416	425	0	2058	0	592	2500
N.S.	1	1.00	1.32	1.34	0.00	6.51	0.00	1.87	7.91
time (sec)	N/A	0.760	6.245	0.431	0.000	7.758	0.000	0.548	10.358

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	250	383	0	1822	0	559	2500
N.S.	1	1.00	0.97	1.48	0.00	7.03	0.00	2.16	9.65
time (sec)	N/A	0.522	4.244	0.832	0.000	7.202	0.000	0.547	12.448

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	158	285	0	903	0	403	378
N.S.	1	1.00	0.71	1.28	0.00	4.07	0.00	1.82	1.70
time (sec)	N/A	0.303	1.061	0.398	0.000	3.461	0.000	0.516	4.413

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	165	294	0	902	0	431	380
N.S.	1	1.00	0.80	1.43	0.00	4.38	0.00	2.09	1.84
time (sec)	N/A	0.244	1.132	0.371	0.000	2.818	0.000	0.583	4.379

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	164	297	0	901	0	431	382
N.S.	1	1.00	0.85	1.55	0.00	4.69	0.00	2.24	1.99
time (sec)	N/A	0.222	1.264	0.298	0.000	2.913	0.000	0.526	4.343

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	163	284	0	905	0	403	378
N.S.	1	1.00	0.89	1.54	0.00	4.92	0.00	2.19	2.05
time (sec)	N/A	0.212	1.204	0.277	0.000	3.097	0.000	0.520	4.320

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	268	365	0	1456	0	532	2500
N.S.	1	1.00	1.11	1.51	0.00	6.02	0.00	2.20	10.33
time (sec)	N/A	0.396	1.498	0.240	0.000	3.095	0.000	0.466	12.790

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	293	399	0	1603	0	564	2500
N.S.	1	1.00	0.98	1.33	0.00	5.36	0.00	1.89	8.36
time (sec)	N/A	0.726	1.683	0.323	0.000	3.315	0.000	0.492	10.384

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	326	439	0	1767	0	615	2500
N.S.	1	1.00	0.84	1.13	0.00	4.57	0.00	1.59	6.46
time (sec)	N/A	0.981	6.378	0.359	0.000	3.673	0.000	0.503	10.803

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	32	47	33	0	30	21
N.S.	1	1.00	0.97	1.03	1.52	1.06	0.00	0.97	0.68
time (sec)	N/A	0.024	0.058	0.059	0.457	2.785	0.000	0.421	0.824

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	73	58	88	73	0	59	52
N.S.	1	1.00	1.30	1.04	1.57	1.30	0.00	1.05	0.93
time (sec)	N/A	0.059	0.188	0.068	0.463	2.579	0.000	0.424	0.859

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	108	74	131	116	0	75	79
N.S.	1	1.00	1.33	0.91	1.62	1.43	0.00	0.93	0.98
time (sec)	N/A	0.082	0.358	0.076	0.464	3.626	0.000	0.463	0.907

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	141	87	171	159	0	88	105
N.S.	1	1.00	1.33	0.82	1.61	1.50	0.00	0.83	0.99
time (sec)	N/A	0.111	0.555	0.089	0.488	3.719	0.000	0.441	1.094

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	69	46	70	52	0	43	21
N.S.	1	1.00	0.99	0.66	1.00	0.74	0.00	0.61	0.30
time (sec)	N/A	0.026	0.064	0.066	0.462	2.963	0.000	0.439	0.875

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	162	76	111	102	0	69	52
N.S.	1	1.00	1.71	0.80	1.17	1.07	0.00	0.73	0.55
time (sec)	N/A	0.067	0.167	0.067	0.457	2.618	0.000	0.427	0.881

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	241	106	155	155	0	85	78
N.S.	1	1.00	2.01	0.88	1.29	1.29	0.00	0.71	0.65
time (sec)	N/A	0.101	0.333	0.073	0.484	2.819	0.000	0.448	0.953

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	344	136	194	208	0	98	105
N.S.	1	1.00	2.37	0.94	1.34	1.43	0.00	0.68	0.72
time (sec)	N/A	0.126	0.533	0.085	0.463	2.659	0.000	0.436	1.099

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	401	1584	0	0	0	0	-1
N.S.	1	1.00	1.37	5.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	30.008	0.918	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	293	914	0	0	0	0	-1
N.S.	1	1.00	1.22	3.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	14.438	0.215	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	232	814	0	0	0	0	-1
N.S.	1	1.00	1.11	3.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.107	9.897	0.181	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	151	215	0	0	0	0	-1
N.S.	1	1.00	1.21	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	1.749	0.247	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	2713	829	0	0	0	0	-1
N.S.	1	1.00	8.22	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	18.360	0.256	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	1173	1254	0	0	0	0	-1
N.S.	1	1.00	2.96	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.374	18.338	0.208	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	550	2522	0	0	0	0	-1
N.S.	1	1.00	1.36	6.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.559	17.962	0.685	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	471	1852	0	0	0	0	-1
N.S.	1	1.00	1.38	5.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	14.218	0.355	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	408	1566	0	0	0	0	-1
N.S.	1	1.00	1.45	5.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	13.390	0.277	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	304	1106	0	0	0	0	-1
N.S.	1	1.00	1.22	4.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	10.256	0.192	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	882	1199	0	0	0	0	-1
N.S.	1	1.00	2.85	3.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.151	17.535	0.181	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	439	1029	0	0	0	0	-1
N.S.	1	1.00	1.31	3.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	12.380	0.203	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	1159	1440	0	0	0	0	-1
N.S.	1	1.00	2.97	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	18.035	0.183	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	615	2806	0	0	0	0	-1
N.S.	1	1.00	1.33	6.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.707	17.212	0.875	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	552	2523	0	0	0	0	-1
N.S.	1	1.00	1.38	6.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	16.562	0.533	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	474	1852	0	0	0	0	-1
N.S.	1	1.00	1.42	5.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	13.928	0.347	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	440	1775	0	0	0	0	-1
N.S.	1	1.00	1.49	6.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	16.447	0.276	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	713	1514	0	0	0	0	-1
N.S.	1	1.00	2.03	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	17.726	0.205	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	454	1640	0	0	0	0	-1
N.S.	1	1.00	1.29	4.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.230	16.362	0.222	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	4588	1646	0	0	0	0	-1
N.S.	1	1.00	11.50	4.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	18.883	0.185	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	1018	1881	0	0	0	0	-1
N.S.	1	1.00	2.21	4.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.615	14.654	0.224	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	1688	2330	0	0	0	0	-1
N.S.	1	1.00	3.18	4.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.864	14.073	0.264	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	1150	2185	0	0	0	0	-1
N.S.	1	1.00	2.85	5.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	13.376	0.304	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	463	1852	0	0	0	0	-1
N.S.	1	1.00	1.29	5.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	11.412	0.378	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	365	1583	0	0	0	0	-1
N.S.	1	1.00	1.21	5.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	8.935	0.309	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	341	919	0	0	0	0	-1
N.S.	1	1.00	1.40	3.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	10.448	0.230	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	238	639	0	0	0	0	-1
N.S.	1	1.00	1.17	3.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.105	11.244	0.188	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	143	0	0	0	0	-1
N.S.	1	1.00	0.94	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.733	0.175	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	138	178	0	0	0	0	-1
N.S.	1	1.00	1.30	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.920	0.184	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	5060	652	0	0	0	0	-1
N.S.	1	1.00	14.97	1.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	19.584	0.213	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	1195	1259	0	0	0	0	-1
N.S.	1	1.00	2.98	3.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	16.926	0.182	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	455	2477	0	0	0	0	-1
N.S.	1	1.00	1.14	6.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.529	10.203	0.539	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	416	1792	0	0	0	0	-1
N.S.	1	1.00	1.28	5.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	11.084	0.282	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	395	1451	0	0	0	0	-1
N.S.	1	1.00	1.54	5.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	8.957	0.221	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	249	837	0	0	0	0	-1
N.S.	1	1.00	1.05	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	5.765	0.187	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	244	817	0	0	0	0	-1
N.S.	1	1.00	1.03	3.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.162	6.070	0.188	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	1249	1209	0	0	0	0	-1
N.S.	1	1.00	3.60	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	16.839	0.188	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	1069	1662	0	0	0	0	-1
N.S.	1	1.00	2.70	4.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	13.006	0.194	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	1745	2298	0	0	0	0	-1
N.S.	1	1.00	3.71	4.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.525	12.824	0.221	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	578	4176	0	0	0	0	-1
N.S.	1	1.00	1.35	9.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.626	13.017	0.508	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	556	3674	0	0	0	0	-1
N.S.	1	1.00	1.54	10.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.396	13.334	0.342	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	503	2733	0	0	0	0	-1
N.S.	1	1.00	1.49	8.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.339	11.089	0.189	0.000	0.000	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	486	2410	0	0	0	0	-1
N.S.	1	1.00	1.53	7.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	10.635	0.183	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	360	1781	0	0	0	0	-1
N.S.	1	1.00	1.18	5.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	5.028	0.155	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	1798	3889	0	0	0	0	-1
N.S.	1	1.00	4.01	8.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.400	12.612	0.182	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	1481	4580	0	0	0	0	-1
N.S.	1	1.00	2.90	8.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.567	14.197	0.257	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	2285	5638	0	0	0	0	-1
N.S.	1	1.00	4.07	10.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.746	13.512	0.342	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	535	2346	7838	0	0	0	0	-1
N.S.	1	1.00	4.39	14.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.596	13.559	0.287	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	97	502	0	188	0	0	-1
N.S.	1	1.00	0.64	3.32	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.213	0.225	0.000	0.700	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	85	396	0	167	0	0	-1
N.S.	1	1.00	0.69	3.22	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.157	0.188	0.000	0.567	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	71	150	0	124	0	0	-1
N.S.	1	1.00	0.73	1.55	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.078	0.124	0.000	0.956	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	107	0	0	-1
N.S.	1	1.00	0.69	2.03	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.050	0.112	0.000	1.039	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	228	0	125	0	0	-1
N.S.	1	1.00	0.75	2.26	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.095	0.095	0.000	0.653	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	88	262	0	145	0	0	-1
N.S.	1	1.00	0.69	2.06	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.237	0.107	0.000	0.618	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	99	290	0	156	0	0	-1
N.S.	1	1.00	0.66	1.92	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.370	0.102	0.000	0.828	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	139	689	0	235	0	0	-1
N.S.	1	1.00	0.70	3.44	0.00	1.18	0.00	0.00	-0.00
time (sec)	N/A	0.108	0.583	0.295	0.000	0.898	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	126	633	0	223	0	0	-1
N.S.	1	1.00	0.72	3.62	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.836	0.251	0.000	0.769	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	93	513	0	190	0	0	-1
N.S.	1	1.00	0.69	3.80	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.216	0.205	0.000	0.748	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	82	202	0	146	0	0	-1
N.S.	1	1.00	0.76	1.87	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.128	0.144	0.000	0.747	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	283	0	147	0	0	-1
N.S.	1	1.00	0.78	2.53	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.125	0.107	0.000	0.654	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	100	357	0	170	0	0	-1
N.S.	1	1.00	0.71	2.53	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.292	0.112	0.000	0.631	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	120	362	0	191	0	0	-1
N.S.	1	1.00	0.69	2.07	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.483	0.127	0.000	0.818	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	177	820	0	270	0	0	-1
N.S.	1	1.00	0.76	3.50	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.170	2.244	0.334	0.000	1.194	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	134	711	0	244	0	0	-1
N.S.	1	1.00	0.71	3.76	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.977	0.260	0.000	1.151	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	106	630	0	214	0	0	-1
N.S.	1	1.00	0.67	3.99	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.323	0.221	0.000	1.179	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	108	303	0	182	0	0	-1
N.S.	1	1.00	0.65	1.83	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.388	0.141	0.000	0.812	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	106	412	0	193	0	0	-1
N.S.	1	1.00	0.68	2.64	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.311	0.128	0.000	0.527	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	132	421	0	216	0	0	-1
N.S.	1	1.00	0.66	2.12	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.624	0.194	0.000	0.858	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	159	470	0	238	0	0	-1
N.S.	1	1.00	0.68	2.01	0.00	1.02	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.853	0.140	0.000	0.736	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	256	1147	0	318	0	0	-1
N.S.	1	1.00	0.89	4.00	0.00	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.285	1.428	0.470	0.000	0.830	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	168	898	0	289	0	0	-1
N.S.	1	1.00	0.68	3.64	0.00	1.17	0.00	0.00	-0.00
time (sec)	N/A	0.259	1.017	0.362	0.000	0.521	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	146	880	0	264	0	0	-1
N.S.	1	1.00	0.70	4.21	0.00	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.252	1.430	0.296	0.000	0.859	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	130	777	0	242	0	0	-1
N.S.	1	1.00	0.62	3.74	0.00	1.16	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.733	0.194	0.000	0.495	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	138	432	0	215	0	0	-1
N.S.	1	1.00	0.67	2.09	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.257	0.395	0.163	0.000	0.992	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	142	476	0	234	0	0	-1
N.S.	1	1.00	0.67	2.26	0.00	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.252	0.553	0.138	0.000	0.905	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	168	529	0	257	0	0	-1
N.S.	1	1.00	0.69	2.16	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.270	0.991	0.136	0.000	0.829	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	199	586	0	286	0	0	-1
N.S.	1	1.00	0.69	2.03	0.00	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.304	1.228	0.145	0.000	0.903	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	165	423	0	0	0	0	-1
N.S.	1	1.00	0.88	2.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.346	31.894	0.250	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	83	353	0	0	0	0	-1
N.S.	1	1.00	0.71	3.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	33.053	0.135	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	63	150	0	0	0	0	-1
N.S.	1	1.00	1.29	3.06	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	10.226	0.099	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	47	187	0	0	0	0	-1
N.S.	1	1.00	0.51	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	10.172	0.117	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	176	226	0	0	0	0	-1
N.S.	1	1.00	1.30	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	24.008	0.156	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	194	552	0	0	0	0	-1
N.S.	1	1.00	1.13	3.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	34.192	0.154	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	294	975	0	0	0	0	-1
N.S.	1	1.00	0.86	2.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.633	35.036	0.421	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	351	841	0	0	0	0	-1
N.S.	1	1.00	1.26	3.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	33.785	0.276	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	582	608	0	0	0	0	-1
N.S.	1	1.00	2.72	2.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	36.463	0.165	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	289	707	0	0	0	0	-1
N.S.	1	1.00	1.39	3.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	35.597	0.218	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	251	788	0	0	0	0	-1
N.S.	1	1.00	1.11	3.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	32.839	0.254	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	319	809	0	0	0	0	-1
N.S.	1	1.00	1.31	3.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	34.417	0.259	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	279	1064	0	0	0	0	-1
N.S.	1	1.00	0.92	3.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	34.746	0.280	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	532	1987	0	0	0	0	-1
N.S.	1	1.00	1.37	5.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.654	36.395	0.506	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	335	1203	0	0	0	0	-1
N.S.	1	1.00	1.06	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.475	35.133	0.253	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	428	1760	0	0	0	0	-1
N.S.	1	1.00	1.37	5.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	35.439	0.395	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	429	1858	0	0	0	0	-1
N.S.	1	1.00	1.40	6.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	35.196	0.403	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	286	1936	0	0	0	0	-1
N.S.	1	1.00	0.89	5.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.424	34.431	0.448	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	707	1957	0	0	0	0	-1
N.S.	1	1.00	2.07	5.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	36.566	0.474	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	731	2216	0	0	0	0	-1
N.S.	1	1.00	1.80	5.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.682	36.618	0.513	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	321	789	0	0	0	0	-1
N.S.	1	1.00	1.35	3.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	13.634	1.164	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	96	265	0	0	0	0	-1
N.S.	1	1.00	0.70	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.237	2.188	0.190	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	925	0	355	0	0	-1
N.S.	1	1.00	1.00	13.81	0.00	5.30	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.075	0.320	0.000	1.022	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	156	1021	0	415	0	0	-1
N.S.	1	1.00	0.81	5.32	0.00	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.411	0.270	0.000	0.826	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	203	1736	0	461	0	0	-1
N.S.	1	1.00	0.83	7.11	0.00	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.406	0.590	0.247	0.000	1.134	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	237	2050	0	501	0	0	-1
N.S.	1	1.00	0.78	6.72	0.00	1.64	0.00	0.00	-0.00
time (sec)	N/A	0.559	0.825	0.262	0.000	0.879	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	411	1744	0	0	0	0	-1
N.S.	1	1.00	1.37	5.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.656	14.178	0.211	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	394	1207	0	0	0	0	-1
N.S.	1	1.00	1.58	4.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	14.254	0.225	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	129	1367	0	0	0	0	-1
N.S.	1	1.00	0.62	6.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	2.333	0.214	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	1219	0	415	0	0	-1
N.S.	1	1.00	0.83	6.52	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.461	0.251	0.000	0.937	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	197	1707	0	463	0	0	-1
N.S.	1	1.00	0.82	7.11	0.00	1.93	0.00	0.00	-0.00
time (sec)	N/A	0.425	0.737	0.223	0.000	1.329	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	237	2050	0	501	0	0	-1
N.S.	1	1.00	0.78	6.77	0.00	1.65	0.00	0.00	-0.00
time (sec)	N/A	0.590	1.146	0.243	0.000	0.909	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	602	2295	0	0	0	0	-1
N.S.	1	1.00	1.63	6.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.898	16.409	0.519	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	560	1982	0	0	0	0	-1
N.S.	1	1.00	1.78	6.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.710	16.317	0.194	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	421	1949	0	0	0	0	-1
N.S.	1	1.00	1.60	7.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.518	15.262	0.206	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	409	1661	0	0	0	0	-1
N.S.	1	1.00	1.56	6.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	13.757	0.193	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	200	1931	0	462	0	0	-1
N.S.	1	1.00	0.84	8.08	0.00	1.93	0.00	0.00	-0.00
time (sec)	N/A	0.456	1.001	0.217	0.000	0.945	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	237	2050	0	501	0	0	-1
N.S.	1	1.00	0.78	6.77	0.00	1.65	0.00	0.00	-0.00
time (sec)	N/A	0.627	1.514	0.243	0.000	0.584	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	286	2788	0	541	0	0	-1
N.S.	1	1.00	0.79	7.68	0.00	1.49	0.00	0.00	-0.00
time (sec)	N/A	0.844	1.728	0.316	0.000	1.089	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	397	1755	0	0	0	0	-1
N.S.	1	1.00	1.27	5.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.591	14.052	0.204	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	329	995	0	0	0	0	-1
N.S.	1	1.00	1.34	4.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	15.491	0.223	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	232	0	0	0	0	-1
N.S.	1	1.00	1.00	3.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.077	0.187	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	153	0	146	0	0	-1
N.S.	1	1.00	1.00	2.28	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.053	0.203	0.000	1.542	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	96	736	0	355	0	0	-1
N.S.	1	1.00	0.68	5.18	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.188	1.751	0.226	0.000	1.401	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	147	1024	0	415	0	0	-1
N.S.	1	1.00	0.75	5.25	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.416	0.240	0.000	1.338	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	193	1736	0	464	0	0	-1
N.S.	1	1.00	0.78	6.97	0.00	1.86	0.00	0.00	-0.00
time (sec)	N/A	0.403	0.516	0.219	0.000	1.769	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	478	1501	0	0	0	0	-1
N.S.	1	1.00	1.39	4.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.688	12.831	0.452	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	434	1144	0	0	0	0	-1
N.S.	1	1.00	2.11	5.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	14.205	0.200	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	103	501	0	488	0	0	-1
N.S.	1	1.00	0.82	3.98	0.00	3.87	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.194	0.191	0.000	0.962	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	156	506	0	525	0	0	-1
N.S.	1	1.00	0.78	2.53	0.00	2.62	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.412	0.190	0.000	1.547	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	165	999	0	565	0	0	-1
N.S.	1	1.00	0.77	4.67	0.00	2.64	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.455	0.232	0.000	1.398	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	203	1317	0	632	0	0	-1
N.S.	1	1.00	0.70	4.56	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.465	0.628	0.253	0.000	1.201	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	250	1861	0	692	0	0	-1
N.S.	1	1.00	0.69	5.17	0.00	1.92	0.00	0.00	-0.00
time (sec)	N/A	0.668	0.913	0.219	0.000	1.759	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	561	4591	0	0	0	0	-1
N.S.	1	1.00	1.22	10.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.972	14.817	0.261	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	487	3854	0	0	0	0	-1
N.S.	1	1.00	1.32	10.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.746	13.480	0.211	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	169	1343	0	688	0	0	-1
N.S.	1	1.00	0.61	4.85	0.00	2.48	0.00	0.00	-0.00
time (sec)	N/A	0.453	0.711	0.238	0.000	0.574	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	178	1822	0	759	0	0	-1
N.S.	1	1.00	0.63	6.48	0.00	2.70	0.00	0.00	-0.00
time (sec)	N/A	0.421	0.722	0.197	0.000	1.139	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	196	2070	0	811	0	0	-1
N.S.	1	1.00	0.65	6.85	0.00	2.69	0.00	0.00	-0.00
time (sec)	N/A	0.442	0.837	0.201	0.000	1.862	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	208	3103	0	863	0	0	-1
N.S.	1	1.00	0.66	9.79	0.00	2.72	0.00	0.00	-0.00
time (sec)	N/A	0.492	0.877	0.230	0.000	1.039	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	257	3614	0	949	0	0	-1
N.S.	1	1.00	0.66	9.24	0.00	2.43	0.00	0.00	-0.00
time (sec)	N/A	0.687	1.130	0.242	0.000	0.775	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	292	4586	0	1036	0	0	-1
N.S.	1	1.00	0.62	9.68	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.910	1.360	0.314	0.000	0.787	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	78	405	0	95	0	0	-1
N.S.	1	1.00	0.64	3.32	0.00	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.096	1.633	0.000	0.762	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	374	0	92	0	0	-1
N.S.	1	1.00	0.62	3.43	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.084	1.137	0.000	0.548	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	68	405	0	95	0	0	-1
N.S.	1	1.00	0.63	3.75	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.056	0.621	0.000	0.759	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	78	390	0	87	0	0	-1
N.S.	1	1.00	0.63	3.17	0.00	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.066	0.431	0.000	0.811	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	81	409	0	108	0	0	-1
N.S.	1	1.00	0.64	3.22	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.097	1.099	0.000	0.647	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	72	381	0	108	0	0	-1
N.S.	1	1.00	0.64	3.37	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.083	1.115	0.000	0.832	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	72	370	0	108	0	0	-1
N.S.	1	1.00	0.56	2.87	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.056	0.410	0.000	0.931	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	394	0	108	0	0	-1
N.S.	1	1.00	0.70	3.43	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.070	0.309	0.000	0.530	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	142	0	47	0	0	-1
N.S.	1	1.00	1.00	2.33	0.00	0.77	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.039	0.201	0.000	0.486	0.000	0.000	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	137	0	44	0	0	-1
N.S.	1	1.00	1.00	2.54	0.00	0.81	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.038	0.265	0.000	0.690	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	144	0	47	0	0	-1
N.S.	1	1.00	1.00	2.67	0.00	0.87	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.028	0.284	0.000	0.820	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	139	0	44	0	0	-1
N.S.	1	1.00	1.00	2.28	0.00	0.72	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.032	0.217	0.000	0.576	0.000	0.000	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	145	0	54	0	0	-1
N.S.	1	1.00	1.00	2.38	0.00	0.89	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.039	0.267	0.000	1.298	0.000	0.000	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	138	0	54	0	0	-1
N.S.	1	1.00	1.00	2.56	0.00	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.039	0.198	0.000	0.775	0.000	0.000	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	136	0	52	0	0	-1
N.S.	1	1.00	0.87	2.19	0.00	0.84	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.032	0.190	0.000	1.206	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	61	142	0	52	0	0	-1
N.S.	1	1.00	1.11	2.58	0.00	0.95	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.031	0.198	0.000	0.554	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	7160	0	0	0	0	0	-1
N.S.	1	1.00	68.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	42.729	0.073	0.000	0.000	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	2.792	0.059	0.000	0.000	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	8052	0	0	0	0	0	-1
N.S.	1	1.00	22.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	38.007	0.084	0.000	0.000	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	7783	0	0	0	0	0	-1
N.S.	1	1.00	25.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	37.791	0.075	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	2505	0	0	0	0	0	-1
N.S.	1	1.00	9.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	48.320	0.066	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	7142	0	0	0	0	0	-1
N.S.	1	1.00	68.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	36.396	0.057	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	3.965	0.066	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	7313	0	0	0	0	0	-1
N.S.	1	1.00	67.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	42.928	0.065	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	37.654	0.067	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	28057	0	0	0	0	0	-1
N.S.	1	1.00	68.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	38.673	0.083	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	8065	0	0	0	0	0	-1
N.S.	1	1.00	22.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	38.038	0.076	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	7809	0	0	0	0	0	-1
N.S.	1	1.00	26.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	37.777	0.064	0.000	0.000	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	7321	0	0	0	0	0	-1
N.S.	1	1.00	67.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	36.831	0.058	0.000	0.000	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.010	35.628	0.062	0.000	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	7796	0	0	0	0	0	-1
N.S.	1	1.00	24.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	37.859	0.091	0.000	0.000	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	7195	0	0	0	0	0	-1
N.S.	1	1.00	27.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	36.429	0.080	0.000	0.000	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	2759	0	0	0	0	0	-1
N.S.	1	1.00	12.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	44.459	0.065	0.000	0.000	0.000	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0	-1
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	13.018	0.058	0.000	0.000	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	0.676	0.058	0.000	0.000	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0	-1
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	13.849	0.065	0.000	0.000	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	0.660	0.059	0.000	0.000	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	10343	0	0	0	0	0	-1
N.S.	1	1.00	94.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	38.220	0.061	0.000	0.000	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	40.199	0.059	0.000	0.000	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	8160	0	0	0	0	0	-1
N.S.	1	1.00	21.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.423	41.694	0.090	0.000	0.000	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	7918	0	0	0	0	0	-1
N.S.	1	1.00	25.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	41.394	0.077	0.000	0.000	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	7325	0	0	0	0	0	-1
N.S.	1	1.00	25.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	42.686	0.065	0.000	0.000	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	10363	0	0	0	0	0	-1
N.S.	1	1.00	94.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	44.386	0.060	0.000	0.000	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.008	49.956	0.061	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	4543	0	0	0	0	0	-1
N.S.	1	1.00	26.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	39.541	0.078	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	4544	0	0	0	0	0	-1
N.S.	1	1.00	26.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	39.558	0.081	0.000	0.000	0.000	0.000	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	7542	0	0	0	0	0	-1
N.S.	1	1.00	43.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	126.710	0.076	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	7588	0	0	0	0	0	-1
N.S.	1	1.00	43.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	126.542	0.076	0.000	0.000	0.000	0.000	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	112.035	0.148	0.000	0.000	0.000	0.000	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	113.002	0.135	0.000	0.000	0.000	0.000	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	63.216	0.122	0.000	0.000	0.000	0.000	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	71.383	0.120	0.000	0.000	0.000	0.000	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	7.160	0.118	0.000	0.000	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	22.328	0.115	0.000	0.000	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	26.767	0.118	0.000	0.000	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	34.558	0.118	0.000	0.000	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	79.843	0.123	0.000	0.000	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	92.973	0.121	0.000	0.000	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	104.986	0.118	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	99.663	0.121	0.000	0.000	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	58.409	0.111	0.000	0.000	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	62.126	0.111	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	41.287	0.109	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	92.044	0.116	0.000	0.000	0.000	0.000	0.000

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	51.354	0.111	0.000	0.000	0.000	0.000	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	56.223	0.107	0.000	0.000	0.000	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	90.734	0.112	0.000	0.000	0.000	0.000	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	123.763	0.109	0.000	0.000	0.000	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	112.888	0.115	0.000	0.000	0.000	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	107.049	0.119	0.000	0.000	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	110.163	0.112	0.000	0.000	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	104.060	0.112	0.000	0.000	0.000	0.000	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	90.217	0.111	0.000	0.000	0.000	0.000	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	115.192	0.117	0.000	0.000	0.000	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	103.755	0.113	0.000	0.000	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	116.792	0.112	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	114.247	0.115	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	135.096	0.109	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	45.395	0.116	0.000	0.000	0.000	0.000	0.000

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	61.019	0.119	0.000	0.000	0.000	0.000	0.000

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	3.620	0.115	0.000	0.000	0.000	0.000	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	3.650	0.115	0.000	0.000	0.000	0.000	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.852	0.118	0.000	0.000	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	1.825	0.118	0.000	0.000	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	65.066	0.122	0.000	0.000	0.000	0.000	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	78.240	0.120	0.000	0.000	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	70.544	0.123	0.000	0.000	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	89.114	0.119	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	69.431	0.122	0.000	0.000	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	81.018	0.117	0.000	0.000	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	77.266	0.113	0.000	0.000	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	91.432	0.121	0.000	0.000	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	82.127	0.119	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	95.321	0.117	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	89.796	0.124	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	100.485	0.125	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	75.193	0.122	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	107.541	0.124	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	91.747	0.118	0.000	0.000	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	104.578	0.117	0.000	0.000	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	94.022	0.121	0.000	0.000	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	103.422	0.116	0.000	0.000	0.000	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	100.072	0.118	0.000	0.000	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	107.813	0.119	0.000	0.000	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	107.523	0.122	0.000	0.000	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	117.565	0.118	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	78.546	0.125	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	123.319	0.122	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	231	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.270	0.577	0.185	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	171	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.218	0.074	0.000	0.000	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	107	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.102	0.082	0.000	0.000	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	5280	0	0	0	0	0	-1
N.S.	1	1.00	27.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	24.325	0.088	0.000	0.000	0.000	0.000	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	13816	0	0	0	0	0	-1
N.S.	1	1.00	46.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.321	40.888	0.068	0.000	0.000	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	21.794	0.105	0.000	0.000	0.000	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.321	0.105	0.000	0.000	0.000	0.000	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	2.607	0.103	0.000	0.000	0.000	0.000	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	2.624	0.092	0.000	0.000	0.000	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	1.471	0.110	0.000	0.000	0.000	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	0.369	0.093	0.000	0.000	0.000	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	8899	0	0	0	0	0	-1
N.S.	1	1.00	32.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	25.566	0.081	0.000	0.000	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	5564	0	0	0	0	0	-1
N.S.	1	1.00	25.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	20.437	0.065	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	2828	0	0	0	0	0	-1
N.S.	1	1.00	27.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	13.798	0.090	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.007	1.228	0.078	0.000	0.000	0.000	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	3.643	0.106	0.000	0.000	0.000	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	2.827	0.170	0.000	0.000	0.000	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	90	318	0	159	0	0	87
N.S.	1	1.00	0.67	2.36	0.00	1.18	0.00	0.00	0.64
time (sec)	N/A	0.077	0.227	0.170	0.000	0.590	0.000	0.000	1.310

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	77	290	0	148	0	0	87
N.S.	1	1.00	0.69	2.61	0.00	1.33	0.00	0.00	0.78
time (sec)	N/A	0.067	0.331	0.162	0.000	0.791	0.000	0.000	1.142

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	66	262	0	137	0	0	80
N.S.	1	1.00	0.76	3.01	0.00	1.57	0.00	0.00	0.92
time (sec)	N/A	0.060	0.157	0.157	0.000	0.779	0.000	0.000	1.039

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	228	0	125	0	0	53
N.S.	1	1.00	0.87	3.74	0.00	2.05	0.00	0.00	0.87
time (sec)	N/A	0.050	0.075	0.152	0.000	0.772	0.000	0.000	0.173

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	152	0	107	0	0	33
N.S.	1	1.00	0.91	4.34	0.00	3.06	0.00	0.00	0.94
time (sec)	N/A	0.044	0.051	0.151	0.000	0.486	0.000	0.000	0.229

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	150	0	156	0	0	60
N.S.	1	1.00	0.89	2.63	0.00	2.74	0.00	0.00	1.05
time (sec)	N/A	0.052	0.103	0.148	0.000	0.611	0.000	0.000	1.249

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	396	0	175	0	0	87
N.S.	1	1.00	0.78	4.77	0.00	2.11	0.00	0.00	1.05
time (sec)	N/A	0.060	0.291	0.250	0.000	1.197	0.000	0.000	1.544

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	502	0	188	0	0	87
N.S.	1	1.00	0.86	4.52	0.00	1.69	0.00	0.00	0.78
time (sec)	N/A	0.069	0.210	0.352	0.000	1.132	0.000	0.000	1.696

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	113	398	0	195	0	0	135
N.S.	1	1.00	0.71	2.49	0.00	1.22	0.00	0.00	0.84
time (sec)	N/A	0.151	0.515	0.186	0.000	1.008	0.000	0.000	1.313

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	98	362	0	180	0	0	128
N.S.	1	1.00	0.73	2.68	0.00	1.33	0.00	0.00	0.95
time (sec)	N/A	0.137	0.399	0.169	0.000	0.703	0.000	0.000	1.191

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	79	357	0	162	0	0	102
N.S.	1	1.00	0.78	3.53	0.00	1.60	0.00	0.00	1.01
time (sec)	N/A	0.121	0.205	0.168	0.000	0.954	0.000	0.000	1.110

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	283	0	147	0	0	76
N.S.	1	1.00	0.89	3.93	0.00	2.04	0.00	0.00	1.06
time (sec)	N/A	0.109	0.112	0.168	0.000	0.689	0.000	0.000	1.046

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	202	0	178	0	0	81
N.S.	1	1.00	0.91	2.97	0.00	2.62	0.00	0.00	1.19
time (sec)	N/A	0.107	0.208	0.173	0.000	0.504	0.000	0.000	1.398

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	73	513	0	198	0	0	108
N.S.	1	1.00	0.77	5.40	0.00	2.08	0.00	0.00	1.14
time (sec)	N/A	0.118	0.407	0.276	0.000	0.632	0.000	0.000	1.520

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	633	0	223	0	0	113
N.S.	1	1.00	0.92	4.69	0.00	1.65	0.00	0.00	0.84
time (sec)	N/A	0.130	0.255	0.372	0.000	0.412	0.000	0.000	1.666

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	142	689	0	235	0	0	113
N.S.	1	1.00	0.89	4.31	0.00	1.47	0.00	0.00	0.71
time (sec)	N/A	0.143	0.363	0.464	0.000	0.616	0.000	0.000	1.789

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	137	470	0	227	0	0	178
N.S.	1	1.00	0.71	2.42	0.00	1.17	0.00	0.00	0.92
time (sec)	N/A	0.214	0.641	0.202	0.000	1.249	0.000	0.000	1.344

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	110	421	0	205	0	0	146
N.S.	1	1.00	0.69	2.65	0.00	1.29	0.00	0.00	0.92
time (sec)	N/A	0.204	0.493	0.189	0.000	0.742	0.000	0.000	1.219

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	84	412	0	185	0	0	125
N.S.	1	1.00	0.72	3.55	0.00	1.59	0.00	0.00	1.08
time (sec)	N/A	0.176	0.264	0.170	0.000	0.794	0.000	0.000	1.180

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	87	303	0	214	0	0	124
N.S.	1	1.00	0.69	2.40	0.00	1.70	0.00	0.00	0.98
time (sec)	N/A	0.178	0.408	0.203	0.000	0.709	0.000	0.000	1.241

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	84	630	0	222	0	0	128
N.S.	1	1.00	0.71	5.34	0.00	1.88	0.00	0.00	1.08
time (sec)	N/A	0.169	0.788	0.313	0.000	0.777	0.000	0.000	2.097

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	125	711	0	244	0	0	156
N.S.	1	1.00	0.84	4.77	0.00	1.64	0.00	0.00	1.05
time (sec)	N/A	0.188	0.625	0.385	0.000	0.582	0.000	0.000	2.167

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	177	820	0	270	0	0	147
N.S.	1	1.00	0.91	4.23	0.00	1.39	0.00	0.00	0.76
time (sec)	N/A	0.216	0.552	0.483	0.000	0.682	0.000	0.000	2.240

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	226	668	0	0	0	0	-1
N.S.	1	1.00	1.49	4.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.414	11.047	0.204	0.000	0.000	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	158	552	0	0	0	0	-1
N.S.	1	1.00	1.41	4.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.274	11.094	0.176	0.000	0.000	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	81	226	0	0	0	0	-1
N.S.	1	1.00	1.08	3.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	10.198	0.149	0.000	0.000	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	187	0	0	0	0	-1
N.S.	1	1.00	0.91	3.53	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.135	0.052	0.141	0.000	0.000	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	150	0	0	0	0	-1
N.S.	1	1.00	1.00	5.17	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.093	0.058	0.134	0.000	0.000	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	195	353	0	0	0	0	-1
N.S.	1	1.00	2.53	4.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	11.964	0.175	0.000	0.000	0.000	0.000	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	210	423	0	0	0	0	-1
N.S.	1	1.00	1.64	3.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.372	12.618	0.329	0.000	0.000	0.000	0.000	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	266	1064	0	0	0	0	-1
N.S.	1	1.00	1.09	4.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	11.236	0.369	0.000	0.000	0.000	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	252	809	0	0	0	0	-1
N.S.	1	1.00	1.37	4.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.337	11.179	0.333	0.000	0.000	0.000	0.000	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	194	788	0	0	0	0	-1
N.S.	1	1.00	1.16	4.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.296	12.572	0.317	0.000	0.000	0.000	0.000	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	229	707	0	0	0	0	-1
N.S.	1	1.00	1.55	4.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	12.070	0.291	0.000	0.000	0.000	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	239	608	0	0	0	0	-1
N.S.	1	1.00	1.55	3.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.321	12.314	0.299	0.000	0.000	0.000	0.000	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	278	841	0	0	0	0	-1
N.S.	1	1.00	1.27	3.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	12.026	0.383	0.000	0.000	0.000	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	353	2216	0	0	0	0	-1
N.S.	1	1.00	1.02	6.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.751	12.336	0.710	0.000	0.000	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	311	1957	0	0	0	0	-1
N.S.	1	1.00	1.10	6.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	11.845	0.744	0.000	0.000	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	286	1936	0	0	0	0	-1
N.S.	1	1.00	1.09	7.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	11.841	0.640	0.000	0.000	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	272	1858	0	0	0	0	-1
N.S.	1	1.00	1.11	7.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	11.335	0.612	0.000	0.000	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	289	1760	0	0	0	0	-1
N.S.	1	1.00	1.14	6.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.512	11.875	0.623	0.000	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	297	1203	0	0	0	0	-1
N.S.	1	1.00	1.16	4.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	11.867	0.379	0.000	0.000	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	334	1987	0	0	0	0	-1
N.S.	1	1.00	1.02	6.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.710	12.276	0.758	0.000	0.000	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	340	1726	0	453	0	0	-1
N.S.	1	1.00	1.39	7.07	0.00	1.86	0.00	0.00	-0.00
time (sec)	N/A	0.468	5.706	1.295	0.000	1.077	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	273	1011	0	415	0	0	-1
N.S.	1	1.00	1.42	5.27	0.00	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.305	5.361	0.236	0.000	0.786	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	198	923	0	355	0	0	-1
N.S.	1	1.00	2.96	13.78	0.00	5.30	0.00	0.00	-0.01
time (sec)	N/A	0.109	2.276	0.214	0.000	0.857	0.000	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	14885	257	0	0	0	0	-1
N.S.	1	1.00	107.86	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	26.986	0.187	0.000	0.000	0.000	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	23549	781	0	0	0	0	-1
N.S.	1	1.00	99.36	3.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	28.892	0.222	0.000	0.000	0.000	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	383	2040	0	491	0	0	-1
N.S.	1	1.00	1.26	6.73	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.645	6.168	0.220	0.000	1.794	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	344	1697	0	455	0	0	-1
N.S.	1	1.00	1.43	7.07	0.00	1.90	0.00	0.00	-0.00
time (sec)	N/A	0.485	5.355	0.184	0.000	0.951	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	284	1209	0	415	0	0	-1
N.S.	1	1.00	1.52	6.47	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.329	4.053	0.239	0.000	1.244	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	25369	1361	0	0	0	0	-1
N.S.	1	1.00	121.38	6.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	27.903	0.197	0.000	0.000	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	24604	1205	0	0	0	0	-1
N.S.	1	1.00	98.81	4.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	29.525	0.224	0.000	0.000	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	36737	1742	0	0	0	0	-1
N.S.	1	1.00	122.87	5.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	30.944	0.188	0.000	0.000	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	477	2778	0	531	0	0	-1
N.S.	1	1.00	1.31	7.65	0.00	1.46	0.00	0.00	-0.00
time (sec)	N/A	0.916	11.017	0.283	0.000	1.047	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	386	2040	0	491	0	0	-1
N.S.	1	1.00	1.27	6.73	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.717	8.133	0.224	0.000	0.975	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	358	1921	0	454	0	0	-1
N.S.	1	1.00	1.50	8.04	0.00	1.90	0.00	0.00	-0.00
time (sec)	N/A	0.539	7.797	0.195	0.000	1.195	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	36372	1651	0	0	0	0	-1
N.S.	1	1.00	138.82	6.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.599	31.928	0.176	0.000	0.000	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	20828	1947	0	0	0	0	-1
N.S.	1	1.00	79.19	7.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.580	30.611	0.178	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	52888	1972	0	0	0	0	-1
N.S.	1	1.00	168.43	6.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.795	31.359	0.204	0.000	0.000	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	61979	2285	0	0	0	0	-1
N.S.	1	1.00	167.96	6.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.975	31.612	0.223	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	340	1725	0	456	0	0	-1
N.S.	1	1.00	1.37	6.93	0.00	1.83	0.00	0.00	-0.00
time (sec)	N/A	0.437	5.762	0.196	0.000	1.592	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	265	1014	0	415	0	0	-1
N.S.	1	1.00	1.36	5.20	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.313	4.868	0.226	0.000	0.737	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	216	732	0	355	0	0	-1
N.S.	1	1.00	1.52	5.15	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.232	2.578	0.216	0.000	0.905	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	102	145	0	146	0	0	-1
N.S.	1	1.00	1.52	2.16	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.543	0.190	0.000	0.730	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	5763	222	0	0	0	0	-1
N.S.	1	1.00	84.75	3.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	27.089	0.174	0.000	0.000	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	21698	985	0	0	0	0	-1
N.S.	1	1.00	88.20	4.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	28.263	0.218	0.000	0.000	0.000	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	51323	1744	0	0	0	0	-1
N.S.	1	1.00	164.50	5.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	31.497	0.192	0.000	0.000	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	419	1851	0	702	0	0	-1
N.S.	1	1.00	1.16	5.14	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.740	7.972	0.177	0.000	1.024	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	382	1305	0	644	0	0	-1
N.S.	1	1.00	1.32	4.52	0.00	2.23	0.00	0.00	-0.00
time (sec)	N/A	0.516	5.603	0.243	0.000	0.889	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	330	997	0	585	0	0	-1
N.S.	1	1.00	1.54	4.66	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	0.359	5.872	0.197	0.000	1.381	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	245	498	0	541	0	0	-1
N.S.	1	1.00	1.22	2.49	0.00	2.70	0.00	0.00	-0.00
time (sec)	N/A	0.329	5.101	0.189	0.000	1.049	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	260	491	0	500	0	0	-1
N.S.	1	1.00	2.06	3.90	0.00	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.162	5.086	0.177	0.000	0.926	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	34326	1134	0	0	0	0	-1
N.S.	1	1.00	166.63	5.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	30.668	0.200	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	36944	1492	0	0	0	0	-1
N.S.	1	1.00	107.08	4.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.765	31.255	0.256	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	527	3604	0	971	0	0	-1
N.S.	1	1.00	1.35	9.22	0.00	2.48	0.00	0.00	-0.00
time (sec)	N/A	0.756	11.341	0.214	0.000	3.213	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	427	3101	0	884	0	0	-1
N.S.	1	1.00	1.35	9.78	0.00	2.79	0.00	0.00	-0.00
time (sec)	N/A	0.565	10.998	0.200	0.000	1.716	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	398	2062	0	835	0	0	-1
N.S.	1	1.00	1.32	6.83	0.00	2.76	0.00	0.00	-0.00
time (sec)	N/A	0.503	5.681	0.188	0.000	0.757	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	447	1812	0	782	0	0	-1
N.S.	1	1.00	1.59	6.45	0.00	2.78	0.00	0.00	-0.00
time (sec)	N/A	0.485	9.669	0.191	0.000	2.025	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	311	1333	0	709	0	0	-1
N.S.	1	1.00	1.12	4.81	0.00	2.56	0.00	0.00	-0.00
time (sec)	N/A	0.514	5.926	0.233	0.000	1.370	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	92128	3844	0	0	0	0	-1
N.S.	1	1.00	248.99	10.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.843	32.286	0.180	0.000	0.000	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	222	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	0.466	0.132	0.000	0.000	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	161	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.262	0.109	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	106	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.076	0.092	0.000	0.000	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	5216	0	0	0	0	0	-1
N.S.	1	1.00	26.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	24.510	0.078	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	13974	0	0	0	0	0	-1
N.S.	1	1.00	45.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	42.164	0.102	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [244] had the largest ratio of [28]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	19	0.210
2	A	5	4	1.00	19	0.210
3	A	5	5	1.00	19	0.263
4	A	4	4	1.00	17	0.235
5	A	2	1	1.00	10	0.100
6	A	3	3	1.00	17	0.176
7	A	4	4	1.00	19	0.210
8	A	5	4	1.00	19	0.210
9	A	6	4	1.00	19	0.210
10	A	7	5	1.00	21	0.238
11	A	6	5	1.00	21	0.238
12	A	6	6	1.00	21	0.286
13	A	5	5	1.00	19	0.263
14	A	4	4	1.00	12	0.333
15	A	4	4	1.00	19	0.210
16	A	4	4	1.00	21	0.190
17	A	6	5	1.00	21	0.238
18	A	6	5	1.00	21	0.238
19	A	8	6	1.00	21	0.286
20	A	11	4	1.00	21	0.190
21	A	11	5	1.00	21	0.238
22	A	9	5	1.00	19	0.263
23	A	5	5	1.00	12	0.417
24	A	6	5	1.00	19	0.263
25	A	6	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	7	5	1.00	21	0.238
27	A	10	5	1.00	21	0.238
28	A	11	4	1.00	21	0.190
29	A	13	4	1.00	21	0.190
30	A	15	4	1.00	21	0.190
31	A	13	5	1.00	21	0.238
32	A	12	5	1.00	19	0.263
33	A	6	6	1.00	12	0.500
34	A	8	6	1.00	19	0.316
35	A	8	6	1.00	21	0.286
36	A	8	6	1.00	21	0.286
37	A	10	5	1.00	21	0.238
38	A	12	5	1.00	21	0.238
39	A	15	4	1.00	21	0.190
40	A	15	4	1.00	21	0.190
41	A	17	4	1.00	21	0.190
42	A	6	5	1.00	21	0.238
43	A	6	6	1.00	21	0.286
44	A	4	4	1.00	21	0.190
45	A	3	3	1.00	21	0.143
46	A	1	1	1.00	19	0.053
47	A	2	2	1.00	12	0.167
48	A	4	4	1.00	19	0.210
49	A	5	5	1.00	21	0.238
50	A	6	5	1.00	21	0.238
51	A	7	5	1.00	21	0.238
52	A	7	7	1.00	21	0.333
53	A	6	6	1.00	21	0.286
54	A	4	4	1.00	21	0.190
55	A	2	2	1.00	21	0.095
56	A	2	2	1.00	19	0.105
57	A	3	3	1.00	12	0.250
58	A	5	5	1.00	19	0.263
59	A	6	6	1.00	21	0.286
60	A	7	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	8	7	1.00	21	0.333
62	A	7	7	1.00	21	0.333
63	A	5	5	1.00	21	0.238
64	A	3	3	1.00	21	0.143
65	A	3	3	1.00	21	0.143
66	A	3	2	1.00	19	0.105
67	A	4	4	1.00	12	0.333
68	A	6	5	1.00	19	0.263
69	A	7	6	1.00	21	0.286
70	A	9	7	1.00	21	0.333
71	A	8	7	1.00	21	0.333
72	A	6	6	1.00	21	0.286
73	A	4	4	1.00	21	0.190
74	A	4	4	1.00	21	0.190
75	A	4	3	1.00	21	0.143
76	A	4	2	1.00	19	0.105
77	A	5	4	1.00	12	0.333
78	A	7	5	1.00	19	0.263
79	A	8	6	1.00	21	0.286
80	A	9	7	1.00	21	0.333
81	A	7	6	1.00	21	0.286
82	A	5	4	1.00	21	0.190
83	A	5	5	1.00	21	0.238
84	A	5	4	1.00	21	0.190
85	A	5	3	1.00	21	0.143
86	A	5	2	1.00	19	0.105
87	A	6	4	1.00	12	0.333
88	A	8	5	1.00	19	0.263
89	A	9	6	1.00	21	0.286
90	A	4	4	1.00	23	0.174
91	A	3	3	1.00	23	0.130
92	A	2	2	1.00	23	0.087
93	A	1	1	1.00	21	0.048
94	A	2	2	1.00	14	0.143
95	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	3	1.00	23	0.130
97	A	5	3	1.00	23	0.130
98	A	6	3	1.00	23	0.130
99	A	6	6	1.00	23	0.261
100	A	4	4	1.00	23	0.174
101	A	3	3	1.00	23	0.130
102	A	2	2	1.00	21	0.095
103	A	4	4	1.00	14	0.286
104	A	5	5	1.00	21	0.238
105	A	5	5	1.00	23	0.217
106	A	6	5	1.00	23	0.217
107	A	6	6	1.00	23	0.261
108	A	5	4	1.00	23	0.174
109	A	4	3	1.00	23	0.130
110	A	3	2	1.00	21	0.095
111	A	5	5	1.00	14	0.357
112	A	4	4	1.00	21	0.190
113	A	4	4	1.00	23	0.174
114	A	5	5	1.00	23	0.217
115	A	6	5	1.00	23	0.217
116	A	1	1	1.00	22	0.045
117	A	2	2	1.00	15	0.133
118	A	3	3	1.00	22	0.136
119	A	5	5	1.00	23	0.217
120	A	4	4	1.00	23	0.174
121	A	3	3	1.00	23	0.130
122	A	2	2	1.00	21	0.095
123	A	5	4	1.00	14	0.286
124	A	6	5	1.00	21	0.238
125	A	7	6	1.00	23	0.261
126	A	6	6	1.00	23	0.261
127	A	5	5	1.00	23	0.217
128	A	4	4	1.00	23	0.174
129	A	3	3	1.00	23	0.130
130	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	6	5	1.00	14	0.357
132	A	7	6	1.00	21	0.286
133	A	8	6	1.00	23	0.261
134	A	6	6	1.00	23	0.261
135	A	5	5	1.00	23	0.217
136	A	4	4	1.00	23	0.174
137	A	4	4	1.00	23	0.174
138	A	4	3	1.00	21	0.143
139	A	7	6	1.00	14	0.429
140	A	8	7	1.00	21	0.333
141	A	2	2	1.00	22	0.091
142	A	5	4	1.00	15	0.267
143	A	7	7	1.00	23	0.304
144	A	6	6	1.00	23	0.261
145	A	5	5	1.00	21	0.238
146	A	3	3	1.00	14	0.214
147	A	3	3	1.00	21	0.143
148	A	8	7	1.00	23	0.304
149	A	7	6	1.00	23	0.261
150	A	6	5	1.00	21	0.238
151	A	3	3	1.00	14	0.214
152	A	3	3	1.00	21	0.143
153	A	7	7	1.00	23	0.304
154	A	6	6	1.00	23	0.261
155	A	5	5	1.00	23	0.217
156	A	4	4	1.00	21	0.190
157	A	3	3	1.00	14	0.214
158	A	3	3	1.00	21	0.143
159	A	9	9	1.00	23	0.391
160	A	8	8	1.00	23	0.348
161	A	8	8	1.00	23	0.348
162	A	8	7	1.00	21	0.333
163	A	3	3	1.00	14	0.214
164	A	3	3	1.00	21	0.143
165	A	8	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	7	5	1.00	21	0.238
167	A	6	5	1.00	21	0.238
168	A	5	4	1.00	21	0.190
169	A	6	5	1.00	21	0.238
170	A	7	5	1.00	21	0.238
171	A	8	5	1.00	21	0.238
172	A	9	6	1.00	23	0.261
173	A	8	6	1.00	23	0.261
174	A	7	6	1.00	23	0.261
175	A	4	4	1.00	23	0.174
176	A	6	5	1.00	23	0.217
177	A	7	6	1.00	23	0.261
178	A	8	6	1.00	23	0.261
179	A	16	5	1.00	23	0.217
180	A	14	5	1.00	23	0.217
181	A	12	5	1.00	23	0.217
182	A	12	6	1.00	23	0.261
183	A	12	5	1.00	23	0.217
184	A	14	5	1.00	23	0.217
185	A	16	5	1.00	23	0.217
186	A	21	5	1.00	23	0.217
187	A	18	5	1.00	23	0.217
188	A	16	5	1.00	23	0.217
189	A	15	6	1.00	23	0.261
190	A	15	6	1.00	23	0.261
191	A	16	5	1.00	23	0.217
192	A	18	5	1.00	23	0.217
193	A	21	5	1.00	23	0.217
194	A	8	6	1.00	23	0.261
195	A	7	6	1.00	23	0.261
196	A	6	5	1.00	23	0.217
197	A	6	5	1.00	23	0.217
198	A	6	5	1.00	23	0.217
199	A	7	6	1.00	23	0.261
200	A	8	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	9	7	1.00	23	0.304
202	A	8	7	1.00	23	0.304
203	A	7	6	1.00	23	0.261
204	A	4	4	1.00	23	0.174
205	A	7	6	1.00	23	0.261
206	A	7	6	1.00	23	0.261
207	A	8	7	1.00	23	0.304
208	A	9	7	1.00	23	0.304
209	A	10	7	1.00	23	0.304
210	A	9	7	1.00	23	0.304
211	A	8	6	1.00	23	0.261
212	A	8	7	1.00	23	0.304
213	A	8	7	1.00	23	0.304
214	A	8	7	1.00	23	0.304
215	A	8	6	1.00	23	0.261
216	A	9	7	1.00	23	0.304
217	A	10	7	1.00	23	0.304
218	A	4	3	1.00	25	0.120
219	A	3	3	1.00	25	0.120
220	A	2	2	1.00	25	0.080
221	A	1	1	1.00	25	0.040
222	A	2	2	1.00	25	0.080
223	A	3	2	1.00	25	0.080
224	A	4	2	1.00	25	0.080
225	A	6	5	1.00	25	0.200
226	A	5	5	1.00	25	0.200
227	A	4	4	1.00	25	0.160
228	A	4	4	1.00	25	0.160
229	A	2	2	1.00	25	0.080
230	A	3	3	1.00	25	0.120
231	A	5	4	1.00	25	0.160
232	A	6	4	1.00	25	0.160
233	A	6	5	1.00	25	0.200
234	A	5	5	1.00	25	0.200
235	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	4	4	1.00	25	0.160
237	A	4	4	1.00	25	0.160
238	A	3	2	1.00	25	0.080
239	A	4	3	1.00	25	0.120
240	A	5	4	1.00	25	0.160
241	A	6	4	1.00	25	0.160
242	A	2	2	1.00	25	0.080
243	A	2	2	1.00	25	0.080
244	A	2	2	1.00	28	0.071
245	A	6	6	1.00	25	0.240
246	A	5	5	1.00	25	0.200
247	A	2	2	1.00	25	0.080
248	A	3	3	1.00	25	0.120
249	A	4	4	1.00	25	0.160
250	A	5	5	1.00	25	0.200
251	A	7	7	1.00	25	0.280
252	A	6	6	1.00	25	0.240
253	A	3	3	1.00	25	0.120
254	A	3	3	1.00	25	0.120
255	A	4	4	1.00	25	0.160
256	A	5	5	1.00	25	0.200
257	A	6	5	1.00	25	0.200
258	A	8	8	1.00	25	0.320
259	A	7	7	1.00	25	0.280
260	A	4	3	1.00	25	0.120
261	A	4	4	1.00	25	0.160
262	A	4	4	1.00	25	0.160
263	A	5	5	1.00	25	0.200
264	A	6	6	1.00	25	0.240
265	A	7	6	1.00	23	0.261
266	A	6	5	1.00	23	0.217
267	A	5	4	1.00	23	0.174
268	A	2	2	1.00	23	0.087
269	A	3	3	1.00	23	0.130
270	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	5	5	1.00	23	0.217
272	A	4	4	1.00	27	0.148
273	A	3	3	1.00	27	0.111
274	A	4	4	1.00	27	0.148
275	A	7	6	1.00	27	0.222
276	A	6	6	1.00	27	0.222
277	A	5	5	1.00	27	0.185
278	A	6	6	1.00	27	0.222
279	A	7	6	1.00	27	0.222
280	A	4	4	1.00	27	0.148
281	A	4	4	1.00	27	0.148
282	A	4	4	1.00	27	0.148
283	A	4	4	1.00	27	0.148
284	A	3	3	1.00	25	0.120
285	A	3	3	1.00	25	0.120
286	C	3	3	0.24	25	0.120
287	A	3	3	1.00	25	0.120
288	A	8	6	1.00	21	0.286
289	A	7	5	1.00	21	0.238
290	A	6	4	1.00	21	0.190
291	A	5	3	1.00	19	0.158
292	A	6	4	1.00	21	0.190
293	A	7	5	1.00	21	0.238
294	A	4	4	1.00	21	0.190
295	A	4	4	1.00	21	0.190
296	A	2	2	1.00	21	0.095
297	A	3	3	1.00	21	0.143
298	A	3	3	1.00	21	0.143
299	A	4	4	1.00	23	0.174
300	A	2	2	1.00	23	0.087
301	A	2	2	1.00	23	0.087
302	A	2	2	1.00	23	0.087
303	A	4	4	1.00	23	0.174
304	A	2	2	1.00	23	0.087
305	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	2	2	1.00	23	0.087
307	A	4	4	1.00	23	0.174
308	A	4	4	1.00	23	0.174
309	A	2	2	1.00	23	0.087
310	A	4	4	1.00	23	0.174
311	A	4	4	1.00	23	0.174
312	A	5	5	1.00	25	0.200
313	A	3	3	1.00	25	0.120
314	A	3	3	1.00	25	0.120
315	A	3	3	1.00	25	0.120
316	A	5	5	1.00	25	0.200
317	A	3	3	1.00	25	0.120
318	A	3	3	1.00	25	0.120
319	A	3	3	1.00	25	0.120
320	A	4	4	1.00	26	0.154
321	A	4	4	1.00	26	0.154
322	A	2	2	1.00	26	0.077
323	A	4	4	1.00	26	0.154
324	A	4	4	1.00	26	0.154
325	A	5	5	1.00	24	0.208
326	A	3	3	1.00	24	0.125
327	A	5	5	1.00	26	0.192
328	A	3	3	1.00	26	0.115
329	A	2	2	1.00	19	0.105
330	A	2	2	1.00	21	0.095
331	A	3	3	1.00	21	0.143
332	A	3	3	1.00	22	0.136
333	A	2	2	1.00	21	0.095
334	A	2	2	1.00	23	0.087
335	A	3	3	1.00	23	0.130
336	A	3	3	1.00	24	0.125
337	A	2	2	1.00	21	0.095
338	A	2	2	1.00	23	0.087
339	A	3	3	1.00	23	0.130
340	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	6	6	1.00	21	0.286
342	A	5	5	1.00	21	0.238
343	A	4	4	1.00	21	0.190
344	A	3	3	1.00	19	0.158
345	A	3	3	1.00	12	0.250
346	A	3	3	1.00	19	0.158
347	A	3	3	1.00	25	0.120
348	A	3	3	1.00	25	0.120
349	A	3	3	1.00	25	0.120
350	A	3	3	1.00	25	0.120
351	A	7	5	1.00	21	0.238
352	A	6	5	1.00	21	0.238
353	A	5	5	1.00	21	0.238
354	A	4	4	1.00	21	0.190
355	A	5	5	1.00	21	0.238
356	A	6	5	1.00	21	0.238
357	A	7	5	1.00	21	0.238
358	A	8	5	1.00	21	0.238
359	A	10	7	1.00	23	0.304
360	A	9	7	1.00	23	0.304
361	A	8	7	1.00	23	0.304
362	A	7	6	1.00	23	0.261
363	A	5	5	1.00	23	0.217
364	A	8	7	1.00	23	0.304
365	A	9	7	1.00	23	0.304
366	A	10	7	1.00	23	0.304
367	A	17	6	1.00	23	0.261
368	A	15	6	1.00	23	0.261
369	A	13	6	1.00	23	0.261
370	A	13	7	1.00	23	0.304
371	A	13	6	1.00	23	0.261
372	A	15	6	1.00	23	0.261
373	A	17	6	1.00	23	0.261
374	A	9	7	1.00	23	0.304
375	A	8	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	7	6	1.00	23	0.261
377	A	7	6	1.00	23	0.261
378	A	7	6	1.00	23	0.261
379	A	8	7	1.00	23	0.304
380	A	9	7	1.00	23	0.304
381	A	10	8	1.00	23	0.348
382	A	9	8	1.00	23	0.348
383	A	8	7	1.00	23	0.304
384	A	8	7	1.00	23	0.304
385	A	5	5	1.00	23	0.217
386	A	8	7	1.00	23	0.304
387	A	9	8	1.00	23	0.348
388	A	10	8	1.00	23	0.348
389	A	11	8	1.00	23	0.348
390	A	10	8	1.00	23	0.348
391	A	9	7	1.00	23	0.304
392	A	9	8	1.00	23	0.348
393	A	9	8	1.00	23	0.348
394	A	9	8	1.00	23	0.348
395	A	9	7	1.00	23	0.304
396	A	10	8	1.00	23	0.348
397	A	11	8	1.00	23	0.348
398	A	5	3	1.00	25	0.120
399	A	4	3	1.00	25	0.120
400	A	3	3	1.00	25	0.120
401	A	2	2	1.00	25	0.080
402	A	3	3	1.00	25	0.120
403	A	4	4	1.00	25	0.160
404	A	5	4	1.00	25	0.160
405	A	6	5	1.00	25	0.200
406	A	4	4	1.00	25	0.160
407	A	3	3	1.00	25	0.120
408	A	5	5	1.00	25	0.200
409	A	5	5	1.00	25	0.200
410	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	7	6	1.00	25	0.240
412	A	6	5	1.00	25	0.200
413	A	5	4	1.00	25	0.160
414	A	4	3	1.00	25	0.120
415	A	5	5	1.00	25	0.200
416	A	5	5	1.00	25	0.200
417	A	5	5	1.00	25	0.200
418	A	6	6	1.00	25	0.240
419	A	7	6	1.00	25	0.240
420	A	6	6	1.00	25	0.240
421	A	5	5	1.00	25	0.200
422	A	4	4	1.00	25	0.160
423	A	3	3	1.36	25	0.120
424	A	6	6	1.00	25	0.240
425	A	7	7	1.00	25	0.280
426	A	8	8	1.00	25	0.320
427	A	7	6	1.00	25	0.240
428	A	6	6	1.00	25	0.240
429	A	5	5	1.00	25	0.200
430	A	4	4	1.00	25	0.160
431	A	4	4	1.00	25	0.160
432	A	7	7	1.00	25	0.280
433	A	8	8	1.00	25	0.320
434	A	7	7	1.00	25	0.280
435	A	6	6	1.00	25	0.240
436	A	5	5	1.00	25	0.200
437	A	5	5	1.00	25	0.200
438	A	5	4	1.00	25	0.160
439	A	8	8	1.00	25	0.320
440	A	9	9	1.00	25	0.360
441	A	8	6	1.00	23	0.261
442	A	7	5	1.00	23	0.217
443	A	5	4	1.00	21	0.190
444	A	7	5	1.00	23	0.217
445	A	8	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	6	4	1.00	19	0.210
447	A	5	4	1.00	19	0.210
448	A	5	5	1.00	19	0.263
449	A	4	4	1.00	17	0.235
450	A	2	1	1.00	10	0.100
451	A	3	3	1.00	17	0.176
452	A	4	4	1.00	19	0.210
453	A	5	4	1.00	19	0.210
454	A	6	4	1.00	19	0.210
455	A	6	4	1.00	19	0.210
456	A	7	5	1.00	21	0.238
457	A	6	5	1.00	21	0.238
458	A	6	6	1.00	21	0.286
459	A	5	5	1.00	19	0.263
460	A	4	4	1.00	12	0.333
461	A	4	4	1.00	19	0.210
462	A	4	4	1.00	21	0.190
463	A	6	5	1.00	21	0.238
464	A	6	5	1.00	21	0.238
465	A	8	6	1.00	21	0.286
466	A	8	7	1.00	21	0.333
467	A	7	7	1.00	21	0.333
468	A	6	6	1.00	19	0.316
469	A	5	4	1.00	12	0.333
470	A	5	5	1.00	19	0.263
471	A	5	5	1.00	21	0.238
472	A	5	5	1.00	21	0.238
473	A	7	6	1.00	21	0.286
474	A	7	6	1.00	21	0.286
475	A	9	7	1.00	21	0.333
476	A	9	7	1.00	21	0.333
477	A	8	7	1.00	21	0.333
478	A	7	7	1.00	19	0.368
479	A	6	5	1.00	12	0.417
480	A	6	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	6	6	1.00	21	0.286
482	A	6	6	1.00	21	0.286
483	A	6	6	1.00	21	0.286
484	A	8	7	1.00	21	0.333
485	A	8	7	1.00	21	0.333
486	A	7	6	1.00	12	0.500
487	A	8	8	1.00	21	0.381
488	A	7	7	1.00	21	0.333
489	A	6	6	1.00	21	0.286
490	A	5	5	1.00	21	0.238
491	A	3	3	1.00	19	0.158
492	A	3	3	1.00	12	0.250
493	A	5	5	1.00	19	0.263
494	A	6	6	1.00	21	0.286
495	A	7	6	1.00	21	0.286
496	A	8	6	1.00	21	0.286
497	A	8	8	1.00	21	0.381
498	A	7	7	1.00	21	0.333
499	A	6	6	1.00	21	0.286
500	A	5	5	1.00	21	0.238
501	A	5	5	1.00	19	0.263
502	A	5	5	1.00	12	0.417
503	A	6	6	1.00	19	0.316
504	A	7	6	1.00	21	0.286
505	A	8	6	1.00	21	0.286
506	A	8	8	1.00	21	0.381
507	A	7	7	1.00	21	0.333
508	A	6	6	1.00	21	0.286
509	A	6	6	1.00	21	0.286
510	A	6	6	1.00	19	0.316
511	A	6	6	1.00	12	0.500
512	A	7	7	1.00	19	0.368
513	A	8	7	1.00	21	0.333
514	A	9	9	1.00	21	0.429
515	A	8	8	1.00	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	7	7	1.00	21	0.333
517	A	7	6	1.00	21	0.286
518	A	7	6	1.00	21	0.286
519	A	7	6	1.00	19	0.316
520	A	7	6	1.00	12	0.500
521	A	8	7	1.00	19	0.368
522	A	9	7	1.00	21	0.333
523	A	2	2	1.00	12	0.167
524	A	4	4	1.00	12	0.333
525	A	5	5	1.00	12	0.417
526	A	6	5	1.00	12	0.417
527	A	3	3	1.00	12	0.250
528	A	5	5	1.00	12	0.417
529	A	6	6	1.00	12	0.500
530	A	7	6	1.00	12	0.500
531	A	5	5	1.00	23	0.217
532	A	4	4	1.00	23	0.174
533	A	3	3	1.00	21	0.143
534	A	1	1	1.00	14	0.071
535	A	6	6	1.00	21	0.286
536	A	7	7	1.00	23	0.304
537	A	7	6	1.00	23	0.261
538	A	6	5	1.00	23	0.217
539	A	5	5	1.00	23	0.217
540	A	4	4	1.00	21	0.190
541	A	5	5	1.00	14	0.357
542	A	6	6	1.00	21	0.286
543	A	7	7	1.00	23	0.304
544	A	8	6	1.00	23	0.261
545	A	7	5	1.00	23	0.217
546	A	6	5	1.00	23	0.217
547	A	5	5	1.00	21	0.238
548	A	6	6	1.00	14	0.429
549	A	6	6	1.00	21	0.286
550	A	7	7	1.00	23	0.304

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
551	A	8	7	1.00	23	0.304
552	A	9	7	1.00	23	0.304
553	A	7	7	1.00	14	0.500
554	A	6	6	1.00	23	0.261
555	A	5	5	1.00	23	0.217
556	A	4	4	1.00	23	0.174
557	A	3	3	1.00	23	0.130
558	A	1	1	1.00	21	0.048
559	A	1	1	1.00	14	0.071
560	A	6	6	1.00	21	0.286
561	A	7	7	1.00	23	0.304
562	A	6	6	1.00	23	0.261
563	A	5	5	1.00	23	0.217
564	A	4	4	1.00	23	0.174
565	A	4	4	1.00	23	0.174
566	A	5	5	1.00	21	0.238
567	A	6	6	1.00	14	0.429
568	A	7	7	1.00	21	0.333
569	A	8	8	1.00	23	0.348
570	A	6	6	1.00	23	0.261
571	A	5	5	1.00	23	0.217
572	A	5	5	1.00	23	0.217
573	A	5	5	1.00	23	0.217
574	A	5	5	1.00	21	0.238
575	A	7	7	1.00	14	0.500
576	A	8	8	1.00	21	0.381
577	A	9	8	1.00	23	0.348
578	A	8	7	1.00	14	0.500
579	A	8	5	1.00	21	0.238
580	A	7	5	1.00	21	0.238
581	A	6	5	1.00	21	0.238
582	A	5	4	1.00	21	0.190
583	A	6	5	1.00	21	0.238
584	A	7	5	1.00	21	0.238
585	A	8	5	1.00	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	9	6	1.00	23	0.261
587	A	8	6	1.00	23	0.261
588	A	7	6	1.00	23	0.261
589	A	6	5	1.00	23	0.217
590	A	6	5	1.00	23	0.217
591	A	7	6	1.00	23	0.261
592	A	8	6	1.00	23	0.261
593	A	9	7	1.00	23	0.304
594	A	8	7	1.00	23	0.304
595	A	7	6	1.00	23	0.261
596	A	7	6	1.00	23	0.261
597	A	7	6	1.00	23	0.261
598	A	8	7	1.00	23	0.304
599	A	9	7	1.00	23	0.304
600	A	10	8	1.00	23	0.348
601	A	9	8	1.00	23	0.348
602	A	8	7	1.00	23	0.304
603	A	8	7	1.00	23	0.304
604	A	8	7	1.00	23	0.304
605	A	8	7	1.00	23	0.304
606	A	9	8	1.00	23	0.348
607	A	10	8	1.00	23	0.348
608	A	10	9	1.00	23	0.391
609	A	6	6	1.00	23	0.261
610	A	2	2	1.00	23	0.087
611	A	4	4	1.00	23	0.174
612	A	8	7	1.00	23	0.304
613	A	9	8	1.00	23	0.348
614	A	11	9	1.00	23	0.391
615	A	10	9	1.00	23	0.391
616	A	9	8	1.00	23	0.348
617	A	9	8	1.00	23	0.348
618	A	9	8	1.00	23	0.348
619	A	9	8	1.00	23	0.348
620	A	10	9	1.00	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	11	10	1.00	23	0.435
622	A	10	9	1.00	23	0.391
623	A	10	9	1.00	23	0.391
624	A	10	9	1.00	23	0.391
625	A	10	9	1.00	23	0.391
626	A	10	9	1.00	23	0.391
627	A	11	10	1.00	23	0.435
628	A	12	12	1.00	25	0.480
629	A	7	7	1.00	25	0.280
630	A	3	3	1.00	25	0.120
631	A	8	8	1.00	25	0.320
632	A	9	9	1.00	25	0.360
633	A	10	9	1.00	25	0.360
634	A	13	13	1.00	25	0.520
635	A	12	12	1.00	25	0.480
636	A	11	11	1.00	25	0.440
637	A	8	8	1.00	25	0.320
638	A	9	9	1.00	25	0.360
639	A	10	9	1.00	25	0.360
640	A	14	13	1.00	25	0.520
641	A	13	13	1.00	25	0.520
642	A	12	12	1.00	25	0.480
643	A	12	12	1.00	25	0.480
644	A	9	9	1.00	25	0.360
645	A	10	9	1.00	25	0.360
646	A	11	9	1.00	25	0.360
647	A	13	13	1.00	25	0.520
648	A	12	12	1.00	25	0.480
649	A	3	3	1.00	25	0.120
650	A	3	3	1.00	25	0.120
651	A	7	7	1.00	25	0.280
652	A	8	8	1.00	25	0.320
653	A	9	9	1.00	25	0.360
654	A	13	13	1.00	25	0.520
655	A	9	9	1.00	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	5	5	1.00	25	0.200
657	A	8	8	1.00	25	0.320
658	A	8	8	1.00	25	0.320
659	A	9	9	1.00	25	0.360
660	A	10	9	1.00	25	0.360
661	A	14	14	1.00	25	0.560
662	A	13	13	1.00	25	0.520
663	A	9	9	1.00	25	0.360
664	A	9	9	1.00	25	0.360
665	A	9	9	1.00	25	0.360
666	A	9	9	1.00	25	0.360
667	A	10	10	1.00	25	0.400
668	A	11	10	1.00	25	0.400
669	A	5	5	1.00	25	0.200
670	A	5	5	1.00	25	0.200
671	A	7	7	1.00	25	0.280
672	A	7	7	1.00	25	0.280
673	A	5	5	1.00	25	0.200
674	A	5	5	1.00	25	0.200
675	A	5	5	1.00	25	0.200
676	A	5	5	1.00	25	0.200
677	A	2	2	1.00	25	0.080
678	A	2	2	1.00	25	0.080
679	A	3	3	1.00	25	0.120
680	A	3	3	1.00	25	0.120
681	A	2	2	1.00	25	0.080
682	A	2	2	1.00	25	0.080
683	A	2	2	1.00	25	0.080
684	A	2	2	1.00	25	0.080
685	A	3	3	1.00	21	0.143
686	A	0	0	0.00	0	0.000
687	A	10	7	1.00	23	0.304
688	A	9	6	1.00	23	0.261
689	A	8	5	1.00	23	0.217
690	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
691	A	0	0	0.00	0	0.000
692	A	3	3	1.00	21	0.143
693	A	0	0	0.00	0	0.000
694	A	11	7	1.00	23	0.304
695	A	10	6	1.00	23	0.261
696	A	9	6	1.00	23	0.261
697	A	3	3	1.00	21	0.143
698	A	0	0	0.00	0	0.000
699	A	9	6	1.00	23	0.261
700	A	8	5	1.00	23	0.217
701	A	7	4	1.00	23	0.174
702	A	3	3	1.00	21	0.143
703	A	0	0	0.00	0	0.000
704	A	3	3	1.00	21	0.143
705	A	0	0	0.00	0	0.000
706	A	3	3	1.00	21	0.143
707	A	0	0	0.00	0	0.000
708	A	9	6	1.00	23	0.261
709	A	8	5	1.00	23	0.217
710	A	8	5	1.00	23	0.217
711	A	3	3	1.00	21	0.143
712	A	0	0	0.00	0	0.000
713	A	6	4	1.00	23	0.174
714	A	6	4	1.00	23	0.174
715	A	6	4	1.00	23	0.174
716	A	6	4	1.00	23	0.174
717	A	0	0	0.00	0	0.000
718	A	0	0	0.00	0	0.000
719	A	0	0	0.00	0	0.000
720	A	0	0	0.00	0	0.000
721	A	0	0	0.00	0	0.000
722	A	0	0	0.00	0	0.000
723	A	0	0	0.00	0	0.000
724	A	0	0	0.00	0	0.000
725	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
726	A	0	0	0.00	0	0.000
727	A	0	0	0.00	0	0.000
728	A	0	0	0.00	0	0.000
729	A	0	0	0.00	0	0.000
730	A	0	0	0.00	0	0.000
731	A	0	0	0.00	0	0.000
732	A	0	0	0.00	0	0.000
733	A	0	0	0.00	0	0.000
734	A	0	0	0.00	0	0.000
735	A	0	0	0.00	0	0.000
736	A	0	0	0.00	0	0.000
737	A	0	0	0.00	0	0.000
738	A	0	0	0.00	0	0.000
739	A	0	0	0.00	0	0.000
740	A	0	0	0.00	0	0.000
741	A	0	0	0.00	0	0.000
742	A	0	0	0.00	0	0.000
743	A	0	0	0.00	0	0.000
744	A	0	0	0.00	0	0.000
745	A	0	0	0.00	0	0.000
746	A	0	0	0.00	0	0.000
747	A	0	0	0.00	0	0.000
748	A	0	0	0.00	0	0.000
749	A	0	0	0.00	0	0.000
750	A	0	0	0.00	0	0.000
751	A	0	0	0.00	0	0.000
752	A	0	0	0.00	0	0.000
753	A	0	0	0.00	0	0.000
754	A	0	0	0.00	0	0.000
755	A	0	0	0.00	0	0.000
756	A	0	0	0.00	0	0.000
757	A	0	0	0.00	0	0.000
758	A	0	0	0.00	0	0.000
759	A	0	0	0.00	0	0.000
760	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	0	0	0.00	0	0.000
762	A	0	0	0.00	0	0.000
763	A	0	0	0.00	0	0.000
764	A	0	0	0.00	0	0.000
765	A	0	0	0.00	0	0.000
766	A	0	0	0.00	0	0.000
767	A	0	0	0.00	0	0.000
768	A	0	0	0.00	0	0.000
769	A	0	0	0.00	0	0.000
770	A	0	0	0.00	0	0.000
771	A	0	0	0.00	0	0.000
772	A	0	0	0.00	0	0.000
773	A	0	0	0.00	0	0.000
774	A	0	0	0.00	0	0.000
775	A	0	0	0.00	0	0.000
776	A	0	0	0.00	0	0.000
777	A	7	5	1.00	23	0.217
778	A	6	4	1.00	23	0.174
779	A	5	3	1.00	21	0.143
780	A	6	4	1.00	23	0.174
781	A	9	4	1.00	23	0.174
782	A	0	0	0.00	0	0.000
783	A	0	0	0.00	0	0.000
784	A	0	0	0.00	0	0.000
785	A	0	0	0.00	0	0.000
786	A	0	0	0.00	0	0.000
787	A	0	0	0.00	0	0.000
788	A	8	5	1.00	21	0.238
789	A	7	4	1.00	21	0.190
790	A	3	3	1.00	19	0.158
791	A	0	0	0.00	0	0.000
792	A	0	0	0.00	0	0.000
793	A	0	0	0.00	0	0.000
794	A	8	5	1.00	21	0.238
795	A	7	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	6	5	1.00	21	0.238
797	A	5	5	1.00	21	0.238
798	A	4	4	1.00	21	0.190
799	A	5	5	1.00	21	0.238
800	A	6	5	1.00	21	0.238
801	A	7	5	1.00	21	0.238
802	A	10	7	1.00	23	0.304
803	A	9	7	1.00	23	0.304
804	A	8	7	1.00	23	0.304
805	A	7	6	1.00	23	0.261
806	A	7	6	1.00	23	0.261
807	A	8	7	1.00	23	0.304
808	A	9	7	1.00	23	0.304
809	A	10	7	1.00	23	0.304
810	A	10	8	1.00	23	0.348
811	A	9	8	1.00	23	0.348
812	A	8	7	1.00	23	0.304
813	A	8	7	1.00	23	0.304
814	A	8	7	1.00	23	0.304
815	A	9	8	1.00	23	0.348
816	A	10	8	1.00	23	0.348
817	A	11	10	1.00	23	0.435
818	A	10	9	1.00	23	0.391
819	A	9	8	1.00	23	0.348
820	A	5	5	1.00	23	0.217
821	A	3	3	1.00	23	0.130
822	A	7	7	1.00	23	0.304
823	A	11	10	1.00	23	0.435
824	A	11	10	1.00	23	0.435
825	A	10	9	1.00	23	0.391
826	A	10	9	1.00	23	0.391
827	A	10	9	1.00	23	0.391
828	A	10	9	1.00	23	0.391
829	A	11	10	1.00	23	0.435
830	A	12	11	1.00	23	0.478

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
831	A	11	10	1.00	23	0.435
832	A	11	10	1.00	23	0.435
833	A	11	10	1.00	23	0.435
834	A	11	10	1.00	23	0.435
835	A	11	10	1.00	23	0.435
836	A	12	11	1.00	23	0.478
837	A	10	10	1.00	25	0.400
838	A	9	9	1.00	25	0.360
839	A	4	4	1.00	25	0.160
840	A	8	8	1.00	25	0.320
841	A	13	13	1.00	25	0.520
842	A	11	10	1.00	25	0.400
843	A	10	10	1.00	25	0.400
844	A	9	9	1.00	25	0.360
845	A	12	12	1.00	25	0.480
846	A	13	13	1.00	25	0.520
847	A	14	14	1.00	25	0.560
848	A	12	10	1.00	25	0.400
849	A	11	10	1.00	25	0.400
850	A	10	10	1.00	25	0.400
851	A	13	13	1.00	25	0.520
852	A	13	13	1.00	25	0.520
853	A	14	14	1.00	25	0.560
854	A	15	14	1.00	25	0.560
855	A	10	10	1.00	25	0.400
856	A	9	9	1.00	25	0.360
857	A	8	8	1.00	25	0.320
858	A	4	4	1.00	25	0.160
859	A	4	4	1.00	25	0.160
860	A	13	13	1.00	25	0.520
861	A	14	14	1.00	25	0.560
862	A	11	10	1.00	25	0.400
863	A	10	10	1.00	25	0.400
864	A	9	9	1.00	25	0.360
865	A	9	9	1.00	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	6	6	1.00	25	0.240
867	A	10	10	1.00	25	0.400
868	A	14	14	1.00	25	0.560
869	A	11	11	1.00	25	0.440
870	A	10	10	1.00	25	0.400
871	A	10	10	1.00	25	0.400
872	A	10	10	1.00	25	0.400
873	A	10	10	1.00	25	0.400
874	A	14	14	1.00	25	0.560
875	A	8	6	1.00	23	0.261
876	A	7	5	1.00	23	0.217
877	A	5	4	1.00	21	0.190
878	A	7	5	1.00	23	0.217
879	A	10	5	1.00	23	0.217

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)}{d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1/
4*a*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 3852, 3853, 3855}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec^4(c+dx)\,dx + a\int \sec^5(c+dx)\,dx\\ &= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx - \frac{a\text{Subst}\left(\int (1\right.}{}\\ &= \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+}{4d}\\ &= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx}{8d}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 76, normalized size = 0.89

$$\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{3a\left(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)\right)}{8d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
[Out] (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.15, size = 73, normalized size = 0.86

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
norman	$\frac{\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{31a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{49a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{3a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{3a\ln\left(\tan\left(\frac{d}{}\right.\right.}{8d}$
risch	$-\frac{ia\left(9\,e^{7i(dx+c)}+33\,e^{5i(dx+c)}-48\,e^{4i(dx+c)}-33\,e^{3i(dx+c)}-64\,e^{2i(dx+c)}-9\,e^{i(dx+c)}-16\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}+3a\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 95, normalized size = 1.12

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a-3\,a\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.27, size = 99, normalized size = 1.16

$$\frac{9\,a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,a\cos\left(dx+c\right)^{3}+9\,a\cos\left(dx+c\right)^{2}+8\,a\cos\left(dx+c\right)+6\,a\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*a*cos(d*x
 + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sec^{4}\left(c+dx\right)dx+\int\sec^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

Giac [A]
time = 0.46, size = 110, normalized size = 1.29

$$\frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-39\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 49*a*tan(1/2*d*x + 1/2*c)^5 +
31*a*tan(1/2*d*x + 1/2*c)^3 - 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^4)/d
```

Mupad [B]
time = 3.94, size = 130, normalized size = 1.53

$$\frac{-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{49\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}-\frac{31\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] ((13*a*tan(c/2 + (d*x)/2))/4 - (31*a*tan(c/2 + (d*x)/2)^3)/12 + (49*a*tan(c
/2 + (d*x)/2)^5)/12 - (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 + (d*x)/2
)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^
8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.2 $\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/
3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 3853, 3855, 3852}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = a\int \sec^3(c+dx)\,dx + a\int \sec^4(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{a\text{Subst}\left(\int (1+x\right.}{}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.16, size = 60, normalized size = 0.95

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.10, size = 60, normalized size = 0.95

method	result	size
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	60
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	60
risch	$-\frac{ia\left(3\,e^{5i(dx+c)}-12\,e^{2i(dx+c)}-3\,e^{i(dx+c)}-4\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	95
norman	$\frac{-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2
*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.11

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a-3\,a\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.36, size = 88, normalized size = 1.40

$$\frac{3\,a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+2\,a\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*a)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sec^{3}\left(c+dx\right)\,dx+\int\sec^{4}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

Giac [A]
time = 0.45, size = 96, normalized size = 1.52

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 + 9*
a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 2.51, size = 102, normalized size = 1.62

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2) - (4*a*tan(c/2 +
(d*x)/2)^3)/3 + a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(
c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.3 $\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3872, 3852, 8, 3853, 3855}

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec^2(c+dx)\,dx + a\int \sec^3(c+dx)\,dx\\ &= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{a\text{Subst}(\int 1\,dx, x}{a}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Maple [A]
time = 0.09, size = 47, normalized size = 1.00

method	result	size
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\tan(dx+c)}{d}$	47
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\tan(dx+c)}{d}$	47
norman	$\frac{\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	87
risch	$-\frac{ia\left(e^{3i(dx+c)}-2\,e^{2i(dx+c)}-e^{i(dx+c)}-2\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*tan(d*x+
c))
```

Maxima [A]
time = 0.28, size = 58, normalized size = 1.23

$$-\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,a\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

Fricas [A]
time = 2.94, size = 74, normalized size = 1.57

$$\frac{a\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sec^2\left(c+dx\right)dx+\int\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sec(c + d*x)**2, x) + Integral(sec(c + d*x)**3, x))
```

Giac [A]
time = 0.48, size = 80, normalized size = 1.70

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 1.06, size = 75, normalized size = 1.60

$$\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^2,x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
- 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.4 $\int \sec(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3872, 3855, 3852, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec(c+dx)\,dx + a\int \sec^2(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.04, size = 30, normalized size = 1.25

method	result	size
derivativedivides	$\frac{a\tan(dx+c)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
default	$\frac{a\tan(dx+c)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
risch	$\frac{2ia}{d(e^{2i(dx+c)}+1)} + \frac{a\ln(e^{i(dx+c)}+i)}{d} - \frac{a\ln(e^{i(dx+c)}-i)}{d}$	59
norman	$-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d} - \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*tan(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.29, size = 29, normalized size = 1.21

$$\frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+a\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + a*tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.
time = 2.52, size = 60, normalized size = 2.50

$$\frac{a\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,a\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [A]
time = 2.55, size = 37, normalized size = 1.54

$$\begin{cases}\frac{a\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+a\tan\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x(a\sec\left(c\right)+a)\sec\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + a*tan(c + d*x))/d, Ne(d, 0
)), (x*(a*sec(c) + a)*sec(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(24) = 48$.
time = 0.44, size = 63, normalized size = 2.62

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 0.68, size = 47, normalized size = 1.96

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
(d*x)/2)^2 - 1))
```

3.5 $\int (a+a\sec(c+dx))\,dx$

Optimal. Leaf size=16

$$ax+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*x+a*arctanh(sin(d*x+c))/d
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {3855}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[a + a*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\,dx &= ax+a\int \sec(c+dx)\,dx\\ &= ax+\frac{a\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$ax+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + a*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Maple [A]
time = 0.03, size = 24, normalized size = 1.50

method	result	size
default	$ax+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
derivativedivides	$\frac{(dx+c)a+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
norman	$ax+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	40
risch	$ax-\frac{a\ln(e^{i(dx+c)}-i)}{d}+\frac{a\ln(e^{i(dx+c)}+i)}{d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+a*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+a/d*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 23, normalized size = 1.44

$$ax+\frac{a\log(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + a*log(sec(d*x + c) + tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.
time = 2.81, size = 36, normalized size = 2.25

$$\frac{2\,adx+a\log(\sin(dx+c)+1)-a\log(-\sin(dx+c)+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

Sympy [A]
time = 1.09, size = 41, normalized size = 2.56

$$ax+a\left(\begin{cases}\frac{\log(\tan(c+dx)+\sec(c+dx))}{d} & \text{for}\ d\neq 0\\ \frac{x(\tan(c)\sec(c)+\sec^2(c))}{\tan(c)+\sec(c)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x)
```

```
[Out] a*x + a*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(c
)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(16) = 32$.
time = 0.45, size = 49, normalized size = 3.06

$$ax + \frac{a\left(\log\left(\left|\frac{1}{\sin(dx+c)} + \sin(dx+c) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(dx+c)} + \sin(dx+c) - 2\right|\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/4*a*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*
x + c) + sin(d*x + c) - 2)))/d
```

Mupad [B]
time = 0.61, size = 20, normalized size = 1.25

$$a\,x + \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + a/cos(c + d*x),x)
```

```
[Out] a*x + (2*a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.6 $\int \cos(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{a\sin(c+dx)}{d}$$

```
[Out] a*x+a*sin(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {3872, 2717, 8}

$$\frac{a\sin(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x + (a*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))\,dx &= a\int 1\,dx+a\int \cos(c+dx)\,dx\\ &= ax+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.73

$$ax+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
[Out] a*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

Maple [A]
time = 0.06, size = 21, normalized size = 1.40

method	result	size
risch	$ax+\frac{a\sin(dx+c)}{d}$	16
derivativedivides	$\frac{a\sin(dx+c)+(dx+c)a}{d}$	21
default	$\frac{a\sin(dx+c)+(dx+c)a}{d}$	21
norman	$\frac{ax+ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*sin(d*x+c)+(d*x+c)*a)
```

Maxima [A]
time = 0.28, size = 20, normalized size = 1.33

$$\frac{(dx+c)a+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] ((d*x + c)*a + a*sin(d*x + c))/d
```

Fricas [A]
time = 2.61, size = 17, normalized size = 1.13

$$\frac{adx+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] (a*d*x + a*sin(d*x + c))/d
```

Sympy [A]
time = 1.13, size = 17, normalized size = 1.13

$$ax+a\left(\begin{cases}x\cos(c) & \text{for } d=0\\ \frac{\sin(c+dx)}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*x + a*Piecewise((x*cos(c), Eq(d, 0)), (sin(c + d*x)/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.
time = 0.43, size = 39, normalized size = 2.60

$$\frac{(dx+c)a + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

Mupad [B]
time = 0.60, size = 15, normalized size = 1.00

$$a\,x+\frac{a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x + (a*sin(c + d*x))/d
```

3.7 $\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2715, 8, 2717}

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = a\int \cos(c+dx)\,dx + a\int \cos^2(c+dx)\,dx$$
$$= \frac{a\sin(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx$$
$$= \frac{ax}{2} + \frac{a\sin(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 0.84

$$\frac{a(2(c+dx)+4\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
[Out] (a*(2*(c + d*x) + 4*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.08, size = 38, normalized size = 1.00

method	result	size
risch	$\frac{ax}{2}+\frac{a\sin(dx+c)}{d}+\frac{a\sin(2dx+2c)}{4d}$	32
derivativedivides	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
default	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
norman	$\frac{\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{ax}{2}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 34, normalized size = 0.89

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a+4\,a\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*a*sin(d*x + c))/d
```

Fricas [A]
time = 3.09, size = 29, normalized size = 0.76

$$\frac{adx + (a\cos(dx+c) + 2a)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x + (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cos^2(c+dx)\sec(c+dx)\,dx + \int \cos^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)**2, x)
)
```

Giac [A]
time = 0.41, size = 56, normalized size = 1.47

$$\frac{(dx+c)a + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/2*((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 1.06, size = 50, normalized size = 1.32

$$\frac{a\,x}{2} + \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x)),x)
[Out] (a*x)/2 + (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (
d*x)/2)^2 + 1)^2)
```

3.8 $\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=54

$$\frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] 1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 2713, 2715, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = a\int \cos^2(c+dx)\,dx + a\int \cos^3(c+dx)\,dx$$
$$= \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx - \frac{a\text{Subst}\left(\int (1-x^2)\,dx, x, \right.}{d}$$
$$= \frac{ax}{2} + \frac{a\sin(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d} - \frac{a\sin^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.08, size = 57, normalized size = 1.06

$$\frac{a(c+dx)}{2d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*Si
n[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.09, size = 49, normalized size = 0.91

method	result
risch	$\frac{ax}{2} + \frac{3a\sin(dx+c)}{4d} + \frac{a\sin(3dx+3c)}{12d} + \frac{a\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$
default	$\frac{\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$
norman	$\frac{\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{ax}{2} + \frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d} + \frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2} + \frac{3ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2} + \frac{ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.85

$$-\frac{4\left(\sin\left(dx+c\right)^3 - 3\sin\left(dx+c\right)\right)a - 3\left(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*a)/d
```

Fricas [A]
time = 3.65, size = 42, normalized size = 0.78

$$\frac{3\,adx + \left(2\,a\cos\left(dx+c\right)^{2} + 3\,a\cos\left(dx+c\right) + 4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x + (2*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*a)*sin(d*x + c))/
d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^{3}\left(c+dx\right)\sec\left(c+dx\right)dx + \int\cos^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)**3, x)
)
```

Giac [A]
time = 0.42, size = 72, normalized size = 1.33

$$\frac{3\left(dx+c\right)a + \frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*a + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 + 4*a*tan(1/2*d*x + 1/2*
c)^3 + 9*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 0.65, size = 55, normalized size = 1.02

$$\frac{a\,x}{2} + \frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d} + \frac{a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d} + \frac{a\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*x)/2 + (2*a*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) +
(a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.9 $\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=76

$$\frac{3ax}{8}+\frac{a\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] 3/8*a*x+a*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*sin
(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 2715, 8, 2713}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = a\int \cos^3(c+dx)\,dx + a\int \cos^4(c+dx)\,dx$$
$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{a\text{Subst}(\int$$
$$= \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+$$
$$= \frac{3ax}{8} + \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)}{4d}$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 0.96

$$\frac{3a(c+dx)}{8d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
[Out] (3*a*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*
Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.10, size = 60, normalized size = 0.79

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{3ax}{8}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{31a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{49a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{3a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{9ax\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.75

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

Fricas [A]
time = 3.05, size = 53, normalized size = 0.70

$$\frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^{3}+8\,a\cos\left(dx+c\right)^{2}+9\,a\cos\left(dx+c\right)+16\,a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*a*cos(d*x + c)
 + 16*a)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^{4}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\cos^{4}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**4*sec(c + d*x), x) + Integral(cos(c + d*x)**4, x)
)
```

Giac [A]
time = 0.44, size = 86, normalized size = 1.13

$$\frac{9\left(dx+c\right)a+\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+39\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 + 49*a*tan(1/2*d*x + 1/
2*c)^5 + 31*a*tan(1/2*d*x + 1/2*c)^3 + 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 4.17, size = 79, normalized size = 1.04

$$\frac{3\,a\,x}{8}+\frac{\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{49\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{31\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (31*a*tan(c/2 + (d*x)/2)^3)/12 +
 (49*a*tan(c/2 + (d*x)/2)^5)/12 + (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c/2
 + (d*x)/2)^2 + 1)^4)
```

3.10 $\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=122

$$\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{9a^2\tan(c+dx)}{5d}+\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{a^2\sec$$

[Out] 3/4*a^2*arctanh(sin(d*x+c))/d+9/5*a^2*tan(d*x+c)/d+3/4*a^2*sec(d*x+c)*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a^2*sec(d*x+c)^4*tan(d*x+c)/d+3/5*a^2*tan(d*x+c)^3/d

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3873, 3853, 3855, 4131, 3852}

$$\frac{3a^2\tan^3(c+dx)}{5d}+\frac{9a^2\tan(c+dx)}{5d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{2d}+\frac{3a^2\tan(c+dx)\sec(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]

[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) + (9*a^2*Tan[c + d*x])/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) + (a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (3*a^2*Tan[c + d*x]^3)/(5*d)

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \sec^5(c+dx)\,dx+\int \sec^4(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,d\\ &= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{a^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{2}\left(3a\right.\\ &= \frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{a^2\sec}{}\\ &= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{9a^2\tan(c+dx)}{5d}+\frac{3a^2\sec(c+dx)\tan(}{4d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 487 vs. 2(122) = 244.
time = 1.62, size = 487, normalized size = 3.99

$$-\frac{a^2\sec(c)\sec^5(c+dx)\left(75\cos(2c+3dx)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+75\cos(4c+3dx)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+15\cos(4c+5dx)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+15\cos(6c+5dx)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+150\cos(dx)\left(\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\right)+150\cos(2c+dx)\left(\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\right)-75\cos(2c+3dx)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-75\cos(4c+3dx)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-15\cos(4c+5dx)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-15\cos(6c+5dx)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-400\sin(dx)+80\sin(2c+dx)-140\sin(c+2dx)-140\sin(3c+2dx)-240\sin(2c+3dx)-30\sin(3c+4dx)-30\sin(5c+4dx)-48\sin(4c+5dx)\right)}{640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/640*(a^2*Sec[c]*Sec[c + d*x]^5*(75*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 75*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 15*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1
5*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 150*Cos[d*x]*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]) + 150*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 75*Cos[2*c + 3*d*x]*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] - 75*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 15*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] - 15*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 400
*Sin[d*x] + 80*Sin[2*c + d*x] - 140*Sin[c + 2*d*x] - 140*Sin[3*c + 2*d*x] -
```

```
 240*Sin[2*c + 3*d*x] - 30*Sin[3*c + 4*d*x] - 30*Sin[5*c + 4*d*x] - 48*Sin[
4*c + 5*d*x]))/d
```

Maple [A]
time = 0.12, size = 111, normalized size = 0.91

method	result
derivativedivides	$\frac{-a^{2}\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)+2a^{2}\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)}{8}\right.}{d}$
default	$\frac{-a^{2}\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)+2a^{2}\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)}{8}\right.}{d}$
risch	$-\frac{ia^{2}\left(15\,\mathrm{e}^{9i(dx+c)}+70\,\mathrm{e}^{7i(dx+c)}-40\,\mathrm{e}^{6i(dx+c)}-200\,\mathrm{e}^{4i(dx+c)}-70\,\mathrm{e}^{3i(dx+c)}-120\,\mathrm{e}^{2i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}-24\right)}{10d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}-\frac{3a^{2}\ln(\mathrm{e}}{}$
norman	$\frac{-\frac{13a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{9a^{2}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{72a^{2}\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{7a^{2}\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^{2}\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{3a^{2}\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2*a^2*(-(-1
/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-a
^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 133, normalized size = 1.09

$$\frac{8\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)a^{2}+40\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)a^{2}-15a^{2}\left(\frac{2\left(3\sin\left(dx+c\right)^{3}-5\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\sin\left(dx+c\right)^{2}+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + 40*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 15*a^2*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.10, size = 124, normalized size = 1.02

$$\frac{15\,a^{2}\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\,a^{2}\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(24\,a^{2}\cos\left(dx+c\right)^{4}+15\,a^{2}\cos\left(dx+c\right)^{3}+12\,a^{2}\cos\left(dx+c\right)^{2}+10\,a^{2}\cos\left(dx+c\right)+4\,a^{2}\right)\sin\left(dx+c\right)}{40\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/40*(15*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a^2*cos(d*x + c)^5*l
og(-sin(d*x + c) + 1) + 2*(24*a^2*cos(d*x + c)^4 + 15*a^2*cos(d*x + c)^3 +
12*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \sec^4\left(c+dx\right)dx+\int 2\sec^5\left(c+dx\right)dx+\int \sec^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**4, x) + Integral(2*sec(c + d*x)**5, x) + Integ
ral(sec(c + d*x)**6, x))
```

Giac [A]
time = 0.45, size = 138, normalized size = 1.13

$$\frac{15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+144\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+65\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(15*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 - 70*a^2*tan(1/2*d*x +
1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 - 90*a^2*tan(1/2*d*x + 1/2*c)^3 +
 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 5.70, size = 170, normalized size = 1.39

$$\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}-7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{72\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}-9\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{13\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] (3*a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) - ((72*a^2*tan(c/2 + (d*x)/2)^5)/5
- 9*a^2*tan(c/2 + (d*x)/2)^3 - 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/2
+ (d*x)/2)^9)/2 + (13*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*tan(c/2 + (d*x)/2)^2
 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)
^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.11 $\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=96

$$\frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{2a^2\text{ta}}{}$$

```
[Out] 7/8*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+7/8*a^2*sec(d*x+c)*tan(d*x
+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a^2*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3873, 3852, 4131, 3853, 3855}

$$\frac{2a^2\tan^3(c+dx)}{3d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{7a^2\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*Tan[c + d*x])/d + (7*a^2*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*
a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^4(c+dx)\,dx + \int \sec^3(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,c$$

$$= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(7a^2\right)\int \sec^3(c+dx)\,dx - \frac{\left(2a^2\right)\mathrm{S}}{}$$

$$= \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2\sec^3(c+dx)\,\mathrm{t}}{4d}$$

$$= \frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\sec(c+dx)\tan(}{8d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 877 vs. 2(96) = 192.
time = 6.44, size = 877, normalized size = 9.14

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-7*Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (
d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(32*d) + (7*Cos[c + d*x]^2*Log[Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)
/(32*d) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(64*
d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^2*Sec[c/2 +
(d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(12*d*(Cos[c/2] - Sin[c/2])
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^2*Sec[c/2 + (
d*x)/2]^4*(a + a*Sec[c + d*x])^2*(29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x
]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(3*d*(Cos[c/2
] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c + d*x]^2*
Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(64*d*(Cos[c/2 + (d*x)/2] + Si
n[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
```

```
d*x])^2*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin
[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d
*x])^2*(-29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2
+ (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*
(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(3*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A]
time = 0.07, size = 112, normalized size = 1.17

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-2a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^2\left(\right.}{d}$
default	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-2a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^2\left(\right.}{d}$
norman	$\frac{\frac{25a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{83a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{77a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{7a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{7a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{7a^2\ln\left(\right.}{}$
risch	$-\frac{ia^2\left(21\,e^{7i(dx+c)}+45\,e^{5i(dx+c)}-96\,e^{4i(dx+c)}-45\,e^{3i(dx+c)}-128\,e^{2i(dx+c)}-21\,e^{i(dx+c)}-32\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}-\frac{7a^2\ln\left(e^{i(dx+c)}-i\right)}{8d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))-2*a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*(1/2*sec(d*x+c)*t
an(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 145, normalized size = 1.51

$$\frac{32\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2-3\,a^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.90, size = 111, normalized size = 1.16

$$\frac{21\,a^2\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-21\,a^2\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(32\,a^2\cos\left(dx+c\right)^3+21\,a^2\cos\left(dx+c\right)^2+16\,a^2\cos\left(dx+c\right)+6\,a^2\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(21*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 21*a^2*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(32*a^2*cos(d*x + c)^3 + 21*a^2*cos(d*x + c)^2 +
16*a^2*cos(d*x + c) + 6*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \sec^{3}\left(c+dx\right)dx+\int 2\sec^{4}\left(c+dx\right)dx+\int \sec^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**3, x) + Integral(2*sec(c + d*x)**4, x) + Integ
ral(sec(c + d*x)**5, x))
```

Giac [A]
time = 0.53, size = 122, normalized size = 1.27

$$\frac{21\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-21\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-77\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+83\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-75\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 21*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*a^2*tan(1/2*d*x +
1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 3.96, size = 141, normalized size = 1.47

$$\frac{7\,a^{2}\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{7\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}-\frac{77\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{12}+\frac{83\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{12}-\frac{25\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (7*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((83*a^2*tan(c/2 + (d*x)/2)^3)/12
 - (77*a^2*tan(c/2 + (d*x)/2)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 - (25*
a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.12 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=74

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d}$$

```
[Out] a^2*arctanh(sin(d*x+c))/d+5/3*a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d+
1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3873, 3853, 3855, 4131, 3852, 8}

$$\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x])/(3*d) + (a^2*Sec[c + d
*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^3(c+dx)\,dx + \int \sec^2(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,d$$
$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d} + a^2\int \mathrm{s}$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\sec^2(c+}{}$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^2\tan(c+dx)}{3d} + \frac{a^2\sec(c+dx)\tan(c-}{d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 318 vs. 2(74) = 148.
time = 0.71, size = 318, normalized size = 4.30

$$-\frac{a^2\sec(c)\sec^3(c+dx)\left(3\cos(2c+3dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(4c+3dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+9\cos(dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+9\cos(2c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-3\cos(2c+3dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\cos(4c+3dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-24\sin(dx)+6\sin(2c+dx)-6\sin(c+2dx)-6\sin(3c+2dx)-10\sin(2c+3dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/24*(a^2*Sec[c]*Sec[c + d*x]^3*(3*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 3*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 9*Cos[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) + 9*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Cos[2*c + 3
*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 3*Cos[4*c + 3*d*x]*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] - 24*Sin[d*x] + 6*Sin[2*c + d*x] - 6*Sin[
c + 2*d*x] - 6*Sin[3*c + 2*d*x] - 10*Sin[2*c + 3*d*x]))/d
```

Maple [**A**]
time = 0.07, size = 75, normalized size = 1.01

method	result	si
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2\tan(dx+c)}{d}$	7
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2\tan(dx+c)}{d}$	7
risch	$-\frac{2ia^2\left(3\,e^{5i(dx+c)}-3\,e^{4i(dx+c)}-12\,e^{2i(dx+c)}-3\,e^{i(dx+c)}-5\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}$	1
norman	$\frac{-\frac{6a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{16a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a^2*(1/2*sec(d*x+c)*tan(d*x+
c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 85, normalized size = 1.15

$$\frac{2\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^2-3a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^2\tan\left(dx+c\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*tan
(d*x + c))/d
```

Fricas [A]
time = 2.75, size = 96, normalized size = 1.30

$$\frac{3a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(5a^2\cos\left(dx+c\right)^2+3a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)}{6d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(5*a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + a^2)*si
n(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sec^2\left(c+dx\right)dx+\int 2\sec^3\left(c+dx\right)dx+\int\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**2, x) + Integral(2*sec(c + d*x)**3, x) + Integ
ral(sec(c + d*x)**4, x))
```

Giac [A]
time = 0.47, size = 106, normalized size = 1.43

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*tan(1/2*d*x + 1/2*c
)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 2.47, size = 112, normalized size = 1.51

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{16\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^5 - (16*a^2
*tan(c/2 + (d*x)/2)^3)/3 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/
2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.13 $\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=54

$$\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 3/2*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*tan(d*x
+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {3873, 3852, 8, 4131, 3855}

$$\frac{2a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \sec^2(c+dx)\,dx+\int \sec(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx\\ &= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\left(3a^2\right)\int \sec(c+dx)\,dx-\frac{\left(2a^2\right)\text{Subs}}{}\\ &= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c}{2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 219 vs. 2(54) = 108.

time = 0.65, size = 219, normalized size = 4.06

$$\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{8\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-6*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2)
+ (8*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(16*d)
```

Maple [A]

time = 0.04, size = 70, normalized size = 1.30

method	result	size
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2a^2\tan(dx+c)+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$	70
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2a^2\tan(dx+c)+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$	70
norman	$\frac{\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{3a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{3a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	95
risch	$-\frac{ia^2\left(e^{3i(dx+c)}-4\,e^{2i(dx+c)}-e^{i(dx+c)}-4\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{3a^2\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{3a^2\ln\left(e^{i(dx+c)}-i\right)}{2d}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2*a^2*ta
n(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.28, size = 81, normalized size = 1.50

$$-\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)-8\,a^2\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a^2*tan(d
*x + c))/d
```

Fricas [A]
time = 2.51, size = 83, normalized size = 1.54

$$\frac{3\,a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^2*log(
-sin(d*x + c) + 1) + 2*(4*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x
+ c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sec\left(c+dx\right)dx+\int2\sec^2\left(c+dx\right)dx+\int\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x), x) + Integral(2*sec(c + d*x)**2, x) + Integral
(sec(c + d*x)**3, x))
```

Giac [A]
time = 0.46, size = 90, normalized size = 1.67

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 1.18, size = 83, normalized size = 1.54

$$\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (3*a^2*tan(c/2 + (d*x)/2)^3 - 5*a^2*t
an(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.14 $\int (a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=34

$$a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}$$

[Out] a^2*x+2*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3858, 3855, 3852, 8}

$$\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+a^2x$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[c + d*x])^2,x]

[Out] a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(c+dx))^2\,dx &= a^2x+a^2\int \sec^2(c+dx)\,dx+\left(2a^2\right)\int \sec(c+dx)\,dx \\
&= a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\text{Subst}(\int 1\,dx,x,-\tan(c+dx))}{d} \\
&= a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(34) = 68$.

time = 0.52, size = 171, normalized size = 5.03

$$
\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(dx-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(d*x - 2*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]
/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(4*d)
```

Maple [A]

time = 0.05, size = 44, normalized size = 1.29

method	result	size
derivativedivides	$\frac{a^2\tan(dx+c)+2a^2\ln(\sec(dx+c)+\tan(dx+c))+a^2(dx+c)}{d}$	44
default	$\frac{a^2\tan(dx+c)+2a^2\ln(\sec(dx+c)+\tan(dx+c))+a^2(dx+c)}{d}$	44
risch	$a^2x+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right)}{d}$	71
norman	$\frac{a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2x-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*tan(d*x+c)+2*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*(d*x+c))
```

Maxima [A]

time = 0.28, size = 41, normalized size = 1.21

$$
a^2x+\frac{2\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a^2\tan\left(dx+c\right)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a^2*log(sec(d*x + c) + tan(d*x + c))/d + a^2*tan(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.
time = 2.91, size = 76, normalized size = 2.24

$$\frac{a^2dx\cos(dx+c)+a^2\cos(dx+c)\log(\sin(dx+c)+1)-a^2\cos(dx+c)\log(-\sin(dx+c)+1)+a^2\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cos(d*x + c) + a^2*cos(d*x + c)*log(sin(d*x + c) + 1) - a^2*cos(d*
x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 1\,dx+\int 2\sec(c+dx)\,dx+\int \sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(1, x) + Integral(2*sec(c + d*x), x) + Integral(sec(c + d*x)*
*2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(34) = 68$.
time = 0.42, size = 79, normalized size = 2.32

$$\frac{(dx+c)a^2+2a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-2a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^2*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

Mupad [B]
time = 0.71, size = 56, normalized size = 1.65

$$a^2\,x+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2))/(d
*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.15 $\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=34

$$2a^2x+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}$$

```
[Out] 2*a^2*x+a^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3873, 8, 4130, 3855}

$$\frac{a^2\sin(c+dx)}{d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int 1\,dx + \int \cos(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= 2a^2x + \frac{a^2\sin(c+dx)}{d} + a^2\int \sec(c+dx)\,dx$$
$$= 2a^2x + \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.38

$$2a^2x + \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\cos(dx)\sin(c)}{d} + \frac{a^2\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*Co
s[c]*Sin[d*x])/d
```

Maple [A]
time = 0.07, size = 44, normalized size = 1.29

method	result
derivativedivides	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2a^2(dx+c)+a^2\sin(dx+c)}{d}$
default	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2a^2(dx+c)+a^2\sin(dx+c)}{d}$
risch	$2a^2x-\frac{ia^2e^{i(dx+c)}}{2d}+\frac{ia^2e^{-i(dx+c)}}{2d}+\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}$
norman	$\frac{-2a^2x-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+2a^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*(d*x+c)+a^2*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 52, normalized size = 1.53

$$\frac{4\,(dx+c)a^2 + a^2(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)) + 2\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*a^2 + a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 2.81, size = 53, normalized size = 1.56

$$\frac{4\,a^2dx + a^2\log\left(\sin\left(dx+c\right)+1\right) - a^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) + 2
*a^2*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cos\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x), x) + Integral(cos(c + d*x)*sec(
c + d*x)**2, x) + Integral(cos(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(34) = 68.
time = 0.44, size = 79, normalized size = 2.32

$$\frac{2\left(dx+c\right)a^2 + a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a^2 + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

Mupad [B]
time = 0.70, size = 33, normalized size = 0.97

$$2\,a^2\,x + \frac{a^2\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] 2*a^2*x + (a^2*(2*atanh(tan(c/2 + (d*x)/2)) + sin(c + d*x)))/d
```

3.16 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=45

$$\frac{3a^2x}{2}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 3/2*a^2*x+2*a^2*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3873, 2717, 4130, 8}

$$\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx &= (2a^2)\int \cos(c+dx)\,dx + \int \cos^2(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx \\
&= \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}(3a^2)\int 1\,dx \\
&= \frac{3a^2x}{2} + \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.76

$$\frac{a^2(6(c+dx)+8\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(6*(c + d*x) + 8*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.07, size = 52, normalized size = 1.16

method	result
risch	$\frac{3a^2x}{2}+\frac{2a^2\sin(dx+c)}{d}+\frac{a^2\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2\sin(dx+c)+a^2(dx+c)}{d}$
default	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2\sin(dx+c)+a^2(dx+c)}{d}$
norman	$\frac{-\frac{3a^2x}{2}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3a^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3a^2x\left(\tan^6\cdots\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*sin(d*x+c)+a^2*(d*
x+c))
```

Maxima [A]
time = 0.29, size = 48, normalized size = 1.07

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2+4\,(dx+c)a^2+8\,a^2\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*a^2 + 8*a^2*sin(d*x
 + c))/d
```

Fricas [A]
time = 3.19, size = 36, normalized size = 0.80

$$\frac{3\,a^2dx + \left(a^2\cos\left(dx+c\right) + 4\,a^2\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*d*x + (a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos^2\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2, x))
```

Giac [A]
time = 0.44, size = 64, normalized size = 1.42

$$\frac{3\,(dx+c)a^2 + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 1.07, size = 57, normalized size = 1.27

$$\frac{3\,a^2\,x}{2} + \frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/2 + (3*a^2*tan(c/2 + (d*x)/2)^3 + 5*a^2*tan(c/2 + (d*x)/2))/(d*(t
an(c/2 + (d*x)/2)^2 + 1)^2)
```

3.17 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=57

$$a^2x+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}$$

```
[Out] a^2*x+2*a^2*sin(d*x+c)/d+a^2*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3873, 2715, 8, 4129, 3092}

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/d - (a^2*S
in[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \cos^2(c+dx)\,dx + \int \cos^3(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{d} + a^2\int 1\,dx + \int \cos(c+dx)\left(a^2+a^2\cos$$
$$= a^2x + \frac{a^2\cos(c+dx)\sin(c+dx)}{d} - \frac{\text{Subst}\left(\int \left(2a^2-a^2x^2\right)\,dx, x, -\text{s}\right.}{d}$$
$$= a^2x + \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{d} - \frac{a^2\sin^3(c+dx}{3d}$$

Mathematica [A]
time = 0.09, size = 41, normalized size = 0.72

$$\frac{a^2(12dx+21\sin(c+dx)+6\sin(2(c+dx))+\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
[Out] (a^2*(12*d*x + 21*Sin[c + d*x] + 6*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(1
2*d)
```

Maple [A]
time = 0.09, size = 64, normalized size = 1.12

method	result
risch	$a^2x+\frac{7a^2\sin(dx+c)}{4d}+\frac{a^2\sin(3dx+3c)}{12d}+\frac{a^2\sin(2dx+2c)}{2d}$
derivativedivides	$\frac{\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\sin(dx+c)}{d}$
default	$\frac{\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\sin(dx+c)}{d}$
norman	$\frac{a^2x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2x-\frac{6a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{10a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-2a^2x\left(\tan^2\left(\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+a^2*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 61, normalized size = 1.07

$$-\frac{2\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}-6\,a^{2}\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^2 - 6*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 3.56, size = 49, normalized size = 0.86

$$\frac{3\,a^{2}dx+\left(a^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}\cos\left(dx+c\right)+5\,a^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*x + (a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + 5*a^2)*sin(d*x
+ c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\cos^{3}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cos^{3}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int\cos^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3, x))
```

Giac [A]
time = 0.44, size = 80, normalized size = 1.40

$$\frac{3\left(dx+c\right)a^{2}+\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+8\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 8*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 0.66, size = 61, normalized size = 1.07

$$a^2\,x+\frac{5\,a^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (5*a^2*sin(c + d*x))/(3*d) + (a^2*cos(c + d*x)^2*sin(c + d*x))/(3*d
) + (a^2*cos(c + d*x)*sin(c + d*x))/d
```

3.18 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=87

$$\frac{7a^2x}{8}+\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a^2\sin^3(c+dx)}{3d}$$

```
[Out] 7/8*a^2*x+2*a^2*sin(d*x+c)/d+7/8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*
x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3873, 2713, 4130, 2715, 8}

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*a^2*x)/8 + (2*a^2*Sin[c + d*x])/d + (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a^2*Sin[c + d*x]^3)/(3*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \cos^3(c+dx)\,dx+\int \cos^4(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,d\\ &= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\left(7a^2\right)\int \cos^2(c+dx)\,dx-\frac{\left(2a^2\right)\mathrm{Su}}{}\\ &= \frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin}{4d}\\ &= \frac{7a^2x}{8}+\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+}{}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 53, normalized size = 0.61

$$\frac{a^2(84dx+144\sin(c+dx)+48\sin(2(c+dx))+16\sin(3(c+dx))+3\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
[Out] (a^2*(84*d*x + 144*Sin[c + d*x] + 48*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)]
 + 3*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A]
time = 0.10, size = 90, normalized size = 1.03

method	result
risch	$\frac{7a^2x}{8}+\frac{3a^2\sin(dx+c)}{2d}+\frac{a^2\sin(4dx+4c)}{32d}+\frac{a^2\sin(3dx+3c)}{6d}+\frac{a^2\sin(2dx+2c)}{2d}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$

default	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{-\frac{7a^2x}{8}-\frac{25a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{2a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{14a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{7a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{21a^2x\left(\tan\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a
^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A]
time = 0.29, size = 83, normalized size = 0.95

$$-\frac{64\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{2}-24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*
a^2)/d
```

Fricas [A]
time = 2.91, size = 63, normalized size = 0.72

$$\frac{21\,a^{2}dx+\left(6\,a^{2}\cos\left(dx+c\right)^{3}+16\,a^{2}\cos\left(dx+c\right)^{2}+21\,a^{2}\cos\left(dx+c\right)+32\,a^{2}\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(21*a^2*d*x + (6*a^2*cos(d*x + c)^3 + 16*a^2*cos(d*x + c)^2 + 21*a^2*c
os(d*x + c) + 32*a^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\cos^{4}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\cos^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int\cos^{4}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)**4*sec(c + d*x), x) + Integral(cos(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**4, x))
```

Giac [A]
time = 0.45, size = 96, normalized size = 1.10

$$\frac{21\,(dx+c)a^2 + \frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+83\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*(d*x + c)*a^2 + 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*a^2*tan(1/2*
d*x + 1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 + 75*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 4.19, size = 89, normalized size = 1.02

$$\frac{7\,a^2\,x}{8}+\frac{\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{77\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{83\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (7*a^2*x)/8 + ((83*a^2*tan(c/2 + (d*x)/2)^3)/12 + (77*a^2*tan(c/2 + (d*x)/2
)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 + (25*a^2*tan(c/2 + (d*x)/2))/4)/(
d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.19 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{3a^2x}{4}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{a^2\sin^5(c+dx)}{5d}$$

```
[Out] 3/4*a^2*x+2*a^2*sin(d*x+c)/d+3/4*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*cos(d*
x+c)^3*sin(d*x+c)/d-a^2*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3873, 2715, 8, 4129, 3092, 380}

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/4 + (2*a^2*Sin[c + d*x])/d + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - (a^2*Sin[c + d*x]^3)/d + (a
^2*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \cos^4(c+dx)\,dx+\int \cos^5(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx \\
&= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^2\right)\int \cos^2(c+dx)\,dx+\int \cos^3(c \\
&= \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}+\frac{1}{4}\left(3a^2\right) \\
&= \frac{3a^2x}{4}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}- \\
&= \frac{3a^2x}{4}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 61, normalized size = 0.59

$$\frac{a^2(60dx+110\sin(c+dx)+40\sin(2(c+dx))+15\sin(3(c+dx))+5\sin(4(c+dx))+\sin(5(c+dx)))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(60*d*x + 110*Sin[c + d*x] + 40*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)]
 + 5*Sin[4*(c + d*x)] + Sin[5*(c + d*x)]))/(80*d)
```

Maple [A]
time = 0.13, size = 96, normalized size = 0.93

method	result

risch	$\frac{3a^2x}{4}+\frac{11a^2\sin(dx+c)}{8d}+\frac{a^2\sin(5dx+5c)}{80d}+\frac{a^2\sin(4dx+4c)}{16d}+\frac{3a^2\sin(3dx+3c)}{16d}+\frac{a^2\sin(2dx+2c)}{2d}$
derivativedivides	$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+2a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2\left(2+\cos^2(dx+c)\right)}{3}}{d}$
default	$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+2a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2\left(2+\cos^2(dx+c)\right)}{3}}{d}$
norman	$\frac{-\frac{3a^2x}{4}-\frac{13a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{5a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{27a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{37a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{11a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{3a^2\left(\mathrm{ta}\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a^2*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*(2+cos(d*x+c)^2
)*sin(d*x+c))
```

Maxima [A]

time = 0.29, size = 95, normalized size = 0.92

$$\frac{16\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)a^2-80\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)a^2+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)a^2}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 80
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

Fricas [A]

time = 2.39, size = 76, normalized size = 0.74

$$\frac{15\,a^2dx+\left(4\,a^2\cos\left(dx+c\right)^4+10\,a^2\cos\left(dx+c\right)^3+12\,a^2\cos\left(dx+c\right)^2+15\,a^2\cos\left(dx+c\right)+24\,a^2\right)\sin\left(dx+c\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/20*(15*a^2*d*x + (4*a^2*cos(d*x + c)^4 + 10*a^2*cos(d*x + c)^3 + 12*a^2*c
os(d*x + c)^2 + 15*a^2*cos(d*x + c) + 24*a^2)*sin(d*x + c))/d
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cos^5\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cos^5\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cos^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)**5*sec(c + d*x), x) + Integral(cos(c + d*x)**
5*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**5, x))
```

Giac [A]
time = 0.46, size = 112, normalized size = 1.09

$$\frac{15\,(dx+c)a^2+\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+144\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+65\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(15*(d*x + c)*a^2 + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 + 70*a^2*tan(1/2*
d*x + 1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 + 90*a^2*tan(1/2*d*x + 1/2*
c)^3 + 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 4.42, size = 105, normalized size = 1.02

$$\frac{3\,a^2\,x}{4}+\frac{\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}+7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{72\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{13\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/4 + (9*a^2*tan(c/2 + (d*x)/2)^3 + (72*a^2*tan(c/2 + (d*x)/2)^5)/5
 + 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/2 + (d*x)/2)^9)/2 + (13*a^2*ta
n(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.20 $\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=114

$$\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d}+$$

```
[Out] 13/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+13/8*a^3*sec(d*x+c)*tan(d
*x+c)/d+3/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+5/3*a^3*tan(d*x+c)^3/d+1/5*a^3*ta
n(d*x+c)^5/d
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3876, 3853, 3855, 3852}

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{13a^3\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
```

```
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+3a^3\sec^5(c+dx)+a^3\sec^6(c\right.$$

$$= a^3\int \sec^3(c+dx)\,dx + a^3\int \sec^6(c+dx)\,dx + \left(3a^3\right)\int \sec^4(c+dx$$

$$= \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{2}a^3\int$$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c}{8d}$$

$$= \frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan}{8d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 487 vs. 2(114) = 228.
time = 1.55, size = 487, normalized size = 4.27

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/3840*(a^3*Sec[c]*Sec[c + d*x]^5*(975*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] + 975*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 195*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1950*Cos
[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]) + 1950*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 975*Cos[2*c + 3*d*x]*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 975*Cos[4*c + 3*d*x]*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] - 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] - 195*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 4640*Sin[d*x] + 1440*Sin[2*c + d*x] - 1500*Sin[c + 2*d*x] - 1500*Si
n[3*c + 2*d*x] - 3040*Sin[2*c + 3*d*x] - 390*Sin[3*c + 4*d*x] - 390*Sin[5*c
 + 4*d*x] - 608*Sin[4*c + 5*d*x]))/d
```

Maple [A]
time = 0.10, size = 146, normalized size = 1.28

method	result
risch	$-\frac{ia^3(195\,e^{9i(dx+c)}+750\,e^{7i(dx+c)}-720\,e^{6i(dx+c)}-2320\,e^{4i(dx+c)}-750\,e^{3i(dx+c)}-1520\,e^{2i(dx+c)}-195\,e^{i(dx+c)}-304)}{60d(e^{2i(dx+c)}+1)^5}$
derivativedivides	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+3a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+}{}\right.}{d}$
default	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+3a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+}{}\right.}{d}$
norman	$\frac{-\frac{51a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{133a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{416a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{91a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{13a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{13a^3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+3*a^3*(-(-1
/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-3
*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*
ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]

time = 0.29, size = 179, normalized size = 1.57

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3-45a^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-60a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 45*a^3*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A]

time = 2.76, size = 124, normalized size = 1.09

$$\frac{195a^3\cos(dx+c)^5\log(\sin(dx+c)+1)-195a^3\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(304a^3\cos(dx+c)^4+195a^3\cos(dx+c)^3+152a^3\cos(dx+c)^2+90a^3\cos(dx+c)+24a^3\right)\sin(dx+c)}{240d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(195*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 195*a^3*cos(d*x + c)^
5*log(-sin(d*x + c) + 1) + 2*(304*a^3*cos(d*x + c)^4 + 195*a^3*cos(d*x + c)
```

```
^3 + 152*a^3*cos(d*x + c)^2 + 90*a^3*cos(d*x + c) + 24*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \sec^{3}\left(c+dx\right)dx+\int 3\sec^{4}\left(c+dx\right)dx+\int 3\sec^{5}\left(c+dx\right)dx+\int \sec^{6}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**3, x) + Integral(3*sec(c + d*x)**4, x) + Integ
ral(3*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**6, x))
```

Giac [A]
time = 0.47, size = 138, normalized size = 1.21

$$\frac{195\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-195\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-910\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1664\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1330\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+765\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(195*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 195*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*a^3*tan(1/2*d
*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*a^3*tan(1/2*d*x + 1/
2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 5.48, size = 170, normalized size = 1.49

$$\frac{13\,a^{3}\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{13\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}-\frac{91\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{6}+\frac{416\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{15}-\frac{133\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{6}+\frac{51\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((416*a^3*tan(c/2 + (d*x)/2)^5)/
15 - (133*a^3*tan(c/2 + (d*x)/2)^3)/6 - (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (
13*a^3*tan(c/2 + (d*x)/2)^9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(5*tan(c
/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan
(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.21 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=93

$$\frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^3}{}$$

```
[Out] 15/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+15/8*a^3*sec(d*x+c)*tan(d
*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3876, 3852, 8, 3853, 3855}

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
a^3*Tan[c + d*x]^3)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\sec^2(c+dx)+3a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+a^3\sec^5(c\right.\\ &= a^3\int \sec^2(c+dx)\,dx+a^3\int \sec^5(c+dx)\,dx+\left(3a^3\right)\int \sec^3(c+dx\\ &= \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\left(3a^3\right.\\ &= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\sec(c+dx)\tan(c}{8d}\\ &= \frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\sec(c+dx)\tan}{8d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 877 vs. 2(93) = 186.
time = 6.43, size = 877, normalized size = 9.43

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-15*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(64*d) + (15*Cos[c + d*x]^3*Log[Cos[c/2
+ (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^
3)/(64*d) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(1
28*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^3*Sec[c/2
 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(16*d*(Cos[c/2] - Sin[c/
2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^3*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (3*Cos[c
+ d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(8*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c + d*
x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(128*d*(Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Se
c[c + d*x])^3*Sin[(d*x)/2])/(16*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2]
```

+ Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec [c + d*x])^3*(-19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (3*Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(8*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))

Maple [A]
time = 0.07, size = 123, normalized size = 1.32

method	result
derivativedivides	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-3a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3}{d}$
default	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-3a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3}{d}$
norman	$\frac{\frac{49a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{73a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{55a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{15a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{15a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+15$
risch	$-\frac{ia^3\left(15\,e^{7i(dx+c)}-8\,e^{6i(dx+c)}+23\,e^{5i(dx+c)}-72\,e^{4i(dx+c)}-23\,e^{3i(dx+c)}-88\,e^{2i(dx+c)}-15\,e^{i(dx+c)}-24\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}-\frac{15a^3\ln(e}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))-3*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^3*(1/2*sec(d*x+c)
*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^3*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 156, normalized size = 1.68

$$\frac{16\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^3-a^3\left(\frac{2\left(3\sin\left(dx+c\right)^3-5\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\sin\left(dx+c\right)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-12a^3\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+16a^3\tan\left(dx+c\right)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - a^3*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x + c
))/d
```

Fricas [A]
time = 2.14, size = 111, normalized size = 1.19

$$\frac{15\,a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-15\,a^3\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(24\,a^3\cos\left(dx+c\right)^3+15\,a^3\cos\left(dx+c\right)^2+8\,a^3\cos\left(dx+c\right)+2\,a^3\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(15*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(24*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c)^2 +
8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \sec^2(c+dx)\,dx+\int 3\sec^3(c+dx)\,dx+\int 3\sec^4(c+dx)\,dx+\int \sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**2, x) + Integral(3*sec(c + d*x)**3, x) + Integ
ral(3*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

Giac [A]
time = 0.49, size = 122, normalized size = 1.31

$$\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+73\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-49\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*a^3*tan(1/2*d*x + 1
/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*a^3*tan(1/2*d*x + 1/2*c))/(tan
(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 4.05, size = 141, normalized size = 1.52

$$\frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{55\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{49\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (15*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((73*a^3*tan(c/2 + (d*x)/2)^3)/4
 - (55*a^3*tan(c/2 + (d*x)/2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 - (49*
a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.22 $\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=72

$$\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^3\tan^3(c+dx)}{3d}$$

```
[Out] 5/2*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+3/2*a^3*sec(d*x+c)*tan(d*x
+c)/d+1/3*a^3*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3876, 3855, 3852, 8, 3853}

$$\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\sec(c+dx)+3a^3\sec^2(c+dx)+3a^3\sec^3(c+dx)+a^3\sec^4(c-\right.\\ &= a^3\int \sec(c+dx)\,dx+a^3\int \sec^4(c+dx)\,dx+\left(3a^3\right)\int \sec^2(c+dx)\,d\\ &= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\left(3a^3\right)\int \mathrm{se}\\ &= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\sec(c+dx)\tan(c-}{2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(72) = 144.
time = 5.57, size = 154, normalized size = 2.14

$$-\frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3\left(60\cos^3(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(50\sin(dx)-20\sin(2c+dx)+9\sin(c+2dx)+9\sin(3c+2dx)+22\sin(2c+3dx))-4\cos(c+dx)\tan(c)\right)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/192*(a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(60*Cos[c + d*x]^3*(Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) - Sec[c]*(50*Sin[d*x] - 20*Sin[2*c + d*x] + 9*Sin[c + 2*d*x] + 9*Sin
[3*c + 2*d*x] + 22*Sin[2*c + 3*d*x]) - 4*Cos[c + d*x]*Tan[c]))/d
```

Maple [A]
time = 0.06, size = 94, normalized size = 1.31

method	result
derivativedivides	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\tan(dx+c)+a^3\ln(\sec(dx}{d}$
default	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\tan(dx+c)+a^3\ln(\sec(dx}{d}$
risch	$-\frac{ia^3\left(9\,\mathrm{e}^{5i(dx+c)}-18\,\mathrm{e}^{4i(dx+c)}-48\,\mathrm{e}^{2i(dx+c)}-9\,\mathrm{e}^{i(dx+c)}-22\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{5a^3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{5a^3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$

norman	$\frac{-\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{40a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{5a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^3*(1/2*sec(d*x+c)*tan(d*x+
c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*tan(d*x+c)+a^3*ln(sec(d*x+c)+tan(d*
x+c)))
```

Maxima [A]
time = 0.29, size = 104, normalized size = 1.44

$$\frac{4\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^3-9a^3\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+36a^3\tan\left(dx+c\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3*l
og(sec(d*x + c) + tan(d*x + c)) + 36*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 4.01, size = 98, normalized size = 1.36

$$\frac{15a^3\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-15a^3\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(22a^3\cos\left(dx+c\right)^2+9a^3\cos\left(dx+c\right)+2a^3\right)\sin\left(dx+c\right)}{12d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(15*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^3*l
og(-sin(d*x + c) + 1) + 2*(22*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 2*a
^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\sec\left(c+dx\right)dx+\int3\sec^2\left(c+dx\right)dx+\int3\sec^3\left(c+dx\right)dx+\int\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sec(c + d*x), x) + Integral(3*sec(c + d*x)**2, x) + Integral
(3*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

Giac [A]
time = 0.52, size = 106, normalized size = 1.47

$$\frac{15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+33\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^3*tan(1/2*d*x + 1
/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 2.57, size = 112, normalized size = 1.56

$$\frac{5\,a^{3}\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-\frac{40\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+11\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^5 - (40*a^3
*tan(c/2 + (d*x)/2)^3)/3 + 11*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)
/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.23 $\int (a+a\sec (c+dx))^3 \, dx$

Optimal. Leaf size=66

$$a^3x+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^3\tan(c+dx)}{2d}+\frac{(a^3+a^3\sec(c+dx))\tan(c+dx)}{2d}$$

```
[Out] a^3*x+7/2*a^3*arctanh(sin(d*x+c))/d+5/2*a^3*tan(d*x+c)/d+1/2*(a^3+a^3*sec(d
*x+c))*tan(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3860, 3999, 3852, 8, 3855}

$$\frac{5a^3\tan(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (7*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*Tan[c + d*x])/(2*d) +
((a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\,dx &= \frac{(a^3+a^3\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}a\int (a+a\sec(c+dx))(2a+5a\sec(c+dx\\ &= a^3x+\frac{(a^3+a^3\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}(5a^3)\int \sec^2(c+dx)\,dx+\frac{1}{2}(7a^3)\int\\ &= a^3x+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(a^3+a^3\sec(c+dx))\tan(c+dx)}{2d}-\frac{(5a^3)\,\text{Subs}}{}\\ &= a^3x+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^3\tan(c+dx)}{2d}+\frac{(a^3+a^3\sec(c+dx))\tan(c\,}{2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 235 vs. 2(66) = 132.
time = 0.96, size = 235, normalized size = 3.56

$$\frac{1}{32}a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(4x-\frac{14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{12\sin(dx)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(4*x - (14*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]])/d + (14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
+ 1/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2) + (12*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + S
in[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]))))/32
```

Maple [A]
time = 0.05, size = 80, normalized size = 1.21

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+a^3(dx+c)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+a^3(dx+c)}{d}$

risch	$a^{3}x-\frac{i a^{3}\left(\mathrm{e}^{3 i (d x+c)}-6\,\mathrm{e}^{2 i (d x+c)}-\mathrm{e}^{i (d x+c)}-6\right)}{d \left(\mathrm{e}^{2 i (d x+c)}+1\right)^{2}}-\frac{7 a^{3} \ln\left(\mathrm{e}^{i (d x+c)}-i\right)}{2 d}+\frac{7 a^{3} \ln\left(\mathrm{e}^{i (d x+c)}+i\right)}{2 d}$
norman	$\frac{a^{3} x+a^{3} x \left(\tan^{4}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+\frac{7 a^{3} \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{d}-\frac{5 a^{3} \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}-2 a^{3} x \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{7 a^{3} \ln\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2 d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*ta
n(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*(d*x+c))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 1.38

$$a^{3}x-\frac{a^{3}\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d}+\frac{3\,a^{3}\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{3\,a^{3}\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/4*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1))/d + 3*a^3*log(sec(d*x + c) + tan(d*x + c))/d + 3
*a^3*tan(d*x + c)/d
```

Fricas [A]
time = 4.07, size = 98, normalized size = 1.48

$$\frac{4\,a^{3}dx\cos\left(dx+c\right)^{2}+7\,a^{3}\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-7\,a^{3}\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(6\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + 7*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)
- 7*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^3*cos(d*x + c) + a^3
)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 1\,dx+\int 3\sec\left(c+dx\right)dx+\int 3\sec^{2}\left(c+dx\right)dx+\int\sec^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(1, x) + Integral(3*sec(c + d*x), x) + Integral(3*sec(c + d*x
)**2, x) + Integral(sec(c + d*x)**3, x))
```

Giac [A]
time = 0.47, size = 100, normalized size = 1.52

$$\frac{2\,(dx+c)a^3+7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 + 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 7*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 0.74, size = 88, normalized size = 1.33

$$a^3\,x+\frac{7\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3,x)
```

```
[Out] a^3*x + (7*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^3 -
 7*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^
2 + 1))
```

3.24 $\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=48

$$3a^3x+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}$$

```
[Out] 3*a^3*x+3*a^3*arctanh(sin(d*x+c))/d+a^3*sin(d*x+c)/d+a^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3876, 2717, 3855, 3852, 8}

$$\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Tan
[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
```

```
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(3a^3+a^3\cos(c+dx)+3a^3\sec(c+dx)+a^3\sec^2(c+dx)\right)\,dx\\ &= 3a^3x+a^3\int\cos(c+dx)\,dx+a^3\int\sec^2(c+dx)\,dx+\left(3a^3\right)\int\sec(c\\ &= 3a^3x+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\text{Subst}(\int 1\,dx,\\ &= 3a^3x+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 211 vs. 2(48) = 96.
time = 0.90, size = 211, normalized size = 4.40

$$\frac{1}{8}a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(3x-\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\cos(dx)\sin(c)}{d}+\frac{\cos(c)\sin(dx)}{d}+\frac{\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(3*x - (3*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]])/d + (3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d +
(Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d + Sin[(d*x)/2]/(d*(Cos[c/2] - Sin
[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(d*x)/2]/(d*(Cos[c/2] +
 Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/8
```

Maple [A]
time = 0.07, size = 55, normalized size = 1.15

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+a^3\sin(dx+c)}{d}$
default	$\frac{a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+a^3\sin(dx+c)}{d}$
risch	$3a^3x-\frac{ia^3\mathrm{e}^{i(dx+c)}}{2d}+\frac{ia^3\mathrm{e}^{-i(dx+c)}}{2d}+\frac{2ia^3}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{3a^3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{3a^3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$
norman	$\frac{3a^3x+\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-3a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3a^3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^3x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*tan(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*(d*x+c)+a^3*sin(d
*x+c))
```

Maxima [A]
time = 0.28, size = 64, normalized size = 1.33

$$\frac{6\,(dx+c)a^3+3\,a^3(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,a^3\sin\left(dx+c\right)+2\,a^3\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a^3 + 3*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) + 2*a^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 3.17, size = 91, normalized size = 1.90

$$\frac{6\,a^3dx\cos\left(dx+c\right)+3\,a^3\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,a^3\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,(a^3\cos\left(dx+c\right)+a^3)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*d*x*cos(d*x + c) + 3*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*
a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + a^3)*sin(d*
x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cos\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\cos\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x), x) + Integral(3*cos(c + d*x)*se
c(c + d*x)**2, x) + Integral(cos(c + d*x)*sec(c + d*x)**3, x) + Integral(co
s(c + d*x), x))
```

Giac [A]
time = 0.48, size = 80, normalized size = 1.67

$$\frac{3\,(dx+c)a^3+3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a^3 + 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^3*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^4 - 1))/d
```

Mupad [B]
time = 0.70, size = 57, normalized size = 1.19

$$3\,a^3\,x+\frac{6\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] 3*a^3*x + (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (4*a^3*tan(c/2 + (d*x)/2))/
(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

3.25 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=59

$$\frac{7a^3x}{2}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 7/2*a^3*x+a^3*arctanh(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*cos(d*x+c)*s
in(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3876, 2717, 2715, 8, 3855}

$$\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(3a^3+3a^3\cos(c+dx)+a^3\cos^2(c+dx)+a^3\sec(c+dx)\right)\,dx\\ &= 3a^3x+a^3\int\cos^2(c+dx)\,dx+a^3\int\sec(c+dx)\,dx+\left(3a^3\right)\int\cos(c\\ &= 3a^3x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\,\mathrm{s}}{2d}\\ &= \frac{7a^3x}{2}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\,\mathrm{s}}{2d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 81, normalized size = 1.37

$$\frac{a^3\left(14dx-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\sin(c+dx)+\sin(2(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(14*d*x - 4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + 12*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.08, size = 71, normalized size = 1.20

method	result
derivativedivides	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+3a^3\sin(dx+c)+a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+3a^3\sin(dx+c)+a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{7a^3x}{2}-\frac{3ia^3e^{i(dx+c)}}{2d}+\frac{3ia^3e^{-i(dx+c)}}{2d}+\frac{a^3\ln(e^{i(dx+c)}+i)}{d}-\frac{a^3\ln(e^{i(dx+c)}-i)}{d}+\frac{a^3\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{7a^3x}{2}+\frac{7a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{9a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-7a^3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{7a^3x\left(\tan^8\left(\right.\right.}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*(d*x+c)+3*a^3*sin(d*x+c)+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

Maxima [A]
time = 0.29, size = 74, normalized size = 1.25

$$\frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})a^3+12\,(dx+c)a^3+2\,a^3(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)})+12\,a^3\sin{(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a^3 + 2*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^3*sin(d*x + c))/d

Fricas [A]
time = 3.06, size = 65, normalized size = 1.10

$$\frac{7\,a^3dx+a^3\log{(\sin{(dx+c)}+1)}-a^3\log{(-\sin{(dx+c)}+1)}+(a^3\cos{(dx+c)}+6\,a^3)\sin{(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*(7*a^3*d*x + a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) + (a^3*cos(d*x + c) + 6*a^3)*sin(d*x + c))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cos^2{(c+dx)}\sec{(c+dx)}\,dx+\int 3\cos^2{(c+dx)}\sec^2{(c+dx)}\,dx+\int\cos^2{(c+dx)}\sec^3{(c+dx)}\,dx+\int\cos^2{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3,x)

[Out] a**3*(Integral(3*cos(c + d*x)**2*sec(c + d*x), x) + Integral(3*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2*sec(c + d*x)**3, x) + Integral(cos(c + d*x)**2, x))

Giac [A]
time = 0.48, size = 100, normalized size = 1.69

$$\frac{7\,(dx+c)a^3+2\,a^3\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+1\right|\right)}-2\,a^3\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-1\right|\right)}+\frac{2\left(5\,a^3\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}^3+7\,a^3\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{\left(\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(7*(d*x + c)*a^3 + 2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 0.73, size = 88, normalized size = 1.49

$$\frac{7\,a^3\,x}{2}+\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{5\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (7*a^3*x)/2 + (2*a^3*atanh(tan(c/2 + (d*x)/2)))/d + (5*a^3*tan(c/2 + (d*x)/
2)^3 + 7*a^3*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*
x)/2)^4 + 1))
```

3.26 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=63

$$\frac{5a^3x}{2}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}$$

```
[Out] 5/2*a^3*x+4*a^3*sin(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d-1/3*a^3*sin(d*
x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3876, 2717, 2715, 8, 2713}

$$-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2
*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3+3a^3\cos(c+dx)+3a^3\cos^2(c+dx)+a^3\cos^3(c+dx)\right)\,dx\\ &= a^3x+a^3\int \cos^3(c+dx)\,dx+\left(3a^3\right)\int \cos(c+dx)\,dx+\left(3a^3\right)\int \cos\\ &= a^3x+\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^3\right)\int 1\\ &= \frac{5a^3x}{2}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{a^3\sin^3(c+}{3d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 44, normalized size = 0.70

$$\frac{a^3(30c+30dx+45\sin(c+dx)+9\sin(2(c+dx))+\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
[Out] (a^3*(30*c + 30*d*x + 45*Sin[c + d*x] + 9*Sin[2*(c + d*x)] + Sin[3*(c + d*x
)]))/(12*d)
```

Maple [A]
time = 0.10, size = 74, normalized size = 1.17

method	result
risch	$\frac{5a^3x}{2}+\frac{15a^3\sin(dx+c)}{4d}+\frac{a^3\sin(3dx+3c)}{12d}+\frac{3a^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3\sin(dx+c)+a^3(dx+c)}{d}$
default	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3\sin(dx+c)+a^3(dx+c)}{d}$
norman	$\frac{\frac{5a^3x}{2}+\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{26a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{32a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{10a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{5a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^3x\left(\tan^2\right.}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+3*a^3*sin(d*x+c)+a^3*(d*x+c))
```

Maxima [A]
time = 0.29, size = 71, normalized size = 1.13

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{3}-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{3}-12\left(dx+c\right)a^{3}-36\,a^{3}\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^3 - 12*(d*x + c)*a^3 - 36*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 3.42, size = 50, normalized size = 0.79

$$\frac{15\,a^{3}dx+\left(2\,a^{3}\cos\left(dx+c\right)^{2}+9\,a^{3}\cos\left(dx+c\right)+22\,a^{3}\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(15*a^3*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 22*a^3)*sin(
d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\cos^{3}\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cos^{3}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int\cos^{3}\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int\cos^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)**3*sec(c + d*x), x) + Integral(3*cos(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(cos(c + d*x)**3, x))
```

Giac [A]
time = 0.46, size = 80, normalized size = 1.27

$$\frac{15\left(dx+c\right)a^{3}+\frac{2\left(15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+40\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+33\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*a^3*tan(1/2*d
*x + 1/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3
)/d
```

Mupad [B]
time = 0.67, size = 63, normalized size = 1.00

$$\frac{5\,a^3\,x}{2}+\frac{11\,a^3\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{3\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (5*a^3*x)/2 + (11*a^3*sin(c + d*x))/(3*d) + (a^3*cos(c + d*x)^2*sin(c + d*x
))/(3*d) + (3*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.27 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=85

$$\frac{15a^3x}{8}+\frac{4a^3\sin(c+dx)}{d}+\frac{15a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a^3\sin^3(c+dx)}{d}$$

```
[Out] 15/8*a^3*x+4*a^3*sin(d*x+c)/d+15/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*cos(
d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3876, 2717, 2715, 8, 2713}

$$-\frac{a^3\sin^3(c+dx)}{d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (15*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cos(c+dx)+3a^3\cos^2(c+dx)+3a^3\cos^3(c+dx)+a^3\cos^4(c\right.$$
$$= a^3\int\cos(c+dx)\,dx + a^3\int\cos^4(c+dx)\,dx + \left(3a^3\right)\int\cos^2(c+dx)$$
$$= \frac{a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^3\cos^3(c+dx)\sin(c}{4d}$$
$$= \frac{3a^3x}{2} + \frac{4a^3\sin(c+dx)}{d} + \frac{15a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^3\cos^3(c}{}$$
$$= \frac{15a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{15a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^3\cos^3(c}{}$$

Mathematica [A]
time = 0.13, size = 51, normalized size = 0.60

$$\frac{a^3(60dx+104\sin(c+dx)+32\sin(2(c+dx))+8\sin(3(c+dx))+\sin(4(c+dx)))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
[Out] (a^3*(60*d*x + 104*Sin[c + d*x] + 32*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]
+ Sin[4*(c + d*x)]))/(32*d)
```

Maple [A]
time = 0.11, size = 100, normalized size = 1.18

method	result
risch	$\frac{15a^3x}{8}+\frac{13a^3\sin(dx+c)}{4d}+\frac{a^3\sin(4dx+4c)}{32d}+\frac{a^3\sin(3dx+3c)}{4d}+\frac{a^3\sin(2dx+2c)}{d}$
derivativedivides	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+}{d}$
default	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+}{d}$

norman	$\frac{\frac{15a^3x}{8}+\frac{49a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{25a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{21a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{11a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{25a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{15a^3(}{}}{(1-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(
2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+
a^3*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 94, normalized size = 1.11

$$-\frac{32\,(\sin(dx+c)^3-3\,\sin(dx+c))a^3-(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3-24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3-32\,a^3\sin(dx+c)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^
3 - 32*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 3.49, size = 63, normalized size = 0.74

$$\frac{15\,a^3dx+\left(2\,a^3\cos(dx+c)^3+8\,a^3\cos(dx+c)^2+15\,a^3\cos(dx+c)+24\,a^3\right)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(15*a^3*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^3*cos(d*x + c)^2 + 15*a^3*cos
(d*x + c) + 24*a^3)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cos^4(c+dx)\sec(c+dx)\,dx+\int 3\cos^4(c+dx)\sec^2(c+dx)\,dx+\int \cos^4(c+dx)\sec^3(c+dx)\,dx+\int \cos^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)**4*sec(c + d*x), x) + Integral(3*cos(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(cos(c + d*x)**4, x))
```

Giac [A]
time = 0.49, size = 96, normalized size = 1.13

$$\frac{15\,(dx+c)a^3+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+73\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+49\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 + 55*a^3*tan(1/2*d
*x + 1/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 + 49*a^3*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 4.17, size = 89, normalized size = 1.05

$$\frac{15\,a^3\,x}{8}+\frac{\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{55\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{49\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (15*a^3*x)/8 + ((73*a^3*tan(c/2 + (d*x)/2)^3)/4 + (55*a^3*tan(c/2 + (d*x)/2
)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 + (49*a^3*tan(c/2 + (d*x)/2))/4)/(
d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.28 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=105

$$\frac{13a^3x}{8}+\frac{4a^3\sin(c+dx)}{d}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}+a^3$$

```
[Out] 13/8*a^3*x+4*a^3*sin(d*x+c)/d+13/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/4*a^3*cos(
d*x+c)^3*sin(d*x+c)/d-5/3*a^3*sin(d*x+c)^3/d+1/5*a^3*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3876, 2715, 8, 2713}

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{13a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3)/
(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
```

```
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cos^2(c+dx)+3a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+a^3\cos^5(\right.$$
$$= a^3\int \cos^2(c+dx)\,dx + a^3\int \cos^5(c+dx)\,dx + \left(3a^3\right)\int \cos^3(c+dx$$
$$= \frac{a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{2}a^3\int$$
$$= \frac{a^3x}{2} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c}{}$$
$$= \frac{13a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(}{}$$

Mathematica [A]
time = 0.16, size = 63, normalized size = 0.60

$$\frac{a^3(780dx+1380\sin(c+dx)+480\sin(2(c+dx))+170\sin(3(c+dx))+45\sin(4(c+dx))+6\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
[Out] (a^3*(780*d*x + 1380*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 170*Sin[3*(c + d
*x)] + 45*Sin[4*(c + d*x)] + 6*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A]
time = 0.13, size = 121, normalized size = 1.15

method	result
risch	$\frac{13a^3x}{8}+\frac{23a^3\sin(dx+c)}{8d}+\frac{a^3\sin(5dx+5c)}{80d}+\frac{3a^3\sin(4dx+4c)}{32d}+\frac{17a^3\sin(3dx+3c)}{48d}+\frac{a^3\sin(2dx+2c)}{d}$
derivativedivides	$\frac{\frac{a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^3\left(2+\cos^2(dx+c)\right)}{d}$
default	$\frac{\frac{a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^3\left(2+\cos^2(dx+c)\right)}{d}$

norman	$\frac{\frac{13a^3x}{8}+\frac{51a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{10a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{77a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}-\frac{272a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{13a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{26a^3}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(2+cos(d*x+c)^2)*si
n(d*x+c)+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 117, normalized size = 1.11

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3+45\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)a^3+120\left(2dx+2c+\sin(2dx+2c)\right)a^3}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 48
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A]
time = 3.32, size = 76, normalized size = 0.72

$$\frac{195a^3dx+\left(24a^3\cos(dx+c)^4+90a^3\cos(dx+c)^3+152a^3\cos(dx+c)^2+195a^3\cos(dx+c)+304a^3\right)\sin(dx+c)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(195*a^3*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^3*cos(d*x + c)^3 + 152*a
^3*cos(d*x + c)^2 + 195*a^3*cos(d*x + c) + 304*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 112, normalized size = 1.07

$$\frac{195\,(dx+c)a^3 + \frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1330\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+765\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(195*(d*x + c)*a^3 + 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*a^3*tan(
1/2*d*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*a^3*tan(1/2*d*x
 + 1/2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)
/d
```

Mupad [B]
time = 4.38, size = 105, normalized size = 1.00

$$\frac{13\,a^3\,x}{8}+\frac{\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{91\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{416\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{133\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{51\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (13*a^3*x)/8 + ((133*a^3*tan(c/2 + (d*x)/2)^3)/6 + (416*a^3*tan(c/2 + (d*x)
/2)^5)/15 + (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (13*a^3*tan(c/2 + (d*x)/2)^9)
/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.29 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=129

$$\frac{23a^3x}{16}+\frac{4a^3\sin(c+dx)}{d}+\frac{23a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{23a^3\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^3\cos^5(c+dx)\sin(c}{6d}$$

```
[Out] 23/16*a^3*x+4*a^3*sin(d*x+c)/d+23/16*a^3*cos(d*x+c)*sin(d*x+c)/d+23/24*a^3*
cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-7/3*a^3*sin(d*x
+c)^3/d+3/5*a^3*sin(d*x+c)^5/d
```

Rubi [A]

time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3876, 2713, 2715, 8}

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{23a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{23a^3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^5
)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
```

```
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+3a^3\cos^5(c+dx)+a^3\cos^6(\right.$$
$$= a^3\int \cos^3(c+dx)\,dx + a^3\int \cos^6(c+dx)\,dx + \left(3a^3\right)\int \cos^4(c+dx$$
$$= \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}\left(5a^3\right.$$
$$= \frac{4a^3\sin(c+dx)}{d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{23a^3\cos^3(c+dx)\,\mathrm{s}}{24d}$$
$$= \frac{9a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{23a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{23a^3\cos^3(}{}$$
$$= \frac{23a^3x}{16} + \frac{4a^3\sin(c+dx)}{d} + \frac{23a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{23a^3\cos}{}$$

Mathematica [A]
time = 0.21, size = 73, normalized size = 0.57

$$\frac{a^3(1380dx+2520\sin(c+dx)+945\sin(2(c+dx))+380\sin(3(c+dx))+135\sin(4(c+dx))+36\sin(5(c+dx))+5\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1380*d*x + 2520*Sin[c + d*x] + 945*Sin[2*(c + d*x)] + 380*Sin[3*(c +
d*x)] + 135*Sin[4*(c + d*x)] + 36*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(
960*d)
```

Maple [A]
time = 0.16, size = 143, normalized size = 1.11

method	result
risch	$\frac{23a^3x}{16}+\frac{21a^3\sin(dx+c)}{8d}+\frac{a^3\sin(6dx+6c)}{192d}+\frac{3a^3\sin(5dx+5c)}{80d}+\frac{9a^3\sin(4dx+4c)}{64d}+\frac{19a^3\sin(3dx+3c)}{48d}+\frac{63a^3\,\mathrm{s}}{}$
derivativedivides	$\frac{a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3a}{d}$

default	$\frac{a^{3}\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^{3}\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+}{d}$
norman	$\frac{23a^{3}x}{16}+\frac{105a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{a^{3}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{353a^{3}\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}-\frac{1303a^{3}\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}-\frac{1339a^{3}\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{120d}+\frac{98}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^3*(2+co
s(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 143, normalized size = 1.11

$$\frac{192\left(3\sin(dx+c)^{5}-10\sin(dx+c)^{3}+15\sin(dx+c)\right)a^{3}-5\left(4\sin(2dx+2c)^{3}-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)a^{3}-320\left(\sin(dx+c)^{3}-3\sin(dx+c)\right)a^{3}+90\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)a^{3}}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 90*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A]
time = 3.64, size = 89, normalized size = 0.69

$$\frac{345\,a^{3}dx+\left(40\,a^{3}\cos(dx+c)^{5}+144\,a^{3}\cos(dx+c)^{4}+230\,a^{3}\cos(dx+c)^{3}+272\,a^{3}\cos(dx+c)^{2}+345\,a^{3}\cos(dx+c)+544\,a^{3}\right)\sin(dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(345*a^3*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^3*cos(d*x + c)^4 + 230*
a^3*cos(d*x + c)^3 + 272*a^3*cos(d*x + c)^2 + 345*a^3*cos(d*x + c) + 544*a^
3)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.53, size = 128, normalized size = 0.99

$$\frac{345\,(dx+c)a^3+\frac{2\left(345\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4554\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+5814\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3165\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1575\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(345*(d*x + c)*a^3 + 2*(345*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*a^3*ta
n(1/2*d*x + 1/2*c)^9 + 4554*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*a^3*tan(1/2*d
*x + 1/2*c)^5 + 3165*a^3*tan(1/2*d*x + 1/2*c)^3 + 1575*a^3*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 3.49, size = 121, normalized size = 0.94

$$\frac{23\,a^3\,x}{16}+\frac{\frac{23\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{391\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{759\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{969\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{211\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8}+\frac{105\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (23*a^3*x)/16 + ((211*a^3*tan(c/2 + (d*x)/2)^3)/8 + (969*a^3*tan(c/2 + (d*x
)/2)^5)/20 + (759*a^3*tan(c/2 + (d*x)/2)^7)/20 + (391*a^3*tan(c/2 + (d*x)/2
)^9)/24 + (23*a^3*tan(c/2 + (d*x)/2)^11)/8 + (105*a^3*tan(c/2 + (d*x)/2))/8
)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.30 $\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=136

$$\frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\sec(c+dx)\tan(c+dx)}{16d}+\frac{41a^4\sec^3(c+dx)\tan(c+dx)}{24d}+$$

```
[Out] 49/16*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+49/16*a^4*sec(d*x+c)*tan
(d*x+c)/d+41/24*a^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*x+
c)/d+4*a^4*tan(d*x+c)^3/d+4/5*a^4*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.14, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3876, 3853, 3855, 3852}

$$\frac{4a^4\tan^5(c+dx)}{5d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{41a^4\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{49a^4\tan(c+dx)\sec(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*Se
c[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(24*
d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d + (
4*a^4*Tan[c + d*x]^5)/(5*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
```

```
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec^3(c+dx)+4a^4\sec^4(c+dx)+6a^4\sec^5(c+dx)+4a^4\sec^6\right.$$
$$= a^4\int \sec^3(c+dx)\,dx + a^4\int \sec^7(c+dx)\,dx + \left(4a^4\right)\int \sec^4(c+dx$$
$$= \frac{a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^4\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^4\sec^5}{}$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{11a^4\sec(c+dx)\tan(c}{4d}$$
$$= \frac{11a^4\tanh^{-1}(\sin(c+dx))}{4d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan}{16d}$$
$$= \frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan}{16d}$$

Mathematica [A]
time = 0.84, size = 211, normalized size = 1.55

$$-\frac{a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(23520\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(-11520\sin(c)+3750\sin(dx)+3750\sin(2c+dx)+15360\sin(c+2dx)-1920\sin(3c+2dx)+3845\sin(2c+3dx)+3845\sin(4c+3dx)+6912\sin(3c+4dx)+735\sin(4c+5dx)+735\sin(6c+5dx)+1152\sin(5c+6dx))\right)}{122880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/122880*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(2352
0*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-11520*Sin[c] + 3750*Sin[d*x] + 3750*
Sin[2*c + d*x] + 15360*Sin[c + 2*d*x] - 1920*Sin[3*c + 2*d*x] + 3845*Sin[2*
c + 3*d*x] + 3845*Sin[4*c + 3*d*x] + 6912*Sin[3*c + 4*d*x] + 735*Sin[4*c +
5*d*x] + 735*Sin[6*c + 5*d*x] + 1152*Sin[5*c + 6*d*x]))/d
```

Maple [A]
time = 0.13, size = 204, normalized size = 1.50

method	result
norman	$\frac{\frac{207a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{1471a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{1967a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}-\frac{1617a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}+\frac{833a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{49a^4\left(\text{ta}\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}$

risch	$-\frac{i a^4 (735\,e^{11i(dx+c)}+3845\,e^{9i(dx+c)}-1920\,e^{8i(dx+c)}+3750\,e^{7i(dx+c)}-11520\,e^{6i(dx+c)}-3750\,e^{5i(dx+c)}-15360\,e^{4i(dx+c}}{120d\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$a^4\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}\right.$
default	$a^4\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)
+5/16*ln(sec(d*x+c)+tan(d*x+c)))-4*a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x
+c)^2)*tan(d*x+c)+6*a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8
*ln(sec(d*x+c)+tan(d*x+c)))-4*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^4*(1
/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(126) = 252$.
time = 0.28, size = 270, normalized size = 1.99

$$\frac{128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^4+640\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4-5a^4\left(\frac{2\left(15\sin(dx+c)^5-40\sin(dx+c)^3+33\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)-180a^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-120a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 6
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 5*a^4*(2*(15*sin(d*x + c)^5 - 40
*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*s
in(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) -
 180*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*a^4
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)))/d
```

Fricas [A]
time = 3.29, size = 137, normalized size = 1.01

$$\frac{735\,a^4\cos(dx+c)^6\log(\sin(dx+c)+1)-735\,a^4\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\left(1152\,a^4\cos(dx+c)^5+735\,a^4\cos(dx+c)^4+576\,a^4\cos(dx+c)^3+410\,a^4\cos(dx+c)^2+192\,a^4\cos(dx+c)+40\,a^4\right)\sin(dx+c)}{480\,d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(735*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 735*a^4*cos(d*x + c)^
6*log(-sin(d*x + c) + 1) + 2*(1152*a^4*cos(d*x + c)^5 + 735*a^4*cos(d*x + c
```

```
)^4 + 576*a^4*cos(d*x + c)^3 + 410*a^4*cos(d*x + c)^2 + 192*a^4*cos(d*x + c
) + 40*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{4}\left(\int \sec^{3}{\left(c+dx\right)}\,dx+\int 4\sec^{4}{\left(c+dx\right)}\,dx+\int 6\sec^{5}{\left(c+dx\right)}\,dx+\int 4\sec^{6}{\left(c+dx\right)}\,dx+\int \sec^{7}{\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Integ
ral(6*sec(c + d*x)**5, x) + Integral(4*sec(c + d*x)**6, x) + Integral(sec(c
 + d*x)**7, x))
```

Giac [A]
time = 0.47, size = 154, normalized size = 1.13

$$\frac{735\,a^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-735\,a^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(735\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-4165\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+9702\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-11802\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7355\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3105\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(735*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 735*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*a^4*tan(1/2
*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*a^4*tan(1/2*d*x +
 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*a^4*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 4.65, size = 199, normalized size = 1.46

$$\frac{49\,a^{4}\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\frac{49\,a^{4}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{833\,a^{4}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{1617\,a^{4}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}-\frac{1967\,a^{4}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{1471\,a^{4}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}-\frac{207\,a^{4}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/cos(c + d*x)^3,x)
```

```
[Out] (49*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((1471*a^4*tan(c/2 + (d*x)/2)^3)
/24 - (1967*a^4*tan(c/2 + (d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/
20 - (833*a^4*tan(c/2 + (d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 -
 (207*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 +
(d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.31 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=111

$$\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^4\sec^3(c+dx)\tan(c+dx)}{d}+\frac{8a^4\,\mathrm{t}}{}$$

```
[Out] 7/2*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*x
+c)/d+a^4*sec(d*x+c)^3*tan(d*x+c)/d+8/3*a^4*tan(d*x+c)^3/d+1/5*a^4*tan(d*x+
c)^5/d
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3876, 3852, 8, 3853, 3855}

$$\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+\frac{7a^4\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^4*
Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx &= \int \left(a^4\sec^2(c+dx)+4a^4\sec^3(c+dx)+6a^4\sec^4(c+dx)+4a^4\sec^5\right.\\ &= a^4\int \sec^2(c+dx)\,dx+a^4\int \sec^6(c+dx)\,dx+\left(4a^4\right)\int \sec^3(c+dx\\ &= \frac{2a^4\sec(c+dx)\tan(c+dx)}{d}+\frac{a^4\sec^3(c+dx)\tan(c+dx)}{d}+\left(2a^4\right)\\ &= \frac{2a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\sec(c+dx)\tan(c}{2d}\\ &= \frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\sec(c+dx)\tan(c}{2d}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 498 vs. $2(111) = 222$.
time = 1.54, size = 498, normalized size = 4.49

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/960*(a^4*Sec[c]*Sec[c + d*x]^5*(525*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
+ 105*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1050*Cos[
d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]]) + 1050*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 525*Cos[2*c + 3*d*x]*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] - 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 105*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] - 2360*Sin[d*x] + 960*Sin[2*c + d*x] - 660*Sin[c + 2*d*x] - 660*Sin[3*
c + 2*d*x] - 1600*Sin[2*c + 3*d*x] + 60*Sin[4*c + 3*d*x] - 210*Sin[3*c + 4*
d*x] - 210*Sin[5*c + 4*d*x] - 332*Sin[4*c + 5*d*x]))/d
```

Maple [A]

time = 0.09, size = 157, normalized size = 1.41

method	result
norman	$\frac{-\frac{25a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{158a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{896a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{98a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{7a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{7a^4\ln\left(\mathrm{t}\right.}{}$
risch	$-\frac{ia^4\left(105\,\mathrm{e}^{9i(dx+c)}-30\,\mathrm{e}^{8i(dx+c)}+330\,\mathrm{e}^{7i(dx+c)}-480\,\mathrm{e}^{6i(dx+c)}-1180\,\mathrm{e}^{4i(dx+c)}-330\,\mathrm{e}^{3i(dx+c)}-800\,\mathrm{e}^{2i(dx+c)}-105\,\mathrm{e}^{i(d}\right.}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{-a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+4a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+}{}\right.}{}$
default	$\frac{-a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+4a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+4*a^4*(-(-1
/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-6
*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/
2*ln(sec(d*x+c)+tan(d*x+c)))+a^4*tan(d*x+c))
```

Maxima [A]

time = 0.29, size = 190, normalized size = 1.71

$$\frac{4\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a^4+120\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^4-15\,a^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+60\,a^4\tan\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 15*a^4*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 60*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c))/d
```

Fricas [A]

time = 2.42, size = 124, normalized size = 1.12

$$\frac{105\,a^4\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-105\,a^4\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(166\,a^4\cos\left(dx+c\right)^4+105\,a^4\cos\left(dx+c\right)^3+68\,a^4\cos\left(dx+c\right)^2+30\,a^4\cos\left(dx+c\right)+6\,a^4\right)\sin\left(dx+c\right)}{60\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/60*(105*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^5
*log(-sin(d*x + c) + 1) + 2*(166*a^4*cos(d*x + c)^4 + 105*a^4*cos(d*x + c)^
3 + 68*a^4*cos(d*x + c)^2 + 30*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*c
os(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \sec^2(c+dx)\,dx+\int 4\sec^3(c+dx)\,dx+\int 6\sec^4(c+dx)\,dx+\int 4\sec^5(c+dx)\,dx+\int \sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**2, x) + Integral(4*sec(c + d*x)**3, x) + Integ
ral(6*sec(c + d*x)**4, x) + Integral(4*sec(c + d*x)**5, x) + Integral(sec(c
 + d*x)**6, x))
```

Giac [A]
time = 0.49, size = 138, normalized size = 1.24

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+896\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-790\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+375\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 - 490*a^4*tan(1/2*d*
x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*a^4*tan(1/2*d*x + 1/2*c
)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 5.47, size = 170, normalized size = 1.53

$$\frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{7\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-\frac{98\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{896\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{158\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/cos(c + d*x)^2,x)
```

```
[Out] (7*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((896*a^4*tan(c/2 + (d*x)/2)^5)/15 -
(158*a^4*tan(c/2 + (d*x)/2)^3)/3 - (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*
tan(c/2 + (d*x)/2)^9 + 25*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/2)^
2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2
)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.32 $\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=96

$$\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{8a^4\tan(c+dx)}{d}+\frac{27a^4\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{4a}{}$$

```
[Out] 35/8*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x+c)*tan(d
*x+c)/d+1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+4/3*a^4*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3876, 3855, 3852, 8, 3853}

$$\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
4*a^4*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec(c+dx)+4a^4\sec^2(c+dx)+6a^4\sec^3(c+dx)+4a^4\sec^4(c\right.$$

$$= a^4\int \sec(c+dx)\,dx + a^4\int \sec^5(c+dx)\,dx + \left(4a^4\right)\int \sec^2(c+dx)\,d$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^4\sec(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^3(c+}{}$$

$$= \frac{4a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c}{8d}$$

$$= \frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(}{8d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 877 vs. 2(96) = 192.
time = 6.43, size = 877, normalized size = 9.14

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-35*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 +
(d*x)/2]^8*(a + a*Sec[c + d*x])^4)/(128*d) + (35*Cos[c + d*x]^4*Log[Cos[c/2
 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])
^4)/(128*d) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/
(256*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^4*Sec[c
/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(24*d*(Cos[c/2] - Sin[
c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/
2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(
Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (5*Cos[
c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(12*d*
(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c +
 d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/(256*d*(Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a
*Sec[c + d*x])^4*Sin[(d*x)/2])/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)
```

```
/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*
Sec[c + d*x])^4*(-97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(Cos[c/2] + Sin[c/2])*
(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (5*Cos[c + d*x]^4*Sec[c/2 +
(d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])
*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A]
time = 0.08, size = 142, normalized size = 1.48

method	result
norman	$\frac{\frac{93a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{511a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{385a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{35a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{35a^4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+$
derivativedivides	$\frac{a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+6}{d}$
default	$\frac{a^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+6}{d}$
risch	$-\frac{ia^4\left(81\,\mathrm{e}^{7i(dx+c)}-96\,\mathrm{e}^{6i(dx+c)}+105\,\mathrm{e}^{5i(dx+c)}-480\,\mathrm{e}^{4i(dx+c)}-105\,\mathrm{e}^{3i(dx+c)}-544\,\mathrm{e}^{2i(dx+c)}-81\,\mathrm{e}^{i(dx+c)}-160\right)}{12d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))-4*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*a^4*(1/2*sec(d*x+c)
*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*a^4*tan(d*x+c)+a^4*ln(sec(d*x+
c)+tan(d*x+c)))
```

Maxima [A]
time = 0.29, size = 175, normalized size = 1.82

$$\frac{64\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^4-3a^4\left(\frac{2\left(3\sin\left(dx+c\right)^3-5\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\sin\left(dx+c\right)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-72a^4\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+48a^4\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+192a^4\tan\left(dx+c\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 3*a^4*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^4*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4*log(sec(d
*x + c) + tan(d*x + c)) + 192*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 2.78, size = 111, normalized size = 1.16

$$\frac{105a^4\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-105a^4\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(160a^4\cos\left(dx+c\right)^3+81a^4\cos\left(dx+c\right)^2+32a^4\cos\left(dx+c\right)+6a^4\right)\sin\left(dx+c\right)}{48d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(105*a^4*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^4
*log(-sin(d*x + c) + 1) + 2*(160*a^4*cos(d*x + c)^3 + 81*a^4*cos(d*x + c)^2
 + 32*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \sec(c+dx)\,dx + \int 4\sec^2(c+dx)\,dx + \int 6\sec^3(c+dx)\,dx + \int 4\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sec(c + d*x), x) + Integral(4*sec(c + d*x)**2, x) + Integral
(6*sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Integral(sec(c +
d*x)**5, x))
```

Giac [A]
time = 0.50, size = 122, normalized size = 1.27

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+511\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-279\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*a^4*tan(1/2*d*
x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*a^4*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 3.99, size = 141, normalized size = 1.47

$$\frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{35\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{4}-\frac{385\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{12}+\frac{511\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{12}-\frac{93\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/cos(c + d*x),x)
```

```
[Out] (35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((511*a^4*tan(c/2 + (d*x)/2)^3)/
12 - (385*a^4*tan(c/2 + (d*x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 -
(93*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*
x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.33 $\int (a + a\sec(c + dx))^4 \, dx$

Optimal. Leaf size=91

$$a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4\tan(c+dx)}{d}+\frac{(a^2+a^2\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{4(a^4+a^4\sec(c+dx))}{3d}$$

```
[Out] a^4*x+6*a^4*arctanh(sin(d*x+c))/d+5*a^4*tan(d*x+c)/d+1/3*(a^2+a^2*sec(d*x+c
))^2*tan(d*x+c)/d+4/3*(a^4+a^4*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3860, 4002, 3999, 3852, 8, 3855}

$$\frac{5a^4\tan(c+dx)}{d}+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4\tan(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{3d}+a^4x+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4,x]
```

```
[Out] a^4*x + (6*a^4*ArcTanh[Sin[c + d*x]])/d + (5*a^4*Tan[c + d*x])/d + ((a^2 +
a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + (4*(a^4 + a^4*Sec[c + d*x])*Tan[c
 + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^4\,dx = \frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{1}{3}a\int (a+a\sec(c+dx))^2(3a+8a\sec(c+\ldots$$

$$=\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4\left(a^4+a^4\sec(c+dx)\right)\tan(c+dx)}{3d}+\frac{1}{6}a\int\ldots$$

$$=a^4x+\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4\left(a^4+a^4\sec(c+dx)\right)\tan(c+dx)}{3d}+\ldots$$

$$=a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4(a^4+a^4\ldots}{\ldots}$$

$$=a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4\tan(c+dx)}{d}+\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c\ldots}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 773 vs. 2(91) = 182.
time = 6.27, size = 773, normalized size = 8.49

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (x*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/16 - (3*Cos[
c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^
8*(a + a*Sec[c + d*x])^4)/(8*d) + (3*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2]
```

```
+ Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/(8*d) +
(Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(
96*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (
Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(13*Cos[c/2] - 1
1*Sin[c/2]))/(192*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (
d*x)/2])^2) + (5*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4
*Sin[(d*x)/2])/(12*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
(d*x)/2])) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Si
n[(d*x)/2])/(96*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*
x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(-1
3*Cos[c/2] - 11*Sin[c/2]))/(192*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2])^2) + (5*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*S
ec[c + d*x])^4*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2]))
```

Maple [A]

time = 0.06, size = 104, normalized size = 1.14

method	result
derivativedivides	$\frac{-a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^4\tan(dx+c)+4a^4\ln(\sec(}{d}$
default	$\frac{-a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^4\tan(dx+c)+4a^4\ln(\sec(}{d}$
risch	$a^4x-\frac{4ia^4\left(3\,\mathrm{e}^{5i(dx+c)}-9\,\mathrm{e}^{4i(dx+c)}-21\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}-10\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{6a^4\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{6a^4\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$
norman	$\frac{a^4x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^4x-\frac{18a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{76a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{10a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+3a^4x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3a^4x\left(}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a^4*(1/2*sec(d*x+c)*tan(d*x+
c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+6*a^4*tan(d*x+c)+4*a^4*ln(sec(d*x+c)+tan(
d*x+c))+a^4*(d*x+c))
```

Maxima [A]

time = 0.30, size = 116, normalized size = 1.27

$$a^4x+\frac{\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^4}{3\,d}-\frac{a^4\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{d}+\frac{4\,a^4\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{6\,a^4\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

[Out] a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d + 4*a^4*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^4*tan(d*x + c)/d

Fricas [A]
time = 3.35, size = 110, normalized size = 1.21

$$\frac{3\,a^4dx\cos\left(dx+c\right)^3+9\,a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-9\,a^4\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(20\,a^4\cos\left(dx+c\right)^2+6\,a^4\cos\left(dx+c\right)+a^4\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/3*(3*a^4*d*x*cos(d*x + c)^3 + 9*a^4*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 9*a^4*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (20*a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 1\,dx+\int 4\sec\left(c+dx\right)dx+\int 6\sec^2\left(c+dx\right)dx+\int 4\sec^3\left(c+dx\right)dx+\int \sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**4,x)

[Out] a**4*(Integral(1, x) + Integral(4*sec(c + d*x), x) + Integral(6*sec(c + d*x)**2, x) + Integral(4*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))

Giac [A]
time = 0.44, size = 116, normalized size = 1.27

$$\frac{3\left(dx+c\right)a^4+18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-38\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)*a^4 + 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 - 38*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d

Mupad [B]
time = 0.88, size = 117, normalized size = 1.29

$$a^4\,x+\frac{12\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{10\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{76\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+18\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4,x)
```

```
[Out] a^4*x + (12*a^4*atanh(tan(c/2 + (d*x)/2)))/d - (10*a^4*tan(c/2 + (d*x)/2)^5
 - (76*a^4*tan(c/2 + (d*x)/2)^3)/3 + 18*a^4*tan(c/2 + (d*x)/2))/(d*(3*tan(c
/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.34 $\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=73

$$4a^4x+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{a^4\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 4*a^4*x+13/2*a^4*arctanh(sin(d*x+c))/d+a^4*sin(d*x+c)/d+4*a^4*tan(d*x+c)/d+
1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.316$, Rules used = {3876, 2717, 3855, 3852, 8, 3853}

$$\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] 4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4*
a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(4a^4 + a^4\cos(c+dx) + 6a^4\sec(c+dx) + 4a^4\sec^2(c+dx) + a^4 \text{s}\ldots\right.$$

$$= 4a^4x + a^4\int \cos(c+dx)\,dx + a^4\int \sec^3(c+dx)\,dx + \left(4a^4\right)\int \sec^2\ldots$$

$$= 4a^4x + \frac{6a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^4\sin(c+dx)}{d} + \frac{a^4\sec(c+dx)\,\text{t}\ldots}{2d}$$

$$= 4a^4x + \frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^4\sin(c+dx)}{d} + \frac{4a^4\tan(c+d\ldots}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 272 vs. 2(73) = 146.
time = 1.39, size = 272, normalized size = 3.73

$$\frac{1}{64}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(16x - \frac{26\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{26\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{4\cos(dx)\sin(c)}{d} + \frac{4\cos(c)\sin(dx)}{d} + \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(16*x - (26*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (26*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (4*Cos[d*x]*Sin[c])/d + (4*Cos[c]*Sin[d*x])/d + 1/(d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))))/64
```

Maple [A]
time = 0.09, size = 91, normalized size = 1.25

method	result

derivativedivides	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^4\tan(dx+c)+6a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+a^4\sin(d}{d}$
default	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^4\tan(dx+c)+6a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+a^4\sin(d}{d}$
risch	$4a^4x-\frac{ia^4e^{i(dx+c)}}{2d}+\frac{ia^4e^{-i(dx+c)}}{2d}-\frac{ia^4(e^{3i(dx+c)}-8e^{2i(dx+c)}-e^{i(dx+c)}-8)}{d(e^{2i(dx+c)}+1)^2}+\frac{13a^4\ln(e^{i(dx+c)}+i)}{2d}-\frac{13a^4\ln}{}$
norman	$\frac{-4a^4x-\frac{11a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{13a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+8a^4x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8a^4x\left(\mathrm{ta}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*a^4*ta
n(d*x+c)+6*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^4*(d*x+c)+a^4*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 110, normalized size = 1.51

$$\frac{16\,(dx+c)a^4-a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^4(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+4\,a^4\sin\left(dx+c\right)+16\,a^4\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a^4 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^4*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 3.12, size = 111, normalized size = 1.52

$$\frac{16\,a^4dx\cos\left(dx+c\right)^2+13\,a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-13\,a^4\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,a^4\cos\left(dx+c\right)^2+8\,a^4\cos\left(dx+c\right)+a^4\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a^4*d*x*cos(d*x + c)^2 + 13*a^4*cos(d*x + c)^2*log(sin(d*x + c) + 1
) - 13*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^4*cos(d*x + c)^2
+ 8*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 6\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 4\cos\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\cos\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int\cos\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)*sec(c + d*x), x) + Integral(6*cos(c + d*x)*se
c(c + d*x)**2, x) + Integral(4*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(
cos(c + d*x)*sec(c + d*x)**4, x) + Integral(cos(c + d*x), x))
```

Giac [A]
time = 0.49, size = 129, normalized size = 1.77

$$\frac{8\,(dx+c)a^4 + 13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} - \frac{2\left(7\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a^4 + 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
 + 1/2*c)^2 + 1) - 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 0.91, size = 115, normalized size = 1.58

$$4\,a^4\,x + \frac{13\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{5\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 11\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^4,x)
```

```
[Out] 4*a^4*x + (13*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)^
3 + 5*a^4*tan(c/2 + (d*x)/2)^5 - 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (
d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.35 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=73

$$\frac{13a^4x}{2}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^4\tan(c+dx)}{d}$$

```
[Out] 13/2*a^4*x+4*a^4*arctanh(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+1/2*a^4*cos(d*x+c
)*sin(d*x+c)/d+a^4*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3876, 2717, 2715, 8, 3855, 3852}

$$\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (
a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx &= \int \left(6a^4+4a^4\cos(c+dx)+a^4\cos^2(c+dx)+4a^4\sec(c+dx)+a^4\right.\\ &= 6a^4x+a^4\int\cos^2(c+dx)\,dx+a^4\int\sec^2(c+dx)\,dx+\left(4a^4\right)\int \cos\\ &= 6a^4x+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+dx}{2}\\ &= \frac{13a^4x}{2}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(73) = 146.
time = 1.80, size = 241, normalized size = 3.30

$$\frac{1}{64}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(26x-\frac{16\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{16\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{16\cos(dx)\sin(c)}{d}+\frac{\cos(2dx)\sin(2c)}{d}+\frac{16\cos(c)\sin(dx)}{d}+\frac{\cos(2c)\sin(2dx)}{d}+\frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(26*x - (16*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (16*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (16*Cos[d*x]*Sin[c])/d + (Cos[2*d*x]*Sin[2*c])/d + (16*Cos[c]*Sin[d*x])/
d + (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

Maple [A]
time = 0.11, size = 82, normalized size = 1.12

method	result

derivativedivides	$\frac{a^4\tan(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4(dx+c)+4a^4\sin(dx+c)+a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^4\tan(dx+c)+4a^4\ln(\sec(dx+c)+\tan(dx+c))+6a^4(dx+c)+4a^4\sin(dx+c)+a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{13a^4x}{2}-\frac{ia^4e^{2i(dx+c)}}{8d}-\frac{2ia^4e^{i(dx+c)}}{d}+\frac{2ia^4e^{-i(dx+c)}}{d}+\frac{ia^4e^{-2i(dx+c)}}{8d}+\frac{2ia^4}{d(e^{2i(dx+c)}+1)}-\frac{4a^4\ln(e^{i(dx+c)}-i}{d}$
norman	$\frac{-\frac{13a^4x}{2}-\frac{11a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{20a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{12a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{13a^4x(t}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*tan(d*x+c)+4*a^4*ln(sec(d*x+c)+tan(d*x+c))+6*a^4*(d*x+c)+4*a^4*sin
(d*x+c)+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.30, size = 85, normalized size = 1.16

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+24\,(dx+c)a^4+8\,a^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+16\,a^4\sin(dx+c)+4\,a^4\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^4 + 8*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^4*sin(d*x + c) + 4*a^4*tan
(d*x + c))/d
```

Fricas [A]
time = 2.94, size = 105, normalized size = 1.44

$$\frac{13\,a^4dx\cos(dx+c)+4\,a^4\cos(dx+c)\log(\sin(dx+c)+1)-4\,a^4\cos(dx+c)\log(-\sin(dx+c)+1)+\left(a^4\cos(dx+c)^2+8\,a^4\cos(dx+c)+2\,a^4\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2*(13*a^4*d*x*cos(d*x + c) + 4*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 4
*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + 8*a^4*cos(
d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\cos^2(c+dx)\sec(c+dx)\,dx+\int 6\cos^2(c+dx)\sec^2(c+dx)\,dx+\int 4\cos^2(c+dx)\sec^3(c+dx)\,dx+\int\cos^2(c+dx)\sec^4(c+dx)\,dx+\int\cos^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)**2*sec(c + d*x), x) + Integral(6*cos(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**2*sec(c + d*x)**3, x) +
Integral(cos(c + d*x)**2*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**2, x)
)
```

Giac [A]
time = 0.47, size = 129, normalized size = 1.77

$$\frac{13\,(dx+c)a^4+8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(7\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(13*(d*x + c)*a^4 + 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
+ 1/2*c)^2 - 1) + 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 0.89, size = 117, normalized size = 1.60

$$\frac{13\,a^4\,x}{2}+\frac{8\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-5\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+11\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (13*a^4*x)/2 + (8*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)
/2)^3 - 5*a^4*tan(c/2 + (d*x)/2)^5 + 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2
 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

3.36 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=73

$$6a^4x+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{7a^4\sin(c+dx)}{d}+\frac{2a^4\cos(c+dx)\sin(c+dx)}{d}-\frac{a^4\sin^3(c+dx)}{3d}$$

```
[Out] 6*a^4*x+a^4*arctanh(sin(d*x+c))/d+7*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*sin(d
*x+c)/d-1/3*a^4*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3876, 2717, 2715, 8, 2713, 3855}

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] 6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*C
os[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx &= \int \left(4a^4+6a^4\cos(c+dx)+4a^4\cos^2(c+dx)+a^4\cos^3(c+dx)+a\right.\\ &= 4a^4x+a^4\int \cos^3(c+dx)\,dx+a^4\int \sec(c+dx)\,dx+\left(4a^4\right)\int \cos\\ &= 4a^4x+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{6a^4\sin(c+dx)}{d}+\frac{2a^4\cos(c+dx}{d}\\ &= 6a^4x+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{7a^4\sin(c+dx)}{d}+\frac{2a^4\cos(c+dx}{d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 91, normalized size = 1.25

$$\frac{a^4\left(72dx-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+81\sin(c+dx)+12\sin(2(c+dx))+\sin(3(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
[Out] (a^4*(72*d*x - 12*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + 81*Sin[c + d*x] + 12*Sin[2*(c + d*x)] + Sin
[3*(c + d*x)]))/(12*d)
```

Maple [A]
time = 0.11, size = 93, normalized size = 1.27

method	result
derivativedivides	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+6a^4\sin(dx+c)+4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^4\left(2+\cos^2(dx+c)\right)\sin(dx}{3}}{d}$
default	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+6a^4\sin(dx+c)+4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^4\left(2+\cos^2(dx+c)\right)\sin(dx}{3}}{d}$

risch	$6a^4x - \frac{27ia^4 e^{i(dx+c)}}{8d} + \frac{27ia^4 e^{-i(dx+c)}}{8d} + \frac{a^4 \ln(e^{i(dx+c)}+i)}{d} - \frac{a^4 \ln(e^{i(dx+c)}-i)}{d} + \frac{a^4 \sin(3dx+3c)}{12d} + \frac{a^4 \sin(}{}$
norman	$\frac{-6a^4x - \frac{18a^4 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{86a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d} + \frac{12a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} - \frac{28a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} - \frac{14a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d} + \frac{10a^4(\mathrm{ta}}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^4*(d*x+c)+6*a^4*sin(d*x+c)+4*a^4*(1/
2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^4*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 97, normalized size = 1.33

$$-\frac{2\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)a^4-6\left(2dx+2c+\sin\left(2dx+2c\right)\right)a^4-24\left(dx+c\right)a^4-3a^4\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-36a^4\sin\left(dx+c\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^4 - 24*(d*x + c)*a^4 - 3*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) - 36*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 3.39, size = 80, normalized size = 1.10

$$\frac{36a^4dx+3a^4\log\left(\sin\left(dx+c\right)+1\right)-3a^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a^4\cos\left(dx+c\right)^2+6a^4\cos\left(dx+c\right)+20a^4\right)\sin\left(dx+c\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(36*a^4*d*x + 3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1
) + 2*(a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + 20*a^4)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int 6\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 4\cos^3\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int \cos^3\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int \cos^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)**3*sec(c + d*x), x) + Integral(6*cos(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**3, x) +
```

```
Integral(cos(c + d*x)**3*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**3, x)
)
```

Giac [A]
time = 0.47, size = 116, normalized size = 1.59

$$\frac{18\,(dx+c)a^4+3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+38\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(18*(d*x + c)*a^4 + 3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 + 38*a^
4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 + 1)^3)/d
```

Mupad [B]
time = 0.69, size = 93, normalized size = 1.27

$$6\,a^4\,x+\frac{20\,a^4\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^4,x)
```

```
[Out] 6*a^4*x + (20*a^4*sin(c + d*x))/(3*d) + (2*a^4*atanh(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/d + (a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (2*a^4*cos(
c + d*x)*sin(c + d*x))/d
```

3.37 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=87

$$\frac{35a^4x}{8}+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}$$

```
[Out] 35/8*a^4*x+8*a^4*sin(d*x+c)/d+27/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(
d*x+c)^3*sin(d*x+c)/d-4/3*a^4*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3876, 2717, 2715, 8, 2713}

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4+4a^4\cos(c+dx)+6a^4\cos^2(c+dx)+4a^4\cos^3(c+dx)+a\right.$$

$$= a^4x+a^4\int \cos^4(c+dx)\,dx+\left(4a^4\right)\int \cos(c+dx)\,dx+\left(4a^4\right)\int \mathrm{c}$$

$$= a^4x+\frac{4a^4\sin(c+dx)}{d}+\frac{3a^4\cos(c+dx)\sin(c+dx)}{d}+\frac{a^4\cos^3(c+}{}$$

$$= 4a^4x+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3(c}{}$$

$$= \frac{35a^4x}{8}+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3}{}$$

Mathematica [A]
time = 0.11, size = 56, normalized size = 0.64

$$\frac{a^4(420c+420dx+672\sin(c+dx)+168\sin(2(c+dx))+32\sin(3(c+dx))+3\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(420*c + 420*d*x + 672*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 32*Sin[3*
(c + d*x)] + 3*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A]
time = 0.11, size = 111, normalized size = 1.28

method	result
risch	$\frac{35a^4x}{8}+\frac{7a^4\sin(dx+c)}{d}+\frac{a^4\sin(4dx+4c)}{32d}+\frac{a^4\sin(3dx+3c)}{3d}+\frac{7a^4\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+}{d}$

default	$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a}{d}$
norman	$\frac{-\frac{35a^4x}{8}-\frac{93a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{163a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{311a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{17a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{329a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{35a}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a^4*sin(d*x+c)+a^4*(d*x+c))

Maxima [A]
time = 0.28, size = 104, normalized size = 1.20

$$-\frac{128\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)a^4-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^4-144\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4-96\left(dx+c\right)a^4-384\,a^4\sin\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out] -1/96*(128*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 - 96*(d*x + c)*a^4 - 384*a^4*sin(d*x + c))/d

Fricas [A]
time = 2.29, size = 63, normalized size = 0.72

$$\frac{105\,a^4dx+\left(6\,a^4\cos\left(dx+c\right)^3+32\,a^4\cos\left(dx+c\right)^2+81\,a^4\cos\left(dx+c\right)+160\,a^4\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/24*(105*a^4*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^4*cos(d*x + c)^2 + 81*a^4*cos(d*x + c) + 160*a^4)*sin(d*x + c))/d

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4,x)

[Out] Timed out

Giac [A]
time = 0.46, size = 96, normalized size = 1.10

$$\frac{105\,(dx+c)a^4+\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+511\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+279\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*a^4*tan(1
/2*d*x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 + 279*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 4.12, size = 89, normalized size = 1.02

$$\frac{35\,a^4\,x}{8}+\frac{\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{385\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{511\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{93\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (35*a^4*x)/8 + ((511*a^4*tan(c/2 + (d*x)/2)^3)/12 + (385*a^4*tan(c/2 + (d*x
)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 + (93*a^4*tan(c/2 + (d*x)/2))/
4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.38 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=102

$$\frac{7a^4x}{2}+\frac{8a^4\sin(c+dx)}{d}+\frac{7a^4\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin^5(c+dx)}{5d}$$

```
[Out] 7/2*a^4*x+8*a^4*sin(d*x+c)/d+7/2*a^4*cos(d*x+c)*sin(d*x+c)/d+a^4*cos(d*x+c)
^3*sin(d*x+c)/d-8/3*a^4*sin(d*x+c)^3/d+1/5*a^4*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3876, 2717, 2715, 8, 2713}

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{7a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (7*a^4*x)/2 + (8*a^4*Sin[c + d*x])/d + (7*a^4*Cos[c + d*x]*Sin[c + d*x])/(2
*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d - (8*a^4*Sin[c + d*x]^3)/(3*d) +
(a^4*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\cos(c+dx) + 4a^4\cos^2(c+dx) + 6a^4\cos^3(c+dx) + 4a^4\cos\right.$$
$$= a^4\int \cos(c+dx)\,dx + a^4\int \cos^5(c+dx)\,dx + \left(4a^4\right)\int \cos^2(c+dx$$
$$= \frac{a^4\sin(c+dx)}{d} + \frac{2a^4\cos(c+dx)\sin(c+dx)}{d} + \frac{a^4\cos^3(c+dx)\sin}{d}$$
$$= 2a^4x + \frac{8a^4\sin(c+dx)}{d} + \frac{7a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^3(c}{}$$
$$= \frac{7a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{7a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^3(c}{}$$

Mathematica [A]
time = 0.16, size = 63, normalized size = 0.62

$$\frac{a^4(840dx + 1470\sin(c+dx) + 480\sin(2(c+dx)) + 145\sin(3(c+dx)) + 30\sin(4(c+dx)) + 3\sin(5(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
[Out] (a^4*(840*d*x + 1470*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 145*Sin[3*(c + d
*x)] + 30*Sin[4*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d)
```

Maple [A]
time = 0.15, size = 133, normalized size = 1.30

method	result
risch	$\frac{7a^4x}{2} + \frac{49a^4\sin(dx+c)}{8d} + \frac{a^4\sin(5dx+5c)}{80d} + \frac{a^4\sin(4dx+4c)}{8d} + \frac{29a^4\sin(3dx+3c)}{48d} + \frac{2a^4\sin(2dx+2c)}{d}$
derivativedivides	$\frac{\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5} + 4a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + 2a^4\left(2+\cos^2(dx+\right.}{d}$
default	$\frac{\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5} + 4a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + 2a^4\left(2+\cos^2(dx+\right.}{d}$

norman	$-\frac{7a^4x}{2}-\frac{25a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{67a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{349a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{203a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{533a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-259a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^4*(2+cos(d*x+c)^2)*
sin(d*x+c)+4*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 128, normalized size = 1.25

$$\frac{8\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))a^4-240\,(\sin(dx+c)^3-3\,\sin(dx+c))a^4+15\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4+120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+120\,a^4\sin(dx+c)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 240
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^4 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 1
20*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 2.74, size = 76, normalized size = 0.75

$$\frac{105\,a^4dx+\left(6\,a^4\cos(dx+c)^4+30\,a^4\cos(dx+c)^3+68\,a^4\cos(dx+c)^2+105\,a^4\cos(dx+c)+166\,a^4\right)\sin(dx+c)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(105*a^4*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^4*cos(d*x + c)^3 + 68*a^4*
cos(d*x + c)^2 + 105*a^4*cos(d*x + c) + 166*a^4)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.47, size = 112, normalized size = 1.10

$$\frac{105\,(dx+c)a^4+\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+896\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+790\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+375\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 + 490*a^4*tan(1
/2*d*x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 + 790*a^4*tan(1/2*d*x +
1/2*c)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 4.43, size = 105, normalized size = 1.03

$$\frac{7\,a^4\,x}{2}+\frac{7\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{98\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{896\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{158\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (7*a^4*x)/2 + ((158*a^4*tan(c/2 + (d*x)/2)^3)/3 + (896*a^4*tan(c/2 + (d*x)/
2)^5)/15 + (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*tan(c/2 + (d*x)/2)^9 + 2
5*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.39 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=127

$$\frac{49a^4x}{16}+\frac{8a^4\sin(c+dx)}{d}+\frac{49a^4\cos(c+dx)\sin(c+dx)}{16d}+\frac{41a^4\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^4\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 49/16*a^4*x+8*a^4*sin(d*x+c)/d+49/16*a^4*cos(d*x+c)*sin(d*x+c)/d+41/24*a^4*
cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d*x+c)/d-4*a^4*sin(d*x+c
)^3/d+4/5*a^4*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3876, 2715, 8, 2713}

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{49a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{49a^4x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)/(5
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
```

```
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\cos^2(c+dx)+4a^4\cos^3(c+dx)+6a^4\cos^4(c+dx)+4a^4\cos\right.$$
$$= a^4\int\cos^2(c+dx)\,dx + a^4\int\cos^6(c+dx)\,dx + \left(4a^4\right)\int\cos^3(c+d$$
$$= \frac{a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^4\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos}{}$$
$$= \frac{a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{41a^4\cos^3}{}$$
$$= \frac{11a^4x}{4} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\mathrm{co}}{}$$
$$= \frac{49a^4x}{16} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\mathrm{co}}{}$$

Mathematica [A]
time = 0.21, size = 73, normalized size = 0.57

$$\frac{a^4(2940dx+5280\sin(c+dx)+1905\sin(2(c+dx))+720\sin(3(c+dx))+225\sin(4(c+dx))+48\sin(5(c+dx))+5\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
[Out] (a^4*(2940*d*x + 5280*Sin[c + d*x] + 1905*Sin[2*(c + d*x)] + 720*Sin[3*(c +
 d*x)] + 225*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/
(960*d)
```

Maple [A]
time = 0.09, size = 169, normalized size = 1.33

method	result
risch	$\frac{49a^4x}{16}+\frac{11a^4\sin(dx+c)}{2d}+\frac{a^4\sin(6dx+6c)}{192d}+\frac{a^4\sin(5dx+5c)}{20d}+\frac{15a^4\sin(4dx+4c)}{64d}+\frac{3a^4\sin(3dx+3c)}{4d}+\frac{127a}{}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+}{d}$

default	$\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+6a}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a^4
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a^4*(2+co
s(d*x+c)^2)*sin(d*x+c)+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 165, normalized size = 1.30

$$\frac{256\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))a^4-5\,(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c))a^4-1280\,(\sin(dx+c)^3-3\,\sin(dx+c))a^4+180\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4+240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 180*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 + 240*(2*d*x + 2*c + sin(2*
d*x + 2*c))*a^4)/d
```

Fricas [A]
time = 1.97, size = 89, normalized size = 0.70

$$\frac{735\,a^4dx+\left(40\,a^4\cos(dx+c)^5+192\,a^4\cos(dx+c)^4+410\,a^4\cos(dx+c)^3+576\,a^4\cos(dx+c)^2+735\,a^4\cos(dx+c)+1152\,a^4\right)\sin(dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(735*a^4*d*x + (40*a^4*cos(d*x + c)^5 + 192*a^4*cos(d*x + c)^4 + 410*
a^4*cos(d*x + c)^3 + 576*a^4*cos(d*x + c)^2 + 735*a^4*cos(d*x + c) + 1152*a
^4)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4,x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep

Giac [A]
time = 0.49, size = 128, normalized size = 1.01

$$\frac{735\,(dx+c)a^4+\frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4165\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+9702\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+11802\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+7355\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] 1/240*(735*(d*x + c)*a^4 + 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 + 4165*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 + 11802*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 + 3105*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d

Mupad [B]
time = 3.42, size = 121, normalized size = 0.95

$$\frac{49\,a^4\,x}{16}+\frac{\frac{49\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{833\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}+\frac{1617\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{20}+\frac{1967\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}+\frac{1471\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}+\frac{207\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6*(a + a/cos(c + d*x))^4,x)

[Out] (49*a^4*x)/16 + ((1471*a^4*tan(c/2 + (d*x)/2)^3)/24 + (1967*a^4*tan(c/2 + (d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/20 + (833*a^4*tan(c/2 + (d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 + (207*a^4*tan(c/2 + (d*x)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)

3.40 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=147

$$\frac{11a^4x}{4}+\frac{8a^4\sin(c+dx)}{d}+\frac{11a^4\cos(c+dx)\sin(c+dx)}{4d}+\frac{11a^4\cos^3(c+dx)\sin(c+dx)}{6d}+\frac{2a^4\cos^5(c+dx)\sin(c}{3d}$$

```
[Out] 11/4*a^4*x+8*a^4*sin(d*x+c)/d+11/4*a^4*cos(d*x+c)*sin(d*x+c)/d+11/6*a^4*cos
(d*x+c)^3*sin(d*x+c)/d+2/3*a^4*cos(d*x+c)^5*sin(d*x+c)/d-16/3*a^4*sin(d*x+c
)^3/d+9/5*a^4*sin(d*x+c)^5/d-1/7*a^4*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3876, 2713, 2715, 8}

$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{9a^4\sin^5(c+dx)}{5d}-\frac{16a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{11a^4\sin(c+dx)\cos^3(c+dx)}{6d}+\frac{11a^4\sin(c+dx)\cos(c+dx)}{4d}+\frac{11a^4x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (11*a^4*x)/4 + (8*a^4*Sin[c + d*x])/d + (11*a^4*Cos[c + d*x]*Sin[c + d*x])/
(4*d) + (11*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(6*d) + (2*a^4*Cos[c + d*x]^5*
Sin[c + d*x])/(3*d) - (16*a^4*Sin[c + d*x]^3)/(3*d) + (9*a^4*Sin[c + d*x]^5
)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
```

```
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\cos^3(c+dx)+4a^4\cos^4(c+dx)+6a^4\cos^5(c+dx)+4a^4\mathrm{co}\right.$$
$$= a^4\int\cos^3(c+dx)\,dx + a^4\int\cos^7(c+dx)\,dx + \left(4a^4\right)\int\cos^4(c+d$$
$$= \frac{a^4\cos^3(c+dx)\sin(c+dx)}{d} + \frac{2a^4\cos^5(c+dx)\sin(c+dx)}{3d} + \left(3a^4\right)$$
$$= \frac{8a^4\sin(c+dx)}{d} + \frac{3a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{11a^4\cos^3(c+dx)}{6d}$$
$$= \frac{3a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{11a^4\mathrm{cos}}{}$$
$$= \frac{11a^4x}{4} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{11a^4\mathrm{co}}{}$$

Mathematica [A]
time = 0.28, size = 83, normalized size = 0.56

$$\frac{a^4(18480dx+33915\sin(c+dx)+13020\sin(2(c+dx))+5495\sin(3(c+dx))+2100\sin(4(c+dx))+651\sin(5(c+dx))+140\sin(6(c+dx))+15\sin(7(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(18480*d*x + 33915*Sin[c + d*x] + 13020*Sin[2*(c + d*x)] + 5495*Sin[3*
(c + d*x)] + 2100*Sin[4*(c + d*x)] + 651*Sin[5*(c + d*x)] + 140*Sin[6*(c +
d*x)] + 15*Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A]
time = 0.12, size = 185, normalized size = 1.26

method	result
risch	$\frac{11a^4x}{4}+\frac{323a^4\sin(dx+c)}{64d}+\frac{a^4\sin(7dx+7c)}{448d}+\frac{a^4\sin(6dx+6c)}{48d}+\frac{31a^4\sin(5dx+5c)}{320d}+\frac{5a^4\sin(4dx+4c)}{16d}+\frac{157}{}$
derivativedivides	$\frac{\frac{a^4\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+4a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+}{6}\right.}{}$

default	$\frac{\frac{a^4\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+4a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+
c)+4*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+6/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^4*(2+co
s(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.30, size = 187, normalized size = 1.27

$$-\frac{48\,(5\,\sin(dx+c)^7-21\,\sin(dx+c)^5+35\,\sin(dx+c)^3-35\,\sin(dx+c))a^4-672\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))a^4+35\,(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c))a^4+560\,(\sin(dx+c)^3-3\,\sin(dx+c))a^4-210\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/1680*(48*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*
sin(d*x + c))*a^4 - 672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x
+ c))*a^4 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) -
 48*sin(2*d*x + 2*c))*a^4 + 560*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 210
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4)/d
```

Fricas [A]
time = 3.50, size = 102, normalized size = 0.69

$$\frac{1155\,a^4dx+\left(60\,a^4\cos(dx+c)^6+280\,a^4\cos(dx+c)^5+576\,a^4\cos(dx+c)^4+770\,a^4\cos(dx+c)^3+908\,a^4\cos(dx+c)^2+1155\,a^4\cos(dx+c)+1816\,a^4\right)\sin(dx+c)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/420*(1155*a^4*d*x + (60*a^4*cos(d*x + c)^6 + 280*a^4*cos(d*x + c)^5 + 576
*a^4*cos(d*x + c)^4 + 770*a^4*cos(d*x + c)^3 + 908*a^4*cos(d*x + c)^2 + 115
5*a^4*cos(d*x + c) + 1816*a^4)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.51, size = 144, normalized size = 0.98

$$\frac{1155\,(dx+c)a^4 + \frac{2\left(1155\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+7700\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+21791\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+33792\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+31521\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+14700\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+5565\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/420*(1155*(d*x + c)*a^4 + 2*(1155*a^4*tan(1/2*d*x + 1/2*c)^13 + 7700*a^4*
tan(1/2*d*x + 1/2*c)^11 + 21791*a^4*tan(1/2*d*x + 1/2*c)^9 + 33792*a^4*tan(
1/2*d*x + 1/2*c)^7 + 31521*a^4*tan(1/2*d*x + 1/2*c)^5 + 14700*a^4*tan(1/2*d
*x + 1/2*c)^3 + 5565*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^7)/d
```

Mupad [B]
time = 3.64, size = 137, normalized size = 0.93

$$\frac{11\,a^4\,x}{4}+\frac{\frac{11\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{2}+\frac{110\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}+\frac{3113\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{30}+\frac{5632\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}+\frac{1501\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{10}+70\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{53\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (11*a^4*x)/4 + (70*a^4*tan(c/2 + (d*x)/2)^3 + (1501*a^4*tan(c/2 + (d*x)/2)^
5)/10 + (5632*a^4*tan(c/2 + (d*x)/2)^7)/35 + (3113*a^4*tan(c/2 + (d*x)/2)^9
)/30 + (110*a^4*tan(c/2 + (d*x)/2)^11)/3 + (11*a^4*tan(c/2 + (d*x)/2)^13)/2
 + (53*a^4*tan(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.41 $\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$

Optimal. Leaf size=156

$$\frac{93a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{16a^5\tan(c+dx)}{d}+\frac{93a^5\sec(c+dx)\tan(c+dx)}{16d}+\frac{85a^5\sec^3(c+dx)\tan(c+dx)}{24d}+$$

```
[Out] 93/16*a^5*arctanh(sin(d*x+c))/d+16*a^5*tan(d*x+c)/d+93/16*a^5*sec(d*x+c)*ta
n(d*x+c)/d+85/24*a^5*sec(d*x+c)^3*tan(d*x+c)/d+5/6*a^5*sec(d*x+c)^5*tan(d*x
+c)/d+28/3*a^5*tan(d*x+c)^3/d+13/5*a^5*tan(d*x+c)^5/d+1/7*a^5*tan(d*x+c)^7/
d
```

Rubi [A]
time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3876, 3853, 3855, 3852}

$$\frac{a^5\tan^7(c+dx)}{7d}+\frac{13a^5\tan^5(c+dx)}{5d}+\frac{28a^5\tan^3(c+dx)}{3d}+\frac{16a^5\tan(c+dx)}{d}+\frac{93a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{5a^5\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{85a^5\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{93a^5\tan(c+dx)\sec(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (93*a^5*ArcTanh[Sin[c + d*x]])/(16*d) + (16*a^5*Tan[c + d*x])/d + (93*a^5*S
ec[c + d*x]*Tan[c + d*x])/(16*d) + (85*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(24
*d) + (5*a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (28*a^5*Tan[c + d*x]^3)/(
3*d) + (13*a^5*Tan[c + d*x]^5)/(5*d) + (a^5*Tan[c + d*x]^7)/(7*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx &= \int \left(a^5\sec^3(c+dx)+5a^5\sec^4(c+dx)+10a^5\sec^5(c+dx)+10a^5 \mathrm{s}\right.\\
&= a^5\int \sec^3(c+dx)\,dx+a^5\int \sec^8(c+dx)\,dx+\left(5a^5\right)\int \sec^4(c+d\\
&= \frac{a^5\sec(c+dx)\tan(c+dx)}{2d}+\frac{5a^5\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{5a^5 \mathrm{s}}{}\\
&= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d}+\frac{16a^5\tan(c+dx)}{d}+\frac{17a^5\sec(c+dx)\tan}{4d}\\
&= \frac{17a^5\tanh^{-1}(\sin(c+dx))}{4d}+\frac{16a^5\tan(c+dx)}{d}+\frac{93a^5\sec(c+dx)\,\mathrm{t}}{16d}\\
&= \frac{93a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{16a^5\tan(c+dx)}{d}+\frac{93a^5\sec(c+dx)\,\mathrm{t}}{16d}
\end{aligned}
$$

Mathematica [A]
time = 1.29, size = 229, normalized size = 1.47

$$
-\frac{a^5(1+\cos(c+dx))^5\sec^{10}\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\left(624960\cos^7(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(374080\sin(dx)-162400\sin(2c+dx)+118825\sin(c+2dx)+118825\sin(3c+2dx)+305088\sin(2c+3dx)-16800\sin(4c+3dx)+62860\sin(3c+4dx)+62860\sin(5c+4dx)+107296\sin(4c+5dx)+9765\sin(5c+6dx)+9765\sin(7c+6dx)+15328\sin(6c+7dx))\right)}{3440640d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -1/3440640*(a^5*(1 + Cos[c + d*x])^5*Sec[(c + d*x)/2]^10*Sec[c + d*x]^7*(62
4960*Cos[c + d*x]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(374080*Sin[d*x] - 162400*Sin[2*c +
 d*x] + 118825*Sin[c + 2*d*x] + 118825*Sin[3*c + 2*d*x] + 305088*Sin[2*c +
3*d*x] - 16800*Sin[4*c + 3*d*x] + 62860*Sin[3*c + 4*d*x] + 62860*Sin[5*c +
4*d*x] + 107296*Sin[4*c + 5*d*x] + 9765*Sin[5*c + 6*d*x] + 9765*Sin[7*c + 6
*d*x] + 15328*Sin[6*c + 7*d*x])))/d
```

Maple [A]
time = 0.15, size = 248, normalized size = 1.59

method	result

risch	$-\frac{i a^5 (9765\,\mathrm{e}^{13 i (d x+c)}+62860\,\mathrm{e}^{11 i (d x+c)}-16800\,\mathrm{e}^{10 i (d x+c)}+118825\,\mathrm{e}^{9 i (d x+c)}-162400\,\mathrm{e}^{8 i (d x+c)}-374080\,\mathrm{e}^{6 i (d x+c)}-1188}{840 d (\mathrm{e}^{2 i (d x+c)}+1)^7}$
norman	$\frac{-\frac{419 a^5 \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{8 d}+\frac{943 a^5 \left(\tan^3\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{6 d}-\frac{37169 a^5 \left(\tan^5\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{120 d}+\frac{11904 a^5 \left(\tan^7\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{35 d}-\frac{8773 a^5 \left(\tan^9\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{40 d}+\frac{155 a}{}}{\left(\tan^2\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^7}$
derivativedivides	$\frac{-a^5 \left(-\frac{16}{35}-\frac{\left(\sec^6(d x+c)\right)}{7}-\frac{6 \left(\sec^4(d x+c)\right)}{35}-\frac{8 \left(\sec^2(d x+c)\right)}{35}\right) \tan(d x+c)+5 a^5 \left(-\left(-\frac{\left(\sec^5(d x+c)\right)}{6}-\frac{5 \left(\sec^3(d x+c)\right)}{24}-\frac{5 \sec(d x}{16}\right.\right.}{}$
default	$\frac{-a^5 \left(-\frac{16}{35}-\frac{\left(\sec^6(d x+c)\right)}{7}-\frac{6 \left(\sec^4(d x+c)\right)}{35}-\frac{8 \left(\sec^2(d x+c)\right)}{35}\right) \tan(d x+c)+5 a^5 \left(-\left(-\frac{\left(\sec^5(d x+c)\right)}{6}-\frac{5 \left(\sec^3(d x+c)\right)}{24}-\frac{5 \sec(d x}{16}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^5*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+5*a^5*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d
*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c)))-10*a^5*(-8/15-1/5*sec(d*x+c)^4-4/15*s
ec(d*x+c)^2)*tan(d*x+c)+10*a^5*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x
+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-5*a^5*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)
+a^5*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(142) = 284$.
time = 0.29, size = 314, normalized size = 2.01

$$\frac{96 \left(5 \tan\left(d x+c\right)^7+21 \tan\left(d x+c\right)^5+35 \tan\left(d x+c\right)^3+35 \tan\left(d x+c\right)\right) a^5+2240 \left(3 \tan\left(d x+c\right)^5+10 \tan\left(d x+c\right)^3+15 \tan\left(d x+c\right)\right) a^5+5600 \left(\tan\left(d x+c\right)^3+3 \tan\left(d x+c\right)\right) a^5-175 a^5 \left(\frac{2 \left(15 \sin(d x+c)^5-40 \sin(d x+c)^3+33 \sin(d x+c)\right)}{\sin(d x+c)^6-3 \sin(d x+c)^4+3 \sin(d x+c)^2-1}-15 \log\left(\sin\left(d x+c\right)+1\right)+15 \log\left(\sin\left(d x+c\right)-1\right)\right)-2100 a^5 \left(\frac{2 \left(3 \sin(d x+c)^3-5 \sin(d x+c)\right)}{\sin(d x+c)^4-2 \sin(d x+c)^2+1}-3 \log\left(\sin\left(d x+c\right)+1\right)+3 \log\left(\sin\left(d x+c\right)-1\right)\right)-840 a^5 \left(\frac{2 \sin(d x+c)}{\sin(d x+c)^2-1}-\log\left(\sin\left(d x+c\right)+1\right)+\log\left(\sin\left(d x+c\right)-1\right)\right)}{3360 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/3360*(96*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*t
an(d*x + c))*a^5 + 2240*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x
+ c))*a^5 + 5600*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^5 - 175*a^5*(2*(15*sin
(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(
d*x + c) - 1)) - 2100*a^5*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 840*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.65, size = 150, normalized size = 0.96

$$\frac{9765 a^5 \cos\left(d x+c\right)^7 \log\left(\sin\left(d x+c\right)+1\right)-9765 a^5 \cos\left(d x+c\right)^7 \log\left(-\sin\left(d x+c\right)+1\right)+2 \left(15328 a^5 \cos\left(d x+c\right)^6+9765 a^5 \cos\left(d x+c\right)^5+7664 a^5 \cos\left(d x+c\right)^4+5950 a^5 \cos\left(d x+c\right)^3+3648 a^5 \cos\left(d x+c\right)^2+1400 a^5 \cos\left(d x+c\right)+240 a^5\right) \sin\left(d x+c\right)}{3360 d \cos\left(d x+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3360*(9765*a^5*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 9765*a^5*cos(d*x +
c)^7*log(-sin(d*x + c) + 1) + 2*(15328*a^5*cos(d*x + c)^6 + 9765*a^5*cos(d*
x + c)^5 + 7664*a^5*cos(d*x + c)^4 + 5950*a^5*cos(d*x + c)^3 + 3648*a^5*cos
(d*x + c)^2 + 1400*a^5*cos(d*x + c) + 240*a^5)*sin(d*x + c))/(d*cos(d*x + c
)^7)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^5\left(\int \sec^3(c+dx)\,dx + \int 5\sec^4(c+dx)\,dx + \int 10\sec^5(c+dx)\,dx + \int 10\sec^6(c+dx)\,dx + \int 5\sec^7(c+dx)\,dx + \int \sec^8(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**5,x)
```

```
[Out] a**5*(Integral(sec(c + d*x)**3, x) + Integral(5*sec(c + d*x)**4, x) + Integ
ral(10*sec(c + d*x)**5, x) + Integral(10*sec(c + d*x)**6, x) + Integral(5*s
ec(c + d*x)**7, x) + Integral(sec(c + d*x)**8, x))
```

Giac [**A**]
time = 0.57, size = 170, normalized size = 1.09

$$\frac{9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(9765\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 65100\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 184233\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 285696\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} + 260183\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 132020\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} + 43995\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^7}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1680*(9765*a^5*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9765*a^5*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) - 2*(9765*a^5*tan(1/2*d*x + 1/2*c)^13 - 65100*a^5*ta
n(1/2*d*x + 1/2*c)^11 + 184233*a^5*tan(1/2*d*x + 1/2*c)^9 - 285696*a^5*tan(
1/2*d*x + 1/2*c)^7 + 260183*a^5*tan(1/2*d*x + 1/2*c)^5 - 132020*a^5*tan(1/2
*d*x + 1/2*c)^3 + 43995*a^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
 1)^7)/d
```

Mupad [**B**]
time = 4.84, size = 228, normalized size = 1.46

$$\frac{93\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d} - \frac{\frac{93\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8} - \frac{155\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{2} + \frac{8773\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{40} - \frac{11904\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35} + \frac{37169\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{120} - \frac{943\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{6} + \frac{419\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} - 7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + 21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} + 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} - 21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} + 7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^5/cos(c + d*x)^3,x)
```

```
[Out] (93*a^5*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((37169*a^5*tan(c/2 + (d*x)/2)^5
)/120 - (943*a^5*tan(c/2 + (d*x)/2)^3)/6 - (11904*a^5*tan(c/2 + (d*x)/2)^7)
/35 + (8773*a^5*tan(c/2 + (d*x)/2)^9)/40 - (155*a^5*tan(c/2 + (d*x)/2)^11)/
2 + (93*a^5*tan(c/2 + (d*x)/2)^13)/8 + (419*a^5*tan(c/2 + (d*x)/2))/8)/(d*(
7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6
- 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)
^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.42 $\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=103

$$-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{4\tan^3(c+dx)}{3ad}$$

```
[Out] -3/2*arctanh(sin(d*x+c))/a/d+4*tan(d*x+c)/a/d-3/2*sec(d*x+c)*tan(d*x+c)/a/d
-sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))+4/3*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3903, 3872, 3853, 3855, 3852}

$$\frac{4\tan^3(c+dx)}{3ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(2*a*d) + (4*Tan[c + d*x])/(a*d) - (3*Sec[c + d*
x]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c +
d*x])) + (4*Tan[c + d*x]^3)/(3*a*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \sec^3(c+dx)(3a-4a\sec(c+dx))\,dx}{a^2}\\ &= -\frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{3\int \sec^3(c+dx)\,dx}{a}+\frac{4\int \sec^4(c+dx)\,dx}{a}\\ &= -\frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{3\int \sec(c+dx)\,dx}{2a}-\frac{4S}{}\\ &= -\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec^3(c+d}{d(a+a}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 374 vs. 2(103) = 206.
time = 3.35, size = 374, normalized size = 3.63

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(6\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{1}{8}\cos\left(\frac{1}{2}(c+dx)\right)\sec(c)\sec^3(c+dx)\left(9\cos(2c+3dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+9\cos(4c+3dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+27\cos(dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+27\cos(2c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-9\cos(2c+3dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-9\cos(4c+3dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+48\sin(dx)-12\sin(2c+dx)-6\sin(c+2dx)-6\sin(3c+2dx)+20\sin(2c+3dx)\right)\right)}{3ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*(6*Sec[c/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2]
*Sec[c]*Sec[c + d*x]^3*(9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 27
*Cos[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) + 27*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 9*Cos[2*c + 3*d*x]*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + 48*Sin[d*x] - 12*Sin[2*c + d*x] - 6*Sin[c + 2*d
*x] - 6*Sin[3*c + 2*d*x] + 20*Sin[2*c + 3*d*x]))/8))/(3*a*d*(1 + Sec[c + d*
x]))
```

Maple [A]
time = 0.09, size = 134, normalized size = 1.30

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+}{da}$
risch	$\frac{i\left(9\,\mathrm{e}^{6i(dx+c)}+9\,\mathrm{e}^{5i(dx+c)}+24\,\mathrm{e}^{4i(dx+c)}+24\,\mathrm{e}^{3i(dx+c)}+39\,\mathrm{e}^{2i(dx+c)}+7\,\mathrm{e}^{i(dx+c)}+16\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2ad}+\frac{3\ln\left(\mathrm{e}^{i(dx}\right.}{2a}$
norman	$\frac{\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{37\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{49\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{9\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}-\frac{3\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(tan(1/2*d*x+1/2*c)-1/3/(tan(1/2*d*x+1/2*c)-1)^3-1/(tan(1/2*d*x+1/2*c
)-1)^2-5/2/(tan(1/2*d*x+1/2*c)-1)+3/2*ln(tan(1/2*d*x+1/2*c)-1)-1/3/(tan(1/2
*d*x+1/2*c)+1)^3+1/(tan(1/2*d*x+1/2*c)+1)^2-5/2/(tan(1/2*d*x+1/2*c)+1)-3/2*
ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(97) = 194$.
time = 0.28, size = 205, normalized size = 1.99

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a-\frac{3a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
+ 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(d*x
 + c) + 1)))/d
```

Fricas [A]
time = 4.57, size = 124, normalized size = 1.20

$$-\frac{9\left(\cos(dx+c)^{4}+\cos(dx+c)^{3}\right)\log\left(\sin(dx+c)+1\right)-9\left(\cos(dx+c)^{4}+\cos(dx+c)^{3}\right)\log\left(-\sin(dx+c)+1\right)-2\left(16\cos(dx+c)^{3}+7\cos(dx+c)^{2}-\cos(dx+c)+2\right)\sin(dx+c)}{12\left(ad\cos(dx+c)^{4}+ad\cos(dx+c)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(9*(cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 9*(cos(d
*x + c)^4 + cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(16*cos(d*x + c)^3 +
 7*cos(d*x + c)^2 - cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a
*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.44, size = 114, normalized size = 1.11

$$-\frac{\frac{9\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{9\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-16\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a - 6*tan(1/2*d*x + 1/2*c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^5 - 16*t
an(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 -
1)^3*a))/d
```

Mupad [B]
time = 0.93, size = 96, normalized size = 0.93

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(a*d) - (3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - (3*tan(c/2
 + (d*x)/2) - (16*tan(c/2 + (d*x)/2)^3)/3 + 5*tan(c/2 + (d*x)/2)^5)/(a*d*(t
an(c/2 + (d*x)/2)^2 - 1)^3)
```

3.43 $\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{2\tan(c+dx)}{ad}+\frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] 3/2*arctanh(sin(d*x+c))/a/d-2*tan(d*x+c)/a/d+3/2*sec(d*x+c)*tan(d*x+c)/a/d-
sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3903, 3872, 3852, 8, 3853, 3855}

$$-\frac{2\tan(c+dx)}{ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*Tan[c + d*x])/(a*d) + (3*Sec[c + d*x
]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d
*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^2(c+dx)(2a-3a\sec(c+dx))\,dx}{a^2} \\
&= -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{2\int \sec^2(c+dx)\,dx}{a} + \frac{3\int \sec^3(c+dx)\,dx}{a} \\
&= \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{3\int \sec(c+dx)\,dx}{2a} + \frac{2\mathrm{Sub}}{} \\
&= \frac{3\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{2\tan(c+dx)}{ad} + \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx}{d(a+a\,\mathrm{se}}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 250 vs. 2(85) = 170.
time = 1.45, size = 250, normalized size = 2.94

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-4\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{4\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)\right)}{2ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*(-4*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]
*(-6*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^(-2) - (4*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])))))/(2*a*d*(1 + Sec[c + d*x]))
```

Maple [**A**]
time = 0.07, size = 108, normalized size = 1.27

method	result
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{da}$
risch	$-\frac{i\left(3\,e^{4i(dx+c)}+3\,e^{3i(dx+c)}+5\,e^{2i(dx+c)}+e^{i(dx+c)}+4\right)}{da\left(e^{2i(dx+c)}+1\right)^{2}\left(e^{i(dx+c)}+1\right)}-\frac{3\ln\left(e^{i(dx+c)}-i\right)}{2ad}+\frac{3\ln\left(e^{i(dx+c)}+i\right)}{2ad}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-tan(1/2*d*x+1/2*c)+1/2/(tan(1/2*d*x+1/2*c)-1)^2+3/2/(tan(1/2*d*x+1/
2*c)-1)-3/2*ln(tan(1/2*d*x+1/2*c)-1)-1/2/(tan(1/2*d*x+1/2*c)+1)^2+3/2/(tan(
1/2*d*x+1/2*c)+1)+3/2*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.29, size = 162, normalized size = 1.91

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) + 1)
))/d
```

Fricas [A]
time = 3.02, size = 112, normalized size = 1.32

$$\frac{3\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(4\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)}{4\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*x
 + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c)^2 + co
s(d*x + c) - 1)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.44, size = 101, normalized size = 1.19

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/a - 2*tan(1/2*d*x + 1/2*c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

Mupad [B]
time = 0.73, size = 95, normalized size = 1.12

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - tan(c/2 + (d*x)/2)/(a*d) - (tan(c/2 +
 (d*x)/2) - 3*tan(c/2 + (d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*ta
n(c/2 + (d*x)/2)^4))
```

3.44 $\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] -arctanh(sin(d*x+c))/a/d+tan(d*x+c)/a/d+tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3875, 3874, 3855, 3879}

$$\frac{\tan(c+dx)}{ad}-\frac{\tanh^{-1}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a*d)) + Tan[c + d*x]/(a*d) + Tan[c + d*x]/(d*(a + 
a*Sec[c + d*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\tan(c+dx)}{ad} - \int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= \frac{\tan(c+dx)}{ad} - \frac{\int \sec(c+dx)\,dx}{a} + \int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= -\frac{\tanh^{-1}(\sin(c+dx))}{ad} + \frac{\tan(c+dx)}{ad} + \frac{\tan(c+dx)}{d(a+a\sec(c+dx))}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 194 vs. $2(51) = 102$.
time = 0.83, size = 194, normalized size = 3.80

$$
\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)\right)}{ad(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]*(Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(a*d
*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.05, size = 74, normalized size = 1.45

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	74
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	74
risch	$\frac{2i\left(e^{2i(dx+c)}+e^{i(dx+c)}+2\right)}{da\left(e^{i(dx+c)}+1\right)\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}$	98
norman	$\frac{\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 1/d/a*(tan(1/2*d*x+1/2*c)-1/(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)-1)
-1/(tan(1/2*d*x+1/2*c)+1)-ln(tan(1/2*d*x+1/2*c)+1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(51) = 102$.
time = 0.30, size = 119, normalized size = 2.33

$$-\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.72, size = 97, normalized size = 1.90

$$-\frac{(\cos(dx+c)^2+\cos(dx+c))\log(\sin(dx+c)+1)-(\cos(dx+c)^2+\cos(dx+c))\log(-\sin(dx+c)+1)-2(2\cos(dx+c)+1)\sin(dx+c)}{2(ad\cos(dx+c)^2+ad\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - (cos(d*x + c)
^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) + 1)*sin(d*x
+ c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.46, size = 84, normalized size = 1.65

$$-\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))
/a - tan(1/2*d*x + 1/2*c)/a + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a))/d
```

Mupad [B]
time = 0.68, size = 67, normalized size = 1.31

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))),x)
```

```
[Out] (2*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2
+ (d*x)/2)))/(a*d) + tan(c/2 + (d*x)/2)/(a*d)
```

3.45 $\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/a/d-tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3874, 3855, 3879}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int \sec(c+dx)\,dx}{a}-\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(38) = 76.
time = 0.19, size = 109, normalized size = 2.87

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right)}{ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*Sec[c + d*x]*(Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*S
in[(d*x)/2]))/(a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.04, size = 46, normalized size = 1.21

method	result	size
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
risch	$-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}$	65
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-tan(1/2*d*x+1/2*c)-ln(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)+1
))
```

Maxima [A]
time = 0.27, size = 75, normalized size = 1.97

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.71, size = 65, normalized size = 1.71

$$\frac{(\cos(dx+c)+1)\log(\sin(dx+c)+1)-(\cos(dx+c)+1)\log(-\sin(dx+c)+1)-2\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.44, size = 54, normalized size = 1.42

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))/
a - tan(1/2*d*x + 1/2*c)/a)/d
```

Mupad [B]
time = 0.65, size = 31, normalized size = 0.82

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)) - tan(c/2 + (d*x)/2))/(a*d)
```

3.46 $\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{3879\}$

$$\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [A]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] Tan[(c + d*x)/2]/(a*d)
```

Maple [A]
time = 0.04, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
norman	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
risch	$\frac{2i}{da\left(e^{i(dx+c)}+1\right)}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)
```

Maxima [A]
time = 0.29, size = 23, normalized size = 1.05

$$\frac{\sin(dx+c)}{ad(\cos(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(a*d*(cos(d*x + c) + 1))
```

Fricas [A]
time = 2.31, size = 22, normalized size = 1.00

$$\frac{\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.44, size = 16, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] tan(1/2*d*x + 1/2*c)/(a*d)
```

Mupad [B]
time = 0.59, size = 16, normalized size = 0.73

$$\frac{\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(a*d)
```

3.47 $\int \frac{1}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{a}-\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] x/a-tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3862, 8}

$$\frac{x}{a}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-1),x]
```

```
[Out] x/a - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\sec(c+dx)}\,dx &= -\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [A]

time = 0.14, size = 58, normalized size = 2.00

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(dx\cos\left(\frac{dx}{2}\right)+dx\cos\left(c+\frac{dx}{2}\right)-2\sin\left(\frac{dx}{2}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-1),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(d*x*Cos[(d*x)/2] + d*x*Cos[c + (d*x)/2] - 2*Sin
[(d*x)/2]))/(2*a*d)
```

Maple [A]
time = 0.04, size = 32, normalized size = 1.10

method	result	size
norman	$\frac{x}{a}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	24
risch	$\frac{x}{a}-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}$	29
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	32
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-tan(1/2*d*x+1/2*c)+2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.49, size = 49, normalized size = 1.69

$$\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c
) + 1)))/d
```

Fricas [A]
time = 2.80, size = 37, normalized size = 1.28

$$\frac{dx\cos(dx+c)+dx-\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cos(d*x + c) + d*x - sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.44, size = 28, normalized size = 0.97

$$\frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a)/d
```

Mupad [B]
time = 0.64, size = 23, normalized size = 0.79

$$\frac{x}{a} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x)),x)
```

```
[Out] x/a - tan(c/2 + (d*x)/2)/(a*d)
```

3.48 $\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=44

$$-\frac{x}{a}+\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] -x/a+2*sin(d*x+c)/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3904, 3872, 2717, 8}

$$\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (2*Sin[c + d*x])/(a*d) - Sin[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos(c+dx)(-2a+a\sec(c+dx))\,dx}{a^2} \\
&= -\frac{\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int 1\,dx}{a} + \frac{2\int \cos(c+dx)\,dx}{a} \\
&= -\frac{x}{a} + \frac{2\sin(c+dx)}{ad} - \frac{\sin(c+dx)}{d(a+a\sec(c+dx))}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(44) = 88.
time = 0.24, size = 89, normalized size = 2.02

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-2dx\cos\left(\frac{dx}{2}\right)-2dx\cos\left(c+\frac{dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)+\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{3dx}{2}\right)\right)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-2*d*x*Cos[(d*x)/2] - 2*d*x*Cos[c + (d*x)/2] +
5*Sin[(d*x)/2] + Sin[c + (d*x)/2] + Sin[c + (3*d*x)/2] + Sin[2*c + (3*d*x)/
2]))/(4*a*d)
```

Maple [A]
time = 0.06, size = 56, normalized size = 1.27

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	56
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	56
risch	$-\frac{x}{a}-\frac{i e^{i(dx+c)}}{2ad}+\frac{i e^{-i(dx+c)}}{2ad}+\frac{2i}{da\left(e^{i(dx+c)}+1\right)}$	66
norman	$\frac{\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x}{a}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(tan(1/2*d*x+1/2*c)+2*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2*a
rctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.
time = 0.49, size = 92, normalized size = 2.09

$$-\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos
(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.88, size = 46, normalized size = 1.05

$$-\frac{dx\cos(dx+c) + dx - (\cos(dx+c)+2)\sin(dx+c)}{ad\cos(dx+c) + ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) + d*x - (cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x +
c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.46, size = 58, normalized size = 1.32

$$-\frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d
*x + 1/2*c)^2 + 1)*a))/d
```

Mupad [B]
time = 0.66, size = 66, normalized size = 1.50

$$\frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(-c-d\,x\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x)),x)
```

```
[Out] (sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)*(c + d*x) + 2*cos(c/2 + (d*x)/2)^2
*sin(c/2 + (d*x)/2))/(a*d*cos(c/2 + (d*x)/2))
```

3.49 $\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{3x}{2a}-\frac{2\sin(c+dx)}{ad}+\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] 3/2*x/a-2*sin(d*x+c)/a/d+3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)*sin(d*x+c
)/d/(a+a*sec(d*x+c))
```

Rubi [A]

time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {3904, 3872, 2715, 8, 2717}

$$-\frac{2\sin(c+dx)}{ad}+\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sin[c + d*x])/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) - (Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \cos^2(c+dx)(-3a+2a\sec(c+dx))\,dx}{a^2}\\ &= -\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{2\int \cos(c+dx)\,dx}{a}+\frac{3\int \cos^2(c+dx)\,dx}{a}\\ &= -\frac{2\sin(c+dx)}{ad}+\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{3\int 1\,dx}{2a}\\ &= \frac{3x}{2a}-\frac{2\sin(c+dx)}{ad}+\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 117, normalized size = 1.58

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(12dx\cos\left(\frac{dx}{2}\right)+12dx\cos\left(c+\frac{dx}{2}\right)-20\sin\left(\frac{dx}{2}\right)-4\sin\left(c+\frac{dx}{2}\right)-3\sin\left(c+\frac{3dx}{2}\right)-3\sin\left(2c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{5dx}{2}\right)\right)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(12*d*x*Cos[(d*x)/2] + 12*d*x*Cos[c + (d*x)/2] -
 20*Sin[(d*x)/2] - 4*Sin[c + (d*x)/2] - 3*Sin[c + (3*d*x)/2] - 3*Sin[2*c +
(3*d*x)/2] + Sin[2*c + (5*d*x)/2] + Sin[3*c + (5*d*x)/2]))/(16*a*d)
```

Maple [A]
time = 0.07, size = 74, normalized size = 1.00

method	result	size
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	74
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	74
risch	$\frac{3x}{2a}+\frac{i e^{i(dx+c)}}{2ad}-\frac{i e^{-i(dx+c)}}{2ad}-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}+\frac{\sin(2dx+2c)}{4ad}$	83

norman	$\frac{\frac{3x}{2a}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-tan(1/2*d*x+1/2*c)+2*(-3/2*tan(1/2*d*x+1/2*c)^3-1/2*tan(1/2*d*x+1/2
*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+3*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.50, size = 133, normalized size = 1.80

$$-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.32, size = 57, normalized size = 0.77

$$\frac{3\,dx\cos\left(dx+c\right)+3\,dx+\left(\cos\left(dx+c\right)^2-\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*d*x*cos(d*x + c) + 3*d*x + (cos(d*x + c)^2 - cos(d*x + c) - 4)*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^2\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.45, size = 73, normalized size = 0.99

$$\frac{\frac{3\,(dx+c)}{a}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a - 2*tan(1/2*d*x + 1/2*c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

Mupad [B]
time = 0.72, size = 89, normalized size = 1.20

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(c+d\,x)}{2}+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] -(sin(c/2 + (d*x)/2) - (3*cos(c/2 + (d*x)/2)*(c + d*x))/2 + 3*cos(c/2 + (d*
x)/2)^2*sin(c/2 + (d*x)/2) - 2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(a*
d*cos(c/2 + (d*x)/2))
```

3.50 $\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{3x}{2a}+\frac{4\sin(c+dx)}{ad}-\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{4\sin^3(c+dx)}{3ad}$$

```
[Out] -3/2*x/a+4*sin(d*x+c)/a/d-3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)^2*sin(d*
x+c)/d/(a+a*sec(d*x+c))-4/3*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3904, 3872, 2713, 2715, 8}

$$-\frac{4\sin^3(c+dx)}{3ad}+\frac{4\sin(c+dx)}{ad}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sin[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*
d) - (Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) - (4*Sin[c + d*
x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \cos^3(c+dx)(-4a+3a\sec(c+dx))\,dx}{a^2}\\ &= -\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{3\int \cos^2(c+dx)\,dx}{a}+\frac{4\int \cos^3(c+dx)\,dx}{a}\\ &= -\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{3\int 1\,dx}{2a}-\frac{4\text{Subst}\left(\int (1\right.}{}\\ &= -\frac{3x}{2a}+\frac{4\sin(c+dx)}{ad}-\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{4}{}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 143, normalized size = 1.52

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-36dx\cos\left(\frac{dx}{2}\right)-36dx\cos\left(c+\frac{dx}{2}\right)+69\sin\left(\frac{dx}{2}\right)+21\sin\left(c+\frac{dx}{2}\right)+18\sin\left(c+\frac{3dx}{2}\right)+18\sin\left(2c+\frac{3dx}{2}\right)-2\sin\left(2c+\frac{5dx}{2}\right)-2\sin\left(3c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{7dx}{2}\right)\right)}{48ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2]
+ 69*Sin[(d*x)/2] + 21*Sin[c + (d*x)/2] + 18*Sin[c + (3*d*x)/2] + 18*Sin[2*
c + (3*d*x)/2] - 2*Sin[2*c + (5*d*x)/2] - 2*Sin[3*c + (5*d*x)/2] + Sin[3*c
+ (7*d*x)/2] + Sin[4*c + (7*d*x)/2]))/(48*a*d)
```

Maple [A]
time = 0.07, size = 85, normalized size = 0.90

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$

default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
risch	$-\frac{3x}{2a}-\frac{7i e^{i(dx+c)}}{8ad}+\frac{7i e^{-i(dx+c)}}{8ad}+\frac{2i}{da\left(e^{i(dx+c)}+1\right)}+\frac{\sin(3dx+3c)}{12ad}-\frac{\sin(2dx+2c)}{4ad}$
norman	$\frac{\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3x}{2a}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{25\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{9x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{9x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3x\left(\tan^6\left(\right.\right.}{2a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(tan(1/2*d*x+1/2*c)-8*(-5/8*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*
c)^3-3/8*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^3-3*arctan(tan(1/2*d*
x+1/2*c)))
```

Maxima [A]
time = 0.50, size = 176, normalized size = 1.87

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3
*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 3.10, size = 70, normalized size = 0.74

$$-\frac{9\,dx\cos\left(dx+c\right)+9\,dx-\left(2\,\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}+7\,\cos\left(dx+c\right)+16\right)\sin\left(dx+c\right)}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(9*d*x*cos(d*x + c) + 9*d*x - (2*cos(d*x + c)^3 - cos(d*x + c)^2 + 7*c
os(d*x + c) + 16)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 88, normalized size = 0.94

$$-\frac{\frac{9\,(dx+c)}{a}-\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)/a - 6*tan(1/2*d*x + 1/2*c)/a - 2*(15*tan(1/2*d*x + 1/2*c)
^5 + 16*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/
2*c)^2 + 1)^3*a))/d
```

Mupad [B]
time = 0.91, size = 70, normalized size = 0.74

$$\frac{\frac{15\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{3\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}-\frac{\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{12}+\frac{\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{24}}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{3\,x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

```
[Out] ((15*sin(c/2 + (d*x)/2))/8 + (3*sin((3*c)/2 + (3*d*x)/2))/4 - sin((5*c)/2 +
 (5*d*x)/2)/12 + sin((7*c)/2 + (7*d*x)/2)/24)/(a*d*cos(c/2 + (d*x)/2)) - (3
*x)/(2*a)
```

3.51 $\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)} \, dx$

Optimal. Leaf size=118

$$\frac{15x}{8a}-\frac{4\sin(c+dx)}{ad}+\frac{15\cos(c+dx)\sin(c+dx)}{8ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{4\sin^3}{}$$

```
[Out] 15/8*x/a-4*sin(d*x+c)/a/d+15/8*cos(d*x+c)*sin(d*x+c)/a/d+5/4*cos(d*x+c)^3*s
in(d*x+c)/a/d-cos(d*x+c)^3*sin(d*x+c)/d/(a+a*sec(d*x+c))+4/3*sin(d*x+c)^3/a
/d
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3904, 3872, 2715, 8, 2713}

$$\frac{4\sin^3(c+dx)}{3ad}-\frac{4\sin(c+dx)}{ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{15\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}+\frac{15x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (15*x)/(8*a) - (4*Sin[c + d*x])/(a*d) + (15*Cos[c + d*x]*Sin[c + d*x])/(8*a
*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (Cos[c + d*x]^3*Sin[c + d*x
])/(d*(a + a*Sec[c + d*x])) + (4*Sin[c + d*x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos^4(c+dx)(-5a+4a\sec(c+dx))\,dx}{a^2} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{4\int \cos^3(c+dx)\,dx}{a} + \frac{5\int \cos^4(c+dx)\,dx}{a} \\
&= \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{15\int \cos^2(c+dx)\,dx}{4a} + \\
&= -\frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^3}{d(} \\
&= \frac{15x}{8a} - \frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} -
\end{aligned}$$

Mathematica [A]

time = 0.32, size = 173, normalized size = 1.47

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(360dx\cos\left(\frac{dx}{2}\right)+360dx\cos\left(c+\frac{dx}{2}\right)-552\sin\left(\frac{dx}{2}\right)-168\sin\left(c+\frac{dx}{2}\right)-120\sin\left(c+\frac{3dx}{2}\right)-120\sin\left(2c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{5dx}{2}\right)+40\sin\left(3c+\frac{5dx}{2}\right)-5\sin\left(3c+\frac{7dx}{2}\right)-5\sin\left(4c+\frac{7dx}{2}\right)+3\sin\left(4c+\frac{9dx}{2}\right)+3\sin\left(5c+\frac{9dx}{2}\right)\right)}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(360*d*x*Cos[(d*x)/2] + 360*d*x*Cos[c + (d*x)/2]
 - 552*Sin[(d*x)/2] - 168*Sin[c + (d*x)/2] - 120*Sin[c + (3*d*x)/2] - 120*S
in[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c + (5*d*x)/2] - 5
*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + 3*Sin[4*c + (9*d*x)/2] + 3
*Sin[5*c + (9*d*x)/2]))/(384*a*d)
```

Maple [A]

time = 0.08, size = 100, normalized size = 0.85

method	result

derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-\frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{115\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-\frac{109\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{15\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-\frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{115\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-\frac{109\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{15\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
risch	$\frac{15x}{8a}+\frac{7i\mathrm{e}^{i(dx+c)}}{8ad}-\frac{7i\mathrm{e}^{-i(dx+c)}}{8ad}-\frac{2i}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{\sin(4dx+4c)}{32ad}-\frac{\sin(3dx+3c)}{12ad}+\frac{\sin(2dx+2c)}{2ad}$
norman	$\frac{\frac{15x}{8a}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{157\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{187\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{41\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{15x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{45x}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-tan(1/2*d*x+1/2*c)+2*(-25/8*tan(1/2*d*x+1/2*c)^7-115/24*tan(1/2*d*x
+1/2*c)^5-109/24*tan(1/2*d*x+1/2*c)^3-7/8*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*
x+1/2*c)^2)^4+15/4*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.51, size = 217, normalized size = 1.84

$$-\frac{\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\,\sin(dx+c)}{a(\cos(dx+c)+1)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.66, size = 79, normalized size = 0.67

$$\frac{45\,dx\cos\left(dx+c\right)+45\,dx+\left(6\,\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^3+13\,\cos\left(dx+c\right)^2-19\,\cos\left(dx+c\right)-64\right)\sin\left(dx+c\right)}{24\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(45*d*x*cos(d*x + c) + 45*d*x + (6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 +
 13*cos(d*x + c)^2 - 19*cos(d*x + c) - 64)*sin(d*x + c))/(a*d*cos(d*x + c)
+ a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.43, size = 101, normalized size = 0.86

$$\frac{\frac{45\,(dx+c)}{a} - \frac{24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+115\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+109\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(45*(d*x + c)/a - 24*tan(1/2*d*x + 1/2*c)/a - 2*(75*tan(1/2*d*x + 1/2*
c)^7 + 115*tan(1/2*d*x + 1/2*c)^5 + 109*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

Mupad [B]
time = 2.45, size = 98, normalized size = 0.83

$$\frac{15\,x}{8\,a} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{115\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] (15*x)/(8*a) - tan(c/2 + (d*x)/2)/(a*d) - ((7*tan(c/2 + (d*x)/2))/4 + (109*
tan(c/2 + (d*x)/2)^3)/12 + (115*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 + (d
*x)/2)^7)/4)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.52 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sec^3(c+dx)}{3d(a+a\sec}$$

```
[Out] 7/2*arctanh(sin(d*x+c))/a^2/d-16/3*tan(d*x+c)/a^2/d+7/2*sec(d*x+c)*tan(d*x+
c)/a^2/d-8/3*sec(d*x+c)^2*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^3*
tan(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3901, 4104, 3872, 3852, 8, 3853, 3855}

$$-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{8\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (16*Tan[c + d*x])/(3*a^2*d) + (7*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*d) - (8*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d
*(1 + Sec[c + d*x])) - (Sec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*
x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \frac{\sec^3(c+dx)(3a-5a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}\\
&= -\frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \sec^2(c+dx)\,(16a\ldots}{\ldots}\\
&= -\frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{16\int \sec^2(c+dx)\,d\ldots}{3a^2}\\
&= \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sec^3(c+dx)\tan(c+\ldots}{3d(a+a\sec(c+d\ldots}\\
&= \frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{8\sec^2(\ldots}{3a^2d\ldots}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 300 vs. 2(123) = 246.
time = 2.00, size = 300, normalized size = 2.44

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-40\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+3\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{8\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)-2\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{3a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^2*(-2*Sec[c/2]*Sin[(d*x)/2] - 40*Cos[(c + d*
x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]^3*(-14*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] + 14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^(-2) - (8*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) -
2*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.09, size = 120, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{\tan(}}{2d\,a^2}$
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{\tan(}}{2d\,a^2}$
risch	$-\frac{i\left(21\,e^{6i(dx+c)}+63\,e^{5i(dx+c)}+98\,e^{4i(dx+c)}+126\,e^{3i(dx+c)}+97\,e^{2i(dx+c)}+75\,e^{i(dx+c)}+32\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^2\left(e^{i(dx+c)}+1\right)^3}+\frac{7\ln\left(e^{i(dx+c)}+i\right)}{2a^2d}-\frac{7\ln}{}$
norman	$\frac{-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{149\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{100\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{18\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{17\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4a}-\frac{7\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*x+1/2*c)+1/(tan(1/2*d*x+1/
2*c)-1)^2+5/(tan(1/2*d*x+1/2*c)-1)-7*ln(tan(1/2*d*x+1/2*c)-1)-1/(tan(1/2*d*
x+1/2*c)+1)^2+5/(tan(1/2*d*x+1/2*c)+1)+7*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.28, size = 190, normalized size = 1.54

$$-\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A]
time = 3.75, size = 162, normalized size = 1.32

$$\frac{21\left(\cos\left(dx+c\right)^{4}+2\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-21\left(\cos\left(dx+c\right)^{4}+2\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(32\cos\left(dx+c\right)^{3}+43\cos\left(dx+c\right)^{2}+6\cos\left(dx+c\right)-3\right)\sin\left(dx+c\right)}{12\left(a^{2}d\cos\left(dx+c\right)^{4}+2a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x +
 c) + 1) - 21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin
(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 + 43*cos(d*x + c)^2 + 6*cos(d*x + c)
- 3)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*c
os(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{5}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.51, size = 122, normalized size = 0.99

$$\frac{\frac{21\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{2}}-\frac{21\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^{2}}+\frac{6\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a^{2}}-\frac{a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+21a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{6}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 21*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/a^2 + 6*(5*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((t
```

```
an(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 21*a^4*ta
n(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 0.74, size = 122, normalized size = 0.99

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{6\,a^2\,d}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a^2\right)}-\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^2),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (3
*tan(c/2 + (d*x)/2) - 5*tan(c/2 + (d*x)/2)^3)/(d*(a^2*tan(c/2 + (d*x)/2)^4
- 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (7*tan(c/2 + (d*x)/2))/(2*a^2*d)
```

3.53 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=89

$$-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{4\tan(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] -2*arctanh(sin(d*x+c))/a^2/d+4/3*tan(d*x+c)/a^2/d+2*tan(d*x+c)/a^2/d/(1+sec
(d*x+c))-1/3*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3901, 4093, 3872, 3855, 3852, 8}

$$\frac{4\tan(c+dx)}{3a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2\tan(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*ArcTanh[Sin[c + d*x]])/(a^2*d) + (4*Tan[c + d*x])/(3*a^2*d) + (2*Tan[c
+ d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a
+ a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(2a-4a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sec(c+dx)\left(-6a^2+4a^2 \text{s}\right.}{3a^4}$$
$$= \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{4\int \sec^2(c+dx)\,dx}{3a^2} - \frac{2\int \text{se}}{}$$
$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} -$$
$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{4\tan(c+dx)}{3a^2d} + \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx}{3d(a+a\,\text{se}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 247 vs. 2(89) = 178.
time = 1.26, size = 247, normalized size = 2.78

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+14\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{3a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^2*(Sec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x
)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*(2*Log[Cos[(c + d*x)/2]
```

```
 - Sin[(c + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x
]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/
2]))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.07, size = 92, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^2}$
risch	$\frac{4i\left(3\,e^{4i(dx+c)}+9\,e^{3i(dx+c)}+11\,e^{2i(dx+c)}+12\,e^{i(dx+c)}+5\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+1\right)}+\frac{2\ln\left(e^{i(dx+c)}-i\right)}{a^2d}-\frac{2\ln\left(e^{i(dx+c)}+i\right)}{a^2d}$
norman	$\frac{-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{34\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3a}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}-2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+5*tan(1/2*d*x+1/2*c)-2/(tan(1/2*d*x+1/2
*c)-1)+4*ln(tan(1/2*d*x+1/2*c)-1)-2/(tan(1/2*d*x+1/2*c)+1)-4*ln(tan(1/2*d*x
+1/2*c)+1))
```

Maxima [A]
time = 0.28, size = 145, normalized size = 1.63

$$\frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.95, size = 146, normalized size = 1.64

$$-\frac{3\left(\cos(dx+c)^3+2\,\cos(dx+c)^2+\cos(dx+c)\right)\log\left(\sin(dx+c)+1\right)-3\left(\cos(dx+c)^3+2\,\cos(dx+c)^2+\cos(dx+c)\right)\log\left(-\sin(dx+c)+1\right)-\left(10\,\cos(dx+c)^2+14\,\cos(dx+c)+3\right)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^3+2\,a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c)
 + 1) - 3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x +
 c) + 1) - (10*cos(d*x + c)^2 + 14*cos(d*x + c) + 3)*sin(d*x + c))/(a^2*d*c
os(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.46, size = 106, normalized size = 1.19

$$-\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^
2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 0.69, size = 92, normalized size = 1.03

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{6a^2d}-\frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^2\right)}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (2
*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2)) + (5*tan(c/2 + (d
*x)/2))/(2*a^2*d)
```

3.54 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)}{3a^2d(1+\sec(c+dx))}+\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] arctanh(sin(d*x+c))/a^2/d-5/3*tan(d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*tan(d*x+c
)/d/(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3884, 4083, 3855, 3879}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^2*d) - (5*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x]
)) + Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
```

```
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\sec(c+dx)(-2a+3a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}\\
&= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \sec(c+dx)\,dx}{a^2}-\frac{5\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{5\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(66) = 132.
time = 0.38, size = 160, normalized size = 2.42

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+8\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{3a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*Sec[c + d*x]^2*(6*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[
c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d
*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.07, size = 62, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^2}$
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^2}$
risch	$-\frac{2i\left(3\,e^{2i(dx+c)}+9\,e^{i(dx+c)}+4\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{a^2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{a^2d}$
norman	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{17\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2a}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3-3*tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+
1/2*c)-1)+2*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.28, size = 98, normalized size = 1.48

$$-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A]
time = 3.12, size = 114, normalized size = 1.73

$$\frac{3\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(4\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{6\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 3*(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c
) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.47, size = 77, normalized size = 1.17

$$\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a^2 - (a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

Mupad [B]
time = 0.64, size = 43, normalized size = 0.65

$$-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(9*tan(c/2 + (d*x)/2) - 12*atanh(tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)^
3)/(6*a^2*d)
```

3.55 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{2\tan(c+dx)}{3d\,(a^2+a^2\sec(c+dx))}$$

```
[Out] -1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+2/3*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3882, 3879}

$$\frac{2\tan(c+dx)}{3d\,(a^2\sec(c+dx)+a^2)}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(3*d*(a^2 +
 a^2*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{2\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= -\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{2\tan(c+dx)}{3d\,(a^2+a^2\sec(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 0.82

$$\frac{\sec^3\left(\frac{1}{2}(c+dx)\right)\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
[Out] (Sec[(c + d*x)/2]^3*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(12*a^2*d)
```

Maple [A]
time = 0.05, size = 32, normalized size = 0.58

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$	32
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$	32
risch	$\frac{2i\left(1+3\,\mathrm{e}^{i(dx+c)}\right)}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$	36
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.84

$$\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

Fricas [A]
time = 3.30, size = 49, normalized size = 0.89

$$\frac{(\cos(dx+c)+2)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c) + 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
 + c) + a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [**A**]
time = 0.49, size = 31, normalized size = 0.56

$$\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

Mupad [**B**]
time = 0.60, size = 30, normalized size = 0.55

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 3))/(6*a^2*d)
```

3.56 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

```
[Out] 1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3881, 3879}

$$\frac{\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*d*(a^2 + a^2*Se
c[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 60, normalized size = 1.09

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(3\sin\left(\frac{dx}{2}\right)-3\sin\left(c+\frac{dx}{2}\right)+2\sin\left(c+\frac{3dx}{2}\right)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(3*Sin[(d*x)/2] - 3*Sin[c + (d*x)/2] + 2*Sin[c
 + (3*d*x)/2]))/(12*a^2*d)
```

Maple [A]
time = 0.05, size = 32, normalized size = 0.58

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$	32
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$	32
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}}{a}$	42
risch	$\frac{2i\left(3\,e^{2i(dx+c)}+3\,e^{i(dx+c)}+2\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.85

$$\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

Fricas [A]
time = 2.41, size = 51, normalized size = 0.93

$$\frac{(2\,\cos(dx+c)+1)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c) + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.46, size = 31, normalized size = 0.56

$$-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

Mupad [B]
time = 0.60, size = 30, normalized size = 0.55

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 3))/(6*a^2*d)
```

3.57 $\int \frac{1}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$\frac{x}{a^2}-\frac{4\tan(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] x/a^2-4/3*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))
^2
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {3862, 4004, 3879}

$$-\frac{4\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x}{a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-2),x]
```

```
[Out] x/a^2 - (4*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - Tan[c + d*x]/(3*d*(
a + a*Sec[c + d*x])^2)
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = -\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-3a+a\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= \frac{x}{a^2} - \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{4\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}$$
$$= \frac{x}{a^2} - \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{4\tan(c+dx)}{3d\,(a^2+a^2\sec(c+dx))}$$

Mathematica [A]
time = 0.30, size = 112, normalized size = 1.96

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(9dx\cos\left(\frac{dx}{2}\right)+9dx\cos\left(c+\frac{dx}{2}\right)+3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\cos\left(2c+\frac{3dx}{2}\right)-18\sin\left(\frac{dx}{2}\right)+12\sin\left(c+\frac{dx}{2}\right)-10\sin\left(c+\frac{3dx}{2}\right)\right)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-2),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] + 9*d*x*Cos[c + (d*x)/2] +
 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 18*Sin[(d*x)/2] +
12*Sin[c + (d*x)/2] - 10*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A]
time = 0.05, size = 46, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	46
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	46
norman	$\frac{\frac{x}{a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}}{a}$	47
risch	$\frac{x}{a^2}-\frac{2i\left(6\,e^{2i(dx+c)}+9\,e^{i(dx+c)}+5\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3-3*tan(1/2*d*x+1/2*c)+4*arctan(tan(1/2*d
*x+1/2*c)))
```

Maxima [A]
time = 0.50, size = 72, normalized size = 1.26

$$-\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 2.99, size = 80, normalized size = 1.40

$$\frac{3\,dx\cos\left(dx+c\right)^{2}+6\,dx\cos\left(dx+c\right)+3\,dx-\left(5\,\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)}{3\,\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (5*cos(d*x + c) +
4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.44, size = 50, normalized size = 0.88

$$\frac{\frac{6\,(dx+c)}{a^{2}}+\frac{a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/a^2 + (a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/a^6)/d
```

Mupad [B]
time = 0.63, size = 35, normalized size = 0.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,d\,x}{6\,a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3 - 9*tan(c/2 + (d*x)/2) + 6*d*x)/(6*a^2*d)
```

3.58 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=72

$$-\frac{2x}{a^2}+\frac{10\sin(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] -2*x/a^2+10/3*sin(d*x+c)/a^2/d-2*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*
x+c)/d/(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3902, 4105, 3872, 2717, 8}

$$\frac{10\sin(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{2x}{a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 + (10*Sin[c + d*x])/(3*a^2*d) - (2*Sin[c + d*x])/(a^2*d*(1 + Sec
[c + d*x])) - Sin[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
```

```
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)(-4a+2a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2} \\ &= -\frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos(c+dx)\,(-10a^2+6a}{3a^4} \\ &= -\frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{2\int 1\,dx}{a^2} + \frac{10\int \cos(c+dx}{3a^2} \\ &= -\frac{2x}{a^2} + \frac{10\sin(c+dx)}{3a^2d} - \frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(72) = 144.
time = 0.51, size = 151, normalized size = 2.10

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-36dx\cos\left(\frac{dx}{2}\right)-36dx\cos\left(c+\frac{dx}{2}\right)-12dx\cos\left(c+\frac{3dx}{2}\right)-12dx\cos\left(2c+\frac{3dx}{2}\right)+66\sin\left(\frac{dx}{2}\right)-30\sin\left(c+\frac{dx}{2}\right)+41\sin\left(c+\frac{3dx}{2}\right)+9\sin\left(2c+\frac{3dx}{2}\right)+3\sin\left(2c+\frac{5dx}{2}\right)+3\sin\left(3c+\frac{5dx}{2}\right)\right)}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2
] - 12*d*x*Cos[c + (3*d*x)/2] - 12*d*x*Cos[2*c + (3*d*x)/2] + 66*Sin[(d*x)/
2] - 30*Sin[c + (d*x)/2] + 41*Sin[c + (3*d*x)/2] + 9*Sin[2*c + (3*d*x)/2] +
 3*Sin[2*c + (5*d*x)/2] + 3*Sin[3*c + (5*d*x)/2]))/(48*a^2*d)
```

Maple [**A**]
time = 0.07, size = 72, normalized size = 1.00

method	result	size

derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	72
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	72
risch	$-\frac{2x}{a^2}-\frac{i\mathrm{e}^{i(dx+c)}}{2a^2d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2a^2d}+\frac{2i\left(9\,\mathrm{e}^{2i(dx+c)}+15\,\mathrm{e}^{i(dx+c)}+8\right)}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$	90
norman	$\frac{-\frac{2x}{a}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3+5*tan(1/2*d*x+1/2*c)+4*tan(1/2*d*x+1/2
*c)/(1+tan(1/2*d*x+1/2*c)^2)-8*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.50, size = 118, normalized size = 1.64

$$\frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x + c)
/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.81, size = 90, normalized size = 1.25

$$-\frac{6\,dx\cos\left(dx+c\right)^2+12\,dx\cos\left(dx+c\right)+6\,dx-\left(3\,\cos\left(dx+c\right)^2+14\,\cos\left(dx+c\right)+10\right)\sin\left(dx+c\right)}{3\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*d*x*cos(d*x + c)^2 + 12*d*x*cos(d*x + c) + 6*d*x - (3*cos(d*x + c)^
2 + 14*cos(d*x + c) + 10)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos{\left(c+dx \right)}}{\sec^{2}{\left(c+dx \right)}+2\sec{\left(c+dx \right)}+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.46, size = 79, normalized size = 1.10

$$-\frac{\frac{12\,(dx+c)}{a^2}-\frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*(d*x + c)/a^2 - 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 +
 1)*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/
d
```

Mupad [B]
time = 0.70, size = 91, normalized size = 1.26

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(c+d\,x)}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(sin(c/2 + (d*x)/2) - 16*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 12*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6
*a^2*d*cos(c/2 + (d*x)/2)^3)
```

3.59 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=110

$$\frac{7x}{2a^2}-\frac{16\sin(c+dx)}{3a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] 7/2*x/a^2-16/3*sin(d*x+c)/a^2/d+7/2*cos(d*x+c)*sin(d*x+c)/a^2/d-8/3*cos(d*x
+c)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*
x+c))^2
```

Rubi [A]
time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3902, 4105, 3872, 2715, 8, 2717}

$$-\frac{16\sin(c+dx)}{3a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{8\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7x}{2a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*x)/(2*a^2) - (16*Sin[c + d*x])/(3*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])
/(2*a^2*d) - (8*Cos[c + d*x]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (
Cos[c + d*x]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(-5a+3a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2} \\
&= -\frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos^2(c+dx)\left(-21a\right.}{} \\
&= -\frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{16\int \cos(c+dx)\,dx}{3a^2} + \\
&= -\frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\mathrm{co}}{3a} \\
&= \frac{7x}{2a^2} - \frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 177, normalized size = 1.61

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(252dx\cos\left(\frac{dx}{2}\right)+252dx\cos\left(c+\frac{dx}{2}\right)+84dx\cos\left(c+\frac{3dx}{2}\right)+84dx\cos\left(2c+\frac{3dx}{2}\right)-381\sin\left(\frac{dx}{2}\right)+147\sin\left(c+\frac{dx}{2}\right)-239\sin\left(c+\frac{3dx}{2}\right)-63\sin\left(2c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{5dx}{2}\right)-15\sin\left(3c+\frac{5dx}{2}\right)+3\sin\left(3c+\frac{7dx}{2}\right)+3\sin\left(4c+\frac{7dx}{2}\right)\right)}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(252*d*x*Cos[(d*x)/2] + 252*d*x*Cos[c + (d*x)/2] + 84*d*x*Cos[c + (3*d*x)/2] + 84*d*x*Cos[2*c + (3*d*x)/2] - 381*Sin[(d*x)/2] + 147*Sin[c + (d*x)/2] - 239*Sin[c + (3*d*x)/2] - 63*Sin[2*c + (3*d*x)/2] - 15*Sin[2*c + (5*d*x)/2] - 15*Sin[3*c + (5*d*x)/2] + 3*Sin[3*c + (7*d*x)/2] + 3*Sin[4*c + (7*d*x)/2]))/(192*a^2*d)

Maple [A]
time = 0.08, size = 88, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-10\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+14\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	88
default	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-10\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+14\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	88
risch	$\frac{7x}{2a^{2}}-\frac{i\mathrm{e}^{2i(dx+c)}}{8a^{2}d}+\frac{i\mathrm{e}^{i(dx+c)}}{a^{2}d}-\frac{i\mathrm{e}^{-i(dx+c)}}{a^{2}d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^{2}d}-\frac{2i\left(12\,\mathrm{e}^{2i(dx+c)}+21\,\mathrm{e}^{i(dx+c)}+11\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	126
norman	$\frac{\frac{7x}{2a}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{71\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{19\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}+\frac{7x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{7x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a}$	135

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*x+1/2*c)+8*(-5/4*tan(1/2*d*x+1/2*c)^3-3/4*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+14*arctan(tan(1/2*d*x+1/2*c)))

Maxima [A]
time = 0.51, size = 164, normalized size = 1.49

$$-\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{2}+\frac{2a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d

Fricas [A]
time = 2.48, size = 99, normalized size = 0.90

$$\frac{21\,dx\cos(dx+c)^{2}+42\,dx\cos(dx+c)+21\,dx+\left(3\,\cos(dx+c)^{3}-6\,\cos(dx+c)^{2}-43\,\cos(dx+c)-32\right)\sin(dx+c)}{6\left(a^{2}d\cos(dx+c)^{2}+2\,a^{2}d\cos(dx+c)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(21*d*x*cos(d*x + c)^2 + 42*d*x*cos(d*x + c) + 21*d*x + (3*cos(d*x + c)^3 - 6*cos(d*x + c)^2 - 43*cos(d*x + c) - 32)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**2,x)

[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2

Giac [A]
time = 0.44, size = 95, normalized size = 0.86

$$\frac{\frac{21\,(dx+c)}{a^2}-\frac{6\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] 1/6*(21*(d*x + c)/a^2 - 6*(5*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 21*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d

Mupad [B]
time = 0.73, size = 113, normalized size = 1.03

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-22\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-30\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3(c+d\,x)}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^2,x)

[Out] (sin(c/2 + (d*x)/2) - 22*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 30*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) + 21*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)

3.60 $\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{5x}{a^2}+\frac{12\sin(c+dx)}{a^2d}-\frac{5\cos(c+dx)\sin(c+dx)}{a^2d}-\frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}-4\,\mathrm{si}$$

```
[Out] -5*x/a^2+12*sin(d*x+c)/a^2/d-5*cos(d*x+c)*sin(d*x+c)/a^2/d-10/3*cos(d*x+c)^
2*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*
x+c))^2-4*sin(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3902, 4105, 3872, 2713, 2715, 8}

$$-\frac{4\sin^3(c+dx)}{a^2d}+\frac{12\sin(c+dx)}{a^2d}-\frac{5\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{10\sin(c+dx)\cos^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{5x}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-5*x)/a^2 + (12*Sin[c + d*x])/(a^2*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(a^2
*d) - (10*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Cos[
c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*Sin[c + d*x]^3)/
(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^3(c+dx)(-6a+4a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2} \\
&= -\frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos^3(c+dx)\,(-3
\\
&= -\frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{10\int \cos^2(c+dx)\,c}{a^2}
\\
&= -\frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c}{3d(a+a\sec(c+}
\\
&= -\frac{5x}{a^2} + \frac{12\sin(c+dx)}{a^2d} - \frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^2(c+dx)\sin(c+d}{3a^2d(1+\sec(c+dx))}
\end{aligned}$$

Mathematica [A]

time = 0.36, size = 199, normalized size = 1.60

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-360dx\cos\left(\frac{dx}{2}\right)-360dx\cos\left(c+\frac{dx}{2}\right)-120dx\cos\left(c+\frac{3dx}{2}\right)-120dx\cos\left(2c+\frac{3dx}{2}\right)+516\sin\left(\frac{dx}{2}\right)-156\sin\left(c+\frac{dx}{2}\right)+342\sin\left(c+\frac{3dx}{2}\right)+118\sin\left(2c+\frac{3dx}{2}\right)+30\sin\left(2c+\frac{5dx}{2}\right)+30\sin\left(3c+\frac{5dx}{2}\right)-3\sin\left(3c+\frac{7dx}{2}\right)-3\sin\left(4c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)+\sin\left(5c+\frac{9dx}{2}\right)\right)}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-360*d*x*Cos[(d*x)/2] - 360*d*x*Cos[c + (d*x)
/2] - 120*d*x*Cos[c + (3*d*x)/2] - 120*d*x*Cos[2*c + (3*d*x)/2] + 516*Sin[(
d*x)/2] - 156*Sin[c + (d*x)/2] + 342*Sin[c + (3*d*x)/2] + 118*Sin[2*c + (3*
d*x)/2] + 30*Sin[2*c + (5*d*x)/2] + 30*Sin[3*c + (5*d*x)/2] - 3*Sin[3*c + (
7*d*x)/2] - 3*Sin[4*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2] + Sin[5*c + (9*d*
x)/2]))/(192*a^2*d)
```

Maple [A]
time = 0.08, size = 101, normalized size = 0.81

method	result
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$
risch	$-\frac{5x}{a^2}+\frac{i\,\mathrm{e}^{2i(dx+c)}}{4a^2d}-\frac{15i\,\mathrm{e}^{i(dx+c)}}{8a^2d}+\frac{15i\,\mathrm{e}^{-i(dx+c)}}{8a^2d}-\frac{i\,\mathrm{e}^{-2i(dx+c)}}{4a^2d}+\frac{2i\left(15\,\mathrm{e}^{2i(dx+c)}+27\,\mathrm{e}^{i(dx+c)}+14\right)}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}+\frac{\sin(3dx+}{12a^2}$
norman	$\frac{-\frac{5x}{a}+\frac{21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{80\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{23\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{15x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{15x(\mathrm{t}}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3+9*tan(1/2*d*x+1/2*c)-8*(-5/2*tan(1/2*d
*x+1/2*c)^5-10/3*tan(1/2*d*x+1/2*c)^3-3/2*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*
x+1/2*c)^2)^3-20*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.49, size = 207, normalized size = 1.67

$$\frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*s
```

```
in(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1) -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^2)/d
```

Fricas [A]
time = 2.48, size = 108, normalized size = 0.87

$$-\frac{15\,dx\cos\left(dx+c\right)^{2}+30\,dx\cos\left(dx+c\right)+15\,dx-\left(\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+33\,\cos\left(dx+c\right)+24\right)\sin\left(dx+c\right)}{3\,\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(15*d*x*cos(d*x + c)^2 + 30*d*x*cos(d*x + c) + 15*d*x - (cos(d*x + c)^
4 - cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 33*cos(d*x + c) + 24)*sin(d*x + c))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 108, normalized size = 0.87

$$-\frac{\frac{30\,(dx+c)}{a^{2}}-\frac{4\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a^{2}}+\frac{a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-27\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(30*(d*x + c)/a^2 - 4*(15*tan(1/2*d*x + 1/2*c)^5 + 20*tan(1/2*d*x + 1/
2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (a^
4*tan(1/2*d*x + 1/2*c)^3 - 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 0.78, size = 135, normalized size = 1.09

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-60\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+30\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\left(c+d\,x\right)}{6\,a^{2}\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a/cos(c + d*x))^2,x)

[Out] -(sin(c/2 + (d*x)/2) - 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 60*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*
x)/2) - 16*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 30*cos(c/2 + (d*x)/2)^
3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

3.61 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=162

$$\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{11\sec^3(c$$

```
[Out] 13/2*arctanh(sin(d*x+c))/a^3/d-152/15*tan(d*x+c)/a^3/d+13/2*sec(d*x+c)*tan(
d*x+c)/a^3/d-1/5*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-11/15*sec(d*x
+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-76/15*sec(d*x+c)^2*tan(d*x+c)/d/(a^
3+a^3*sec(d*x+c))
```

Rubi [A]

time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3901, 4104, 3872, 3852, 8, 3853, 3855}

$$-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{76\tan(c+dx)\sec^2(c+dx)}{15d\,(a^3\sec(c+dx)+a^3)}+\frac{13\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{11\tan(c+dx)\sec^3(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (152*Tan[c + d*x])/(15*a^3*d) + (13*
Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - (Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) - (11*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Se
c[c + d*x])^2) - (76*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c +
d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^4(c+dx)(4a-7a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^3(c+dx)(33a^2-43}{a+a\sec(c+}}{15a^4}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\sec^2(c+dx)\,\mathrm{ta}}{15d\,(a^3+a^3\sec(}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\sec^2(c+dx)\,\mathrm{ta}}{15d\,(a^3+a^3\sec(}$$

$$= \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(}{15ad(a+a\sec(c+}$$

$$= \frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{152\tan(c+dx)}{15a^3d} + \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\mathrm{sec}}{5a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(162) = 324.
time = 1.02, size = 351, normalized size = 2.17

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(24960\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\sec(c)\sec^2(c+dx)\left(-1235\sin\left(\frac{dx}{2}\right)+3805\sin\left(\frac{3dx}{2}\right)-4329\sin\left(c-\frac{dx}{2}\right)+1989\sin\left(c+\frac{dx}{2}\right)-3575\sin\left(2c+\frac{dx}{2}\right)-475\sin\left(c+\frac{3dx}{2}\right)+2005\sin\left(2c+\frac{3dx}{2}\right)-2275\sin\left(3c+\frac{3dx}{2}\right)+2673\sin\left(c+\frac{5dx}{2}\right)+105\sin\left(2c+\frac{5dx}{2}\right)+1593\sin\left(3c+\frac{5dx}{2}\right)-975\sin\left(4c+\frac{5dx}{2}\right)+1325\sin\left(2c+\frac{7dx}{2}\right)+255\sin\left(3c+\frac{7dx}{2}\right)+875\sin\left(4c+\frac{7dx}{2}\right)-195\sin\left(5c+\frac{7dx}{2}\right)+304\sin\left(3c+\frac{9dx}{2}\right)+90\sin\left(4c+\frac{9dx}{2}\right)+214\sin\left(5c+\frac{9dx}{2}\right)\right)\right)}{480a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/480*(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(24960*Cos[(c + d*x)/2]^5*(Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) + Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-1235*Sin[(d*x)/2] + 3805*Sin[(3*d*x)/2
] - 4329*Sin[c - (d*x)/2] + 1989*Sin[c + (d*x)/2] - 3575*Sin[2*c + (d*x)/2]
 - 475*Sin[c + (3*d*x)/2] + 2005*Sin[2*c + (3*d*x)/2] - 2275*Sin[3*c + (3*d
*x)/2] + 2673*Sin[c + (5*d*x)/2] + 105*Sin[2*c + (5*d*x)/2] + 1593*Sin[3*c
+ (5*d*x)/2] - 975*Sin[4*c + (5*d*x)/2] + 1325*Sin[2*c + (7*d*x)/2] + 255*S
in[3*c + (7*d*x)/2] + 875*Sin[4*c + (7*d*x)/2] - 195*Sin[5*c + (7*d*x)/2] +
 304*Sin[3*c + (9*d*x)/2] + 90*Sin[4*c + (9*d*x)/2] + 214*Sin[5*c + (9*d*x)
/2])))/(a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.12, size = 135, normalized size = 0.83

method	result
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-26\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-}{4d\,a^3}$

default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-26\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{}{(\tan}}{4d\,a^3}$
risch	$-\frac{i\left(195\,e^{8i(dx+c)}+975\,e^{7i(dx+c)}+2275\,e^{6i(dx+c)}+3575\,e^{5i(dx+c)}+4329\,e^{4i(dx+c)}+3805\,e^{3i(dx+c)}+2673\,e^{2i(dx+c)}+1325\,e\right.}{15d\,a^3\left(e^{2i(dx+c)}+1\right)^2\left(e^{i(dx+c)}+1\right)^5}$
norman	$\frac{\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{721\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}+\frac{6613\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60ad}-\frac{1165\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}+\frac{475\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{59\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5-8/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d*
x+1/2*c)+2/(tan(1/2*d*x+1/2*c)-1)^2+14/(tan(1/2*d*x+1/2*c)-1)-26*ln(tan(1/2
*d*x+1/2*c)-1)-2/(tan(1/2*d*x+1/2*c)+1)^2+14/(tan(1/2*d*x+1/2*c)+1)+26*ln(t
an(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.29, size = 211, normalized size = 1.30

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x + c
)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [A]
time = 2.25, size = 206, normalized size = 1.27

$$\frac{195\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-195\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)-2\left(304\cos(dx+c)^4+717\cos(dx+c)^3+479\cos(dx+c)^2+45\cos(dx+c)-15\right)\sin(dx+c)}{60\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+a^3d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x +
 c)^2)*log(sin(d*x + c) + 1) - 195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*c
os(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(304*cos(d*x + c
)^4 + 717*cos(d*x + c)^3 + 479*cos(d*x + c)^2 + 45*cos(d*x + c) - 15)*sin(d
```

*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.45, size = 139, normalized size = 0.86

$$\frac{\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{60\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}-\frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+40a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+465a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 390*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3 + 60*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 40
*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 0.68, size = 141, normalized size = 0.87

$$\frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{20\,a^3\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\,a^3\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^3\right)}-\frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (13*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^5/(20*a^3*d) -
(2*tan(c/2 + (d*x)/2)^3)/(3*a^3*d) - (5*tan(c/2 + (d*x)/2) - 7*tan(c/2 + (d
*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^3))
 - (31*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

3.62 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=128

$$-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{9\tan(c+dx)}{5a^3d}-\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{3\tan(c}{d\left(a^3+a^3\mathrm{se}\right.}$$

```
[Out] -3*arctanh(sin(d*x+c))/a^3/d+9/5*tan(d*x+c)/a^3/d-1/5*sec(d*x+c)^3*tan(d*x+
c)/d/(a+a*sec(d*x+c))^3-3/5*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+
3*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3901, 4104, 4093, 3872, 3855, 3852, 8}

$$\frac{9\tan(c+dx)}{5a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{3\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{3\tan(c+dx)\sec^2(c+dx)}{5ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + (9*Tan[c + d*x])/(5*a^3*d) - (Sec[c +
d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (3*Sec[c + d*x]^2*Tan[c
 + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (3*Tan[c + d*x])/(d*(a^3 + a^3*Se
c[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)(3a-6a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(18a^2-27a^2\,\mathrm{s}}{a+a\sec(c+dx}}{15a^4}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{3\tan(c+dx)}{d\,(a^3+a^3\sec(c+dx}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{3\tan(c+dx)}{d\,(a^3+a^3\sec(c+dx}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx}{5ad(a+a\sec(c+dx))^2}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{9\tan(c+dx)}{5a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c}{5ad(a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 294 vs. $2(128) = 256$.

time = 1.33, size = 294, normalized size = 2.30

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+8\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+76\cos^4\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+20\cos^5\left(\frac{1}{2}(c+dx)\right)\left(3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)+8\cos^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{5a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(Sec[c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)
/2]^2*Sec[c/2]*Sin[(d*x)/2] + 76*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] +
 20*Cos[(c + d*x)/2]^5*(3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos
[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/2] + 8*Cos[(c + d*x)/2]^3*Tan
[c/2]))/(5*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]

time = 0.09, size = 105, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\right.}{4d\,a^3}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\right.}{4d\,a^3}$

risch	$\frac{2i\left(15\,\mathrm{e}^{6i(dx+c)}+75\,\mathrm{e}^{5i(dx+c)}+160\,\mathrm{e}^{4i(dx+c)}+200\,\mathrm{e}^{3i(dx+c)}+189\,\mathrm{e}^{2i(dx+c)}+105\,\mathrm{e}^{i(dx+c)}+24\right)}{5d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^{3}d}+$
norman	$\frac{\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{45\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{591\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}-\frac{81\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{51\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10ad}+\tan^{13}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^3+17*tan(1/2*d*x+1
/2*c)-4/(tan(1/2*d*x+1/2*c)-1)+12*ln(tan(1/2*d*x+1/2*c)-1)-4/(tan(1/2*d*x+1
/2*c)+1)-12*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.28, size = 165, normalized size = 1.29

$$\frac{\frac{40\,\sin\left(dx+c\right)}{\left(a^{3}-\frac{a^{3}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}\right)\left(\cos\left(dx+c\right)+1\right)}+\frac{\frac{85\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{10\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}}{a^{3}}-\frac{60\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^{3}}+\frac{60\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^{3}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)/a^3)/d
```

Fricas [A]
time = 3.96, size = 190, normalized size = 1.48

$$-\frac{15\left(\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(24\,\cos\left(dx+c\right)^{3}+57\,\cos\left(dx+c\right)^{2}+39\,\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{10\left(a^{3}d\cos\left(dx+c\right)^{4}+3\,a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+a^{3}d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/10*(15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x +
 c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(24*cos(d*x + c)^3 +
57*cos(d*x + c)^2 + 39*cos(d*x + c) + 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{5}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.52, size = 122, normalized size = 0.95

$$-\frac{\frac{60 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^3} - \frac{60 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right|\right)}{a^3} + \frac{40 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-1\right)a^3} - \frac{a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5+10 a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+85 a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{15}}}{20 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^3 + 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a
^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 + 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85*a^
12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 0.68, size = 111, normalized size = 0.87

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20\,a^3\,d}-\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(2*a^3*d) + tan(c/2 + (d*x)/2)^5/(20*a^3*d) - (6*atanh
(tan(c/2 + (d*x)/2)))/(a^3*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d
*x)/2)^2 - a^3)) + (17*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

3.63 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{29\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/a^3/d-1/5*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+
7/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-29/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+
c))
```

Rubi [A]
time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3901, 4093, 4083, 3855, 3879}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{29\tan(c+dx)}{15d\,(a^3\sec(c+dx)+a^3)}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^3*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(5*d*(a + a*S
ec[c + d*x])^3) + (7*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (29*Ta
n[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{\int \frac{\sec^2(c+dx)(2a-5a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}\\ &= -\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{\int \frac{\sec(c+dx)(-14a^2+15a^2\,\mathrm{se}}{a+a\sec(c+dx)}}{15a^4}\\ &= -\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{\int \sec(c+dx)\,dx}{a^3}-\frac{29}{}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}-\end{aligned}$$

Mathematica [A]
time = 0.51, size = 209, normalized size = 1.99

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(60\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+14\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+88\cos^4\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+3\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)+14\cos^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{15a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(60*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*S
ec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 88*Cos
[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]*Tan[c/2] + 14*Co
s[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.08, size = 75, normalized size = 0.71

method	result
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d\,a^3}$
default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d\,a^3}$
risch	$-\frac{2i(15\,\mathrm{e}^{4i(dx+c)}+75\,\mathrm{e}^{3i(dx+c)}+145\,\mathrm{e}^{2i(dx+c)}+95\,\mathrm{e}^{i(dx+c)}+22)}{15d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^5}-\frac{\ln(\mathrm{e}^{i(dx+c)}-i)}{a^3d}+\frac{\ln(\mathrm{e}^{i(dx+c)}+i)}{a^3d}$
norman	$\frac{\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{59\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}+\frac{43\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10ad}-\frac{9\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10ad}-\frac{11\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3a^2}+\frac{\ln\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*x
+1/2*c)-4*ln(tan(1/2*d*x+1/2*c)-1)+4*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.30, size = 119, normalized size = 1.13

$$-\frac{\frac{\frac{105\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{20\,\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{3\,\sin\left(dx+c\right)^5}{\left(\cos\left(dx+c\right)+1\right)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
/a^3)/d
```

Fricas [A]
time = 2.71, size = 158, normalized size = 1.50

$$\frac{15\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(22\,\cos\left(dx+c\right)^2+51\,\cos\left(dx+c\right)+32\right)\sin\left(dx+c\right)}{30\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(sin(d
*x + c) + 1) - 15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
```

```
log(-sin(d*x + c) + 1) - 2*(22*cos(d*x + c)^2 + 51*cos(d*x + c) + 32)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.50, size = 94, normalized size = 0.90

$$\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}-\frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+20a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+105a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*a^12*tan(1/2*d*x + 1
/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 0.69, size = 58, normalized size = 0.55

$$-\frac{105\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-120\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(105*tan(c/2 + (d*x)/2) - 120*atanh(tan(c/2 + (d*x)/2)) + 20*tan(c/2 + (d*
x)/2)^3 + 3*tan(c/2 + (d*x)/2)^5)/(60*a^3*d)
```

3.64 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{7\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx))}$$

```
[Out] 1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+
7/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3884, 4085, 3879}

$$\frac{7\tan(c+dx)}{15d\,(a^3\sec(c+dx)+a^3)}-\frac{8\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(15*a*d*(a + a
*Sec[c + d*x])^2) + (7*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\int \frac{\sec(c+dx)(-3a+5a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}\\ &= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{7\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}\\ &= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{7\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 57, normalized size = 0.69

$$\frac{\sec^5\left(\frac{1}{2}(c+dx)\right)\left(10\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)}{120a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
[Out] (Sec[(c + d*x)/2]^5*(10*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(120*a^3*d)
```

Maple [A]
time = 0.06, size = 45, normalized size = 0.54

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	45
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	45
risch	$\frac{4i\left(10\,\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}+1\right)}{15d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^5}$	47
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{30ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2a^2}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 67, normalized size = 0.81

$$\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

Fricas [A]
time = 2.88, size = 75, normalized size = 0.90

$$\frac{\left(2\,\cos\left(dx+c\right)^2+6\,\cos\left(dx+c\right)+7\right)\sin\left(dx+c\right)}{15\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(2*cos(d*x + c)^2 + 6*cos(d*x + c) + 7)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.49, size = 46, normalized size = 0.55

$$\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 + 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

Mupad [B]
time = 0.62, size = 45, normalized size = 0.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15\right)}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(10*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + 15)
)/(60*a^3*d)
```

3.65 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$-\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\tan(c+dx)}{5d\left(a^3+a^3\sec(c+dx)\right)}$$

```
[Out] -1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+1/5*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+
1/5*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3882, 3881, 3879}

$$\frac{\tan(c+dx)}{5d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\tan(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/5*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^3) + Tan[c + d*x]/(5*a*d*(a + a*S
ec[c + d*x])^2) + Tan[c + d*x]/(5*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{3\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a}$$
$$= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{5a^2}$$
$$= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\tan(c+dx)}{5d\,(a^3+a^3\sec(c+dx))}$$

Mathematica [A]
time = 0.18, size = 71, normalized size = 0.86

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(5\sin\left(\frac{dx}{2}\right)-5\sin\left(c+\frac{dx}{2}\right)+5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)}{80a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(5*Sin[(d*x)/2] - 5*Sin[c + (d*x)/2] + 5*Sin[c
 + (3*d*x)/2] + Sin[2*c + (5*d*x)/2]))/(80*a^3*d)
```

Maple [A]
time = 0.08, size = 32, normalized size = 0.39

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	32
default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	32
risch	$\frac{2i\left(5\,{e}^{3i(dx+c)}+5\,{e}^{2i(dx+c)}+5\,{e}^{i(dx+c)}+1\right)}{5d\,a^3\left({e}^{i(dx+c)}+1\right)^5}$	58
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.57

$$\frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{20\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)
^5)/(a^3*d)
```

Fricas [A]
time = 2.88, size = 73, normalized size = 0.88

$$\frac{\left(\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{5\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sin(d*x + c)/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^{2}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.48, size = 31, normalized size = 0.37

$$-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{20\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

Mupad [B]
time = 0.60, size = 30, normalized size = 0.36

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-5\right)}{20\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^4 - 5))/(20*a^3*d)
```

3.66 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{2\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

```
[Out] 1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+2/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+
2/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3881, 3879}

$$\frac{2\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{2\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*Tan[c + d*x])/(15*a*d*(a + a
*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a}$$
$$= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{2\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}$$
$$= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{2\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx)}$$

Mathematica [A]
time = 0.24, size = 86, normalized size = 1.04

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(40\sin\left(\frac{dx}{2}\right)-30\sin\left(c+\frac{dx}{2}\right)+20\sin\left(c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)\right)}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(40*Sin[(d*x)/2] - 30*Sin[c + (d*x)/2] + 20*Si
n[c + (3*d*x)/2] - 15*Sin[2*c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2]))/(240*
a^3*d)
```

Maple [A]
time = 0.06, size = 45, normalized size = 0.54

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	45
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$	45
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}}{a^2}$	61
risch	$\frac{2i\left(15\,\mathrm{e}^{4i(dx+c)}+30\,\mathrm{e}^{3i(dx+c)}+40\,\mathrm{e}^{2i(dx+c)}+20\,\mathrm{e}^{i(dx+c)}+7\right)}{15d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^5}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/
2*c))
```

Maxima [A]
time = 0.29, size = 67, normalized size = 0.81

$$\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

Fricas [**A**]
time = 2.50, size = 75, normalized size = 0.90

$$\frac{\left(7\,\cos\left(dx+c\right)^{2}+6\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{15\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(7*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [**A**]
time = 0.48, size = 46, normalized size = 0.55

$$\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 - 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

Mupad [**B**]
time = 0.62, size = 45, normalized size = 0.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+15\right)}{60\,a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(3*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 15)
)/(60*a^3*d)
```

3.67 $\int \frac{1}{(a+a\sec(c+dx))^3} \, dx$

Optimal. Leaf size=88

$$\frac{x}{a^3}-\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{22\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx))}$$

```
[Out] x/a^3-1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-7/15*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^2-22/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3862, 4007, 4004, 3879}

$$-\frac{22\tan(c+dx)}{15d\,(a^3\sec(c+dx)+a^3)}+\frac{x}{a^3}-\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-3),x]
```

```
[Out] x/a^3 - Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - (7*Tan[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - (22*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]
))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-5a+2a\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}\\ &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{15a^2-7a^2\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^4}\\ &= \frac{x}{a^3} - \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{22\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}\\ &= \frac{x}{a^3} - \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{22\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 162, normalized size = 1.84

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(60dx\cos^5\left(\frac{1}{2}(c+dx)\right)-3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+26\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-128\cos^4\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-3\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)+26\cos^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{15a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-3),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(60*d*x*Cos[(c + d*x)/2]^5 - 3*Sec[c/2]*
Sin[(d*x)/2] + 26*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 128*Cos[(c + d
*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] - 3*Cos[(c + d*x)/2]*Tan[c/2] + 26*Cos[(c +
d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.06, size = 59, normalized size = 0.67

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$	59

default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$	59
norman	$\frac{\frac{x}{a}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}}{a^2}$	66
risch	$\frac{x}{a^3}-\frac{2i\left(45\,e^{4i(dx+c)}+135\,e^{3i(dx+c)}+185\,e^{2i(dx+c)}+115\,e^{i(dx+c)}+32\right)}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5+4/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*x
+1/2*c)+8*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.50, size = 92, normalized size = 1.05

$$-\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 1.89, size = 116, normalized size = 1.32

$$\frac{15\,dx\cos(dx+c)^3+45\,dx\cos(dx+c)^2+45\,dx\cos(dx+c)+15\,dx-\left(32\,\cos(dx+c)^2+51\,\cos(dx+c)+22\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (32*cos(d*x + c)^2 + 51*cos(d*x + c) + 22)*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a
**3
```

Giac [A]
time = 0.43, size = 68, normalized size = 0.77

$$\frac{\frac{60\,(dx+c)}{a^3} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*a^12*tan(1/2*d
*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 0.69, size = 81, normalized size = 0.92

$$\frac{x}{a^3} - \frac{\frac{32\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15} - \frac{13\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{30} + \frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{20}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 - (sin(c/2 + (d*x)/2)/20 - (13*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
))/30 + (32*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/15)/(a^3*d*cos(c/2 + (
d*x)/2)^5)
```

3.68 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{3x}{a^3}+\frac{24\sin(c+dx)}{5a^3d}-\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}-\frac{3\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)}$$

```
[Out] -3*x/a^3+24/5*sin(d*x+c)/a^3/d-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-3/5*sin(
d*x+c)/a/d/(a+a*sec(d*x+c))^2-3*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3902, 4105, 3872, 2717, 8}

$$\frac{24\sin(c+dx)}{5a^3d}-\frac{3\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{3x}{a^3}-\frac{3\sin(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 + (24*Sin[c + d*x])/(5*a^3*d) - Sin[c + d*x]/(5*d*(a + a*Sec[c +
 d*x])^3) - (3*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (3*Sin[c + d*
x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
```

```
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{\int \frac{\cos(c+dx)(-6a+3a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}\\
&= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}-\frac{\int \frac{\cos(c+dx)\left(-27a^2+18a^2\sec(\right.}{a+a\sec(c+dx)}}{15a^4}\\
&= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}-\frac{3\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)}\\
&= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}-\frac{3\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)}\\
&= -\frac{3x}{a^3}+\frac{24\sin(c+dx)}{5a^3d}-\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}-
\end{aligned}$$

Mathematica [A]
time = 0.57, size = 169, normalized size = 1.64

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-12\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+96\cos^4\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+20\cos^5\left(\frac{1}{2}(c+dx)\right)(-3dx+\sin(c+dx))+\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)-12\cos^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{5a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(Sec[c/2]*Sin[(d*x)/2] - 12*Cos[(c + d*x
)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 96*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2]
+ 20*Cos[(c + d*x)/2]^5*(-3*d*x + Sin[c + d*x]) + Cos[(c + d*x)/2]*Tan[c/2]
 - 12*Cos[(c + d*x)/2]^3*Tan[c/2]))/(5*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.08, size = 85, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-24\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$	85
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-24\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$	85
risch	$-\frac{3x}{a^3}-\frac{i\,e^{i(dx+c)}}{2a^3d}+\frac{i\,e^{-i(dx+c)}}{2a^3d}+\frac{4i\left(15\,e^{4i(dx+c)}+50\,e^{3i(dx+c)}+70\,e^{2i(dx+c)}+45\,e^{i(dx+c)}+12\right)}{5d\,a^3\left(e^{i(dx+c)}+1\right)^5}$	112
norman	$\frac{-\frac{3x}{a}+\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}-\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2*tan(1/2*d*x+1/2*c)^3+17*tan(1/2*d*x+1
/2*c)+8*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-24*arctan(tan(1/2*d*x+1
/2*c)))
```

Maxima [A]

time = 0.49, size = 137, normalized size = 1.33

$$\frac{\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]

time = 2.03, size = 126, normalized size = 1.22

$$-\frac{15\,dx\cos(dx+c)^3+45\,dx\cos(dx+c)^2+45\,dx\cos(dx+c)+15\,dx-\left(5\,\cos(dx+c)^3+39\,\cos(dx+c)^2+57\,\cos(dx+c)+24\right)\sin(dx+c)}{5\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (5*cos(d*x + c)^3 + 39*cos(d*x + c)^2 + 57*cos(d*x + c) + 24)*sin
```

```
(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [A]
time = 0.49, size = 96, normalized size = 0.93

$$-\frac{\frac{60\,(dx+c)}{a^3}-\frac{40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}-\frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+85\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*(d*x + c)/a^3 - 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
+ 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 - 10*a^12*tan(1/2*d*x + 1/2*c)^3 +
 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 0.73, size = 113, normalized size = 1.10

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+96\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-60\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5(c+d\,x)}{20\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (sin(c/2 + (d*x)/2) - 12*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 96*cos(c
/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x
)/2) - 60*cos(c/2 + (d*x)/2)^5*(c + d*x))/(20*a^3*d*cos(c/2 + (d*x)/2)^5)
```

3.69 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=147

$$\frac{13x}{2a^3}-\frac{152\sin(c+dx)}{15a^3d}+\frac{13\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{76\,c}{15a}$$

```
[Out] 13/2*x/a^3-152/15*sin(d*x+c)/a^3/d+13/2*cos(d*x+c)*sin(d*x+c)/a^3/d-1/5*cos
(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-11/15*cos(d*x+c)*sin(d*x+c)/a/d/(a+
a*sec(d*x+c))^2-76/15*cos(d*x+c)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]

time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3902, 4105, 3872, 2715, 8, 2717}

$$-\frac{152\sin(c+dx)}{15a^3d}+\frac{13\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{76\sin(c+dx)\cos(c+dx)}{15d\,(a^3\sec(c+dx)+a^3)}+\frac{13x}{2a^3}-\frac{11\sin(c+dx)\cos(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*x)/(2*a^3) - (152*Sin[c + d*x])/(15*a^3*d) + (13*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d) - (Cos[c + d*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) -
 (11*Cos[c + d*x]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (76*Cos[c
 + d*x]*Sin[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(-7a+4a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(-43a^2+33a}{a+a\sec(c+d}}{15a^4}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\cos(c+dx)\sin(c}{15d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\cos(c+dx)\sin(c}{15d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{152\sin(c+dx)}{15a^3d} + \frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{1}{1}$$

$$= \frac{13x}{2a^3} - \frac{152\sin(c+dx)}{15a^3d} + \frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}$$

Mathematica [A]
time = 0.58, size = 181, normalized size = 1.23

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+46\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-508\cos^4\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+15\cos^5\left(\frac{1}{2}(c+dx)\right)(26dx-12\sin(c+dx)+\sin(2(c+dx)))-3\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)+46\cos^3\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{15a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(-3*Sec[c/2]*Sin[(d*x)/2] + 46*Cos[(c +
d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 508*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)
/2] + 15*Cos[(c + d*x)/2]^5*(26*d*x - 12*Sin[c + d*x] + Sin[2*(c + d*x)]) -
 3*Cos[(c + d*x)/2]*Tan[c/2] + 46*Cos[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(
1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.09, size = 101, normalized size = 0.69

method	result
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-28\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+52\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-28\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+52\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
risch	$\frac{13x}{2a^3}-\frac{ie^{2i(dx+c)}}{8a^3d}+\frac{3ie^{i(dx+c)}}{2a^3d}-\frac{3ie^{-i(dx+c)}}{2a^3d}+\frac{ie^{-2i(dx+c)}}{8a^3d}-\frac{2i\left(150\,e^{4i(dx+c)}+525\,e^{3i(dx+c)}+745\,e^{2i(dx+c)}+48\right.}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$
norman	$\frac{\frac{13x}{2a}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{131\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{97\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{17\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}+\frac{13x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{13x(}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5+8/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d*
x+1/2*c)+16*(-7/4*tan(1/2*d*x+1/2*c)^3-5/4*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d
*x+1/2*c)^2)^2+52*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.52, size = 184, normalized size = 1.25

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin
```

```
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 2.64, size = 135, normalized size = 0.92

$$\frac{195\,dx\cos(dx+c)^3+585\,dx\cos(dx+c)^2+585\,dx\cos(dx+c)+195\,dx+\left(15\,\cos(dx+c)^4-45\,\cos(dx+c)^3-479\,\cos(dx+c)^2-717\,\cos(dx+c)-304\right)\sin(dx+c)}{30\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(195*d*x*cos(d*x + c)^3 + 585*d*x*cos(d*x + c)^2 + 585*d*x*cos(d*x + c
) + 195*d*x + (15*cos(d*x + c)^4 - 45*cos(d*x + c)^3 - 479*cos(d*x + c)^2 -
 717*cos(d*x + c) - 304)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.51, size = 113, normalized size = 0.77

$$\frac{\frac{390\,(dx+c)}{a^3}-\frac{60\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+465\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*(d*x + c)/a^3 - 60*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5
 - 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 0.76, size = 137, normalized size = 0.93

$$-\frac{3\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-46\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+508\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-390\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5(c+d\,x)}{60\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(3*sin(c/2 + (d*x)/2) - 46*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 508*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 +
 (d*x)/2) - 120*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) - 390*cos(c/2 + (d*
x)/2)^5*(c + d*x))/(60*a^3*d*cos(c/2 + (d*x)/2)^5)
```

3.70 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=193

$$\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\sec(c+dx)\tan(c+dx)}{2a^4d}-\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2}-\frac{288\,\mathrm{se}}{35a}$$

```
[Out] 21/2*arctanh(sin(d*x+c))/a^4/d-576/35*tan(d*x+c)/a^4/d+21/2*sec(d*x+c)*tan(
d*x+c)/a^4/d-43/35*sec(d*x+c)^3*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-288/35*se
c(d*x+c)^2*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(d*x+c)^5*tan(d*x+c)/d/(a
+a*sec(d*x+c))^4-2/5*sec(d*x+c)^4*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A]

time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3901, 4104, 3872, 3852, 8, 3853, 3855}

$$-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{43\tan(c+dx)\sec^3(c+dx)}{35a^4d(\sec(c+dx)+1)^2}-\frac{288\tan(c+dx)\sec^2(c+dx)}{35a^4d(\sec(c+dx)+1)}+\frac{21\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{2\tan(c+dx)\sec^4(c+dx)}{5ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (21*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (576*Tan[c + d*x])/(35*a^4*d) + (21*
Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - (43*Sec[c + d*x]^3*Tan[c + d*x])/(35
*a^4*d*(1 + Sec[c + d*x])^2) - (288*Sec[c + d*x]^2*Tan[c + d*x])/(35*a^4*d*
(1 + Sec[c + d*x])) - (Sec[c + d*x]^5*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x
])^4) - (2*Sec[c + d*x]^4*Tan[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^5(c+dx)(5a-9a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^4(c+dx)(56a^2-73a}{(a+a\sec(c+d}}{35a^4}$$

$$= -\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan}{5ad(a+a\sec(c}$$

$$= -\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan}{5ad(a+a\sec(c}$$

$$= -\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan}{5ad(a+a\sec(c}$$

$$= \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(}{7d(a+a\sec(c+}$$

$$= \frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{576\tan(c+dx)}{35a^4d} + \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43}{3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 403 vs. 2(193) = 386.
time = 1.50, size = 403, normalized size = 2.09

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(376320\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\sec(c)\sec^2(c+dx)\left(-24402\sin\left(\frac{dx}{2}\right)+55556\sin\left(\frac{3dx}{2}\right)-61054\sin\left(c-\frac{dx}{2}\right)+33614\sin\left(c+\frac{dx}{2}\right)-51842\sin\left(2c+\frac{dx}{2}\right)-12460\sin\left(c+\frac{3dx}{2}\right)+33716\sin\left(2c+\frac{3dx}{2}\right)-34300\sin\left(3c+\frac{3dx}{2}\right)+39788\sin\left(c+\frac{5dx}{2}\right)-2940\sin\left(2c+\frac{5dx}{2}\right)+26068\sin\left(3c+\frac{5dx}{2}\right)-16660\sin\left(4c+\frac{5dx}{2}\right)+21351\sin\left(2c+\frac{7dx}{2}\right)+1295\sin\left(3c+\frac{7dx}{2}\right)+14911\sin\left(4c+\frac{7dx}{2}\right)-5145\sin\left(5c+\frac{7dx}{2}\right)+7329\sin\left(3c+\frac{9dx}{2}\right)+1225\sin\left(4c+\frac{9dx}{2}\right)+5369\sin\left(5c+\frac{9dx}{2}\right)-735\sin\left(6c+\frac{9dx}{2}\right)+1152\sin\left(4c+\frac{11dx}{2}\right)+280\sin\left(5c+\frac{11dx}{2}\right)+872\sin\left(6c+\frac{11dx}{2}\right)\right)\right)}{2240a^4d(1+\sec(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/2240*(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(376320*Cos[(c + d*x)/2]^7*(Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-24402*Sin[(d*x)/2] + 55556*Sin[(3*d*
x)/2] - 61054*Sin[c - (d*x)/2] + 33614*Sin[c + (d*x)/2] - 51842*Sin[2*c + (
d*x)/2] - 12460*Sin[c + (3*d*x)/2] + 33716*Sin[2*c + (3*d*x)/2] - 34300*Sin
[3*c + (3*d*x)/2] + 39788*Sin[c + (5*d*x)/2] - 2940*Sin[2*c + (5*d*x)/2] +
26068*Sin[3*c + (5*d*x)/2] - 16660*Sin[4*c + (5*d*x)/2] + 21351*Sin[2*c + (
7*d*x)/2] + 1295*Sin[3*c + (7*d*x)/2] + 14911*Sin[4*c + (7*d*x)/2] - 5145*S
in[5*c + (7*d*x)/2] + 7329*Sin[3*c + (9*d*x)/2] + 1225*Sin[4*c + (9*d*x)/2]
 + 5369*Sin[5*c + (9*d*x)/2] - 735*Sin[6*c + (9*d*x)/2] + 1152*Sin[4*c + (1
1*d*x)/2] + 280*Sin[5*c + (11*d*x)/2] + 872*Sin[6*c + (11*d*x)/2])))/(a^4*d
*(1 + Sec[c + d*x])^4)
```

Maple [A]
time = 0.07, size = 148, normalized size = 0.77

method	result
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-84\ln\left(\mathrm{ta}\right.}{8d\,a^4}$
default	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-84\ln\left(\mathrm{ta}\right.}{8d\,a^4}$
risch	$-\frac{i(735\,\mathrm{e}^{10i(dx+c)}+5145\,\mathrm{e}^{9i(dx+c)}+16660\,\mathrm{e}^{8i(dx+c)}+34300\,\mathrm{e}^{7i(dx+c)}+51842\,\mathrm{e}^{6i(dx+c)}+61054\,\mathrm{e}^{5i(dx+c)}+55556\,\mathrm{e}^{4i(dx+c}}{35d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^2\left(\mathrm{e}^{i(dx+c)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-9/5*tan(1/2*d*x+1/2*c)^5-13*tan(1/2*d*
x+1/2*c)^3-111*tan(1/2*d*x+1/2*c)+4/(tan(1/2*d*x+1/2*c)-1)^2+36/(tan(1/2*d*
x+1/2*c)-1)-84*ln(tan(1/2*d*x+1/2*c)-1)-4/(tan(1/2*d*x+1/2*c)+1)^2+36/(tan(
1/2*d*x+1/2*c)+1)+84*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]

time = 0.28, size = 231, normalized size = 1.20

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) + 455
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c)/(co
s(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^
4)/d
```

Fricas [A]

time = 2.99, size = 250, normalized size = 1.30

$$\frac{735\left(\cos(dx+c)^6+4\cos(dx+c)^5+6\cos(dx+c)^4+4\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-735\left(\cos(dx+c)^6+4\cos(dx+c)^5+6\cos(dx+c)^4+4\cos(dx+c)^3+\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)-2\left(1152\cos(dx+c)^5+3873\cos(dx+c)^4+4548\cos(dx+c)^3+2012\cos(dx+c)^2+140\cos(dx+c)-35\right)\sin(dx+c)}{140\left(a^4d\cos(dx+c)^6+4a^4d\cos(dx+c)^5+6a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+a^4d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/140*(735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*
x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 735*(cos(d*x + c)^6 + 4*
```

```
cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*log(
-sin(d*x + c) + 1) - 2*(1152*cos(d*x + c)^5 + 3873*cos(d*x + c)^4 + 4548*co
s(d*x + c)^3 + 2012*cos(d*x + c)^2 + 140*cos(d*x + c) - 35)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^7(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**7/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.49, size = 155, normalized size = 0.80

$$\frac{\frac{2940\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{2940\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{280\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^4}-\frac{5a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+63a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+455a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3885a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{28}}}{280d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 2940*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^4 + 280*(9*tan(1/2*d*x + 1/2*c)^3 - 7*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (5*a^24*tan(1/2*d*x + 1/2*c)^7
+ 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 + 3885*a
^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 0.75, size = 160, normalized size = 0.83

$$\frac{21\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^4\,d}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,a^4\,d}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^4\right)}-\frac{111\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (21*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (9*tan(c/2 + (d*x)/2)^5)/(40*a^4*d
) - tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (13*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) -
 (7*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^3)/(d*(a^4*tan(c/2 + (d*x)/2)
^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) - (111*tan(c/2 + (d*x)/2))/(8*a^4*d
)
```

3.71 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=159

$$-\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{244\tan(c+dx)}{105a^4d}-\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}+\frac{4\tan(c+dx)}{a^4d(1+\sec(c+dx))}-\frac{\sec^4(c+dx)}{7d(a+a\,\mathrm{se}}$$

```
[Out] -4*arctanh(sin(d*x+c))/a^4/d+244/105*tan(d*x+c)/a^4/d-88/105*sec(d*x+c)^2*t
an(d*x+c)/a^4/d/(1+sec(d*x+c))^2+4*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(
d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-12/35*sec(d*x+c)^3*tan(d*x+c)/a/d/
(a+a*sec(d*x+c))^3
```

Rubi [A]
time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3901, 4104, 4093, 3872, 3855, 3852, 8}

$$\frac{244\tan(c+dx)}{105a^4d}-\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{88\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{4\tan(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{12\tan(c+dx)\sec^3(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-4*ArcTanh[Sin[c + d*x]])/(a^4*d) + (244*Tan[c + d*x])/(105*a^4*d) - (88*S
ec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) + (4*Tan[c + d
*x])/(a^4*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a
*Sec[c + d*x])^4) - (12*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c +
 d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^4(c+dx)(4a-8a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)(36a^2-52a}{(a+a\sec(c+d}}{35a^4}$$

$$= -\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan}{35ad(a+a\sec(c-}$$

$$= -\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan}{35ad(a+a\sec(c-}$$

$$= -\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan}{35ad(a+a\sec(c-}$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+}{7d(a+a\sec(c+dx)}$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{244\tan(c+dx)}{105a^4d} - \frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec}{7}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 349 vs. 2(159) = 318.
time = 1.26, size = 349, normalized size = 2.19

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(107520\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\sec(c)\sec(c+dx)\left(-10780\sin\left(\frac{dx}{2}\right)+18788\sin\left(\frac{3dx}{2}\right)-20524\sin\left(c-\frac{dx}{2}\right)+14644\sin\left(c+\frac{dx}{2}\right)-16660\sin\left(2c+\frac{dx}{2}\right)-4690\sin\left(c+\frac{3dx}{2}\right)+14378\sin\left(2c+\frac{3dx}{2}\right)-9100\sin\left(3c+\frac{3dx}{2}\right)+11668\sin\left(c+\frac{5dx}{2}\right)-630\sin\left(2c+\frac{5dx}{2}\right)+9358\sin\left(3c+\frac{5dx}{2}\right)-2940\sin\left(4c+\frac{5dx}{2}\right)+4228\sin\left(2c+\frac{7dx}{2}\right)+315\sin\left(3c+\frac{7dx}{2}\right)+3493\sin\left(4c+\frac{7dx}{2}\right)-420\sin\left(5c+\frac{7dx}{2}\right)+664\sin\left(3c+\frac{9dx}{2}\right)+105\sin\left(4c+\frac{9dx}{2}\right)+559\sin\left(5c+\frac{9dx}{2}\right)\right)\right)}{1680a^4d(1+\sec(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^4*(107520*Cos[(c + d*x)/2]^7*(Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Se
c[c/2]*Sec[c]*Sec[c + d*x]*(-10780*Sin[(d*x)/2] + 18788*Sin[(3*d*x)/2] - 20
524*Sin[c - (d*x)/2] + 14644*Sin[c + (d*x)/2] - 16660*Sin[2*c + (d*x)/2] -
4690*Sin[c + (3*d*x)/2] + 14378*Sin[2*c + (3*d*x)/2] - 9100*Sin[3*c + (3*d*
x)/2] + 11668*Sin[c + (5*d*x)/2] - 630*Sin[2*c + (5*d*x)/2] + 9358*Sin[3*c
+ (5*d*x)/2] - 2940*Sin[4*c + (5*d*x)/2] + 4228*Sin[2*c + (7*d*x)/2] + 315*
Sin[3*c + (7*d*x)/2] + 3493*Sin[4*c + (7*d*x)/2] - 420*Sin[5*c + (7*d*x)/2]
 + 664*Sin[3*c + (9*d*x)/2] + 105*Sin[4*c + (9*d*x)/2] + 559*Sin[5*c + (9*d
*x)/2])))/(1680*a^4*d*(1 + Sec[c + d*x])^4)
```

Maple [A]
time = 0.06, size = 118, normalized size = 0.74

---

method	result
derivativedivides	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{}{\tan(}}{8d\,a^{4}}$
default	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{}{\tan(}}{8d\,a^{4}}$
risch	$\frac{8i\left(105\,e^{8i(dx+c)}+735\,e^{7i(dx+c)}+2275\,e^{6i(dx+c)}+4165\,e^{5i(dx+c)}+5131\,e^{4i(dx+c)}+4697\,e^{3i(dx+c)}+2917\,e^{2i(dx+c)}+1057\right.}{105d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}\left(e^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+7/5*tan(1/2*d*x+1/2*c)^5+23/3*tan(1/2*d
*x+1/2*c)^3+49*tan(1/2*d*x+1/2*c)-8/(tan(1/2*d*x+1/2*c)-1)+32*ln(tan(1/2*d*
x+1/2*c)-1)-8/(tan(1/2*d*x+1/2*c)+1)-32*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]

time = 0.29, size = 186, normalized size = 1.17

$$\frac{\frac{1680\sin(dx+c)}{\left(a^{4}-\frac{a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{147\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{4}}-\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{4}}+\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{4}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Fricas [A]

time = 3.29, size = 234, normalized size = 1.47

$$-\frac{210\left(\cos(dx+c)^{5}+4\cos(dx+c)^{4}+6\cos(dx+c)^{3}+4\cos(dx+c)^{2}+\cos(dx+c)\right)\log(\sin(dx+c)+1)-210\left(\cos(dx+c)^{5}+4\cos(dx+c)^{4}+6\cos(dx+c)^{3}+4\cos(dx+c)^{2}+\cos(dx+c)\right)\log(-\sin(dx+c)+1)-\left(664\cos(dx+c)^{4}+2236\cos(dx+c)^{3}+2636\cos(dx+c)^{2}+1184\cos(dx+c)+105\right)\sin(dx+c)}{105\left(a^{4}d\cos(dx+c)^{5}+4a^{4}d\cos(dx+c)^{4}+6a^{4}d\cos(dx+c)^{3}+4a^{4}d\cos(dx+c)^{2}+a^{4}d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d
*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(cos(d*x + c)^5 + 4*c
os(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(-si
n(d*x + c) + 1) - (664*cos(d*x + c)^4 + 2236*cos(d*x + c)^3 + 2636*cos(d*x
+ c)^2 + 1184*cos(d*x + c) + 105)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a
```

```
^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4
*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**6/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.47, size = 139, normalized size = 0.87

$$-\frac{\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4}-\frac{15a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+147a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+805a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+5145a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{28}}}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^4 + 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*a^4) - (15*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*a^24*tan(1/2*d*x + 1/2*c
)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*a^24*tan(1/2*d*x + 1/2*c))/a^2
8)/d
```

Mupad [B]
time = 0.70, size = 130, normalized size = 0.82

$$\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24a^4d}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{40a^4d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56a^4d}-\frac{8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^4\right)}+\frac{49\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (23*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) + (7*tan(c/2 + (d*x)/2)^5)/(40*a^4*d)
+ tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (8*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) -
 (2*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 - a^4)) + (49*tan(c/2
+ (d*x)/2))/(8*a^4*d)
```

3.72 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=136

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2}-\frac{43\tan(c+dx)}{21a^4d(1+\sec(c+dx))}-\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{2\sec^2(}{7ad(}$$

```
[Out] arctanh(sin(d*x+c))/a^4/d+11/21*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-43/21*tan
(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))
^4-2/7*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A]
time = 0.22, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3901, 4104, 4093, 4083, 3855, 3879}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{43\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}+\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{2\tan(c+dx)\sec^2(c+dx)}{7ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) + (11*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*
x])^2) - (43*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^3*
Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*Sec[c + d*x]^2*Tan[c + d*x]
)/(7*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx &= -\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{\int \frac{\sec^3(c+dx)(3a-7a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}\\
&= -\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{2\sec^2(c+dx)\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}-\\
&= \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2}-\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\\
&= \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2}-\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2}-\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-
\end{aligned}
$$

Mathematica [A]

time = 0.91, size = 193, normalized size = 1.42

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(1344\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\left(686\sin\left(\frac{dx}{2}\right)-434\sin\left(c+\frac{dx}{2}\right)+525\sin\left(c+\frac{3dx}{2}\right)-147\sin\left(2c+\frac{3dx}{2}\right)+203\sin\left(2c+\frac{5dx}{2}\right)-21\sin\left(3c+\frac{5dx}{2}\right)+32\sin\left(3c+\frac{7dx}{2}\right)\right)\right)}{84a^4d(1+\sec(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/84*(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(1344*Cos[(c + d*x)/2]^7*(Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
+ Sec[c/2]*(686*Sin[(d*x)/2] - 434*Sin[c + (d*x)/2] + 525*Sin[c + (3*d*x)/2
] - 147*Sin[2*c + (3*d*x)/2] + 203*Sin[2*c + (5*d*x)/2] - 21*Sin[3*c + (5*d
*x)/2] + 32*Sin[3*c + (7*d*x)/2])))/(a^4*d*(1 + Sec[c + d*x])^4)
```

Maple [A]
time = 0.10, size = 88, normalized size = 0.65

method	result
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8d\,a^4}$
default	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8d\,a^4}$
risch	$-\frac{2i\left(21\,\mathrm{e}^{6i(dx+c)}+147\,\mathrm{e}^{5i(dx+c)}+434\,\mathrm{e}^{4i(dx+c)}+686\,\mathrm{e}^{3i(dx+c)}+525\,\mathrm{e}^{2i(dx+c)}+203\,\mathrm{e}^{i(dx+c)}+32\right)}{21d\,a^4\left(\mathrm{e}^{i(dx+c)}+1\right)^7}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^4d}-$
norman	$\frac{-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{169\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{229\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{293\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}-\frac{121\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{168ad}-\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{168ad}-}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-tan(1/2*d*x+1/2*c)^5-11/3*tan(1/2*d*x+
1/2*c)^3-15*tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c)-1)+8*ln(tan(1/2*d*x+
1/2*c)+1))
```

Maxima [A]
time = 0.30, size = 139, normalized size = 1.02

$$-\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos
```

```
(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 +
168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Fricas [A]
time = 4.54, size = 202, normalized size = 1.49

$$\frac{21\left(\cos\left(dx+c\right)^{4}+4\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-21\left(\cos\left(dx+c\right)^{4}+4\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)+1\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(32\cos\left(dx+c\right)^{3}+107\cos\left(dx+c\right)^{2}+124\cos\left(dx+c\right)+52\right)\sin\left(dx+c\right)}{42\left(a^{4}d\cos\left(dx+c\right)^{4}+4a^{4}d\cos\left(dx+c\right)^{3}+6a^{4}d\cos\left(dx+c\right)^{2}+4a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/42*(21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x
+ c) + 1)*log(sin(d*x + c) + 1) - 21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6
*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(32*cos(d*
x + c)^3 + 107*cos(d*x + c)^2 + 124*cos(d*x + c) + 52)*sin(d*x + c))/(a^4*d
*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d
*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{5}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.52, size = 110, normalized size = 0.81

$$\frac{\frac{168\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{4}}-\frac{168\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^{4}}-\frac{3a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+21a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+77a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+315a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{28}}}{168d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 168*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^4 - (3*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*a^24*tan(1/2*d*x
+ 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 + 315*a^24*tan(1/2*d*x + 1/2*c)
)/a^28)/d
```

Mupad [B]
time = 0.67, size = 83, normalized size = 0.61

$$-\frac{\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24\,a^{4}}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{8\,a^{4}}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{56\,a^{4}}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^{4}}+\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^{4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^4),x)
```

```
[Out] -((11*tan(c/2 + (d*x)/2)^3)/(24*a^4) + tan(c/2 + (d*x)/2)^5/(8*a^4) + tan(c
/2 + (d*x)/2)^7/(56*a^4) - (2*atanh(tan(c/2 + (d*x)/2)))/a^4 + (15*tan(c/2
+ (d*x)/2))/(8*a^4))/d
```

3.73 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{\tan(c+dx)}{5d\left(a^4+a^4\sec(c+dx)\right)}$$

```
[Out] 1/7*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+3/35*tan(d*x+c)/a/d/(a+a*s
ec(d*x+c))^3-8/35*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+1/5*tan(d*x+c)/d/(a^4
+a^4*sec(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3895, 3884, 4085, 3879}

$$\frac{\tan(c+dx)}{5d\left(a^4\sec(c+dx)+a^4\right)}-\frac{8\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*Tan[c + d*x
])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(35*d*(a^2 + a^2*Sec[
c + d*x])^2) + Tan[c + d*x]/(5*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
```

```
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx &= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}\\ &= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{3\int \frac{\sec(c+dx)(-3a+5a\sec(c}{(a+a\sec(c+dx))^2}}{35a^3}\\ &= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{35d\left(a^2+a^2\sec(c+d\right.}\\ &= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{35d\left(a^2+a^2\sec(c+d\right.}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 69, normalized size = 0.58

$$\frac{\sec^7\left(\frac{1}{2}(c+dx)\right)\left(35\sin\left(\frac{1}{2}(c+dx)\right)+21\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)}{1120a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
[Out] (Sec[(c + d*x)/2]^7*(35*Sin[(c + d*x)/2] + 21*Sin[(3*(c + d*x))/2] + 7*Sin[
(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))/(1120*a^4*d)
```

Maple [A]
time = 0.08, size = 56, normalized size = 0.47

method	result

derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$
risch	$\frac{4i\left(35\,e^{3i(dx+c)}+21\,e^{2i(dx+c)}+7\,e^{i(dx+c)}+1\right)}{35d\,a^4\left(e^{i(dx+c)}+1\right)^7}$
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40ad}-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70ad}-\frac{13\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280ad}+\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140ad}+\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3 a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/
2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 87, normalized size = 0.72

$$\frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

Fricas [A]
time = 9.54, size = 99, normalized size = 0.82

$$\frac{\left(2\,\cos\left(dx+c\right)^3+8\,\cos\left(dx+c\right)^2+13\,\cos\left(dx+c\right)+12\right)\sin\left(dx+c\right)}{35\,\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(2*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 13*cos(d*x + c) + 12)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^4\left(c+dx\right)}{\sec^4\left(c+dx\right)+4\sec^3\left(c+dx\right)+6\sec^2\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**4,x)

[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4

Giac [A]
time = 0.49, size = 59, normalized size = 0.49

$$\frac{5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{280\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] 1/280*(5*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d*x + 1/2*c)^3 + 35*tan(1/2*d*x + 1/2*c))/(a^4*d)

Mupad [B]
time = 0.67, size = 58, normalized size = 0.48

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+35\right)}{280\,a^{4}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^4),x)

[Out] (tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 5*tan(c/2 + (d*x)/2)^6 + 35))/(280*a^4*d)

3.74 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4} dx$

Optimal. Leaf size=112

$$\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{13\tan(c+dx)}{105d\,(a^2+a^2\sec(c+dx))^2}+\frac{13\tan(c+dx)}{105d\,(a^4+a^4\sec(c+dx))}$$

```
[Out] 1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-11/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
+13/105*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+13/105*tan(d*x+c)/d/(a^4+a^4*se
c(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3884, 4085, 3881, 3879}

$$\frac{13\tan(c+dx)}{105d\,(a^4\sec(c+dx)+a^4)}+\frac{13\tan(c+dx)}{105d\,(a^2\sec(c+dx)+a^2)^2}-\frac{11\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - (11*Tan[c + d*x])/(35*a*d*(a +
a*Sec[c + d*x])^3) + (13*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) +
 (13*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
```

```
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec(c+dx)(-4a+7a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$
$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^2}$$
$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx\right.}$$
$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx\right.}$$

Mathematica [A]
time = 0.24, size = 87, normalized size = 0.78

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(35\sin\left(\frac{dx}{2}\right)-35\sin\left(c+\frac{dx}{2}\right)+2\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)\right)}{1680a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(35*Sin[(d*x)/2] - 35*Sin[c + (d*x)/2] + 2*(21
*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*c + (7*d*x)/2])))/(168
0*a^4*d)
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.52

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58

default	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58
risch	$\frac{8i\left(35\,e^{4i(dx+c)}+35\,e^{3i(dx+c)}+42\,e^{2i(dx+c)}+14\,e^{i(dx+c)}+2\right)}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$	69
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{60ad}+\frac{31\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{420ad}+\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2 a^3}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5+1/3*tan(1/2*d
*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.78

$$\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

Fricas [A]
time = 4.04, size = 99, normalized size = 0.88

$$\frac{\left(8\,\cos\left(dx+c\right)^3+32\,\cos\left(dx+c\right)^2+52\,\cos\left(dx+c\right)+13\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(8*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 52*cos(d*x + c) + 13)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^3\left(c+dx\right)}{\sec^4\left(c+dx\right)+4\sec^3\left(c+dx\right)+6\sec^2\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.48, size = 59, normalized size = 0.53

$$-\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(15*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*
d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B]
time = 0.66, size = 58, normalized size = 0.52

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+105\right)}{840\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 - 15
*tan(c/2 + (d*x)/2)^6 + 105))/(840*a^4*d)
```

3.75 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$-\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{8\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{8\tan(c+dx)}{105d\left(a^4+a^4\sec(c+dx)\right)}$$

```
[Out] -1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+4/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
+8/105*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+8/105*tan(d*x+c)/d/(a^4+a^4*sec(
d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3882, 3881, 3879}

$$\frac{8\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{8\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/7*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(35*a*d*(a
+ a*Sec[c + d*x])^3) + (8*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2)
+ (8*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$

$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{8\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^2}$$

$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{8\tan(c+dx)}{105d\,(a^2+a^2\sec(c+ \cdots}$$

$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{8\tan(c+dx)}{105d\,(a^2+a^2\sec(c+ \cdots}$$

Mathematica [A]
time = 0.27, size = 99, normalized size = 0.88

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(280\sin\left(\frac{dx}{2}\right)-175\sin\left(c+\frac{dx}{2}\right)+168\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+91\sin\left(2c+\frac{5dx}{2}\right)+13\sin\left(3c+\frac{7dx}{2}\right)\right)}{6720a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(280*Sin[(d*x)/2] - 175*Sin[c + (d*x)/2] + 168
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 91*Sin[2*c + (5*d*x)/2] +
13*Sin[3*c + (7*d*x)/2]))/(6720*a^4*d)
```

Maple [A]
time = 0.08, size = 58, normalized size = 0.52

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58
risch	$\frac{2i\left(105\,e^{5i(dx+c)}+175\,e^{4i(dx+c)}+280\,e^{3i(dx+c)}+168\,e^{2i(dx+c)}+91\,e^{i(dx+c)}+13\right)}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$	80
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{60ad}-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{56ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^3}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d*
x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.78

$$\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

Fricas [A]
time = 2.20, size = 99, normalized size = 0.88

$$\frac{\left(13 \cos\left(dx+c\right)^{3} + 52 \cos\left(dx+c\right)^{2} + 32 \cos\left(dx+c\right) + 8\right) \sin\left(dx+c\right)}{105 \left(a^{4} d \cos\left(dx+c\right)^{4} + 4 a^{4} d \cos\left(dx+c\right)^{3} + 6 a^{4} d \cos\left(dx+c\right)^{2} + 4 a^{4} d \cos\left(dx+c\right) + a^{4} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(13*cos(d*x + c)^3 + 52*cos(d*x + c)^2 + 32*cos(d*x + c) + 8)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{2}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\, dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.45, size = 59, normalized size = 0.53

$$\frac{15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{7} - 21 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{5} - 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{3} + 105 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{840 a^{4} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(15*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*d
*x + 1/2*c)^3 + 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B]
time = 0.67, size = 58, normalized size = 0.52

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-105\right)}{840\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^4),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 - 1
5*tan(c/2 + (d*x)/2)^6 - 105))/(840*a^4*d)
```

3.76 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{2\tan(c+dx)}{35d\left(a^4+a^4\sec(c+dx)\right)}$$

```
[Out] 1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+3/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3+
2/35*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+2/35*tan(d*x+c)/d/(a^4+a^4*sec(d*x
+c))
```

Rubi [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3881, 3879}

$$\frac{2\tan(c+dx)}{35d\left(a^4\sec(c+dx)+a^4\right)}+\frac{2\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) + (3*Tan[c + d*x])/(35*a*d*(a + a
*Sec[c + d*x])^3) + (2*Tan[c + d*x])/(35*d*(a^2 + a^2*Sec[c + d*x])^2) + (2
*Tan[c + d*x])/(35*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$
$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{6\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^2}$$
$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{35d\,(a^2+a^2\sec(c+dx)}$$
$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{35d\,(a^2+a^2\sec(c+dx)}$$

Mathematica [A]
time = 0.26, size = 112, normalized size = 1.00

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(210\sin\left(\frac{dx}{2}\right)-210\sin\left(c+\frac{dx}{2}\right)+147\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+49\sin\left(2c+\frac{5dx}{2}\right)-35\sin\left(3c+\frac{5dx}{2}\right)+12\sin\left(3c+\frac{7dx}{2}\right)\right)}{2240a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(210*Sin[(d*x)/2] - 210*Sin[c + (d*x)/2] + 147
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 49*Sin[2*c + (5*d*x)/2] -
35*Sin[3*c + (5*d*x)/2] + 12*Sin[3*c + (7*d*x)/2]))/(2240*a^4*d)
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.52

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58
default	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$	58
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56ad}}{a^3}$	80
risch	$\frac{2i\left(35\,e^{6i(dx+c)}+105\,e^{5i(dx+c)}+210\,e^{4i(dx+c)}+210\,e^{3i(dx+c)}+147\,e^{2i(dx+c)}+49\,e^{i(dx+c)}+12\right)}{35d\,a^4\left(e^{i(dx+c)}+1\right)^7}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1
/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.29, size = 87, normalized size = 0.78

$$\frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

Fricas [A]
time = 5.49, size = 99, normalized size = 0.88

$$\frac{\left(12\,\cos\left(dx+c\right)^3+13\,\cos\left(dx+c\right)^2+8\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{35\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(12*cos(d*x + c)^3 + 13*cos(d*x + c)^2 + 8*cos(d*x + c) + 2)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec\left(c+dx\right)}{\sec^4\left(c+dx\right)+4\sec^3\left(c+dx\right)+6\sec^2\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)
**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.50, size = 59, normalized size = 0.53

$$-\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/280*(5*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d
*x + 1/2*c)^3 - 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B]
time = 0.67, size = 58, normalized size = 0.52

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-35\right)}{280\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^4),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 - 35))/(280*a^4*d)
```

3.77 $\int \frac{1}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=111

$$\frac{x}{a^4}-\frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2}-\frac{32\tan(c+dx)}{21a^4d(1+\sec(c+dx))}-\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}$$

```
[Out] x/a^4-11/21*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-32/21*tan(d*x+c)/a^4/d/(1+sec
(d*x+c))-1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-2/7*tan(d*x+c)/a/d/(a+a*sec(d*
x+c))^3
```

Rubi [A]
time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3862, 4007, 4004, 3879}

$$-\frac{32\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}-\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}+\frac{x}{a^4}-\frac{2\tan(c+dx)}{7ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-4),x]
```

```
[Out] x/a^4 - (11*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*x])^2) - (32*Tan[c + d*x
])/(21*a^4*d*(1 + Sec[c + d*x])) - Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4
) - (2*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))^4}\,dx &= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{\int \frac{-7a+3a\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a^2}\\ &= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}+\frac{\int \frac{35a^2-20a^2\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^4}\\ &= -\frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2}-\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))}\\ &= \frac{x}{a^4}-\frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2}-\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{2\tan(c+dx)}{7ad(a+a\sec(c+\end{aligned}$$

$$= \frac{x}{a^4}-\frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2}-\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{2\tan(c+dx)}{7ad(a+a\sec(c+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 224 vs. 2(111) = 222.
time = 0.40, size = 224, normalized size = 2.02

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(735dx\cos\left(\frac{dx}{2}\right)+735dx\cos\left(c+\frac{dx}{2}\right)+441dx\cos\left(c+\frac{3dx}{2}\right)+441dx\cos\left(2c+\frac{3dx}{2}\right)+147dx\cos\left(2c+\frac{5dx}{2}\right)+147dx\cos\left(3c+\frac{5dx}{2}\right)+21dx\cos\left(3c+\frac{7dx}{2}\right)+21dx\cos\left(4c+\frac{7dx}{2}\right)-1988\sin\left(\frac{dx}{2}\right)+1652\sin\left(c+\frac{dx}{2}\right)-1428\sin\left(c+\frac{3dx}{2}\right)+756\sin\left(2c+\frac{3dx}{2}\right)-560\sin\left(2c+\frac{5dx}{2}\right)+168\sin\left(3c+\frac{5dx}{2}\right)-104\sin\left(3c+\frac{7dx}{2}\right)\right)}{2688a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-4),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(735*d*x*Cos[(d*x)/2] + 735*d*x*Cos[c + (d*x)/
2] + 441*d*x*Cos[c + (3*d*x)/2] + 441*d*x*Cos[2*c + (3*d*x)/2] + 147*d*x*Co
s[2*c + (5*d*x)/2] + 147*d*x*Cos[3*c + (5*d*x)/2] + 21*d*x*Cos[3*c + (7*d*x
)/2] + 21*d*x*Cos[4*c + (7*d*x)/2] - 1988*Sin[(d*x)/2] + 1652*Sin[c + (d*x)
/2] - 1428*Sin[c + (3*d*x)/2] + 756*Sin[2*c + (3*d*x)/2] - 560*Sin[2*c + (5
*d*x)/2] + 168*Sin[3*c + (5*d*x)/2] - 104*Sin[3*c + (7*d*x)/2]))/(2688*a^4*
d)
```

Maple [A]
time = 0.06, size = 72, normalized size = 0.65

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}$	72
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}$	72
norman	$\frac{\frac{x}{a}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56ad}}{a^3}$	85
risch	$\frac{x}{a^4}-\frac{4i\left(42\,e^{6i(dx+c)}+189\,e^{5i(dx+c)}+413\,e^{4i(dx+c)}+497\,e^{3i(dx+c)}+357\,e^{2i(dx+c)}+140\,e^{i(dx+c)}+26\right)}{21d\,a^4\left(e^{i(dx+c)}+1\right)^7}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-tan(1/2*d*x+1/2*c)^5+11/3*tan(1/2*d*x+1
/2*c)^3-15*tan(1/2*d*x+1/2*c)+16*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.51, size = 112, normalized size = 1.01

$$-\frac{\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d
```

Fricas [A]
time = 3.10, size = 152, normalized size = 1.37

$$\frac{21\,dx\cos(dx+c)^4+84\,dx\cos(dx+c)^3+126\,dx\cos(dx+c)^2+84\,dx\cos(dx+c)+21\,dx-\left(52\,\cos(dx+c)^3+124\,\cos(dx+c)^2+107\,\cos(dx+c)+32\right)\sin(dx+c)}{21\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/21*(21*d*x*cos(d*x + c)^4 + 84*d*x*cos(d*x + c)^3 + 126*d*x*cos(d*x + c)^
2 + 84*d*x*cos(d*x + c) + 21*d*x - (52*cos(d*x + c)^3 + 124*cos(d*x + c)^2
+ 107*cos(d*x + c) + 32)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(
d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(1/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec
(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.46, size = 83, normalized size = 0.75

$$\frac{\frac{168\,(dx+c)}{a^4}+\frac{3\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-315\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*(d*x + c)/a^4 + (3*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*a^24*tan(1/2
*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 - 315*a^24*tan(1/2*d*x + 1
/2*c))/a^28)/d
```

Mupad [B]
time = 0.72, size = 102, normalized size = 0.92

$$\frac{x}{a^4}+\frac{-\frac{52\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{21}+\frac{16\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{21}-\frac{5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{28}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^4,x)
```

```
[Out] x/a^4 + (sin(c/2 + (d*x)/2)/56 - (5*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)
)/28 + (16*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/21 - (52*cos(c/2 + (d*x
)/2)^6*sin(c/2 + (d*x)/2))/21)/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.78 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=126

$$-\frac{4x}{a^4}+\frac{664\sin(c+dx)}{105a^4d}-\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{4\sin(c+dx)}{a^4d(1+\sec(c+dx))}-\frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{12\,\mathrm{s}}{35ad(a+}$$

```
[Out] -4*x/a^4+664/105*sin(d*x+c)/a^4/d-88/105*sin(d*x+c)/a^4/d/(1+sec(d*x+c))^2-
4*sin(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-12/35
*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A]
time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3902, 4105, 3872, 2717, 8}

$$\frac{664\sin(c+dx)}{105a^4d}-\frac{4\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{88\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{4x}{a^4}-\frac{12\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-4*x)/a^4 + (664*Sin[c + d*x])/(105*a^4*d) - (88*Sin[c + d*x])/(105*a^4*d*
(1 + Sec[c + d*x])^2) - (4*Sin[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) - Sin[c
 + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - (12*Sin[c + d*x])/(35*a*d*(a + a*Sec
[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
```

```
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos(c+dx)(-8a+4a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(-52a^2+36a^2\sec}{(a+a\sec(c+dx))^2}}{35a^4}$$

$$= -\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx}$$

$$= -\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx}$$

$$= -\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx}$$

$$= -\frac{4x}{a^4} + \frac{664\sin(c+dx)}{105a^4d} - \frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 263 vs. 2(126) = 252.
time = 0.46, size = 263, normalized size = 2.09

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(29400dx\cos\left(\frac{dx}{2}\right)+29400dx\cos\left(c+\frac{dx}{2}\right)+17640dx\cos\left(c+\frac{3dx}{2}\right)+17640dx\cos\left(2c+\frac{3dx}{2}\right)+5880dx\cos\left(2c+\frac{5dx}{2}\right)+5880dx\cos\left(3c+\frac{5dx}{2}\right)+840dx\cos\left(3c+\frac{7dx}{2}\right)+840dx\cos\left(4c+\frac{7dx}{2}\right)-60830\sin\left(\frac{dx}{2}\right)+46130\sin\left(c+\frac{dx}{2}\right)-46116\sin\left(c+\frac{3dx}{2}\right)+18060\sin\left(2c+\frac{3dx}{2}\right)-19292\sin\left(2c+\frac{5dx}{2}\right)+2100\sin\left(3c+\frac{5dx}{2}\right)-3791\sin\left(3c+\frac{7dx}{2}\right)-735\sin\left(4c+\frac{7dx}{2}\right)-105\sin\left(4c+\frac{9dx}{2}\right)-105\sin\left(5c+\frac{9dx}{2}\right)\right)}{26880a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/26880*(Sec[c/2]*Sec[(c + d*x)/2]^7*(29400*d*x*Cos[(d*x)/2] + 29400*d*x*C
os[c + (d*x)/2] + 17640*d*x*Cos[c + (3*d*x)/2] + 17640*d*x*Cos[2*c + (3*d*x
```

```
)/2] + 5880*d*x*Cos[2*c + (5*d*x)/2] + 5880*d*x*Cos[3*c + (5*d*x)/2] + 840*
d*x*Cos[3*c + (7*d*x)/2] + 840*d*x*Cos[4*c + (7*d*x)/2] - 60830*Sin[(d*x)/2
] + 46130*Sin[c + (d*x)/2] - 46116*Sin[c + (3*d*x)/2] + 18060*Sin[2*c + (3*
d*x)/2] - 19292*Sin[2*c + (5*d*x)/2] + 2100*Sin[3*c + (5*d*x)/2] - 3791*Sin
[3*c + (7*d*x)/2] - 735*Sin[4*c + (7*d*x)/2] - 105*Sin[4*c + (9*d*x)/2] - 1
05*Sin[5*c + (9*d*x)/2]))/(a^4*d)
```

Maple [A]

time = 0.10, size = 98, normalized size = 0.78

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}-64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$
default	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}-64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$
risch	$-\frac{4x}{a^{4}}-\frac{i\,e^{i(dx+c)}}{2a^{4}d}+\frac{i\,e^{-i(dx+c)}}{2a^{4}d}+\frac{4i\left(525\,e^{6i(dx+c)}+2625\,e^{5i(dx+c)}+5950\,e^{4i(dx+c)}+7420\,e^{3i(dx+c)}+5397\,e^{2i(dx+c)}+\right.}{105d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{-\frac{4x}{a}+\frac{65\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{31\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{47\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60ad}+\frac{11\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70ad}-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56ad}-\frac{4x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7+7/5*tan(1/2*d*x+1/2*c)^5-23/3*tan(1/2*
d*x+1/2*c)^3+49*tan(1/2*d*x+1/2*c)+16*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2
*c)^2)-64*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]

time = 0.54, size = 158, normalized size = 1.25

$$\frac{\frac{1680\sin(dx+c)}{\left(a^{4}+\frac{a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{147\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{4}}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{4}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4)/d
```

Fricas [A]

time = 6.68, size = 162, normalized size = 1.29

$$-\frac{420\,dx\cos(dx+c)^{4}+1680\,dx\cos(dx+c)^{3}+2520\,dx\cos(dx+c)^{2}+1680\,dx\cos(dx+c)+420\,dx-\left(105\cos(dx+c)^{4}+1184\cos(dx+c)^{3}+2636\cos(dx+c)^{2}+2236\cos(dx+c)+664\right)\sin(dx+c)}{105\left(a^{4}d\cos(dx+c)^{4}+4\,a^{4}d\cos(dx+c)^{3}+6\,a^{4}d\cos(dx+c)^{2}+4\,a^{4}d\cos(dx+c)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(420*d*x*cos(d*x + c)^4 + 1680*d*x*cos(d*x + c)^3 + 2520*d*x*cos(d*x
 + c)^2 + 1680*d*x*cos(d*x + c) + 420*d*x - (105*cos(d*x + c)^4 + 1184*cos(
d*x + c)^3 + 2636*cos(d*x + c)^2 + 2236*cos(d*x + c) + 664)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos{\left(c+dx \right)}}{\sec^{4}{\left(c+dx \right)}+4\sec^{3}{\left(c+dx \right)}+6\sec^{2}{\left(c+dx \right)}+4\sec{\left(c+dx \right)}+1}\, dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)
**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.46, size = 112, normalized size = 0.89

$$-\frac{\frac{3360\,(dx+c)}{a^4}-\frac{1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}+\frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5145\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*(d*x + c)/a^4 - 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a^4) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*a^24*tan(1/2*d*x + 1
/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

Mupad [B]
time = 0.75, size = 137, normalized size = 1.09

$$-\frac{15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-192\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1144\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6112\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1680\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3360\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7(c+d\,x)}{840\,a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^4,x)
```

```
[Out] -(15*sin(c/2 + (d*x)/2) - 192*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 114
4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 6112*cos(c/2 + (d*x)/2)^6*sin(c
/2 + (d*x)/2) - 1680*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 3360*cos(c/2
 + (d*x)/2)^7*(c + d*x))/(840*a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.79 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=176

$$\frac{21x}{2a^4}-\frac{576\sin(c+dx)}{35a^4d}+\frac{21\cos(c+dx)\sin(c+dx)}{2a^4d}-\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2}-\frac{288\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))}-$$

```
[Out] 21/2*x/a^4-576/35*sin(d*x+c)/a^4/d+21/2*cos(d*x+c)*sin(d*x+c)/a^4/d-43/35*c
os(d*x+c)*sin(d*x+c)/a^4/d/(1+sec(d*x+c))^2-288/35*cos(d*x+c)*sin(d*x+c)/a^
4/d/(1+sec(d*x+c))-1/7*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-2/5*cos(d
*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A]
time = 0.27, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3902, 4105, 3872, 2715, 8, 2717}

$$-\frac{576\sin(c+dx)}{35a^4d}+\frac{21\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{288\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)}-\frac{43\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)^2}+\frac{21x}{2a^4}-\frac{2\sin(c+dx)\cos(c+dx)}{5ad(a\sec(c+dx)+a)^3}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (21*x)/(2*a^4) - (576*Sin[c + d*x])/(35*a^4*d) + (21*Cos[c + d*x]*Sin[c + d
*x])/(2*a^4*d) - (43*Cos[c + d*x]*Sin[c + d*x])/(35*a^4*d*(1 + Sec[c + d*x]
)^2) - (288*Cos[c + d*x]*Sin[c + d*x])/(35*a^4*d*(1 + Sec[c + d*x])) - (Cos
[c + d*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*Cos[c + d*x]*Sin[
c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)(-9a+5a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(-73a^2+56a^2}{(a+a\sec(c+dx))}}{35a^4}$$

$$= -\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+}{5ad(a+a\sec(c+d}$$

$$= -\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+}{5ad(a+a\sec(c+d}$$

$$= -\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+}{5ad(a+a\sec(c+d}$$

$$= -\frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} -$$

$$= \frac{21x}{2a^4} - \frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos(c+dx)\sin(c+}{35a^4d(1+\sec(c+dx)}$$

Mathematica [A]

time = 0.56, size = 289, normalized size = 1.64

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(102900dx\cos\left(\frac{dx}{2}\right)+102900dx\cos\left(c+\frac{dx}{2}\right)+61740dx\cos\left(c+\frac{3dx}{2}\right)+61740dx\cos\left(2c+\frac{3dx}{2}\right)+20580dx\cos\left(2c+\frac{5dx}{2}\right)+20580dx\cos\left(3c+\frac{5dx}{2}\right)+2940dx\cos\left(3c+\frac{7dx}{2}\right)+2940dx\cos\left(4c+\frac{7dx}{2}\right)-179830\sin\left(\frac{dx}{2}\right)+128730\sin\left(c+\frac{dx}{2}\right)-140826\sin\left(c+\frac{3dx}{2}\right)+44310\sin\left(2c+\frac{3dx}{2}\right)-60487\sin\left(2c+\frac{5dx}{2}\right)+1225\sin\left(3c+\frac{5dx}{2}\right)-12001\sin\left(3c+\frac{7dx}{2}\right)-3185\sin\left(4c+\frac{7dx}{2}\right)-315\sin\left(4c+\frac{9dx}{2}\right)-315\sin\left(5c+\frac{9dx}{2}\right)+35\sin\left(5c+\frac{11dx}{2}\right)+35\sin\left(6c+\frac{11dx}{2}\right)\right)}{35840a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(102900*d*x*Cos[(d*x)/2] + 102900*d*x*Cos[c +
(d*x)/2] + 61740*d*x*Cos[c + (3*d*x)/2] + 61740*d*x*Cos[2*c + (3*d*x)/2] +
20580*d*x*Cos[2*c + (5*d*x)/2] + 20580*d*x*Cos[3*c + (5*d*x)/2] + 2940*d*x*
Cos[3*c + (7*d*x)/2] + 2940*d*x*Cos[4*c + (7*d*x)/2] - 179830*Sin[(d*x)/2]
+ 128730*Sin[c + (d*x)/2] - 140826*Sin[c + (3*d*x)/2] + 44310*Sin[2*c + (3*
d*x)/2] - 60487*Sin[2*c + (5*d*x)/2] + 1225*Sin[3*c + (5*d*x)/2] - 12001*Si
n[3*c + (7*d*x)/2] - 3185*Sin[4*c + (7*d*x)/2] - 315*Sin[4*c + (9*d*x)/2] -
 315*Sin[5*c + (9*d*x)/2] + 35*Sin[5*c + (11*d*x)/2] + 35*Sin[6*c + (11*d*x
)/2]))/(35840*a^4*d)
```

Maple [A]

time = 0.10, size = 114, normalized size = 0.65

method	result
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-72\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+168\arctan}{8d\,a^4}$
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-72\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+168\arctan}{8d\,a^4}$
risch	$\frac{21x}{2a^4}-\frac{i\mathrm{e}^{2i(dx+c)}}{8a^4d}+\frac{2i\mathrm{e}^{i(dx+c)}}{a^4d}-\frac{2i\mathrm{e}^{-i(dx+c)}}{a^4d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^4d}-\frac{2i\left(700\,\mathrm{e}^{6i(dx+c)}+3675\,\mathrm{e}^{5i(dx+c)}+8505\,\mathrm{e}^{4i(dx+c)}+\right.}{35d\,a^4\left(\mathrm{e}\right.}$
norman	$\frac{\frac{21x}{2a}-\frac{167\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{281\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{217\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{167\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140ad}-\frac{53\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280ad}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56ad}+\frac{2}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-9/5*tan(1/2*d*x+1/2*c)^5+13*tan(1/2*d*x
+1/2*c)^3-111*tan(1/2*d*x+1/2*c)+16*(-9/2*tan(1/2*d*x+1/2*c)^3-7/2*tan(1/2*
d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+168*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]

time = 0.50, size = 204, normalized size = 1.16

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) - 455
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a^4)/d
```

Fricas [**A**]
time = 8.73, size = 171, normalized size = 0.97

$$\frac{735\,dx\cos\left(dx+c\right)^{4}+2940\,dx\cos\left(dx+c\right)^{3}+4410\,dx\cos\left(dx+c\right)^{2}+2940\,dx\cos\left(dx+c\right)+735\,dx+\left(35\,\cos\left(dx+c\right)^{5}-140\,\cos\left(dx+c\right)^{4}-2012\,\cos\left(dx+c\right)^{3}-4548\,\cos\left(dx+c\right)^{2}-3873\,\cos\left(dx+c\right)-1152\right)\sin\left(dx+c\right)}{70\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/70*(735*d*x*cos(d*x + c)^4 + 2940*d*x*cos(d*x + c)^3 + 4410*d*x*cos(d*x +
 c)^2 + 2940*d*x*cos(d*x + c) + 735*d*x + (35*cos(d*x + c)^5 - 140*cos(d*x
+ c)^4 - 2012*cos(d*x + c)^3 - 4548*cos(d*x + c)^2 - 3873*cos(d*x + c) - 11
52)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*
cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{2}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [**A**]
time = 0.48, size = 128, normalized size = 0.73

$$\frac{\frac{2940\,(dx+c)}{a^{4}}-\frac{280\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{4}}+\frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-63\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+455\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3885\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*(d*x + c)/a^4 - 280*(9*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (5*a^24*tan(1/2*d*x + 1/2*c
)^7 - 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 - 38
85*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 0.81, size = 159, normalized size = 0.90

$$\frac{5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-78\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+596\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4408\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2520\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+560\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2940\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7(c+d\,x)}{280\,a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^4,x)
```

```
[Out] (5*sin(c/2 + (d*x)/2) - 78*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 596*co
s(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 4408*cos(c/2 + (d*x)/2)^6*sin(c/2 +
 (d*x)/2) - 2520*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 560*cos(c/2 + (d
*x)/2)^10*sin(c/2 + (d*x)/2) + 2940*cos(c/2 + (d*x)/2)^7*(c + d*x))/(280*a^
4*d*cos(c/2 + (d*x)/2)^7)
```

3.80 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=200

$$-\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{181\tan(c+dx)}{63a^5d}-\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4}-\frac{29\sec^3(c}{63a^2d(c}$$

```
[Out] -5*arctanh(sin(d*x+c))/a^5/d+181/63*tan(d*x+c)/a^5/d-1/9*sec(d*x+c)^5*tan(d
*x+c)/d/(a+a*sec(d*x+c))^5-5/21*sec(d*x+c)^4*tan(d*x+c)/a/d/(a+a*sec(d*x+c)
)^4-29/63*sec(d*x+c)^3*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-67/63*sec(d*x+c)
^2*tan(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2+5*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A]

time = 0.32, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3901, 4104, 4093, 3872, 3855, 3852, 8}

$$\frac{181\tan(c+dx)}{63a^5d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{5\tan(c+dx)}{d\,(a^5\sec(c+dx)+a^5)}-\frac{67\tan(c+dx)\sec^2(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}-\frac{29\tan(c+dx)\sec^3(c+dx)}{63a^2d(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{5\tan(c+dx)\sec^4(c+dx)}{21ad(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^5*d) + (181*Tan[c + d*x])/(63*a^5*d) - (Sec[c
 + d*x]^5*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) - (5*Sec[c + d*x]^4*Ta
n[c + d*x])/(21*a*d*(a + a*Sec[c + d*x])^4) - (29*Sec[c + d*x]^3*Tan[c + d*
x])/(63*a^2*d*(a + a*Sec[c + d*x])^3) - (67*Sec[c + d*x]^2*Tan[c + d*x])/(6
3*a^3*d*(a + a*Sec[c + d*x])^2) + (5*Tan[c + d*x])/(d*(a^5 + a^5*Sec[c + d*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx &= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\sec^5(c+dx)(5a-10a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2} \\
&= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^4(c+dx)(60a^2-85a}{(a+a\sec(c+d}}{63a^4} \\
&= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan}{63a^2d(a+a\sec(c} \\
&= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan}{63a^2d(a+a\sec(c} \\
&= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan}{63a^2d(a+a\sec(c} \\
&= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan}{63a^2d(a+a\sec(c} \\
&= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+}{21ad(a+a\sec(c+dx)} \\
&= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{181\tan(c+dx)}{63a^5d} - \frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec}{21a}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(200) = 400.
time = 1.85, size = 401, normalized size = 2.00

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^5*(322560*Cos[(c + d*x)/2]^9*(Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Se
c[c/2]*Sec[c]*Sec[c + d*x]*(-33978*Sin[(d*x)/2] + 52002*Sin[(3*d*x)/2] - 56
952*Sin[c - (d*x)/2] + 43722*Sin[c + (d*x)/2] - 47208*Sin[2*c + (d*x)/2] -
18144*Sin[c + (3*d*x)/2] + 41796*Sin[2*c + (3*d*x)/2] - 28350*Sin[3*c + (3*
d*x)/2] + 34578*Sin[c + (5*d*x)/2] - 5691*Sin[2*c + (5*d*x)/2] + 28719*Sin[
3*c + (5*d*x)/2] - 11550*Sin[4*c + (5*d*x)/2] + 15517*Sin[2*c + (7*d*x)/2]
- 504*Sin[3*c + (7*d*x)/2] + 13186*Sin[4*c + (7*d*x)/2] - 2835*Sin[5*c + (7
*d*x)/2] + 4149*Sin[3*c + (9*d*x)/2] + 252*Sin[4*c + (9*d*x)/2] + 3582*Sin[
5*c + (9*d*x)/2] - 315*Sin[6*c + (9*d*x)/2] + 496*Sin[4*c + (11*d*x)/2] + 6
3*Sin[5*c + (11*d*x)/2] + 433*Sin[6*c + (11*d*x)/2])))/(2016*a^5*d*(1 + Sec
[c + d*x])^5)
```

Maple [A]
time = 0.07, size = 131, normalized size = 0.66

method	result
derivativedivides	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+80\ln\left(\tan\right.}{16d\,a^5}$
default	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+80\ln\left(\tan\right.}{16d\,a^5}$
risch	$\frac{2i\left(315\,e^{10i(dx+c)}+2835\,e^{9i(dx+c)}+11550\,e^{8i(dx+c)}+28350\,e^{7i(dx+c)}+47208\,e^{6i(dx+c)}+56952\,e^{5i(dx+c)}+52002\,e^{4i(dx+c)}\right.}{63d\,a^5\left(e^{i(dx+c)}+1\right)^9\left(e^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7+6*tan(1/2*d*x
+1/2*c)^5+24*tan(1/2*d*x+1/2*c)^3+129*tan(1/2*d*x+1/2*c)-16/(tan(1/2*d*x+1/
2*c)-1)+80*ln(tan(1/2*d*x+1/2*c)-1)-16/(tan(1/2*d*x+1/2*c)+1)-80*ln(tan(1/2
*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.28, size = 206, normalized size = 1.03

$$\frac{\frac{2016\sin(dx+c)}{\left(a^5-\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\sin(dx+c)}{\cos(dx+c)+1}+\frac{1512\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{72\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^5}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 - a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) + 1512*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

Fricas [A]
time = 6.53, size = 278, normalized size = 1.39

$$-\frac{315\left(\cos(dx+c)^6+5\cos(dx+c)^5+10\cos(dx+c)^4+10\cos(dx+c)^3+5\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-315\left(\cos(dx+c)^6+5\cos(dx+c)^5+10\cos(dx+c)^4+10\cos(dx+c)^3+5\cos(dx+c)^2+\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2\left(496\cos(dx+c)^5+2165\cos(dx+c)^4+3633\cos(dx+c)^3+2840\cos(dx+c)^2+946\cos(dx+c)+63\right)\sin(dx+c)}{126\left(a^5d\cos(dx+c)^6+5a^5d\cos(dx+c)^5+10a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+5a^5d\cos(dx+c)^2+a^5d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/126*(315*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos
(d*x + c)^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 315*
```

```
(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 5*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(496*cos(d*x
+ c)^5 + 2165*cos(d*x + c)^4 + 3633*cos(d*x + c)^3 + 2840*cos(d*x + c)^2 +
946*cos(d*x + c) + 63)*sin(d*x + c))/(a^5*d*cos(d*x + c)^6 + 5*a^5*d*cos(d*
x + c)^5 + 10*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 5*a^5*d*cos(
d*x + c)^2 + a^5*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^7(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**7/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A]
time = 0.53, size = 155, normalized size = 0.78

$$-\frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^5}-\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^5}+\frac{2016\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^5}-\frac{7a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+72a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+378a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1512a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8127a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{45}}}{1008d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^5 + 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^
2 - 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 + 72*a^40*tan(1/2*d*x + 1/2*c)
^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 + 1512*a^40*tan(1/2*d*x + 1/2*c)^3 + 8
127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

Mupad [B]
time = 0.72, size = 149, normalized size = 0.74

$$\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2a^5d}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8a^5d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{14a^5d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{144a^5d}-\frac{10\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^5d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^5\right)}+\frac{129\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a/cos(c + d*x))^5),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^3)/(2*a^5*d) + (3*tan(c/2 + (d*x)/2)^5)/(8*a^5*d) + t
an(c/2 + (d*x)/2)^7/(14*a^5*d) + tan(c/2 + (d*x)/2)^9/(144*a^5*d) - (10*ata
nh(tan(c/2 + (d*x)/2)))/(a^5*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^5*tan(c/2 +
(d*x)/2)^2 - a^5)) + (129*tan(c/2 + (d*x)/2))/(16*a^5*d)
```

3.81 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=177

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}-\frac{34\sec^2(c+dx)\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}+$$

```
[Out] arctanh(sin(d*x+c))/a^5/d-1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-
13/63*sec(d*x+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4-34/105*sec(d*x+c)^2*ta
n(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+173/315*tan(d*x+c)/a^3/d/(a+a*sec(d*x+c))
^2-661/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A]
time = 0.29, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3901, 4104, 4093, 4083, 3855, 3879}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{661\tan(c+dx)}{315d\,(a^5\sec(c+dx)+a^5)}+\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{34\tan(c+dx)\sec^2(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{13\tan(c+dx)\sec^3(c+dx)}{63ad(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^5*d) - (Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*S
ec[c + d*x])^5) - (13*Sec[c + d*x]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d
*x])^4) - (34*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*x])^
3) + (173*Tan[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (661*Tan[c + d
*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
```

```
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\sec^4(c+dx)(4a-9a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^3(c+dx)(39a^2-63a}{(a+a\sec(c+d}}{63a^4}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan}{105a^2d(a+a\sec(c}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan}{105a^2d(a+a\sec(c}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan}{105a^2d(a+a\sec(c}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}$$

Mathematica [A]
time = 1.86, size = 219, normalized size = 1.24

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(80640\cos^9\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\left(35973\sin\left(\frac{dx}{2}\right)-25515\sin\left(c+\frac{dx}{2}\right)+29757\sin\left(c+\frac{3dx}{2}\right)-11235\sin\left(2c+\frac{3dx}{2}\right)+14733\sin\left(2c+\frac{5dx}{2}\right)-2835\sin\left(3c+\frac{5dx}{2}\right)+4077\sin\left(3c+\frac{7dx}{2}\right)-315\sin\left(4c+\frac{7dx}{2}\right)+488\sin\left(4c+\frac{9dx}{2}\right)\right)\right)}{2520a^5d(1+\sec(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -1/2520*(Cos[(c + d*x)/2]*Sec[c + d*x]^5*(80640*Cos[(c + d*x)/2]^9*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) + Sec[c/2]*(35973*Sin[(d*x)/2] - 25515*Sin[c + (d*x)/2] + 29757*Sin[c +
(3*d*x)/2] - 11235*Sin[2*c + (3*d*x)/2] + 14733*Sin[2*c + (5*d*x)/2] - 2835
*Sin[3*c + (5*d*x)/2] + 4077*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c + (7*d*x)/2
] + 488*Sin[4*c + (9*d*x)/2])))/(a^5*d*(1 + Sec[c + d*x])^5)
```

Maple [A]
time = 0.07, size = 101, normalized size = 0.57

method	result
derivativedivides	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{26\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+16}{16d\,a^5}$
default	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{26\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+16}{16d\,a^5}$
risch	$-\frac{2i\left(315\,e^{8i(dx+c)}+2835\,e^{7i(dx+c)}+11235\,e^{6i(dx+c)}+25515\,e^{5i(dx+c)}+35973\,e^{4i(dx+c)}+29757\,e^{3i(dx+c)}+14733\,e^{2i(dx+c}\right.}{315d\,a^5\left(e^{i(dx+c)}+1\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-6/7*tan(1/2*d*x+1/2*c)^7-16/5*tan(1/2
*d*x+1/2*c)^5-26/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d*x+1/2*c)-16*ln(tan(1/2
*d*x+1/2*c)-1)+16*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.29, size = 159, normalized size = 0.90

$$-\frac{\frac{\frac{9765 \sin(dx+c)}{\cos(dx+c)+1}+\frac{2730 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{5040 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^5}}{5040\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) + 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 50
40*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)/(co
s(d*x + c) + 1) - 1)/a^5)/d
```

Fricas [A]
time = 7.02, size = 246, normalized size = 1.39

$$\frac{315 \left(\cos(dx+c)^5+5 \cos(dx+c)^4+10 \cos(dx+c)^3+10 \cos(dx+c)^2+5 \cos(dx+c)+1\right) \log\left(\sin(dx+c)+1\right)-315 \left(\cos(dx+c)^5+5 \cos(dx+c)^4+10 \cos(dx+c)^3+10 \cos(dx+c)^2+5 \cos(dx+c)+1\right) \log\left(-\sin(dx+c)+1\right)-2 \left(488 \cos(dx+c)^4+2125 \cos(dx+c)^3+3549 \cos(dx+c)^2+2740 \cos(dx+c)+863\right) \sin(dx+c)}{630 \left(a^5 d \cos(dx+c)^5+5 a^5 d \cos(dx+c)^4+10 a^5 d \cos(dx+c)^3+10 a^5 d \cos(dx+c)^2+5 a^5 d \cos(dx+c)+a^5 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/630*(315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(
d*x + c)^2 + 5*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 315*(cos(d*x + c)^
5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x +
c) + 1)*log(-sin(d*x + c) + 1) - 2*(488*cos(d*x + c)^4 + 2125*cos(d*x + c)^
3 + 3549*cos(d*x + c)^2 + 2740*cos(d*x + c) + 863)*sin(d*x + c))/(a^5*d*cos
(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*c
os(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^6(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\, dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**6/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A]
time = 0.52, size = 126, normalized size = 0.71

$$\frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^5}-\frac{5040\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^5}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2730\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9765\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 + 270*a^40*tan(1/2
*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 + 2730*a^40*tan(1/2*d*x
+ 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

Mupad [B]
time = 0.69, size = 99, normalized size = 0.56

$$-\frac{\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^5}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5\,a^5}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^5}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{144\,a^5}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^5}+\frac{31\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^5),x)
```

```
[Out] -((13*tan(c/2 + (d*x)/2)^3)/(24*a^5) + tan(c/2 + (d*x)/2)^5/(5*a^5) + (3*ta
n(c/2 + (d*x)/2)^7)/(56*a^5) + tan(c/2 + (d*x)/2)^9/(144*a^5) - (2*atanh(ta
n(c/2 + (d*x)/2)))/a^5 + (31*tan(c/2 + (d*x)/2))/(16*a^5))/d
```

3.82 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=159

$$\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}-\frac{32\tan(c+dx)}{315ad\,(a^2+a^2\sec(c+$$

```
[Out] 1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+4/63*sec(d*x+c)^3*tan(d*x+
c)/a/d/(a+a*sec(d*x+c))^4+4/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-32/315*
tan(d*x+c)/a/d/(a^2+a^2*sec(d*x+c))^2+4/45*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c)
)
```

Rubi [A]
time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3895, 3884, 4085, 3879}

$$\frac{4\tan(c+dx)}{45d\,(a^5\sec(c+dx)+a^5)}-\frac{32\tan(c+dx)}{315ad\,(a^2\sec(c+dx)+a^2)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{4\tan(c+dx)\sec^3(c+dx)}{63ad(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) + (4*Sec[c + d*x
]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(105*a
^2*d*(a + a*Sec[c + d*x])^3) - (32*Tan[c + d*x])/(315*a*d*(a^2 + a^2*Sec[c
+ d*x])^2) + (4*Tan[c + d*x])/(45*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
```

```
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx &= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}\\ &= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}\\ &= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+\ldots}\\ &= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+\ldots}\\ &= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+\ldots}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 97, normalized size = 0.61

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(126\sin\left(\frac{1}{2}(c+dx)\right)+84\sin\left(\frac{3}{2}(c+dx)\right)+36\sin\left(\frac{5}{2}(c+dx)\right)+9\sin\left(\frac{7}{2}(c+dx)\right)+\sin\left(\frac{9}{2}(c+dx)\right)\right)}{315a^5d(1+\sec(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^5*(126*Sin[(c + d*x)/2] + 84*Sin[(3*(c + d*x
))/2] + 36*Sin[(5*(c + d*x))/2] + 9*Sin[(7*(c + d*x))/2] + Sin[(9*(c + d*x)
)/2]))/(315*a^5*d*(1 + Sec[c + d*x])^5)
```

Maple [A]
time = 0.05, size = 71, normalized size = 0.45

method	result	size
risch	$\frac{16i\left(126\,e^{4i(dx+c)}+84\,e^{3i(dx+c)}+36\,e^{2i(dx+c)}+9\,e^{i(dx+c)}+1\right)}{315d\,a^{5}\left(e^{i(dx+c)}+1\right)^{9}}$	69
derivativedivides	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	71
default	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5+4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.29, size = 107, normalized size = 0.67

$$\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{420\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{378\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{180\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{5040\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) + 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 180*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Fricas [A]
time = 4.10, size = 123, normalized size = 0.77

$$\frac{\left(8\,\cos\left(dx+c\right)^{4}+40\,\cos\left(dx+c\right)^{3}+84\,\cos\left(dx+c\right)^{2}+100\,\cos\left(dx+c\right)+83\right)\sin\left(dx+c\right)}{315\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+c\right)+a^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 100*cos(d
*x + c) + 83)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^{5}\left(c+dx\right)}{\sec^{5}\left(c+dx\right)+5\sec^{4}\left(c+dx\right)+10\sec^{3}\left(c+dx\right)+10\sec^{2}\left(c+dx\right)+5\sec\left(c+dx\right)+1}\,dx}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A]
time = 0.50, size = 72, normalized size = 0.45

$$\frac{35 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^9 + 180 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^7 + 378 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 420 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 315 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{5040\,a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 + 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/
2*d*x + 1/2*c)^5 + 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(
a^5*d)
```

Mupad [B]
time = 0.76, size = 127, normalized size = 0.80

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+180\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\right)}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^5),x)
```

```
[Out] (sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 + 1
80*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^4 + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(5040*a
^5*d*cos(c/2 + (d*x)/2)^9)
```

3.83 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=159

$$-\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{63ad\left(a^2+a^2\sec(c+\right.}$$

```
[Out] -1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+5/63*sec(d*x+c)^3*tan(d*x
+c)/a/d/(a+a*sec(d*x+c))^4+1/21*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-8/63*ta
n(d*x+c)/a/d/(a^2+a^2*sec(d*x+c))^2+1/9*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3896, 3895, 3884, 4085, 3879}

$$\frac{\tan(c+dx)}{9d\left(a^5\sec(c+dx)+a^5\right)}-\frac{8\tan(c+dx)}{63ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{5\tan(c+dx)\sec^3(c+dx)}{63ad(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -1/9*(Sec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^5) + (5*Sec[c +
d*x]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) + Tan[c + d*x]/(21*a^2
*d*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(63*a*d*(a^2 + a^2*Sec[c + d*
x])^2) + Tan[c + d*x]/(9*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
```

```
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3896

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx &= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a} \\
&= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{5\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2} \\
&= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+ \\
&= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+ \\
&= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 97, normalized size = 0.61

$$\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(63\sin\left(\frac{dx}{2}\right)-63\sin\left(c+\frac{dx}{2}\right)+84\sin\left(c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)\right)}{8064a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(63*Sin[(d*x)/2] - 63*Sin[c + (d*x)/2] + 84*Si
n[c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Sin[3*c + (7*d*x)/2] + Sin[4
*c + (9*d*x)/2]))/(8064*a^5*d)
```

Maple [A]
time = 0.10, size = 58, normalized size = 0.36

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
default	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
risch	$\frac{4i\left(63\,\mathrm{e}^{5i(dx+c)}+63\,\mathrm{e}^{4i(dx+c)}+84\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+1\right)}{63d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{112ad}-\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1008ad}+\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{336ad}+\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right.}{336ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-2/7*tan(1/2*d*x+1/2*c)^7+2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.55

$$\frac{\frac{63\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{18\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{7\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{1008\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9)/(a^5*d)
```

Fricas [A]
time = 4.04, size = 123, normalized size = 0.77

$$\frac{\left(2\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+21\,\cos\left(dx+c\right)^{2}+25\,\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{63\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+c\right)+a^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 25*cos(d*x
 + c) + 5)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A]
time = 0.53, size = 59, normalized size = 0.37

$$-\frac{7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+18\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-42\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-63\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{1008\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 + 18*tan(1/2*d*x + 1/2*c)^7 - 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

Mupad [B]
time = 0.72, size = 58, normalized size = 0.36

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+42\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+63\right)}{1008\,a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^5),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 - 7*
tan(c/2 + (d*x)/2)^8 + 63))/(1008*a^5*d)
```

3.84 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=139

$$\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{45a^3d(a+a\sec(c+dx))^2}+$$

```
[Out] 1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-2/9*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4+1
/15*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+2/45*tan(d*x+c)/a^3/d/(a+a*sec(d*x+
c))^2+2/45*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3884, 4085, 3881, 3879}

$$\frac{2\tan(c+dx)}{45d\,(a^5\sec(c+dx)+a^5)}+\frac{2\tan(c+dx)}{45a^3d(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{15a^2d(a\sec(c+dx)+a)^3}-\frac{2\tan(c+dx)}{9ad(a\sec(c+dx)+a)^4}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - (2*Tan[c + d*x])/(9*a*d*(a + a*
Sec[c + d*x])^4) + Tan[c + d*x]/(15*a^2*d*(a + a*Sec[c + d*x])^3) + (2*Tan[
c + d*x])/(45*a^3*d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(45*d*(a^5 +
 a^5*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
```

```
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx &= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{\int \frac{\sec(c+dx)(-5a+9a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}\\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4}+\frac{\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{3a^2}\\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3}\\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3}\\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 110, normalized size = 0.79

$$\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(81\sin\left(\frac{dx}{2}\right)-45\sin\left(c+\frac{dx}{2}\right)+54\sin\left(c+\frac{3dx}{2}\right)-30\sin\left(2c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)\right)}{5760a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(81*Sin[(d*x)/2] - 45*Sin[c + (d*x)/2] + 54*Si
n[c + (3*d*x)/2] - 30*Sin[2*c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Si
n[3*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2]))/(5760*a^5*d)
```

Maple [A]
time = 0.10, size = 45, normalized size = 0.32

method	result	siz
derivativedivides	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$	45
default	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$	45
risch	$\frac{4i\left(30\,\mathrm{e}^{6i(dx+c)}+45\,\mathrm{e}^{5i(dx+c)}+81\,\mathrm{e}^{4i(dx+c)}+54\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+1\right)}{45d\,a^5\left(\mathrm{e}^{i(dx+c)}+1\right)^9}$	91
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}-\frac{13\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{720ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{72ad}+\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2a^4}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-2/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1
/2*c))
```

Maxima [A]
time = 0.28, size = 67, normalized size = 0.48

$$\frac{\frac{45\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{18\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{720\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/720*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Fricas [A]
time = 3.07, size = 123, normalized size = 0.88

$$\frac{\left(2\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3+21\,\cos\left(dx+c\right)^2+10\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{45\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx+c\right)^4+10\,a^5d\cos\left(dx+c\right)^3+10\,a^5d\cos\left(dx+c\right)^2+5\,a^5d\cos\left(dx+c\right)+a^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/45*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{3}(c+dx)}{\sec^{5}(c+dx)+5\sec^{4}(c+dx)+10\sec^{3}(c+dx)+10\sec^{2}(c+dx)+5\sec(c+dx)+1}\,dx}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A]
time = 0.49, size = 46, normalized size = 0.33

$$\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{720\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/720*(5*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^5 + 45*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

Mupad [B]
time = 0.66, size = 45, normalized size = 0.32

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+45\right)}{720\,a^{5}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^5),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(5*tan(c/2 + (d*x)/2)^8 - 18*tan(c/2 + (d*x)/2)^4 + 45))/(720*a^5*d)
```

3.85 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=143

$$-\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{63ad\,(a^2+a^2\sec(c+dx)}$$

```
[Out] -1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+5/63*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4
+1/21*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+2/63*tan(d*x+c)/a/d/(a^2+a^2*sec(
d*x+c))^2+2/63*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3882, 3881, 3879}

$$\frac{2\tan(c+dx)}{63d\,(a^5\sec(c+dx)+a^5)}+\frac{2\tan(c+dx)}{63ad\,(a^2\sec(c+dx)+a^2)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}+\frac{5\tan(c+dx)}{63ad(a\sec(c+dx)+a)^4}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -1/9*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^5) + (5*Tan[c + d*x])/(63*a*d*(a
+ a*Sec[c + d*x])^4) + Tan[c + d*x]/(21*a^2*d*(a + a*Sec[c + d*x])^3) + (2*
Tan[c + d*x])/(63*a*d*(a^2 + a^2*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(63*d*
(a^5 + a^5*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
```

```
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx &= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}\\ &= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}\\ &= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx)}\\ &= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx)}\\ &= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx)}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 125, normalized size = 0.87

$$\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(315\sin\left(\frac{dx}{2}\right)-315\sin\left(c+\frac{dx}{2}\right)+273\sin\left(c+\frac{3dx}{2}\right)-147\sin\left(2c+\frac{3dx}{2}\right)+117\sin\left(2c+\frac{5dx}{2}\right)-63\sin\left(3c+\frac{5dx}{2}\right)+45\sin\left(3c+\frac{7dx}{2}\right)+5\sin\left(4c+\frac{9dx}{2}\right)\right)}{16128a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(315*Sin[(d*x)/2] - 315*Sin[c + (d*x)/2] + 273
*Sin[c + (3*d*x)/2] - 147*Sin[2*c + (3*d*x)/2] + 117*Sin[2*c + (5*d*x)/2] -
 63*Sin[3*c + (5*d*x)/2] + 45*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)/2]
))/(16128*a^5*d)
```

Maple [A]
time = 0.08, size = 58, normalized size = 0.41

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$	58
default	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$	58
risch	$\frac{2i\left(63\,e^{7i(dx+c)}+147\,e^{6i(dx+c)}+315\,e^{5i(dx+c)}+315\,e^{4i(dx+c)}+273\,e^{3i(dx+c)}+117\,e^{2i(dx+c)}+45\,e^{i(dx+c)}+5\right)}{63d\,a^5\left(e^{i(dx+c)}+1\right)^9}$	102

norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56ad}+\frac{25\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1008ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^4}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9+2/7*tan(1/2*d*x+1/2*c)^7-2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.30, size = 87, normalized size = 0.61

$$\frac{\frac{63 \sin(dx+c)}{\cos(dx+c)+1}-\frac{42 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{18 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{7 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\, a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) - 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9)/(a^5*d)
```

Fricas [A]
time = 15.42, size = 123, normalized size = 0.86

$$\frac{\left(5\,\cos\left(dx+c\right)^4+25\,\cos\left(dx+c\right)^3+21\,\cos\left(dx+c\right)^2+10\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{63\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx+c\right)^4+10\,a^5d\cos\left(dx+c\right)^3+10\,a^5d\cos\left(dx+c\right)^2+5\,a^5d\cos\left(dx+c\right)+a^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(5*cos(d*x + c)^4 + 25*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2\left(c+dx\right)}{\sec^5\left(c+dx\right)+5\sec^4\left(c+dx\right)+10\sec^3\left(c+dx\right)+10\sec^2\left(c+dx\right)+5\sec\left(c+dx\right)+1}\, dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A]
time = 0.50, size = 59, normalized size = 0.41

$$-\frac{7 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-18 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+42 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-63 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{1008\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^7 + 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

Mupad [B]
time = 0.69, size = 58, normalized size = 0.41

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+42\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-63\right)}{1008\,a^{5}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^5),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 + 7
*tan(c/2 + (d*x)/2)^8 - 63))/(1008*a^5*d)
```

3.86 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=143

$$\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}+\frac{8\tan(c+dx)}{315ad\left(a^2+a^2\sec(c+dx)\right)^2}$$

```
[Out] 1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+4/63*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4+
4/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+8/315*tan(d*x+c)/a/d/(a^2+a^2*sec
(d*x+c))^2+8/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3881, 3879}

$$\frac{8\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{8\tan(c+dx)}{315ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{4\tan(c+dx)}{63ad(a\sec(c+dx)+a)^4}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) + (4*Tan[c + d*x])/(63*a*d*(a + a
*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*x])^3) + (
8*Tan[c + d*x])/(315*a*d*(a^2 + a^2*Sec[c + d*x])^2) + (8*Tan[c + d*x])/(31
5*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx &= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a} \\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2} \\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))} \\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))} \\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))}
\end{aligned}
$$

Mathematica [A]

time = 0.29, size = 138, normalized size = 0.97

$$
\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(5418\sin\left(\frac{dx}{2}\right)-5040\sin\left(c+\frac{dx}{2}\right)+3612\sin\left(c+\frac{3dx}{2}\right)-3360\sin\left(2c+\frac{3dx}{2}\right)+1728\sin\left(2c+\frac{5dx}{2}\right)-1260\sin\left(3c+\frac{5dx}{2}\right)+432\sin\left(3c+\frac{7dx}{2}\right)-315\sin\left(4c+\frac{7dx}{2}\right)+83\sin\left(4c+\frac{9dx}{2}\right)\right)}{80640a^5d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(5418*Sin[(d*x)/2] - 5040*Sin[c + (d*x)/2] + 3
612*Sin[c + (3*d*x)/2] - 3360*Sin[2*c + (3*d*x)/2] + 1728*Sin[2*c + (5*d*x)
/2] - 1260*Sin[3*c + (5*d*x)/2] + 432*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c +
(7*d*x)/2] + 83*Sin[4*c + (9*d*x)/2]))/(80640*a^5*d)
```

Maple [A]

time = 0.07, size = 71, normalized size = 0.50

method	result
derivativedivides	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$
default	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{28ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{144ad}}{a^4}$
risch	$\frac{2i\left(315\,\mathrm{e}^{8i(dx+c)}+1260\,\mathrm{e}^{7i(dx+c)}+3360\,\mathrm{e}^{6i(dx+c)}+5040\,\mathrm{e}^{5i(dx+c)}+5418\,\mathrm{e}^{4i(dx+c)}+3612\,\mathrm{e}^{3i(dx+c)}+1728\,\mathrm{e}^{2i(dx+c)}+432\,\mathrm{e}^{i}\right.}{315d\,a^5\left(\mathrm{e}^{i(dx+c)}+1\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 107, normalized size = 0.75

$$\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{420\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{180\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 180*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Fricas [A]
time = 4.59, size = 123, normalized size = 0.86

$$\frac{\left(83\cos\left(dx+c\right)^4+100\cos\left(dx+c\right)^3+84\cos\left(dx+c\right)^2+40\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{315\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx+c\right)^4+10\,a^5d\cos\left(dx+c\right)^3+10\,a^5d\cos\left(dx+c\right)^2+5\,a^5d\cos\left(dx+c\right)+a^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(83*cos(d*x + c)^4 + 100*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 40*cos(
d*x + c) + 8)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec\left(c+dx\right)}{\sec^5\left(c+dx\right)+5\sec^4\left(c+dx\right)+10\sec^3\left(c+dx\right)+10\sec^2\left(c+dx\right)+5\sec\left(c+dx\right)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x
)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A]
time = 0.51, size = 72, normalized size = 0.50

$$\frac{35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-180\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+378\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-420\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+315\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{5040\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")

[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 - 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/2*d*x + 1/2*c)^5 - 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(a^5*d)

Mupad [B]
time = 0.76, size = 127, normalized size = 0.89

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-180\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\right)}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^5),x)

[Out] (sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 - 180*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 - 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(5040*a^5*d*cos(c/2 + (d*x)/2)^9)

3.87 $\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=144

$$\frac{x}{a^5}-\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}-\frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}-\frac{173\tan(c+dx)}{315a^3d(a+a\sec(c+$$

```
[Out] x/a^5-1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-13/63*tan(d*x+c)/a/d/(a+a*sec(d*x
+c))^4-34/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-173/315*tan(d*x+c)/a^3/d/
(a+a*sec(d*x+c))^2-488/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3862, 4007, 4004, 3879}

$$-\frac{488\tan(c+dx)}{315d\,(a^5\sec(c+dx)+a^5)}+\frac{x}{a^5}-\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{34\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}-\frac{13\tan(c+dx)}{63ad(a\sec(c+dx)+a)^4}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-5),x]
```

```
[Out] x/a^5 - Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - (13*Tan[c + d*x])/(63*a
*d*(a + a*Sec[c + d*x])^4) - (34*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*
x])^3) - (173*Tan[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (488*Tan[c
 + d*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))^5}\,dx &= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{\int \frac{-9a+4a\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a^2}\\
&= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\int \frac{63a^2-39a^2\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{63a^4}\\
&= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}-\frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx}\\
&= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}-\frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx}\\
&= \frac{x}{a^5}-\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}-\frac{34\tan(c+dx}{105a^2d(a+a\sec(c}\\
&= \frac{x}{a^5}-\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}-\frac{34\tan(c+dx}{105a^2d(a+a\sec(c}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 280, normalized size = 1.94

$$\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(39690dx\cos\left(\frac{dx}{2}\right)+39690dx\cos\left(c+\frac{dx}{2}\right)+26460dx\cos\left(c+\frac{3dx}{2}\right)+26460dx\cos\left(2c+\frac{3dx}{2}\right)+11340dx\cos\left(2c+\frac{5dx}{2}\right)+11340dx\cos\left(3c+\frac{5dx}{2}\right)+2835dx\cos\left(3c+\frac{7dx}{2}\right)+2835dx\cos\left(4c+\frac{7dx}{2}\right)+315dx\cos\left(4c+\frac{9dx}{2}\right)+315dx\cos\left(5c+\frac{9dx}{2}\right)-116676\sin\left(\frac{dx}{2}\right)+100800\sin\left(c+\frac{dx}{2}\right)-88284\sin\left(c+\frac{3dx}{2}\right)+56700\sin\left(2c+\frac{3dx}{2}\right)-43236\sin\left(2c+\frac{5dx}{2}\right)+18900\sin\left(3c+\frac{5dx}{2}\right)-12384\sin\left(3c+\frac{7dx}{2}\right)+3150\sin\left(4c+\frac{7dx}{2}\right)-1726\sin\left(4c+\frac{9dx}{2}\right)\right)}{161280a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-5),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(39690*d*x*Cos[(d*x)/2] + 39690*d*x*Cos[c + (d
*x)/2] + 26460*d*x*Cos[c + (3*d*x)/2] + 26460*d*x*Cos[2*c + (3*d*x)/2] + 11
340*d*x*Cos[2*c + (5*d*x)/2] + 11340*d*x*Cos[3*c + (5*d*x)/2] + 2835*d*x*Co
s[3*c + (7*d*x)/2] + 2835*d*x*Cos[4*c + (7*d*x)/2] + 315*d*x*Cos[4*c + (9*d
*x)/2] + 315*d*x*Cos[5*c + (9*d*x)/2] - 116676*Sin[(d*x)/2] + 100800*Sin[c
+ (d*x)/2] - 88284*Sin[c + (3*d*x)/2] + 56700*Sin[2*c + (3*d*x)/2] - 43236*
Sin[2*c + (5*d*x)/2] + 18900*Sin[3*c + (5*d*x)/2] - 12384*Sin[3*c + (7*d*x)
```

```
/2] + 3150*Sin[4*c + (7*d*x)/2] - 1726*Sin[4*c + (9*d*x)/2]))/(161280*a^5*d
)
```

Maple [A]
time = 0.07, size = 85, normalized size = 0.59

method	result
derivativedivides	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{26\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\,a^5}$
default	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{26\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\,a^5}$
norman	$\frac{\frac{x}{a}-\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5ad}+\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{144ad}}{a^4}$
risch	$\frac{x}{a^5}-\frac{2i\left(1575\,e^{8i(dx+c)}+9450\,e^{7i(dx+c)}+28350\,e^{6i(dx+c)}+50400\,e^{5i(dx+c)}+58338\,e^{4i(dx+c)}+44142\,e^{3i(dx+c)}+21618\,e\right.}{315d\,a^5\left(e^{i(dx+c)}+1\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9+6/7*tan(1/2*d*x+1/2*c)^7-16/5*tan(1/2
*d*x+1/2*c)^5+26/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d*x+1/2*c)+32*arctan(tan
(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.50, size = 132, normalized size = 0.92

$$-\frac{\frac{\frac{9765\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{2730\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{270\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{10080\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) - 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 10
080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

Fricas [A]
time = 3.74, size = 188, normalized size = 1.31

$$\frac{315\,dx\cos(dx+c)^5+1575\,dx\cos(dx+c)^4+3150\,dx\cos(dx+c)^3+3150\,dx\cos(dx+c)^2+1575\,dx\cos(dx+c)+315\,dx-\left(863\,\cos(dx+c)^4+2740\,\cos(dx+c)^3+3549\,\cos(dx+c)^2+2125\,\cos(dx+c)+488\right)\sin(dx+c)}{315\left(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+10\,a^5d\cos(dx+c)^3+10\,a^5d\cos(dx+c)^2+5\,a^5d\cos(dx+c)+a^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (863*c
os(d*x + c)^4 + 2740*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2125*cos(d*x +
c) + 488)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(1/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x)**3 + 10*s
ec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A]
time = 0.47, size = 100, normalized size = 0.69

$$\frac{\frac{5040\,(dx+c)}{a^5}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2730\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9765\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(5040*(d*x + c)/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 270*a^40*tan
(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 - 2730*a^40*tan(1/2*
d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

Mupad [B]
time = 0.80, size = 125, normalized size = 0.87

$$\frac{x}{a^5}-\frac{\frac{863\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{315}-\frac{356\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{315}+\frac{169\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{420}-\frac{41\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{504}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{144}}{a^5\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^5,x)
```

```
[Out] x/a^5 - (sin(c/2 + (d*x)/2)/144 - (41*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/
2))/504 + (169*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/420 - (356*cos(c/2
+ (d*x)/2)^6*sin(c/2 + (d*x)/2))/315 + (863*cos(c/2 + (d*x)/2)^8*sin(c/2 +
(d*x)/2))/315)/(a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.88 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=159

$$-\frac{5x}{a^5}+\frac{496\sin(c+dx)}{63a^5d}-\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4}-\frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3}-\frac{63c}{}$$

```
[Out] -5*x/a^5+496/63*sin(d*x+c)/a^5/d-1/9*sin(d*x+c)/d/(a+a*sec(d*x+c))^5-5/21*s
in(d*x+c)/a/d/(a+a*sec(d*x+c))^4-29/63*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-
67/63*sin(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2-5*sin(d*x+c)/d/(a^5+a^5*sec(d*x+c
))
```

Rubi [A]
time = 0.27, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3902, 4105, 3872, 2717, 8}

$$\frac{496\sin(c+dx)}{63a^5d}-\frac{5\sin(c+dx)}{d\left(a^5\sec(c+dx)+a^5\right)}-\frac{5x}{a^5}-\frac{67\sin(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}-\frac{29\sin(c+dx)}{63a^2d(a\sec(c+dx)+a)^3}-\frac{5\sin(c+dx)}{21ad(a\sec(c+dx)+a)^4}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (-5*x)/a^5 + (496*Sin[c + d*x])/(63*a^5*d) - Sin[c + d*x]/(9*d*(a + a*Sec[c
 + d*x])^5) - (5*Sin[c + d*x])/(21*a*d*(a + a*Sec[c + d*x])^4) - (29*Sin[c
+ d*x])/(63*a^2*d*(a + a*Sec[c + d*x])^3) - (67*Sin[c + d*x])/(63*a^3*d*(a
+ a*Sec[c + d*x])^2) - (5*Sin[c + d*x])/(d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\cos(c+dx)(-10a+5a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos(c+dx)(-85a^2+60a^2\sec(}{(a+a\sec(c+dx))^3}}{63a^4}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx)}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx)}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx)}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx)}$$

$$= -\frac{5x}{a^5} + \frac{496\sin(c+dx)}{63a^5d} - \frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} -$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(159) = 318.
time = 0.68, size = 319, normalized size = 2.01

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(79380dx\cos\left(\frac{dx}{2}\right)+79380dx\cos\left(c+\frac{dx}{2}\right)+52920dx\cos\left(c+\frac{3dx}{2}\right)+52920dx\cos\left(2c+\frac{3dx}{2}\right)+22680dx\cos\left(2c+\frac{5dx}{2}\right)+22680dx\cos\left(3c+\frac{5dx}{2}\right)+5670dx\cos\left(3c+\frac{7dx}{2}\right)+5670dx\cos\left(4c+\frac{7dx}{2}\right)+630dx\cos\left(4c+\frac{9dx}{2}\right)+630dx\cos\left(5c+\frac{9dx}{2}\right)-175014\sin\left(\frac{dx}{2}\right)+143010\sin\left(c+\frac{dx}{2}\right)-138726\sin\left(c+\frac{3dx}{2}\right)+73290\sin\left(2c+\frac{3dx}{2}\right)-70389\sin\left(2c+\frac{5dx}{2}\right)+20475\sin\left(3c+\frac{5dx}{2}\right)-21141\sin\left(3c+\frac{7dx}{2}\right)+1575\sin\left(4c+\frac{7dx}{2}\right)-3091\sin\left(4c+\frac{9dx}{2}\right)-567\sin\left(5c+\frac{9dx}{2}\right)-63\sin\left(5c+\frac{11dx}{2}\right)-63\sin\left(6c+\frac{11dx}{2}\right)\right)}{64512a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -1/64512*(Sec[c/2]*Sec[(c + d*x)/2]^9*(79380*d*x*Cos[(d*x)/2] + 79380*d*x*C
os[c + (d*x)/2] + 52920*d*x*Cos[c + (3*d*x)/2] + 52920*d*x*Cos[2*c + (3*d*x
)/2] + 22680*d*x*Cos[2*c + (5*d*x)/2] + 22680*d*x*Cos[3*c + (5*d*x)/2] + 56
70*d*x*Cos[3*c + (7*d*x)/2] + 5670*d*x*Cos[4*c + (7*d*x)/2] + 630*d*x*Cos[4
*c + (9*d*x)/2] + 630*d*x*Cos[5*c + (9*d*x)/2] - 175014*Sin[(d*x)/2] + 1430
10*Sin[c + (d*x)/2] - 138726*Sin[c + (3*d*x)/2] + 73290*Sin[2*c + (3*d*x)/2
] - 70389*Sin[2*c + (5*d*x)/2] + 20475*Sin[3*c + (5*d*x)/2] - 21141*Sin[3*c
 + (7*d*x)/2] + 1575*Sin[4*c + (7*d*x)/2] - 3091*Sin[4*c + (9*d*x)/2] - 567
*Sin[5*c + (9*d*x)/2] - 63*Sin[5*c + (11*d*x)/2] - 63*Sin[6*c + (11*d*x)/2]
))/(a^5*d)
```

Maple [A]
time = 0.10, size = 111, normalized size = 0.70

method	result
derivativedivides	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-160\,\mathrm{arc}}{16d\,a^5}$
default	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-160\,\mathrm{arc}}{16d\,a^5}$
norman	$\frac{-\frac{5x}{a}+\frac{161\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{105\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{17\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}-\frac{65\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1008ad}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144ad}-\frac{5}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}$
risch	$-\frac{5x}{a^5}-\frac{i\mathrm{e}^{i(dx+c)}}{2a^5d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2a^5d}+\frac{2i\left(945\,\mathrm{e}^{8i(dx+c)}+6300\,\mathrm{e}^{7i(dx+c)}+19740\,\mathrm{e}^{6i(dx+c)}+36414\,\mathrm{e}^{5i(dx+c)}+43092\,\mathrm{e}^{4i(d}\right.}{63d\,a^5\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-8/7*tan(1/2*d*x+1/2*c)^7+6*tan(1/2*d*x
+1/2*c)^5-24*tan(1/2*d*x+1/2*c)^3+129*tan(1/2*d*x+1/2*c)+32*tan(1/2*d*x+1/2
*c)/(1+tan(1/2*d*x+1/2*c)^2)-160*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.52, size = 178, normalized size = 1.12

$$\frac{\frac{2016\sin(dx+c)}{\left(a^5+\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\sin(dx+c)}{\cos(dx+c)+1}-\frac{1512\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{72\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{10080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 + a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) - 1512*sin(d*x
```

```
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

Fricas [A]
time = 2.54, size = 198, normalized size = 1.25

$$-\frac{315\,dx\cos\left(dx+c\right)^{5}+1575\,dx\cos\left(dx+c\right)^{4}+3150\,dx\cos\left(dx+c\right)^{3}+3150\,dx\cos\left(dx+c\right)^{2}+1575\,dx\cos\left(dx+c\right)+315\,dx-\left(63\,\cos\left(dx+c\right)^{5}+946\,\cos\left(dx+c\right)^{4}+2840\,\cos\left(dx+c\right)^{3}+3633\,\cos\left(dx+c\right)^{2}+2165\,\cos\left(dx+c\right)+496\right)\sin\left(dx+c\right)}{63\,\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+c\right)+a^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/63*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (63*co
s(d*x + c)^5 + 946*cos(d*x + c)^4 + 2840*cos(d*x + c)^3 + 3633*cos(d*x + c)
^2 + 2165*cos(d*x + c) + 496)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d
*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5
*d*cos(d*x + c) + a^5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos\left(c+dx\right)}{\sec^{5}\left(c+dx\right)+5\sec^{4}\left(c+dx\right)+10\sec^{3}\left(c+dx\right)+10\sec^{2}\left(c+dx\right)+5\sec\left(c+dx\right)+1}\,dx}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x
)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A]
time = 0.48, size = 129, normalized size = 0.81

$$-\frac{\frac{5040\,(dx+c)}{a^{5}}-\frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{5}}-\frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+378\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1512\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8127\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*(d*x + c)/a^5 - 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 - 72*a^40*tan(1/2*d*x + 1/
2*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 - 1512*a^40*tan(1/2*d*x + 1/2*c)^3
 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

Mupad [B]
time = 0.83, size = 159, normalized size = 1.00

$$\frac{7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-100\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+636\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2512\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+10096\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2016\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5040\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\left(c+d\,x\right)}{1008\,a^{5}\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^5,x)
```

```
[Out] (7*sin(c/2 + (d*x)/2) - 100*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 636*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 2512*cos(c/2 + (d*x)/2)^6*sin(c/2
+ (d*x)/2) + 10096*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 2016*cos(c/2 +
 (d*x)/2)^10*sin(c/2 + (d*x)/2) - 5040*cos(c/2 + (d*x)/2)^9*(c + d*x))/(100
8*a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.89 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=215

$$\frac{31x}{2a^5}-\frac{7664\sin(c+dx)}{315a^5d}+\frac{31\cos(c+dx)\sin(c+dx)}{2a^5d}-\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4}-\frac{28}{45}$$

```
[Out] 31/2*x/a^5-7664/315*sin(d*x+c)/a^5/d+31/2*cos(d*x+c)*sin(d*x+c)/a^5/d-1/9*c
os(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^5-17/63*cos(d*x+c)*sin(d*x+c)/a/d/(
a+a*sec(d*x+c))^4-28/45*cos(d*x+c)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-577/
315*cos(d*x+c)*sin(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2-3832/315*cos(d*x+c)*sin(
d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A]
time = 0.35, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3902, 4105, 3872, 2715, 8, 2717}

$$-\frac{7664\sin(c+dx)}{315a^5d}+\frac{31\sin(c+dx)\cos(c+dx)}{2a^5d}-\frac{3832\sin(c+dx)\cos(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{31x}{2a^5}-\frac{577\sin(c+dx)\cos(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{28\sin(c+dx)\cos(c+dx)}{45a^2d(a\sec(c+dx)+a)^3}-\frac{17\sin(c+dx)\cos(c+dx)}{63ad(a\sec(c+dx)+a)^4}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (31*x)/(2*a^5) - (7664*Sin[c + d*x])/(315*a^5*d) + (31*Cos[c + d*x]*Sin[c +
 d*x])/(2*a^5*d) - (Cos[c + d*x]*Sin[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5)
 - (17*Cos[c + d*x]*Sin[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) - (28*Cos
[c + d*x]*Sin[c + d*x])/(45*a^2*d*(a + a*Sec[c + d*x])^3) - (577*Cos[c + d*
x]*Sin[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (3832*Cos[c + d*x]*Si
n[c + d*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\cos^2(c+dx)(-11a+6a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)(-111a^2+85a^2}{(a+a\sec(c+dx))}}{63a^4}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+}{45a^2d(a+a\sec(c+d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+}{45a^2d(a+a\sec(c+d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+}{45a^2d(a+a\sec(c+d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+}{45a^2d(a+a\sec(c+d}$$

$$= -\frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} - \frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17}{6}$$

$$= \frac{31x}{2a^5} - \frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} - \frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5}$$

Mathematica [**A**]
time = 0.76, size = 345, normalized size = 1.60

$$\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(4921560dx\cos\left(\frac{dx}{2}\right)+4921560dx\cos\left(c+\frac{dx}{2}\right)+3281040dx\cos\left(c+\frac{3dx}{2}\right)+3281040dx\cos\left(2c+\frac{3dx}{2}\right)+1406160dx\cos\left(2c+\frac{5dx}{2}\right)+1406160dx\cos\left(3c+\frac{5dx}{2}\right)+351540dx\cos\left(3c+\frac{7dx}{2}\right)+351540dx\cos\left(4c+\frac{7dx}{2}\right)+39060dx\cos\left(4c+\frac{9dx}{2}\right)+39060dx\cos\left(5c+\frac{9dx}{2}\right)-9163224\sin\left(\frac{dx}{2}\right)+7194600\sin\left(c+\frac{dx}{2}\right)-7472241\sin\left(c+\frac{3dx}{2}\right)+3432975\sin\left(2c+\frac{3dx}{2}\right)-3871989\sin\left(2c+\frac{5dx}{2}\right)+801675\sin\left(3c+\frac{5dx}{2}\right)-1186056\sin\left(3c+\frac{7dx}{2}\right)-17640\sin\left(4c+\frac{7dx}{2}\right)-175184\sin\left(4c+\frac{9dx}{2}\right)-45360\sin\left(5c+\frac{9dx}{2}\right)-3465\sin\left(5c+\frac{11dx}{2}\right)-3465\sin\left(6c+\frac{11dx}{2}\right)+315\sin\left(6c+\frac{13dx}{2}\right)+315\sin\left(7c+\frac{13dx}{2}\right)\right)}{1290240a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(4921560*d*x*Cos[(d*x)/2] + 4921560*d*x*Cos[c
+ (d*x)/2] + 3281040*d*x*Cos[c + (3*d*x)/2] + 3281040*d*x*Cos[2*c + (3*d*x)
/2] + 1406160*d*x*Cos[2*c + (5*d*x)/2] + 1406160*d*x*Cos[3*c + (5*d*x)/2] +
 351540*d*x*Cos[3*c + (7*d*x)/2] + 351540*d*x*Cos[4*c + (7*d*x)/2] + 39060*
d*x*Cos[4*c + (9*d*x)/2] + 39060*d*x*Cos[5*c + (9*d*x)/2] - 9163224*Sin[(d*
x)/2] + 7194600*Sin[c + (d*x)/2] - 7472241*Sin[c + (3*d*x)/2] + 3432975*Sin
[2*c + (3*d*x)/2] - 3871989*Sin[2*c + (5*d*x)/2] + 801675*Sin[3*c + (5*d*x)
/2] - 1186056*Sin[3*c + (7*d*x)/2] - 17640*Sin[4*c + (7*d*x)/2] - 175184*Si
n[4*c + (9*d*x)/2] - 45360*Sin[5*c + (9*d*x)/2] - 3465*Sin[5*c + (11*d*x)/2
] - 3465*Sin[6*c + (11*d*x)/2] + 315*Sin[6*c + (13*d*x)/2] + 315*Sin[7*c +
(13*d*x)/2]))/(1290240*a^5*d)
```

Maple [**A**]
time = 0.12, size = 127, normalized size = 0.59

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{10\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{48\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+50\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-176\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}{\left(1+\tan^{2}\left(\frac{dx}{2}+\right.\right.}}{16d\,a^{5}}$
default	$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{10\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{48\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+50\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-176\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}{\left(1+\tan^{2}\left(\frac{dx}{2}+\right.\right.}}{16d\,a^{5}}$
norman	$\frac{\frac{31x}{2a}-\frac{495\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{207\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{1303\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80ad}+\frac{141\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70ad}-\frac{2159\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5040ad}+\frac{19\left(\tan^{11}\left(\frac{dx}{2}\right.\right.}{252ad}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a^{4}}$
risch	$\frac{31x}{2a^{5}}-\frac{i\mathrm{e}^{2i(dx+c)}}{8a^{5}d}+\frac{5i\mathrm{e}^{i(dx+c)}}{2a^{5}d}-\frac{5i\mathrm{e}^{-i(dx+c)}}{2a^{5}d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^{5}d}-\frac{2i\left(11025\,\mathrm{e}^{8i(dx+c)}+77175\,\mathrm{e}^{7i(dx+c)}+247695\,\mathrm{e}^{6i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9+10/7*tan(1/2*d*x+1/2*c)^7-48/5*tan(1/
2*d*x+1/2*c)^5+50*tan(1/2*d*x+1/2*c)^3-351*tan(1/2*d*x+1/2*c)+32*(-11/2*tan
(1/2*d*x+1/2*c)^3-9/2*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+496*ar
ctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]

time = 0.51, size = 224, normalized size = 1.04

$$-\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{5}+\frac{2a^{5}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{5}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}-\frac{15750\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3024\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{450\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{a^{5}}-\frac{156240\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{5}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3)/(a^5 + 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) + 1)
- 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9)/a^5 - 156240*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^5)/d
```

Fricas [A]

time = 2.49, size = 207, normalized size = 0.96

$$\frac{9765\,dx\cos(dx+c)^{5}+48825\,dx\cos(dx+c)^{4}+97650\,dx\cos(dx+c)^{3}+97650\,dx\cos(dx+c)^{2}+48825\,dx\cos(dx+c)+9765\,dx+\left(315\cos(dx+c)^{6}-1575\cos(dx+c)^{5}-28828\cos(dx+c)^{4}-87440\cos(dx+c)^{3}-112119\cos(dx+c)^{2}-66875\cos(dx+c)-15328\right)\sin(dx+c)}{630\left(a^{5}d\cos(dx+c)^{5}+5a^{5}d\cos(dx+c)^{4}+10a^{5}d\cos(dx+c)^{3}+10a^{5}d\cos(dx+c)^{2}+5a^{5}d\cos(dx+c)+a^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

[Out] 1/630*(9765*d*x*cos(d*x + c)^5 + 48825*d*x*cos(d*x + c)^4 + 97650*d*x*cos(d*x + c)^3 + 97650*d*x*cos(d*x + c)^2 + 48825*d*x*cos(d*x + c) + 9765*d*x + (315*cos(d*x + c)^6 - 1575*cos(d*x + c)^5 - 28828*cos(d*x + c)^4 - 87440*cos(d*x + c)^3 - 112119*cos(d*x + c)^2 - 66875*cos(d*x + c) - 15328)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**5,x)

[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5

Giac [A]
time = 0.51, size = 145, normalized size = 0.67

$$\frac{\frac{78120\,(dx+c)}{a^5}-\frac{5040\left(11\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^5}-\frac{35a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-450a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3024a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15750a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+110565a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")

[Out] 1/5040*(78120*(d*x + c)/a^5 - 5040*(11*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5) - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2*c)^5 - 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d

Mupad [B]
time = 0.95, size = 181, normalized size = 0.84

$$-\frac{35\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-590\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+4584\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-23288\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+129824\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+55440\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-10080\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-78120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9(c+dx)}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^5,x)

[Out] -(35*sin(c/2 + (d*x)/2) - 590*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 4584*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 23288*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) + 129824*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 55440*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 10080*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2) - 78120*cos(c/2 + (d*x)/2)^9*(c + d*x))/(5040*a^5*d*cos(c/2 + (d*x)/2)^9)

3.90 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{4a\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}-\frac{8\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{35d}+\frac{12(a+a\sec(c+$$

```
[Out] 12/35*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+4/5*a*tan(d*x+c)/d/(a+a*sec(d*x
+c))^(1/2)+2/7*a*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-8/35*(a+a
*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3888, 3885, 4086, 3877}

$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{8\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{35d}+\frac{4a\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*a*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sec[c + d*x]^3*Tan
[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (8*Sqrt[a + a*Sec[c + d*x]]*Tan
[c + d*x])/(35*d) + (12*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
```

```
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{6}{7}\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}$$
$$= \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{12(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35ad}$$
$$= \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{8\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{35d} +$$
$$= \frac{4a\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{8\sqrt{a+a}}{}$$

Mathematica [A]
time = 0.14, size = 58, normalized size = 0.48

$$\frac{2a(16+8\sec(c+dx)+6\sec^2(c+dx)+5\sec^3(c+dx))\tan(c+dx)}{35d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*(16 + 8*Sec[c + d*x] + 6*Sec[c + d*x]^2 + 5*Sec[c + d*x]^3)*Tan[c + d*
x])/(35*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.32, size = 82, normalized size = 0.67

method	result	size

default	$-\frac{2\left(16\left(\cos^4(dx+c)\right)-8\left(\cos^3(dx+c)\right)-2\left(\cos^2(dx+c)\right)-\cos(dx+c)-5\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{35d\cos(dx+c)^3\sin(dx+c)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/d*(16*cos(d*x+c)^4-8*cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c)-5)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 16/35*(35*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*
c) + d)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*sqrt(a)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*
x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d
*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*
d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2
*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos
(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(1
0*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*si
n(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*s
in(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*
sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*
c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2
*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x +
8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x +
 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2
*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x +
2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d
*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))/(((2*(4*cos(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4
*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*co
```

```
s(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) +
16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*
x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*
c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*
d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + si
n(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x
 + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c)
+ sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x
 + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(4*cos(8*d*x + 8
*c) + 6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*
x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*
c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos
(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2
+ 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x +
 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) +
 sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d
*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*si
n(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)), x) - (7*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(2*d*x + 2*c) - (7*cos(2*d*x + 2*c) + 2)*sin(7/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))/((d*cos(2*d*x + 2*c)^2 + d*
sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4))
```

Fricas [A]
time = 2.67, size = 82, normalized size = 0.67

$$\frac{2 \left(16 \cos \left(dx+c\right)^{3}+8 \cos \left(dx+c\right)^{2}+6 \cos \left(dx+c\right)+5\right) \sqrt{\frac{a \cos \left(dx+c\right)+a}{\cos \left(dx+c\right)}}\, \sin \left(dx+c\right)}{35 \left(d \cos \left(dx+c\right)^{4}+d \cos \left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x +
c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\sec \left(c+dx\right)+1\right)}\, \sec^{4}\left(c+dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**4, x)
```

Giac [A]
time = 0.83, size = 120, normalized size = 0.98

$$-\frac{2\,\sqrt{2}\,\left(35\,a^{4}-\left(35\,a^{4}+\left(9\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-49\,a^{4}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\mathrm{sgn}(\cos\left(dx+c\right))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{35\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*sqrt(2)*(35*a^4 - (35*a^4 + (9*a^4*tan(1/2*d*x + 1/2*c)^2 - 49*a^4)*t
an(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*sgn(cos(d*x + c))*tan(1/2*d*
x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)*d)
```

Mupad [B]
time = 5.93, size = 331, normalized size = 2.71

$$-\frac{e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,32i}{35\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)}-\frac{\left(\frac{16i}{7\,d}+\frac{e^{c\,1i+d\,x\,1i}\,16i}{7\,d}\right)\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)^{3}}+\frac{\left(\frac{16i}{5\,d}+\frac{e^{c\,1i+d\,x\,1i}\,128i}{35\,d}\right)\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)^{2}}-\frac{e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,16i}{35\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

```
[Out] ((16i/(5*d) + (exp(c*1i + d*x*1i)*128i)/(35*d))*(a + a/(exp(- c*1i - d*x*1i
)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d
*x*2i) + 1)^2) - ((16i/(7*d) + (exp(c*1i + d*x*1i)*16i)/(7*d))*(a + a/(exp(
- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1
)*(exp(c*2i + d*x*2i) + 1)^3) - (exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*32i)/(35*d*(exp(c*1i + d*x*1i) + 1))
 - (exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2))^(1/2)*16i)/(35*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1))
```

3.91 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{14a\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}-\frac{4\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad}$$

```
[Out] 2/5*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+14/15*a*tan(d*x+c)/d/(a+a*sec(d*x
+c))^(1/2)-4/15*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3885, 4086, 3877}

$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}-\frac{4\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{14a\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (14*a*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (4*Sqrt[a + a*Sec[c +
 d*x]]*Tan[c + d*x])/(15*d) + (2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(
5*a*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
```

```
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad} + \frac{2\int \sec(c+dx)\left(\frac{3a}{2}-a\sec(\right.}{}$$
$$= -\frac{4\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2(a+a\sec(c+dx))^{3/2}\tan}{5ad}$$
$$= \frac{14a\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} - \frac{4\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2}{}$$

Mathematica [A]
time = 0.12, size = 48, normalized size = 0.56

$$\frac{2a(8+4\sec(c+dx)+3\sec^2(c+dx))\tan(c+dx)}{15d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (2*a*(8 + 4*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[a*(1
+ Sec[c + d*x])])
```

Maple [A]
time = 0.12, size = 72, normalized size = 0.84

method	result	size
default	$-\frac{2\left(8\left(\cos^3(dx+c)\right)-4\left(\cos^2(dx+c)\right)-\cos(dx+c)-3\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{15d\cos(dx+c)^2\sin(dx+c)}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/15/d*(8*cos(d*x+c)^3-4*cos(d*x+c)^2-cos(d*x+c)-3)*(a*(1+cos(d*x+c))/cos(
d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 8/15*(15*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c
) + d)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sqrt(a)*integrate((((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x +
 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x +
2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x +
 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(8*d*x +
 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x
+ 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x +
 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x
+ 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x +
 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x
+ 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(8*d*x +
 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x
+ 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x +
 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(3*c
os(6*d*x + 6*c) + 3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) +
 cos(8*d*x + 8*c)^2 + 6*(3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x +
 6*c) + 9*cos(6*d*x + 6*c)^2 + 9*cos(4*d*x + 4*c)^2 + 6*cos(4*d*x + 4*c)*co
s(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(3*sin(6*d*x + 6*c) + 3*sin(4*d*x +
 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 6*(3*sin(
4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 9*sin(6*d*x + 6*c)^2 +
9*sin(4*d*x + 4*c)^2 + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*
c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(3*co
s(6*d*x + 6*c) + 3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) +
cos(8*d*x + 8*c)^2 + 6*(3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x +
6*c) + 9*cos(6*d*x + 6*c)^2 + 9*cos(4*d*x + 4*c)^2 + 6*cos(4*d*x + 4*c)*cos
(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(3*sin(6*d*x + 6*c) + 3*sin(4*d*x +
4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 6*(3*sin(4
*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 9*sin(6*d*x + 6*c)^2 + 9
*sin(4*d*x + 4*c)^2 + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c
)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) - (5*co
s(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(2*d*x + 2*c) - (
5*cos(2*d*x + 2*c) + 2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)))*sqrt(a))/((d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d
*x + 2*c) + d)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c

```
) + 1)^(1/4))
```

Fricas [A]
time = 2.68, size = 72, normalized size = 0.84

$$\frac{2\,\left(8\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+3\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{15\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**3, x)
```

Giac [A]
time = 0.84, size = 101, normalized size = 1.17

$$\frac{2\,\sqrt{2}\,\left(15\,a^{3}+\left(7\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,a^{3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\mathrm{sgn}(\cos\left(dx+c\right))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{15\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(2)*(15*a^3 + (7*a^3*tan(1/2*d*x + 1/2*c)^2 - 10*a^3)*tan(1/2*d*x
+ 1/2*c)^2)*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)
^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 4.46, size = 115, normalized size = 1.34

$$\frac{8\,\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,5\mathrm{i}-\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,5\mathrm{i}-\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,2\mathrm{i}+2\mathrm{i}\right)}{15\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] (8*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*2i
+ d*x*2i)*5i - exp(c*3i + d*x*3i)*5i - exp(c*5i + d*x*5i)*2i + 2i))/(15*d*(
exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

3.92 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{2a\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}$$

```
[Out] 2/3*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*(a+a*sec(d*x+c))^(1/2)*tan(d*
x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3883, 3877}

$$\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a + a*Sec[c + d
*x]]*Tan[c + d*x])/(3*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx &= \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)\sqrt{a+a\sec(\\ &= \frac{2a\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 36, normalized size = 0.64

$$\frac{2a(2+\sec(c+dx))\tan(c+dx)}{3d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*(2 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.10, size = 62, normalized size = 1.11

method	result	size
default	$-\frac{2\left(2\left(\cos^{2}(dx+c)\right)-\cos(dx+c)-1\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{3d\sin(dx+c)\cos(dx+c)}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(2*cos(d*x+c)^2-cos(d*x+c)-1)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/si
n(d*x+c)/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
3/4)*sqrt(a)*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x
 + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x
+ 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x
+ 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x +
 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x
+ 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x +
 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x +
6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2
```

```
*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(2*cos
(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 4
*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c
)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*
x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + s
in(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
^2 + (2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*
x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + c
os(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)), x) + sqrt(a)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(3/4)*d)
```

Fricas [A]
time = 4.02, size = 60, normalized size = 1.07

$$\frac{2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(2\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x +
c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.79, size = 82, normalized size = 1.46

$$\frac{2\sqrt{2}\left(a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-3a^{2}\right)\mathrm{sgn}(\cos\left(dx+c\right))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2)*sgn(cos(d*x + c))*tan(1/2*
d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)*d)
```

Mupad [B]
time = 1.39, size = 108, normalized size = 1.93

$$\frac{4\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(3\,\sin\left(c+d\,x\right)+4\,\sin\left(2\,c+2\,d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{3\,d\,\left(12\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+4\,\cos\left(3\,c+3\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

```
[Out] (4*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(3*sin(c + d*x) + 4*sin(2*c
+ 2*d*x) + 3*sin(3*c + 3*d*x) + sin(4*c + 4*d*x)))/(3*d*(12*cos(c + d*x) +
8*cos(2*c + 2*d*x) + 4*cos(3*c + 3*d*x) + cos(4*c + 4*d*x) + 7))
```

3.93 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3877}

$$\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 29, normalized size = 1.12

$$\frac{2\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

Maple [A]
time = 0.08, size = 42, normalized size = 1.62

method	result	size
default	$-\frac{2\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))}{d\sin(dx+c)}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [A]
time = 3.75, size = 41, normalized size = 1.58

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a(\sec(c+dx)+1)}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.
time = 0.75, size = 62, normalized size = 2.38

$$-\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\ a\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))*tan(1/2*
d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*d)
```

Mupad [B]
time = 0.19, size = 41, normalized size = 1.58

$$\frac{2\sin(c+dx)\sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}}}{d(\cos(c+dx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] (2*sin(c + d*x)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(d*(cos(c + d*
x) + 1))
```

3.94 $\int \sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3859, 209}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]],x]
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 1.62

$$\frac{\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/
2]*Sqrt[a*(1 + Sec[c + d*x])])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(31) = 62$.
time = 0.13, size = 89, normalized size = 2.41

method	result	size
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}}{d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(31) = 62$.
time = 0.57, size = 146, normalized size = 3.95

$$\frac{\sqrt{a}\arctan\left(\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c)+1\right)\right)+\sin(dx+c),\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan\left(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c)+1\right)\right)+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

Fricas [A]
time = 3.55, size = 133, normalized size = 3.59

$$\left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d},\ -\frac{2\,\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) +
 1))/d, -2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sec(c + d*x) + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(31) = 62.
time = 0.96, size = 130, normalized size = 3.51

$$-\frac{\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(dx+c))}{d|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x +
```

```
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*
sgn(cos(d*x + c))/(d*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2), x)
```

3.95 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

[Out] arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3890, 3859, 209}

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]

[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d), Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\int \sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{a\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+d}}\right.}{d}$$
$$= \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 0.21, size = 62, normalized size = 1.00

$$\frac{a\left(\cos(c+dx)+\frac{\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (a*(Cos[c + d*x] + ArcTanh[Sqrt[1 - Sec[c + d*x]]]/Sqrt[1 - Sec[c + d*x]])*
Tan[c + d*x])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(54) = 108$.
time = 0.28, size = 123, normalized size = 1.98

method	result
default	$-\frac{\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)+2\left(\cos^2(dx+c)\right)-2\cos(dx+c)\right)\sqrt{\frac{a(1}{c}}}{2d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/d*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*cos(
d*x+c)^2-2*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(54) = 108$.
time = 0.59, size = 791, normalized size = 12.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
 - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

Fricas [A]
time = 3.23, size = 242, normalized size = 3.90

$$\left[\frac{\sqrt{-a}\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\left(d\cos\left(dx+c\right)+d\right)},-\frac{\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x +
```

```
 c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c))/(d*cos(d*x + c) + d), -(sqrt(a)*(cos(d*x + c) + 1)
*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d
*x + c))) - sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c))/(d*cos(d*x + c) + d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(54) = 108.
time = 0.81, size = 282, normalized size = 4.55

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{8\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\sqrt{-a}\,a-\sqrt{-a}\,a^{2}\right)}{\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}a+a^{2}}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/abs(a) + 8*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a - sqrt(-a)*a^2)/((sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))*sgn(cos
(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2), x)
```

3.96 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=102

$$\frac{3\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 3/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+3/4*a*sin(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c
))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3890, 3859, 209}

$$\frac{3\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{3a\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d) +
 (3*a*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]*Sin[c
+ d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{3}{4}\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}$$
$$= \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{3}{8}\int \sqrt{a+}$$
$$= \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}-\frac{(3a)\text{Subst}(}{}$$
$$= \frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 47, normalized size = 0.46

$$\frac{2\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (2*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])]*Tan[(c + d*x)/2])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(86) = 172.
time = 0.17, size = 221, normalized size = 2.17

method	result
default	$\frac{\left(3\sin(dx+c)\cos(dx+c)\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+3\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(}{1+\cos(}}}{2}\right.\right.}{16d\cos(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(3*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)+3*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*cos
(d*x+c)^4-4*cos(d*x+c)^3+12*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/
2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(86) = 172$.
time = 0.63, size = 1059, normalized size = 10.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
```

```
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 1)))/d
```

Fricas [A]
time = 3.29, size = 270, normalized size = 2.65

$$\left[\frac{3\,\sqrt{-a}\,(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,\left(2\,\cos(dx+c)^2+3\,\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{8\,(d\cos(dx+c)+d)},\ -\frac{3\,\sqrt{a}\,(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\,\cos(dx+c)^2+3\,\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{4\,(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x
 + c) - a)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/
4*(3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 + 3*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) +
 d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\,(\sec(c+dx)+1)}\,\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(86) = 172.
time = 1.02, size = 378, normalized size = 3.71

$$-\frac{\sqrt{2}\left(\frac{3\,\sqrt{2}\,\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan(\frac{1}{2}dx+\frac{1}{2}c)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan(\frac{1}{2}dx+\frac{1}{2}c)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\,\sqrt{2}\,|a|-6\,a\right|}\right)}{|a|}-\frac{8\left(5\left(\sqrt{-a}\tan(\frac{1}{2}dx+\frac{1}{2}c)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6\sqrt{-a}\,a+19\left(\sqrt{-a}\tan(\frac{1}{2}dx+\frac{1}{2}c)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4\sqrt{-a}\,a^2-17\left(\sqrt{-a}\tan(\frac{1}{2}dx+\frac{1}{2}c)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}\,a^3+\sqrt{-a}\,a^4\right)}{\left(\left(\sqrt{-a}\tan(\frac{1}{2}dx+\frac{1}{2}c)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan(\frac{1}{2}dx+\frac{1}{2}c)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)^2}\right)\mathrm{sgn}(\cos(dx+c))}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4
*sqrt(2)*abs(a) - 6*a))/abs(a) - 8*(5*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a + 19*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^2 - 17*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^3
 + sqrt(-a)*a^4)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*a + a^2)^2)*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

3.97 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{5\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 5/8*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+5/8*a*sin(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)+5/12*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+
c))^(1/2)+1/3*a*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3890, 3859, 209}

$$\frac{5\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{5a\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{5a\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (5*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d) +
 (5*a*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (5*a*Cos[c + d*x]*Sin[
c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]^2*Sin[c + d*x])
/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{5}{6}\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}$$

$$= \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{5}{8}\int \cos($$

$$= \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+}{3d\sqrt{a+}}$$

$$= \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+}{3d\sqrt{a+}}$$

$$= \frac{5\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.10, size = 47, normalized size = 0.34

$$\frac{2\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (2*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])]*Tan[(c + d*x)/2])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(118) = 236$.
time = 0.20, size = 310, normalized size = 2.25

method	result
default	$-\frac{\left(15(\cos^2(dx+c))\sin(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{2}+30\cos(dx+c)\sin(dx+c)\,\mathrm{arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/d*(15*cos(d*x+c)^2*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)*2^(1/2)+30*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)*2^(1/2)+15*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)
+64*cos(d*x+c)^6+16*cos(d*x+c)^5+40*cos(d*x+c)^4-120*cos(d*x+c)^3)*(a*(1+co
s(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(118) = 236$.
time = 0.75, size = 1921, normalized size = 13.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arct
an2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
```

```
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))/d
```

Fricas [A]

time = 2.36, size = 290, normalized size = 2.10

$$\left[\frac{15\sqrt{-a}\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{2a\cos\left(dx+c\right)^2-2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)+2\left(8\cos\left(dx+c\right)^3+10\cos\left(dx+c\right)^2+15\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)+d\right)},\ -\frac{15\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)-\left(8\cos\left(dx+c\right)^3+10\cos\left(dx+c\right)^2+15\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{24\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*cos(d*x + c)^3 + 10*cos(d*x + c)^2
+ 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c) + d), -1/24*(15*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*cos(
d*x + c)^3 + 10*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(118) = 236.
time = 1.02, size = 475, normalized size = 3.44

$$-\frac{\sqrt{2}\left(\frac{15\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{8\left(63\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^{10}\sqrt{-a}\,a-369\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^8\sqrt{-a}\,a^2+1638\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6\sqrt{-a}\,a^3-1074\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4\sqrt{-a}\,a^4+171\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}\,a^5-13\sqrt{-a}\,a^6\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)^3}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(15*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/abs(a) + 8*(63*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(-a)*a - 369*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^2 + 1638*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-
```

```
a)*a^3 - 1074*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*sqrt(-a)*a^4 + 171*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^5 - 13*sqrt(-a)*a^6)/((sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)*sgn(c
os(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^3\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

3.98 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{35\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}}+\frac{7a\cos^2(c-}{24d\sqrt{a+}}$$

```
[Out] 35/64*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+35/64*a*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+35/96*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec
(d*x+c))^(1/2)+7/24*a*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*
a*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3890, 3859, 209}

$$\frac{35\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{35a\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{35a\sin(c+dx)\cos(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (35*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d)
 + (35*a*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (35*a*Cos[c + d*x]
*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (7*a*Cos[c + d*x]^2*Sin[c
+ d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(
4*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
```

```
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{7}{8}\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}$$

$$= \frac{7a\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{35}{48}\int \mathrm{co}$$

$$= \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^3}{4d\sqrt{}}$$

$$= \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c}{24d\sqrt{a}}$$

$$= \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c}{24d\sqrt{a}}$$

$$= \frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.10, size = 47, normalized size = 0.27

$$\frac{2\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])]*Tan[(c + d*x)/2])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(150) = 300.
time = 0.23, size = 399, normalized size = 2.29

method	result

default	$-\frac{\left(-105\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)(\cos^{3}(dx+c))\sin(dx+c)\sqrt{2}-315\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3072/d*(-105*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)-315*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-315*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)-105*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)+768*cos(d*x+c)^8+128*cos(d*x+c)^7+224*cos(d*x+c)^6+560*cos(d*x+c)^5-1680*cos(d*x+c)^4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 6638 vs. $2(150) = 300$.
time = 0.87, size = 6638, normalized size = 38.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/768*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 9*sin(4*d*x + 4*c)^3 + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arc

```
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*
cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) - 9)*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(
64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) +
7*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
9*cos(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
36*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
^2 + sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (9*cos(4*d*x +
4*c)^3 + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c
)^2 - 10*cos(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) + 8)*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*
c)^2 + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^
2 + 26*cos(4*d*x + 4*c)^2 + 25*cos(4*d*x + 4*c) + 8)*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 - (32*(cos(4*d*x +
4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*
c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x
 + 4*c)^2 - 7*cos(4*d*x + 4*c) - 9)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 7*sin(4*d*x + 4*c))*sin(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 9*cos(4*d*x + 4*c))*cos(3/4*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(
4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c)^2 - 8*cos(4*d*x + 4
*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 9*(2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x
+ 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*
x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(9*co
s(4*d*x + 4*c) + 8)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*si
n(4*d*x + 4*c) + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c))*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((64*(cos(4*d
*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 20*(sin(4*d*x + 4*c)^3 + (cos(4*d*
x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c) + 8*(cos(4*d*x + 4*c)
^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 5*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 5*sin(4*d*x + 4*c)^3 + 4
*(5*sin(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c)^2 + 10*cos(4*d*x + 4*c) - 11)*
sin(4*d*x + 4*c) - 64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*
```

```
sin(4*d*x + 4*c) + 40*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*
x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 10*(2*sin(4*d*x + 4*c)^3
+ 2*(cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + cos(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (16*cos(4*d*x +
 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 17*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*a...
```

Fricas [**A**]
time = 2.83, size = 310, normalized size = 1.78

$$\left[\frac{105\sqrt{-a}\,(\cos(dx+c)+1)\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48\cos(dx+c)^4+56\cos(dx+c)^3+70\cos(dx+c)^2+105\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{384\,(d\cos(dx+c)+d)},\ -\frac{105\sqrt{a}\,(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(48\cos(dx+c)^4+56\cos(dx+c)^3+70\cos(dx+c)^2+105\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{192\,(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*cos(d*x + c)^4 + 56*cos(d*x + c)
^3 + 70*cos(d*x + c)^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(105*sqrt(a)*(cos(d*x +
c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) - (48*cos(d*x + c)^4 + 56*cos(d*x + c)^3 + 70*cos(d*x + c)
^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,(\sec(c+dx)+1)}\,\cos^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(150) = 300.
time = 0.99, size = 571, normalized size = 3.28

$$-\frac{\sqrt{2}\left(\frac{105\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{8\left(279X^{14}\sqrt{-a}\,a+285X^{12}\sqrt{-a}\,a^2-4605X^{10}\sqrt{-a}\,a^3+37281X^{8}\sqrt{-a}\,a^4-35643X^{6}\sqrt{-a}\,a^5+9175X^{4}\sqrt{-a}\,a^6-1311X^{2}\sqrt{-a}\,a^7+63\sqrt{-a}\,a^8\right)}{\left(X^4-6X^2a+a^2\right)^4}\right)\operatorname{sgn}(\cos(dx+c))}{768\,d}$$

where $X=\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/768*sqrt(2)*(105*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 8*(279*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*sqrt(-a)*a + 285*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*sqrt(-a)*a^2 - 4605
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*s
qrt(-a)*a^3 + 37281*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^8*sqrt(-a)*a^4 - 35643*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a^5 + 9175*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^6 - 1311*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-
a)*a^7 + 43*sqrt(-a)*a^8)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^4\sqrt{a+\frac{a}{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

3.99 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{68a^2\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}}+\frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}-\frac{136a\sqrt{a+a\sec(c}{31}$$

```
[Out] 68/105*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+68/45*a^2*tan(d*x+c)/d/(a+a*sec(
d*x+c))^(1/2)+34/63*a^2*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/
9*a^2*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-136/315*a*(a+a*sec(d
*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.19, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3899, 21, 3888, 3885, 4086, 3877}

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{68a^2\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{68\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105d}-\frac{136a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (68*a^2*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (34*a^2*Sec[c + d*x
]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sec[c + d*x]^4*T
an[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (136*a*Sqrt[a + a*Sec[c + d*x
]]*Tan[c + d*x])/(315*d) + (68*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(10
5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
```

```
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(2a)\int \frac{\sec^4(c+dx)\left(\frac{17a}{2}+\frac{17}{2}a\right.}{\sqrt{a+a\sec(c+}}$$
$$= \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(17a)\int \sec^4(c+dx)\sqrt{a+a\,\mathrm{se}}$$
$$= \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} +$$
$$= \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} +$$
$$= \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} -$$
$$= \frac{68a^2\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2}{9}$$

Mathematica [A]
time = 0.50, size = 70, normalized size = 0.43

$$\frac{2a^2(272+136\sec(c+dx)+102\sec^2(c+dx)+85\sec^3(c+dx)+35\sec^4(c+dx))\tan(c+dx)}{315d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
[Out] (2*a^2*(272 + 136*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 85*Sec[c + d*x]^3 + 3
5*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.12, size = 93, normalized size = 0.57

method	result	size
default	$-\frac{2\left(272\left(\cos^5(dx+c)\right)-136\left(\cos^4(dx+c)\right)-34\left(\cos^3(dx+c)\right)-17\left(\cos^2(dx+c)\right)-50\cos(dx+c)-35\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a}{315d\cos(dx+c)^4\sin(dx+c)}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/315/d*(272*cos(d*x+c)^5-136*cos(d*x+c)^4-34*cos(d*x+c)^3-17*cos(d*x+c)^2
-50*cos(d*x+c)-35)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x
+c)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 16/315*(315*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*((a*d*cos(2*d*x + 2*c)^4 + a*d*sin(2*d*x + 2*c)^4 + 4*a*d*cos(2*d
*x + 2*c)^3 + 6*a*d*cos(2*d*x + 2*c)^2 + 4*a*d*cos(2*d*x + 2*c) + 2*(a*d*co
s(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*sin(2*d*x + 2*c)^2 + a*d)*
integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d
*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*
d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*s
in(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) +
4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) +
 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c)
- 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c)
 - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8
*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x +
4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x +
 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x
+ 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x
+ 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*
x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x
 + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*
d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/((co
s(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c)^2 + 16*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c)^2 + 36*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x
 + 6*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(10*d*x + 10*c)^2 + 16*(cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*
c)^2 + 36*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(6*d*x + 6*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
```

```
2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*s
in(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*cos(8*d*x + 8*c) + 6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*
c)^2 + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 8*(cos(2*d*x + 2*c)^3 + cos(2
*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*
c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 12*(cos(2*d*x + 2*c)^3 + cos(2*
d*x + 2*c)*sin(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d
*x + 2*c))*cos(6*d*x + 6*c) + 8*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c)
+ cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 4*(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c) + 6*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 4*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*
x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*
sin(10*d*x + 10*c) + 8*(sin(2*d*x + 2*c)^3 + 6*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 4*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c
) + 12*(sin(2*d*x + 2*c)^3 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sin(2*d*x + 2*c)^3 + (
cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x +
4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(2*d
*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x +...
```

Fricas [A]
time = 2.64, size = 98, normalized size = 0.60

$$\frac{2\left(272\,a\cos\left(dx+c\right)^{4}+136\,a\cos\left(dx+c\right)^{3}+102\,a\cos\left(dx+c\right)^{2}+85\,a\cos\left(dx+c\right)+35\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(272*a*cos(d*x + c)^4 + 136*a*cos(d*x + c)^3 + 102*a*cos(d*x + c)^2 +
 85*a*cos(d*x + c) + 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}} \sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**4, x)
```

Giac [A]
time = 1.09, size = 180, normalized size = 1.11

$$\frac{4\left(315\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c)) - \left(525\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c)) - \left(819\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c)) + 47\left(2\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 9\sqrt{2}a^6\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{315\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^4\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 4/315*(315*sqrt(2)*a^6*sgn(cos(d*x + c)) - (525*sqrt(2)*a^6*sgn(cos(d*x + c
)) - (819*sqrt(2)*a^6*sgn(cos(d*x + c)) + 47*(2*sqrt(2)*a^6*sgn(cos(d*x + c
))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2
*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d
)
```

Mupad [B]
time = 6.70, size = 429, normalized size = 2.65

$$\frac{\left(\frac{a\,32\mathrm{i}}{9\,d}-\frac{a\,e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,32\mathrm{i}}{9\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}-\frac{\left(\frac{a\,80\mathrm{i}}{7\,d}-\frac{a\,e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,176\mathrm{i}}{63\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}+\frac{\left(\frac{a\,48\mathrm{i}}{5\,d}+\frac{a\,e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,352\mathrm{i}}{105\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2}-\frac{a\,e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\sqrt{a+\frac{a}{\frac{e^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,544\mathrm{i}}{315\,d\left(e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)}-\frac{a\,e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\sqrt{a+\frac{a}{\frac{e^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,272\mathrm{i}}{315\,d\left(e^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

```
[Out] (((a*32i)/(9*d) - (a*exp(c*1i + d*x*1i)*32i)/(9*d))*(a + a/(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i
 + d*x*2i) + 1)^4) - (((a*80i)/(7*d) - (a*exp(c*1i + d*x*1i)*176i)/(63*d))*
(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i +
 d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + (((a*48i)/(5*d) + (a*exp(c*1i +
 d*x*1i)*352i)/(105*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(
c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2
)*544i)/(315*d*(exp(c*1i + d*x*1i) + 1)) - (a*exp(c*1i + d*x*1i)*(a + a/(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*272i)/(315*d*(exp(c*1i
+ d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1))
```

3.100 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{152a^2\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{38a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{105d}-\frac{4(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d}+\frac{2(a$$

```
[Out] -4/35*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+a*sec(d*x+c))^(5/2)*tan(d*
x+c)/a/d+152/105*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+38/105*a*(a+a*sec(
d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3885, 4086, 3878, 3877}

$$\frac{152a^2\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}-\frac{4\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{38a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (152*a^2*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (38*a*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(105*d) - (4*(a + a*Sec[c + d*x])^(3/2)*Tan[c +
 d*x])/(35*d) + (2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
```

```
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad}+\frac{2\int \sec(c+dx)\left(\frac{5a}{2}-a\sec(\right.}{}\\ &= -\frac{4(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d}+\frac{2(a+a\sec(c+dx))^{5/2}\tan}{7ad}\\ &= \frac{38a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{105d}-\frac{4(a+a\sec(c+dx))^{3/2}\tan}{35d}\\ &= \frac{152a^2\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{38a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{105d}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 60, normalized size = 0.52

$$\frac{2a^2(104+52\sec(c+dx)+39\sec^2(c+dx)+15\sec^3(c+dx))\tan(c+dx)}{105d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(104 + 52*Sec[c + d*x] + 39*Sec[c + d*x]^2 + 15*Sec[c + d*x]^3)*Tan[
c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.11, size = 83, normalized size = 0.72

method	result	size
default	$-\frac{2\left(104\left(\cos^4(dx+c)\right)-52\left(\cos^3(dx+c)\right)-13\left(\cos^2(dx+c)\right)-24\cos(dx+c)-15\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a}{105d\cos(dx+c)^3\sin(dx+c)}$	83

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/105/d*(104*cos(d*x+c)^4-52*cos(d*x+c)^3-13*cos(d*x+c)^2-24*cos(d*x+c)-15)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)*a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 8/105*(105*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(3*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*

```
cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*
d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 3*(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 6*(cos(2*d*
x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x
 + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3 + co
s(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c)
)*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 3*(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c
) + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si
n(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
2*c))*sin(8*d*x + 8*c) + 6*(sin(2*d*x + 2*c)^3 + 3*(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(si
n(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x
+ 2*c))*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c)^2 + 9*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c)^2 + 9*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 9
*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d
*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2
*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
 6*c) + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8
*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*
d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 +
 cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2
*c)^3 + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(6*d*x + 6*c) + 3*(cos(2*d*x + 2*c)^2 + si...
```

Fricas [A]

time = 2.63, size = 87, normalized size = 0.75

$$\frac{2\left(104\,a\cos\left(dx+c\right)^{3}+52\,a\cos\left(dx+c\right)^{2}+39\,a\cos\left(dx+c\right)+15\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{105\,\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 +
 d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**3, x)
```

Giac [A]
time = 1.16, size = 151, normalized size = 1.30

$$-\frac{4\left(105\sqrt{2}\,a^5\mathrm{sgn}(\cos(dx+c))-\left(140\sqrt{2}\,a^5\mathrm{sgn}(\cos(dx+c))+19\left(2\sqrt{2}\,a^5\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7\sqrt{2}\,a^5\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{105\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -4/105*(105*sqrt(2)*a^5*sgn(cos(d*x + c)) - (140*sqrt(2)*a^5*sgn(cos(d*x +
c)) + 19*(2*sqrt(2)*a^5*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2
)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*ta
n(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a)*d)
```

Mupad [B]
time = 5.08, size = 346, normalized size = 2.98

$$-\frac{\left(\frac{a\,16\mathrm{i}}{7\,d}+\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}+\frac{\left(\frac{a\,8\mathrm{i}}{3\,d}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,104\mathrm{i}}{105\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}+\frac{\left(\frac{a\,8\mathrm{i}}{5\,d}+\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,184\mathrm{i}}{35\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,208\mathrm{i}}{105\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] (((a*8i)/(3*d) - (a*exp(c*1i + d*x*1i)*104i)/(105*d))*(a + a/(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*
2i + d*x*2i) + 1)) - (((a*16i)/(7*d) + (a*exp(c*1i + d*x*1i)*16i)/(7*d))*(a
 + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d
*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + (((a*8i)/(5*d) + (a*exp(c*1i + d*
x*1i)*184i)/(35*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))
^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(c*1i
 + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*20
8i)/(105*d*(exp(c*1i + d*x*1i) + 1))
```

3.101 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=86

$$\frac{8a^2\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{5d}+\frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

```
[Out] 2/5*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+8/5*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c
))^(1/2)+2/5*a*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3883, 3878, 3877}

$$\frac{8a^2\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{5d}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(5*d) + (2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/
(5*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{3}{5}\int \sec(c+dx)(a+a\sec$$
$$= \frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{5d} + \frac{2(a+a\sec(c+dx))^{3/2}\tan}{5d}$$
$$= \frac{8a^2\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{5d} +$$

Mathematica [A]
time = 0.14, size = 48, normalized size = 0.56

$$\frac{2a^2(6+3\sec(c+dx)+\sec^2(c+dx))\tan(c+dx)}{5d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(6 + 3*Sec[c + d*x] + Sec[c + d*x]^2)*Tan[c + d*x])/(5*d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

Maple [A]
time = 0.09, size = 73, normalized size = 0.85

method	result	size
default	$-\frac{2\left(6\left(\cos^3(dx+c)\right)-3\left(\cos^2(dx+c)\right)-2\cos(dx+c)-1\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\;a}{5d\cos(dx+c)^2\sin(dx+c)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(6*cos(d*x+c)^3-3*cos(d*x+c)^2-2*cos(d*x+c)-1)*(a*(1+cos(d*x+c))/cos
(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 4/5*(5*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*((a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x +
2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*
c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c
) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c
) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c)
 - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c
) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c)
)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*
cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2
 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/((cos(2*d*x + 2*
c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^
3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(
6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*
x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*
x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^
2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (c
os(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4
*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(2*d*
x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x +
 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*si
n(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*co
s(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x +
 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
```

```
2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^
3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d
*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2), x)
+ 3*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*
c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
+ 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c)
+ 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) -
 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c)
- cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*co
s(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 +
 sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*s
in(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/((cos(2*d*x + 2*c)
^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2...
```

Fricas [A]
time = 3.34, size = 74, normalized size = 0.86

$$\frac{2\left(6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(\sec\left(c+dx\right)+1)\right)^{\frac{3}{2}}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**2, x)

Giac [A]
time = 0.98, size = 121, normalized size = 1.41

$$\frac{4\left(5\sqrt{2}\,a^4\mathrm{sgn}(\cos(dx+c)) + \left(2\sqrt{2}\,a^4\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 5\sqrt{2}\,a^4\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{5\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] 4/5*(5*sqrt(2)*a^4*sgn(cos(d*x + c)) + (2*sqrt(2)*a^4*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)

Mupad [B]
time = 4.36, size = 116, normalized size = 1.35

$$\frac{4\,a\,\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,5\mathrm{i}-\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,5\mathrm{i}-\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)}{5\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)

[Out] (4*a*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*2i + d*x*2i)*5i - exp(c*3i + d*x*3i)*5i - exp(c*5i + d*x*5i)*3i + 3i))/(5*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)

3.102 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{8a^2\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}$$

```
[Out] 8/3*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*(a+a*sec(d*x+c))^(1/2)*ta
n(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3878, 3877}

$$\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= \frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{1}{3}(4a)\int \sec(c+dx)\sqrt{a+}\\ &= \frac{8a^2\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 38, normalized size = 0.64

$$\frac{2a^2(5+\sec(c+dx))\tan(c+dx)}{3d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
[Out] (2*a^2*(5 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.09, size = 63, normalized size = 1.07

method	result	size
default	$-\frac{2\left(5\left(\cos^{2}\left(dx+c\right)\right)-4\cos\left(dx+c\right)-1\right)\sqrt{\frac{a\left(1+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\,a}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/d*(5*cos(d*x+c)^2-4*cos(d*x+c)-1)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/
sin(d*x+c)/cos(d*x+c)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [A]
time = 4.47, size = 61, normalized size = 1.03

$$\frac{2\left(5\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(5*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}} \sec(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x), x)
```

Giac [A]
time = 0.93, size = 93, normalized size = 1.58

$$\frac{4\left(2\sqrt{2}\,a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\sqrt{2}\,a^3\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 4/3*(2*sqrt(2)*a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 3*sqrt(2)*a^3
*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 1.24, size = 111, normalized size = 1.88

$$\frac{2\,a\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(12\,\sin\left(c+d\,x\right)+14\,\sin\left(2\,c+2\,d\,x\right)+12\,\sin\left(3\,c+3\,d\,x\right)+5\,\sin\left(4\,c+4\,d\,x\right)\right)}{3\,d\,\left(12\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+4\,\cos\left(3\,c+3\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

```
[Out] (2*a*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(12*sin(c + d*x) + 14*sin(
2*c + 2*d*x) + 12*sin(3*c + 3*d*x) + 5*sin(4*c + 4*d*x)))/(3*d*(12*cos(c +
d*x) + 8*cos(2*c + 2*d*x) + 4*cos(3*c + 3*d*x) + cos(4*c + 4*d*x) + 7))
```

3.103 $\int (a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3860, 21, 3859, 209}

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^{3/2}\,dx &= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(2a)\int\frac{\frac{a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx\\ &= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+a\int\sqrt{a+a\sec(c+dx)}\,dx\\ &= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(2a^2)\,\text{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 75, normalized size = 1.14

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(58) = 116$.
time = 0.11, size = 181, normalized size = 2.74

method	result
default	$\frac{\left(-\cos(dx+c)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)-\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{arctanh}\left(\right.\right.}{d(1+\cos(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(-cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+2*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(1+cos(d*x+c))*a

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(58) = 116$.
time = 0.61, size = 997, normalized size = 15.11

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co

```
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*d)
```

Fricas [A]
time = 3.75, size = 235, normalized size = 3.56

$$\left[\frac{(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{d\cos(dx+c)+d}, -\frac{2\left((a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)\right)}{d\cos(dx+c)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [((a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) + 2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c) + d), -2*((a*cos(d*x + c) + a)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(c+dx)+a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(58) = 116.
time = 1.15, size = 195, normalized size = 2.95

$$-\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,a^2\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}+\frac{\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}\,|a|-6\,a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -(2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2*sgn(cos(d*x + c))*tan(1
/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + sqrt(-a)*a^2*log(abs(2*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sq
rt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/abs(a))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a+\frac{a}{\cos \left(c+d\,x\right)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(3/2), x)
```

3.104 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{3a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 3*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*sin(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3899, 21, 3890, 3859, 209}

$$\frac{3a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a^
2*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= -\frac{2a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-(2a)\int \frac{\cos(c+dx)\left(-\frac{3a}{2}-\frac{3}{2}a\sec(c+d\right.}{\sqrt{a+a\sec(c+dx)}}\\ &= -\frac{2a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(3a)\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\\ &= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}(3a)\int \sqrt{a+a\sec(c+dx)}\,dx\\ &= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+d}{\sqrt{a+a\sec(c}}\right.}{d}\\ &= \frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 89, normalized size = 1.37

$$\frac{a\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(3\sqrt{2}\,\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2
]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2])
)/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(57) = 114$.
time = 0.11, size = 125, normalized size = 1.92

method	result
default	$-\frac{\left(3\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)+2\left(\cos^2(dx+c)\right)-2\cos(dx+c)\right)\sqrt{\frac{a(}{}}}{2d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*co
s(d*x+c)^2-2*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(57) = 114$.
time = 0.63, size = 803, normalized size = 12.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
```

```
1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

Fricas [A]
time = 3.71, size = 248, normalized size = 3.82

$$\left[\frac{2\,a\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(a\cos\left(dx+c\right)+a\right)\sqrt{-a}\log\left(\frac{2\,a\cos\left(dx+c\right)^{2}-2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)}{2\left(d\cos\left(dx+c\right)+d\right)},\ \frac{a\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)}{d\cos\left(dx+c\right)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + 3*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x
+ c) - a)/(cos(d*x + c) + 1)))/(d*cos(d*x + c) + d), (a*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*(a*cos(d*x + c) + a)*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))))/(d*cos(d*x + c) + d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\cos\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(57) = 114.
time = 1.14, size = 278, normalized size = 4.28

$$-\frac{\sqrt{2}\sqrt{-a}\,a^{3}\left(\frac{3\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}\left|a\right|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}\left|a\right|-6\,a\right|}\right)}{a\left|a\right|}+\frac{8\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-a\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}a+a^{2}\right)a}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(-a)*a^3*(3*sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/(a*abs(a)) + 8*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a))*sgn(cos(d*x + c))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2), x)
```

3.105 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=106

$$\frac{7a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 7/4*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+7/4*a^2*sin
(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d
*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3898, 21, 3890, 3859, 209}

$$\frac{7a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d) +
 (7*a^2*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]*Si
n[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}a\int \frac{\cos(c+dx)\left(\frac{7a}{2}+\frac{7}{2}a\sec(c+\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}(7a)\int \cos(c+dx)\sqrt{a+a\sec(c-}$$

$$= \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}(7a)\int$$

$$= \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{(7a^2)\,\mathrm{Sul}}{}$$

$$= \frac{7a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A]
time = 0.38, size = 108, normalized size = 1.02

$$\frac{a\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{1-\sec(c+dx)}\,(7\sin(c+dx)+\sin(2(c+dx)))+7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)\right)}{4d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(7*Sin[c
 + d*x] + Sin[2*(c + d*x)]) + 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x
]))/(4*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(90) = 180$.
time = 0.13, size = 222, normalized size = 2.09

method	result
default	$\frac{\left(7\sin(dx+c)\cos(dx+c)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+7\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx\text{-}}{1+\cos(dx}}}{2\,\text{co}}\right.\right.}{16d\cos(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(7*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)+7*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*cos
(d*x+c)^4-20*cos(d*x+c)^3+28*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1
/2)/cos(d*x+c)/sin(d*x+c)*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.75, size = 278, normalized size = 2.62

$$\left[\frac{7\left(a\cos\left(dx+c\right)+a\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\,a\cos\left(dx+c\right)^{2}+7\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{8\left(d\cos\left(dx+c\right)+d\right)},\ -\frac{7\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)}{\sqrt{a}\,\sin(dx+c)}\right)-\left(2\,a\cos\left(dx+c\right)^{2}+7\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{4\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a*cos(d*x + c)^2 + 7*a*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d
), -1/4*(7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*a*cos(d*x + c)^2 + 7*
a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c) + d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

3.106 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{11a^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 11/8*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+11/8*a^2*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+11/12*a^2*cos(d*x+c)*sin(d*x+c)/d/(a+a*s
ec(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3898, 21, 3890, 3859, 209}

$$\frac{11a^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{11a^2\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
+ (11*a^2*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Cos[c + d*
x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^2*Sin[
c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{1}{3}a\int\frac{\cos^2(c+dx)\left(\frac{11a}{2}+\frac{11}{2}a\sec(\ldots\right.}{\sqrt{a+a\sec(c+dx)}}\ldots\\
&= \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{1}{6}(11a)\int\cos^2(c+dx)\sqrt{a+a\sec\ldots}\\
&= \frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{1}{8}(\ldots\\
&= \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2\ldots}{3d\sqrt{\ldots}}\\
&= \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2\ldots}{3d\sqrt{\ldots}}\\
&= \frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}\ldots
\end{aligned}$$

Mathematica [A]
time = 0.55, size = 120, normalized size = 0.83

$$\frac{a\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{1-\sec(c+dx)}\,(35\sin(c+dx)+11\sin(2(c+dx))+2\sin(3(c+dx)))+33\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)\right)}{24d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(35*Sin[
c + d*x] + 11*Sin[2*(c + d*x)] + 2*Sin[3*(c + d*x)]) + 33*ArcTanh[Sqrt[1 -
Sec[c + d*x]]]*Tan[c + d*x]))/(24*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(124) = 248$.
time = 0.16, size = 311, normalized size = 2.16

method	result
default	$-\frac{\left(33\left(\cos^{2}(dx+c)\right)\sin(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{2}+66\cos(dx+c)\sin(dx+c)\,\mathrm{arc}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/d*(33*cos(d*x+c)^2*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)*2^(1/2)+66*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)*2^(1/2)+33*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)
+64*cos(d*x+c)^6+112*cos(d*x+c)^5+88*cos(d*x+c)^4-264*cos(d*x+c)^3)*(a*(1+c
os(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.27, size = 300, normalized size = 2.08

$$\left[\frac{33\left(a\cos\left(dx+c\right)+a\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\,a\cos\left(dx+c\right)^{3}+22\,a\cos\left(dx+c\right)^{2}+33\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)+d\right)},\ -\frac{33\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)}{\sqrt{a}\,\sin(dx+c)}\right)-\left(8\,a\cos\left(dx+c\right)^{3}+22\,a\cos\left(dx+c\right)^{2}+33\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{24\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(33*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a*cos(d*x + c)^3 + 22*a*cos(d*x +
 c)^2 + 33*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c) + d), -1/24*(33*(a*cos(d*x + c) + a)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - (8*a*cos(d*x + c)^3 + 22*a*cos(d*x + c)^2 + 33*a*cos(d*x + c))*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(124) = 248.
time = 1.73, size = 535, normalized size = 3.72

$$-\frac{33\sqrt{-a}\,a\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-33\sqrt{-a}\,a\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\frac{4\left(33\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^{10}\sqrt{-a}\,a^2\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-303\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^{8}\sqrt{-a}\,a^3\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+2394\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^{6}\sqrt{-a}\,a^4\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-1806\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^{4}\sqrt{-a}\,a^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+309\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^{2}\sqrt{-a}\,a^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-19\sqrt{2}\sqrt{-a}\,a^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/48*(33*sqrt(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 33*sqrt
(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(33*sqrt(2)*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(-a)*
a^2*sgn(cos(d*x + c)) - 303*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 2394*sqrt
(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6
*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1806*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^5*sgn(cos(d*x + c))
+ 309*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 19*sqrt(2)*sqrt(-a)*a^7*sgn(cos
(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*a + a^2)^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

3.107 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$\frac{284a^3\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}-\frac{568a^2\sqrt{a+a\,\mathrm{se}}}{}$$

```
[Out] 284/231*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+284/99*a^3*tan(d*x+c)/d/(a+a*
sec(d*x+c))^(1/2)+710/693*a^3*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1
/2)+46/99*a^3*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-568/693*a^2*
(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d+2/11*a^2*sec(d*x+c)^4*(a+a*sec(d*x+c))^
(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.27, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3899, 4101, 3888, 3885, 4086, 3877}

$$\frac{46a^3\tan(c+dx)\sec^4(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{284a^3\tan(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}-\frac{568a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{693d}+\frac{284a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{231d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (284*a^3*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) + (710*a^3*Sec[c + d
*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (46*a^3*Sec[c + d*x]
^4*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) - (568*a^2*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(693*d) + (2*a^2*Sec[c + d*x]^4*Sqrt[a + a*Sec[c +
d*x]]*Tan[c + d*x])/(11*d) + (284*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(231*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{11d} + \frac{1}{11}(2a)\int \text{se}$$

$$= \frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\sqrt{a+a\sec(c+}}{11d}$$

$$= \frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{284a^3\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} + \frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{46a}{}$$

Mathematica [**A**]
time = 0.22, size = 80, normalized size = 0.39

$$\frac{2a^3(1136+568\sec(c+dx)+426\sec^2(c+dx)+355\sec^3(c+dx)+224\sec^4(c+dx)+63\sec^5(c+dx))\tan(c+dx)}{693d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(1136 + 568*Sec[c + d*x] + 426*Sec[c + d*x]^2 + 355*Sec[c + d*x]^3 +
 224*Sec[c + d*x]^4 + 63*Sec[c + d*x]^5)*Tan[c + d*x])/(693*d*Sqrt[a*(1 + S
ec[c + d*x])])
```

Maple [**A**]
time = 0.13, size = 105, normalized size = 0.52

method	result
default	$-\frac{2(1136(\cos^6(dx+c))-568(\cos^5(dx+c))-142(\cos^4(dx+c))-71(\cos^3(dx+c))-131(\cos^2(dx+c))-161\cos(dx+c)-63)\sqrt{\frac{a(1+\cos}{\cos(d}}}{693d\cos(dx+c)^5\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/693/d*(1136*cos(d*x+c)^6-568*cos(d*x+c)^5-142*cos(d*x+c)^4-71*cos(d*x+c)
^3-131*cos(d*x+c)^2-161*cos(d*x+c)-63)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/
cos(d*x+c)^5/sin(d*x+c)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 16/693*(693*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*((a^2*d*cos(2*d*x + 2*c)^4 + a^2*d*sin(2*d*x + 2*c)^4 + 4*a^2*d*c
os(2*d*x + 2*c)^3 + 6*a^2*d*cos(2*d*x + 2*c)^2 + 4*a^2*d*cos(2*d*x + 2*c) +
 a^2*d + 2*(a^2*d*cos(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*si
n(2*d*x + 2*c)^2)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(
8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos
(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*si
n(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*s
in(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)
*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*s
in(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c
)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*
c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6
*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(9/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x
+ 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x
 + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*
x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d
*x + 4*c)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x
 + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*
x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin
(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(9/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))/(((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c)^2 +
 16*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c)^2 + 36*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(10*d*x
+ 10*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*sin(8*d*x + 8*c)^2 + 36*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x +
```

```
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3
 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + 6*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 4*(cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*
c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 8*(cos(2
*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 6*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 4*(cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*
c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 12*(cos(2*
d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d
*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 8*(cos(2*d*x + 2*c)^3 +
cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*
c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 4*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8
*c) + 6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 8*(sin(2*d*x + 2*c)^3 + 6*(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c
) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si
n(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
2*c))*sin(8*d*x + 8*c) + 12*(sin(2*d*x + 2*c)^3 + 4*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(s
in(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x
 + 2*c))*sin(4*d*x + 4*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)...
```

Fricas [A]
time = 2.68, size = 121, normalized size = 0.60

$$\frac{2\left(1136\,a^2\cos\left(dx+c\right)^5+568\,a^2\cos\left(dx+c\right)^4+426\,a^2\cos\left(dx+c\right)^3+355\,a^2\cos\left(dx+c\right)^2+224\,a^2\cos\left(dx+c\right)+63\,a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{693\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(1136*a^2*cos(d*x + c)^5 + 568*a^2*cos(d*x + c)^4 + 426*a^2*cos(d*x +
 c)^3 + 355*a^2*cos(d*x + c)^2 + 224*a^2*cos(d*x + c) + 63*a^2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c
)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 1.15, size = 209, normalized size = 1.03

$$-\frac{8\left(693\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c)) - \left(1617\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c)) - \left(3003\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c)) - 25\left(99\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c)) + 4\left(2\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 11\sqrt{2}a^8\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{693\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^5\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] -8/693*(693*sqrt(2)*a^8*sgn(cos(d*x + c)) - (1617*sqrt(2)*a^8*sgn(cos(d*x +
 c)) - (3003*sqrt(2)*a^8*sgn(cos(d*x + c)) - 25*(99*sqrt(2)*a^8*sgn(cos(d*x
 + c)) + 4*(2*sqrt(2)*a^8*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 11*sqr
t(2)*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)
*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*t
an(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 9.32, size = 542, normalized size = 2.67

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,64i}{11\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,64i}{11\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^5}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,16i}{d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,640i}{231\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^2}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,64i}{9\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,2176i}{99\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^4}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,80i}{7\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,12688i}{693\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^3}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,2272i}{693\,d\left(e^{c\,1i+d\,x\,1i}+1\right)}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,1136i}{693\,d\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*64i)/(
11*d) + (a^2*exp(c*1i + d*x*1i)*64i)/(11*d)))/((exp(c*1i + d*x*1i) + 1)*(ex
p(c*2i + d*x*2i) + 1)^5) + ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x
*1i)/2))^(1/2)*((a^2*16i)/d + (a^2*exp(c*1i + d*x*1i)*640i)/(231*d)))/((exp
(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - ((a + a/(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*64i)/(9*d) + (a^2*exp(c*1i + d
*x*1i)*2176i)/(99*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^4
) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*80
i)/(7*d) - (a^2*exp(c*1i + d*x*1i)*12688i)/(693*d)))/((exp(c*1i + d*x*1i) +
 1)*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1
i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*2272i)/(693*d*(exp(c*1i + d*x*
1i) + 1)) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*
1i + d*x*1i)/2))^(1/2)*1136i)/(693*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d
*x*2i) + 1))
```

3.108 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=146

$$\frac{832a^3\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{208a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{315d}+\frac{26a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d}-$$

```
[Out] 26/105*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d-4/63*(a+a*sec(d*x+c))^(5/2)*ta
n(d*x+c)/d+2/9*(a+a*sec(d*x+c))^(7/2)*tan(d*x+c)/a/d+832/315*a^3*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)+208/315*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3885, 4086, 3878, 3877}

$$\frac{832a^3\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}-\frac{4\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{63d}+\frac{26a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (832*a^3*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (208*a^2*Sqrt[a +
 a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (26*a*(a + a*Sec[c + d*x])^(3/2)*T
an[c + d*x])/(105*d) - (4*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) +
 (2*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*a*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
```

```
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx &= \frac{2(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad}+\frac{2\int \sec(c+dx)\left(\frac{7a}{2}-a\sec(\right.}{}\\ &= -\frac{4(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d}+\frac{2(a+a\sec(c+dx))^{7/2}\operatorname{ta}}{9ad}\\ &= \frac{26a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d}-\frac{4(a+a\sec(c+dx))^{5/2}\operatorname{ta}}{63d}\\ &= \frac{208a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{315d}+\frac{26a(a+a\sec(c+dx))^{3/}}{105d}\\ &= \frac{832a^3\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{208a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+da}{315d}\end{aligned}$$

Mathematica [A]
time = 0.52, size = 70, normalized size = 0.48

$$\frac{2a^3(584+292\sec(c+dx)+219\sec^2(c+dx)+130\sec^3(c+dx)+35\sec^4(c+dx))\tan(c+dx)}{315d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(584 + 292*Sec[c + d*x] + 219*Sec[c + d*x]^2 + 130*Sec[c + d*x]^3 +
35*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.11, size = 95, normalized size = 0.65

method	result	size

default	$-\frac{2\left(584\left(\cos^5(dx+c)\right)-292\left(\cos^4(dx+c)\right)-73\left(\cos^3(dx+c)\right)-89\left(\cos^2(dx+c)\right)-95\cos(dx+c)-35\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a^2}{315d\cos(dx+c)^4\sin(dx+c)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*(584*cos(d*x+c)^5-292*cos(d*x+c)^4-73*cos(d*x+c)^3-89*cos(d*x+c)^2
-95*cos(d*x+c)-35)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x
+c)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 8/315*(315*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(5*(a^2*d*cos(2*d*x + 2*c)^4 + a^2*d*sin(2*d*x + 2*c)^4 + 4*a^2*d*
cos(2*d*x + 2*c)^3 + 6*a^2*d*cos(2*d*x + 2*c)^2 + 4*a^2*d*cos(2*d*x + 2*c)
+ a^2*d + 2*(a^2*d*cos(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*s
in(2*d*x + 2*c)^2)*integrate((((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6
*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*
d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2
*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos
(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(8
*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(
4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2
*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(
6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2
*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(8
*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(
4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2
*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((
cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c)^2 + 9*(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 9*(co
```

```
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 9*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 +
 (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(co
s(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 3*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 6*(cos(
2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2
*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3
+ cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x +
2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 3*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x +
 6*c) + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) + 6*(sin(2*d*x + 2*c)^3 + 3*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6
*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*
d*x + 2*c))*sin(4*d*x + 4*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c)^2 + 9*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x
 + 6*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c)^2 + 9*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2
 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin
(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x
 + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3
*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d
*x + 6*c) + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x
 + 8*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x
 + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(co
s(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x
 + 2*c)^3 + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x +...
```

Fricas [A]

time = 3.16, size = 108, normalized size = 0.74

$$\frac{2\left(584\,a^{2}\cos\left(dx+c\right)^{4}+292\,a^{2}\cos\left(dx+c\right)^{3}+219\,a^{2}\cos\left(dx+c\right)^{2}+130\,a^{2}\cos\left(dx+c\right)+35\,a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.21, size = 180, normalized size = 1.23

$$\frac{8\left(315\,\sqrt{2}\,a^{7}\mathrm{sgn}(\cos\left(dx+c\right))-\left(630\,\sqrt{2}\,a^{7}\mathrm{sgn}(\cos\left(dx+c\right))-13\left(63\,\sqrt{2}\,a^{7}\mathrm{sgn}(\cos\left(dx+c\right))+4\left(2\,\sqrt{2}\,a^{7}\mathrm{sgn}(\cos\left(dx+c\right))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,\sqrt{2}\,a^{7}\mathrm{sgn}(\cos\left(dx+c\right))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{315\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{4}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 8/315*(315*sqrt(2)*a^7*sgn(cos(d*x + c)) - (630*sqrt(2)*a^7*sgn(cos(d*x + c
)) - 13*(63*sqrt(2)*a^7*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^7*sgn(cos(d*x +
c))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^7*sgn(cos(d*x + c)))*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/
2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*
d)
```

Mupad [**B**]
time = 8.18, size = 456, normalized size = 3.12

$$\frac{\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^2\,32\mathrm{i}}{9\,d}-\frac{a^2\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,32\mathrm{i}}{9\,d}\right)}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^2\,96\mathrm{i}}{7\,d}-\frac{a^2\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,32\mathrm{i}}{63\,d}\right)}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^2\,8\mathrm{i}}{3\,d}-\frac{a^2\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,584\mathrm{i}}{315\,d}\right)}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}+\frac{\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^2\,56\mathrm{i}}{5\,d}+\frac{a^2\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,904\mathrm{i}}{105\,d}\right)}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{a^2\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,1168\mathrm{i}}{315\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

```
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*32i)/(
9*d) - (a^2*exp(c*1i + d*x*1i)*32i)/(9*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(
c*2i + d*x*2i) + 1)^4) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2)*((a^2*96i)/(7*d) - (a^2*exp(c*1i + d*x*1i)*32i)/(63*d)))/((exp
(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + ((a + a/(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*8i)/(3*d) - (a^2*exp(c*1i + d*
x*1i)*584i)/(315*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)) +
 ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*56i)/
(5*d) + (a^2*exp(c*1i + d*x*1i)*904i)/(105*d)))/((exp(c*1i + d*x*1i) + 1)*(
exp(c*2i + d*x*2i) + 1)^2) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d
*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*1168i)/(315*d*(exp(c*1i + d*x*1i) +
 1))
```

3.109 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{64a^3\tan(c+dx)}{21d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{21d}+\frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{7d}+\frac{2(c}{}$$

```
[Out] 2/7*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+a*sec(d*x+c))^(5/2)*tan(d*
x+c)/d+64/21*a^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*(a+a*sec(d*x
+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3883, 3878, 3877}

$$\frac{64a^3\tan(c+dx)}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Tan[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(7*d) + (2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{5}{7}\int \sec(c+dx)(a+a\sec(c$$
$$= \frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{7d} + \frac{2(a+a\sec(c+dx))^{5/2}\tan}{7d}$$
$$= \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{21d} + \frac{2a(a+a\sec(c+dx))^{3/2}\,t}{7d}$$
$$= \frac{64a^3\tan(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{21d}$$

Mathematica [A]
time = 0.18, size = 60, normalized size = 0.52

$$\frac{2a^3(46+23\sec(c+dx)+12\sec^2(c+dx)+3\sec^3(c+dx))\tan(c+dx)}{21d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(46 + 23*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 3*Sec[c + d*x]^3)*Tan[c
+ d*x])/(21*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.10, size = 85, normalized size = 0.73

method	result	size
default	$-\frac{2\left(46\left(\cos^4(dx+c)\right)-23\left(\cos^3(dx+c)\right)-11\left(\cos^2(dx+c)\right)-9\cos(dx+c)-3\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a^2}{21d\cos(dx+c)^3\sin(dx+c)}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(46*cos(d*x+c)^4-23*cos(d*x+c)^3-11*cos(d*x+c)^2-9*cos(d*x+c)-3)*(a
*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 4/21*(21*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*((a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(
2*d*x + 2*c) + a^2*d)*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*co
s(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin
(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*c
os(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin
(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(
2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin
(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6
*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*
d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((c
os(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(
2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x +
2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2
*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(
2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x
+ 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*
sin(4*d*x + 4*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^
2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 +
2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*
d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d
*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c)
```

```
+ cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(
2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
2*c))*sin(4*d*x + 4*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)), x) + 10*(a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a
^2*d*cos(2*d*x + 2*c) + a^2*d)*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*
c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x +
 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x +
2*c)^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x +
 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x +
6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x +
 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x +
4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
 + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))/(((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + ...
```

Fricas [A]
time = 2.95, size = 95, normalized size = 0.82

$$\frac{2\left(46\,a^{2}\cos\left(dx+c\right)^{3}+23\,a^{2}\cos\left(dx+c\right)^{2}+12\,a^{2}\cos\left(dx+c\right)+3\,a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c) +
 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c
)^4 + d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(\sec\left(c+dx\right)+1)\right)^{\frac{5}{2}}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*sec(c + d*x)**2, x)
```

Giac [A]
time = 1.12, size = 151, normalized size = 1.30

$$-\frac{8\left(21\sqrt{2}\,a^{6}\mathrm{sgn}(\cos(dx+c))-\left(35\sqrt{2}\,a^{6}\mathrm{sgn}(\cos(dx+c))+4\left(2\sqrt{2}\,a^{6}\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-7\sqrt{2}\,a^{6}\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{21\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -8/21*(21*sqrt(2)*a^6*sgn(cos(d*x + c)) - (35*sqrt(2)*a^6*sgn(cos(d*x + c))
 + 4*(2*sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2)*a^
6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/
2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)*d)
```

Mupad [B]
time = 4.58, size = 349, normalized size = 3.01

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^{2}\,20i}{3\,d}-\frac{a^{2}\,e^{c\,1i+d\,x\,1i}\,4i}{21\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^{2}\,16i}{7\,d}+\frac{a^{2}\,e^{c\,1i+d\,x\,1i}\,16i}{7\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^{3}}-\frac{a^{2}\,e^{c\,1i+d\,x\,1i}\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,92i}{21\,d\left(e^{c\,1i+d\,x\,1i}+1\right)}+\frac{a^{2}\,e^{c\,1i+d\,x\,1i}\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,48i}{7\,d\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*20i)/(
3*d) - (a^2*exp(c*1i + d*x*1i)*4i)/(21*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(
c*2i + d*x*2i) + 1)) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2))^(1/2)*((a^2*16i)/(7*d) + (a^2*exp(c*1i + d*x*1i)*16i)/(7*d)))/((exp(c*
1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(c*1i + d*x*1i)*(a
+ a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*92i)/(21*d*(exp(
c*1i + d*x*1i) + 1)) + (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)
/2 + exp(c*1i + d*x*1i)/2))^(1/2)*48i)/(7*d*(exp(c*1i + d*x*1i) + 1)*(exp(c
*2i + d*x*2i) + 1)^2)
```

3.110 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{64a^3\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

```
[Out] 2/5*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+64/15*a^3*tan(d*x+c)/d/(a+a*sec(d
*x+c))^(1/2)+16/15*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3878, 3877}

$$\frac{64a^3\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(5*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{1}{5}(8a)\int \sec(c+dx)(a+a$$
$$= \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{5d}$$
$$= \frac{64a^3\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}$$

Mathematica [**A**]
time = 0.10, size = 50, normalized size = 0.56

$$\frac{2a^3(43+14\sec(c+dx)+3\sec^2(c+dx))\tan(c+dx)}{15d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(43 + 14*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [**A**]
time = 0.10, size = 75, normalized size = 0.84

method	result	size
default	$-\frac{2\left(43\left(\cos^3(dx+c)\right)-29\left(\cos^2(dx+c)\right)-11\cos(dx+c)-3\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a^2}{15d\cos(dx+c)^2\sin(dx+c)}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(43*cos(d*x+c)^3-29*cos(d*x+c)^2-11*cos(d*x+c)-3)*(a*(1+cos(d*x+c))
/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)*a^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [A]
time = 3.53, size = 82, normalized size = 0.92

$$\frac{2\left(43\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(\sec\left(c+dx\right)+1)\right)^{\frac{5}{2}}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*sec(c + d*x), x)
```

Giac [A]
time = 1.06, size = 122, normalized size = 1.37

$$\frac{8\left(15\sqrt{2}\,a^5\mathrm{sgn}(\cos\left(dx+c\right))+4\left(2\sqrt{2}\,a^5\mathrm{sgn}(\cos\left(dx+c\right))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-5\sqrt{2}\,a^5\mathrm{sgn}(\cos\left(dx+c\right))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] 8/15*(15*sqrt(2)*a^5*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^5*sgn(cos(d*x + c))
*tan(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/
2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 4.51, size = 146, normalized size = 1.64

$$-\frac{2\,a^2\,\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,15\mathrm{i}-\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,70\mathrm{i}+\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,70\mathrm{i}-\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,15\mathrm{i}+\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,43\mathrm{i}-43\mathrm{i}\right)}{15\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

```
[Out] -(2*a^2*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(
c*1i + d*x*1i)*15i - exp(c*2i + d*x*2i)*70i + exp(c*3i + d*x*3i)*70i - exp(
c*4i + d*x*4i)*15i + exp(c*5i + d*x*5i)*43i - 43i))/(15*d*(exp(c*1i + d*x*1
i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

3.111 $\int (a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{14a^3\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+14/3*a^3*tan(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3860, 4000, 3859, 209, 3877}

$$\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^3\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (14
*a^3*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\sec(c + dx))^{5/2}\,dx = \frac{2a^2\sqrt{a + a\sec(c + dx)}\,\tan(c + dx)}{3d} + \frac{1}{3}(2a)\int \sqrt{a + a\sec(c + dx)}\,\Big(\frac{3a}{2} + $$

$$= \frac{2a^2\sqrt{a + a\sec(c + dx)}\,\tan(c + dx)}{3d} + a^2\int \sqrt{a + a\sec(c + dx)}\,dx + \frac{1}{3}(7a^2$$

$$= \frac{14a^3\tan(c + dx)}{3d\sqrt{a + a\sec(c + dx)}} + \frac{2a^2\sqrt{a + a\sec(c + dx)}\,\tan(c + dx)}{3d} - \frac{(2a^3)\,\mathrm{Subst}}{}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a + a\sec(c + dx)}}\right)}{d} + \frac{14a^3\tan(c + dx)}{3d\sqrt{a + a\sec(c + dx)}} + \frac{2a^2\sqrt{a + a}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.37, size = 360, normalized size = 3.67

$$\frac{\csc^3\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^{5/2}\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}\left(256\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(\frac{3}{2},2,\frac{7}{2};1,\frac{9}{2};2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sin^6\left(\frac{1}{2}(c+dx)\right)+512\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sin^6\left(\frac{1}{2}(c+dx)\right)\left(2-3\sin^2\left(\frac{1}{2}(c+dx)\right)+\sin^4\left(\frac{1}{2}(c+dx)\right)\right)+\frac{21\sqrt{2}\,\mathrm{ArcSin}\left(\sqrt{2}\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\left(15-10\sin^2\left(\frac{1}{2}(c+dx)\right)+3\sin^4\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}-14\sqrt{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}\left(45+30\sin^2\left(\frac{1}{2}(c+dx)\right)-31\sin^4\left(\frac{1}{2}(c+dx)\right)+12\sin^6\left(\frac{1}{2}(c+dx)\right)\right)\right)}{672d\sec^{\frac{5}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]^5*(a*(1 + Sec[c + d*x]))^(5/2)*Sqrt[(1
 - 2*Sin[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*(256*Cos[(c +
 d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]
*Sin[(c + d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Sin[(c + d*x)/
2]^2]*Sin[(c + d*x)/2]^6*(2 - 3*Sin[(c + d*x)/2]^2 + Sin[(c + d*x)/2]^4) +
```

```
(21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[(c + d*x)/2]^2]]*(15 - 10*Sin[(c + d*x)
/2]^2 + 3*Sin[(c + d*x)/2]^4))/Sqrt[Sin[(c + d*x)/2]^2] - 14*Sqrt[1 - 2*Sin
[(c + d*x)/2]^2]*(45 + 30*Sin[(c + d*x)/2]^2 - 31*Sin[(c + d*x)/2]^4 + 12*S
in[(c + d*x)/2]^6)))/(672*d*Sec[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(84) = 168$.
time = 0.13, size = 214, normalized size = 2.18

method	result
default	$\frac{\left(-3\left(\cos^{2}(dx+c)\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)-3\cos(dx+c)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c}}$ … $}{3d(1+\cos(dx+c))\cos(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(-3*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-3*
cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+16*cos(d*x+c)*
sin(d*x+c)+2*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(1+cos(d*x+c))
/cos(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(84) = 168$.
time = 0.61, size = 1395, normalized size = 14.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
```

```
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 2.62, size = 310, normalized size = 3.16

$$\left[\frac{3\left(a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^{2}-2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\,a^{2}\cos\left(dx+c\right)+a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}, -\frac{2\left(3\left(a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8\,a^{2}\cos\left(dx+c\right)+a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x + c)
 + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)^2 + d*cos(d*x + c)), -2/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
```

```
in(d*x + c))) - (8*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec (c+dx)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2),x)
[Out] Integral((a*sec(c + d*x) + a)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(84) = 168.
time = 1.27, size = 225, normalized size = 2.30

$$-\frac{\frac{3\sqrt{-a}\,a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(dx+c))}{|a|}-\frac{2\left(7\sqrt{2}a^4\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\sqrt{2}a^4\operatorname{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] -1/3*(3*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a)
 - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(7*sqrt(2)*a^4*sgn(cos(d*x + c))*tan(
1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos (c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2), x)
```

3.112 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=94

$$\frac{5a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d}$$

```
[Out] 5*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-a^3*sin(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)+2*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3899, 4100, 3859, 209}

$$\frac{5a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (a^
3*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*
x]]*Sin[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} + (2a)\int \cos(c+dx)\sqrt{a+a\,\mathrm{s}}$$

$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}$$

$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{(}{}$$

$$= \frac{5a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.43, size = 189, normalized size = 2.01

$$\frac{2\cos^{\frac{5}{2}}(c+dx)(a(1+\sec(c+dx)))^{5/2}\left(12\,{}_3F_2\left(\frac{3}{2},2,\frac{5}{2};1,\frac{9}{2};2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sin^2\left(\frac{1}{2}(c+dx)\right)+\frac{1}{8}\sec^4\left(\frac{1}{2}(c+dx)\right)\left(7(89+28\cos(c+dx)+3\cos(2(c+dx)))\,{}_2F_1\left(\frac{1}{2},\frac{3}{2};\frac{7}{2};2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)+24(3+\cos(c+dx))\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{9}{2};2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sin^2(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{105d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2)*(12*HypergeometricPFQ[{3
/2, 2, 5/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2 + (Sec[(c +
 d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F1
[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 24*(3 + Cos[c + d*x])*Hypergeometri
c2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2))/8)*Tan[(c + d*x)
/2])/(105*d)
```

Maple [A]
time = 0.12, size = 128, normalized size = 1.36

method	result
default	$-\frac{\left(5\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)+2(\cos^2(dx+c))+2\cos(dx+c)-4\right)\sqrt{}}{2d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*co
s(d*x+c)^2+2*cos(d*x+c)-4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a
^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1383 vs. $2(84) = 168$.
time = 0.63, size = 1383, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*cos(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x + 2*c)
^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x + 2
*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) + 4*a
^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c) +
(a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c) - a
^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
```

```
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(
a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 2.95, size = 276, normalized size = 2.94

$$\left[\frac{5\,(a^2\cos(dx+c)+a^2)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,(a^2\cos(dx+c)+2\,a^2)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{2\,(d\cos(dx+c)+d)},\ -\frac{5\,(a^2\cos(dx+c)+a^2)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-(a^2\cos(dx+c)+2\,a^2)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{d\cos(dx+c)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*c
os(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a^2*cos(d*x + c) + 2*a^2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -(5*(a
^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (a^2*cos(d*x + c) + 2*a^2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(84) = 168.

time = 1.68, size = 368, normalized size = 3.91

$$-\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}+5\sqrt{-a}\,a^2\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)\mathrm{sgn}(\cos(dx+c))-5\sqrt{-a}\,a^2\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)\mathrm{sgn}(\cos(dx+c))+\frac{4\left(3\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}\,a^3\mathrm{sgn}(\cos(dx+c))-\sqrt{2}\sqrt{-a}\,a^4\mathrm{sgn}(\cos(dx+c))\right)}{\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^3*sgn(cos(d*x + c))*t
an(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 5*sqrt(-a)*a^2*log(abs
((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 5*sqrt(-a)*a^2*log(abs((sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2)
- 3)))*sgn(cos(d*x + c)) + 4*(3*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^3*sgn(cos(d*x + c)) - sqrt(
2)*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2), x)
```

3.113 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=106

$$\frac{19a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+}{2d}$$

```
[Out] 19/4*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+9/4*a^3*si
n(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*cos(d*x+c)*sin(d*x+c)*(a+a*sec(d*
x+c))^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3898, 4100, 3859, 209}

$$\frac{19a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (9*a^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
```

```
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}a\int \cos(c+d$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin}{2d}$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin}{2d}$$

$$= \frac{19a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.59, size = 150, normalized size = 1.42

$$-\frac{a^2\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{1-\sec(c+dx)}\,(\sin(c+dx)+3\sin(2(c+dx)))-7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)-32\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}\,\tan(c+dx)\right)}{4d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/4*(a^2*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(
Sin[c + d*x] + 3*Sin[2*(c + d*x)]) - 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[
c + d*x] - 32*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]]*Tan[c + d*x]))/(d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(90) = 180$.
time = 0.14, size = 224, normalized size = 2.11

method	result
default	$\frac{\left(19\sin(dx+c)\cos(dx+c)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+19\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(\ldots}{1+\cos(\ldots}}}{\ldots}\right.}{16d\cos(dx+\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(19*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)+19*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*c
os(d*x+c)^4-36*cos(d*x+c)^3+44*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^
(1/2)/cos(d*x+c)/sin(d*x+c)*a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 4.16, size = 294, normalized size = 2.77

$$\left[\frac{19\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\,\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2a^2\cos(dx+c)^2+11a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8\left(d\cos(dx+c)+d\right)},\ -\frac{19\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2a^2\cos(dx+c)^2+11a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{4\left(d\cos(dx+c)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(19*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*
cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a^2*cos(d*x + c)^2 + 11*a^2*co
s(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c) + d), -1/4*(19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*a^2*co
s(d*x + c)^2 + 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(90) = 180.
time = 1.27, size = 364, normalized size = 3.43

$$-\frac{\sqrt{2}\sqrt{-a}\,a^{5}\left(\frac{19\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{a^{2}|a|}+\frac{8\left(19\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{6}-171\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}a+89\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}a^{2}-9a^{3}\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}a+a^{2}\right)^{2}a^{2}}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(-a)*a^5*(19*sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ 4*sqrt(2)*abs(a) - 6*a))/(a^2*abs(a)) + 8*(19*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6 - 171*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a + 89*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^2 - 9*a^3)/(((sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2
*a^2))*sgn(cos(d*x + c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

3.114 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=144

$$\frac{25a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+\ldots}{}$$

```
[Out] 25/8*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+25/8*a^3*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+13/12*a^3*cos(d*x+c)*sin(d*x+c)/d/(a+a*s
ec(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3898, 4100, 3890, 3859, 209}

$$\frac{25a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{25a^3\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (25*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
+ (25*a^3*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (13*a^3*Cos[c + d*
x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^2*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}a\int \cos^2(c+$$

$$= \frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+dx)\sqrt{a+a\sec(c+d}}{3d}$$

$$= \frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2}{}$$

$$= \frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2}{}$$

$$= \frac{25a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.82, size = 151, normalized size = 1.05

$$\frac{a^2\left(165\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+(31+159\cos(c+dx)+31\cos(2(c+dx))-2\cos(3(c+dx)))\sqrt{1-\sec(c+dx)}+192\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{72d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(165*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (31 + 159*Cos[c + d*x] + 31*Cos
[2*(c + d*x)] - 2*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]] + 192*Hypergeome
tric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 +
Sec[c + d*x])]*Sin[c + d*x])/(72*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(124) = 248$.
time = 0.16, size = 313, normalized size = 2.17

method	result
default	$-\frac{\left(75\left(\cos^2(dx+c)\right)\sin(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{2}+150\cos(dx+c)\sin(dx+c)\,\mathrm{ar}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/d*(75*cos(d*x+c)^2*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)*2^(1/2)+150*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)*2^(1/2)+75*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c
)+64*cos(d*x+c)^6+208*cos(d*x+c)^5+328*cos(d*x+c)^4-600*cos(d*x+c)^3)*(a*(1
+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2*a^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.81, size = 320, normalized size = 2.22

$$\left[\frac{75\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\,\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8a^2\cos(dx+c)^3+34a^2\cos(dx+c)^2+75a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{48\left(d\cos(dx+c)+d\right)},\ -\frac{75\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8a^2\cos(dx+c)^3+34a^2\cos(dx+c)^2+75a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{24\left(d\cos(dx+c)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(75*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a
*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x + c)^3 + 34*a^2*c
os(d*x + c)^2 + 75*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(75*(a^2*cos(d*x + c) + a^2)*sq
rt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - (8*a^2*cos(d*x + c)^3 + 34*a^2*cos(d*x + c)^2 + 75*a^2*cos
(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(124) = 248.
time = 1.87, size = 539, normalized size = 3.74

$$-\frac{75\sqrt{-a}\,a^{2}\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{2}+3\right)\right|\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-75\sqrt{-a}\,a^{2}\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+a\left(2\sqrt{2}-3\right)\right|\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\frac{4\left(75\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{10}\sqrt{-a}\,a^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-1125\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{8}\sqrt{-a}\,a^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+6174\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{6}\sqrt{-a}\,a^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-4314\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}\sqrt{-a}\,a^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+807\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\sqrt{-a}\,a^{7}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-49\sqrt{2}\sqrt{-a}\,a^{8}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}a+a^{2}\right)^{3}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/48*(75*sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 75*sq
rt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(75*sqrt(2)*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(
-a)*a^3*sgn(cos(d*x + c)) - 1125*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 6174
*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^6*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 4314*sqrt(2)*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^6*sgn(cos(d*x +
 c)) + 807*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 49*sqrt(2)*sqrt(-a)*a^8*sg
n(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
```

```
2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*a + a^2)^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^3 \left(a+\frac{a}{\cos(c+dx)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

3.115 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=182

$$\frac{163a^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}}+\frac{17a^3\,\mathrm{co}\cdots}{24d\sqrt{\cdots}} \text{ (cut off at the page edge)}$$

[Out] 163/64*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+163/64*a^3*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+163/96*a^3*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+17/24*a^3*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A]
time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3898, 4100, 3890, 3859, 209}

$$\frac{163a^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{163a^3\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{17a^3\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)\cos(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]

[Out] (163*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d) + (163*a^3*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (17*a^3*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
```

```
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4100

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^3(c+$$

$$= \frac{17a^3\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^3(c+dx)\sqrt{a+a\sec(c+dx}}{4d}$$

$$= \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3}{24}$$

$$= \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3}{24}$$

$$= \frac{163a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.81, size = 161, normalized size = 0.88

$$\frac{a^2\left(675\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+(231+849\cos(c+dx)+233\cos(2(c+dx))+58\cos(3(c+dx))+2\cos(4(c+dx)))\sqrt{1-\sec(c+dx)}+512\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\sin(c+dx)}{320d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(675*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (231 + 849*Cos[c + d*x] + 233*C
os[2*(c + d*x)] + 58*Cos[3*(c + d*x)] + 2*Cos[4*(c + d*x)])*Sqrt[1 - Sec[c
+ d*x]] + 512*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(320*d*(1 + Cos[c + d*
x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(158) = 316.
time = 0.18, size = 402, normalized size = 2.21

method	result
default	$\frac{\left(489\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)(\cos^3(dx+c))\sin(dx+c)\sqrt{2}+1467\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\text{arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3072/d*(489*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^3*sin(d*
x+c)*2^(1/2)+1467*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*si
n(d*x+c)*2^(1/2)+1467*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*
sin(d*x+c)*2^(1/2)+489*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*si
n(d*x+c)-768*cos(d*x+c)^8-2176*cos(d*x+c)^7-2272*cos(d*x+c)^6-2608*cos(d*x+
c)^5+7824*cos(d*x+c)^4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(
d*x+c)^3*a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.71, size = 346, normalized size = 1.90

$$\left[\frac{489\,\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos\left(dx+c\right)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)+2\,\left(48\,a^2\cos\left(dx+c\right)^4+184\,a^2\cos\left(dx+c\right)^3+326\,a^2\cos\left(dx+c\right)^2+489\,a^2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{384\,\left(d\cos\left(dx+c\right)+d\right)},\ -\frac{489\,\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)}{\sqrt{a}\,\sin\left(dx+c\right)}\right)-\left(48\,a^2\cos\left(dx+c\right)^4+184\,a^2\cos\left(dx+c\right)^3+326\,a^2\cos\left(dx+c\right)^2+489\,a^2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{192\,\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(489*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) +
 a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*a^2*cos(d*x + c)^4 + 184*a
^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*
(489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*a^2*cos(d*x + c)^4 + 1
84*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

3.116 $\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

[Out] -2*a*tan(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3877}

$$-\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]

[Out] (-2*a*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = -\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

Mathematica [A]
time = 0.12, size = 30, normalized size = 1.11

$$\frac{2\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]

[Out] (2*Cot[(c + d*x)/2]*Sqrt[a - a*Sec[c + d*x]])/d

Maple [A]
time = 0.18, size = 42, normalized size = 1.56

method	result	size
default	$-\frac{2\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\,\sin(dx+c)}{d(-1+\cos(dx+c))}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/d*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)/(-1+cos(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*sec(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [A]
time = 2.81, size = 44, normalized size = 1.63

$$\frac{2\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\,(\cos(dx+c)+1)}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*(cos(d*x + c) + 1)/(d*sin(d*x + c
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a(\sec(c+dx)-1)}\,\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(-a*(sec(c + d*x) - 1))*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(25) = 50$.
time = 0.68, size = 57, normalized size = 2.11

$$-\frac{2\sqrt{2}\,a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\mathrm{sgn}(\cos\left(dx+c\right))}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*a*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x
 + c))/(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*d)
```

Mupad [B]
time = 0.79, size = 36, normalized size = 1.33

$$\frac{\sin\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}}{d\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] (sin(c + d*x)*(a - a/cos(c + d*x))^(1/2))/(d*sin(c/2 + (d*x)/2)^2)
```

3.117 $\int \sqrt{a-a\sec(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A]

time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3859, 209}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\sec(c+dx)}\,dx &= \frac{(2a)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 188, normalized size = 4.95

$$-\frac{\left(\tanh^{-1}\left(\frac{e^{idx}}{\sqrt{\cos(c)-i\sin(c)}\sqrt{\cos(c)+e^{2idx}(\cos(c)+i\sin(c))-i\sin(c)}}\right)+\tanh^{-1}\left(\frac{\sqrt{\cos(c)+e^{2idx}(\cos(c)+i\sin(c))-i\sin(c)}}{\sqrt{\cos(c)-i\sin(c)}}\right)\right)\cos(c+dx)\left(i+\cot\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a-a\sec(c+dx)}\sqrt{\cos(c)-i\sin(c)}}{d\sqrt{(1+e^{2idx})\cos(c)+i(-1+e^{2idx})\sin(c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] -(((ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*
(Cos[c] + I*Sin[c]) - I*Sin[c]])] + ArcTanh[Sqrt[Cos[c] + E^((2*I)*d*x)*(Co
s[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]])*Cos[c + d*x]*(I + Co
t[(c + d*x)/2])*Sqrt[a - a*Sec[c + d*x]]*Sqrt[Cos[c] - I*Sin[c]])/(d*Sqrt[(
1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.
time = 0.12, size = 91, normalized size = 2.39

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\,\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)}{d(-1+\cos(dx+c))}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*2^(1/2)*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2))/(-1+cos(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.
time = 0.54, size = 146, normalized size = 3.84

$$\frac{\sqrt{a}\arctan\left(\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)+1)\right)+\sin(dx+c),\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)+1)\right)+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.
time = 2.67, size = 182, normalized size = 4.79

$$\left[\frac{\sqrt{-a}\log\left(-\frac{4\left(2\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)}{2d}, -\frac{\sqrt{a}\arctan\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{(2a\cos(dx+c)+a)\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log(-(4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*
sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (8*a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))/d, -sqrt(a)*arctan(2*(cos(d
*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))/(
(2*a*cos(d*x + c) + a)*sin(d*x + c)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\sec(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sec(c + d*x) + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.
time = 0.68, size = 65, normalized size = 1.71

$$-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\operatorname{sgn}(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))*s
gn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a-\frac{a}{\cos \left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x))^(1/2),x)
[Out] int((a - a/cos(c + d*x))^(1/2), x)
```

3.118 $\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

```
[Out] -arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*sin(d*x+c)/d
/(a-a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3890, 3859, 209}

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d) + (a
*Sin[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}} - \frac{1}{2}\int \sqrt{a-a\sec(c+dx)}\,dx$$
$$= \frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}} - \frac{a\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d} + \frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.91, size = 260, normalized size = 4.00

$$\frac{\cos(c+dx)\sqrt{a-a\sec(c+dx)}\left(\tanh^{-1}\left(\frac{e^{idx}}{\sqrt{\cos(c)-i\sin(c)}\sqrt{\cos(c)+e^{2idx}(\cos(c)+i\sin(c))-i\sin(c)}}\right)\left(i+\cot\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c)-i\sin(c)}+\tanh^{-1}\left(\frac{\sqrt{\cos(c)+e^{2idx}(\cos(c)+i\sin(c))-i\sin(c)}}{\sqrt{\cos(c)-i\sin(c)}}\right)\left(i+\cot\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c)-i\sin(c)}-2\sqrt{2}\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\cos(dx)+i\sin(dx))}\right)}{2d\sqrt{(1+e^{2idx})\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]]*(ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*
Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])]*(I +
Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + ArcTanh[Sqrt[Cos[c] + E^((2*I)*
d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]]*(I + Cot[(c +
 d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] - 2*Sqrt[2]*Cot[(c + d*x)/2]*Sqrt[Cos[c +
 d*x]*(Cos[d*x] + I*Sin[d*x])]))/(2*d*Sqrt[(1 + E^((2*I)*d*x))*Cos[c] + I*(
-1 + E^((2*I)*d*x))*Sin[c]])
```

Maple [A]
time = 0.15, size = 103, normalized size = 1.58

method	result	size
default	$\frac{\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\,\sin(dx+c)\left(\sqrt{2}\,\cos(dx+c)+\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\right)\sqrt{2}}{2d(-1+\cos(dx+c))}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)*(2^(1/2)*cos(d*x+c)+(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*2^(1/2)))/(-1+cos(d*x+c))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(57) = 114$.
time = 0.60, size = 791, normalized size = 12.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c
) - (cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))) - 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(57) = 114$.
time = 3.42, size = 294, normalized size = 4.52

$$\left[\frac{\sqrt{-a}\log\left(\frac{4\left(2\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-4\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{4d\sin(dx+c)}, \frac{\sqrt{a}\arctan\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{(2a\cos(dx+c)+a)\sin(dx+c)}\right)\sin(dx+c)-2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{2d\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a)*log((4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*
sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*
```

```
a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*(cos(d*x +
 c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(d*sin(d*x +
 c)), 1/2*(sqrt(a)*arctan(2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a
*cos(d*x + c) - a)/cos(d*x + c))/((2*a*cos(d*x + c) + a)*sin(d*x + c)))*sin
(d*x + c) - 2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos
(d*x + c)))/(d*sin(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sec\left(c+dx\right)-1\right)}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sec(c + d*x) - 1))*cos(c + d*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(57) = 114.
time = 0.53, size = 134, normalized size = 2.06

$$\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\frac{2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)
^2 - a)/sqrt(a))*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)) - 2*sqr
t(a*tan(1/2*d*x + 1/2*c)^2 - a)*a*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x
+ 1/2*c))/(a*tan(1/2*d*x + 1/2*c)^2 + a))*sgn(cos(d*x + c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2), x)
```

3.119 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}-\frac{2\sqrt{a+}$$

```
[Out] -arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)+28/15*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*sec(d*x+c)^2*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)-2/15*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a/d
```

Rubi [A]

time = 0.21, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3907, 4095, 4086, 3880, 209}

$$-\frac{\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}+\frac{28\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(Sqrt[a]*d)) + (28*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Se
c[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Sqrt[a + a*S
ec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[d^2/(b*(2*n - 3)), Int[(d
```

```
*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^2(c+dx)(4a-a\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15ad} + \frac{2\int \frac{\sec(c+dx)(}{\sqrt{a+}}$$

$$= \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}}{15ad}$$

$$= \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}}{15ad}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(}{5d\sqrt{}}$$

Mathematica [A]

time = 0.23, size = 106, normalized size = 0.76

$$\frac{\left(-15\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+2\sqrt{1-\sec(c+dx)}\,(13-\sec(c+dx)+3\sec^2(c+dx))\right)\tan(c+dx)}{15d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-15*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - Sec[c + 
d*x]]*(13 - Sec[c + d*x] + 3*Sec[c + d*x]^2))*Tan[c + d*x])/(15*d*Sqrt[1 - 
Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(119) = 238$.
time = 0.14, size = 314, normalized size = 2.24

method	result
default	$-\frac{\left(15(\cos^2(dx+c))\sin(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)+30\cos(dx+c)\sin(dx+c)\left(-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/d*(15*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln
(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))+30*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+15*ln
(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)+104*cos(d*x+c)^3-112*cos
(d*x+c)^2+32*cos(d*x+c)-24)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^
2/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 2.57, size = 347, normalized size = 2.48

$$\left[\frac{15\,\sqrt{2}\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\,\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\cos\left(dx+c\right)\sin\left(dx+c\right)+3\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)+4\left(13\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+3\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{30\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}, \frac{2\left(13\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+3\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)+\frac{15\,\sqrt{2}\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)}{\sqrt{a}}}{15\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-1/a)*log((2*s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(
d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) + 4*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^
2), 1/15*(2*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c
)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{4}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 1.17, size = 150, normalized size = 1.07

$$\frac{\sqrt{2}\left(\frac{15\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}}+\frac{2\left(\left(17\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-20\,a^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,a^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}\right)}{15\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)))/sqrt(-a) + 2*((17*a^2*tan(1/2*d*x + 1/2*c)^2 - 20*a^2)
```

```
*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/
2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(d*sgn(cos(d*x + c)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)), x)
```

3.120 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad}$$

```
[Out] arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(
1/2)-4/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*(a+a*sec(d*x+c))^(1/2)*tan
(d*x+c)/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3885, 4086, 3880, 209}

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}-\frac{4\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[a]*d) - (4*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a
+ a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
```

```
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad} + \frac{2\int \frac{\sec(c+dx)\left(\frac{a}{2}-a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= -\frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad} + \int \frac{\sec(\cdots}{\sqrt{a+a\,\mathrm{s}\cdots}}$$

$$= -\frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad} - \frac{2\mathrm{Subst}\left(\int \frac{\cdots}{2\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+\cdots}}{\cdots}$$

Mathematica [A]
time = 0.16, size = 86, normalized size = 0.83

$$-\frac{\left(-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+\frac{2}{3}(1-\sec(c+dx))^{3/2}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(((-(Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]) + (2*(1 - Sec[c + d*
x])^(3/2))/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d
*x])]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(87) = 174$.
time = 0.12, size = 221, normalized size = 2.12

method	result
default	$\frac{\left(-3\cos(dx+c)\sin(dx+c)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-3\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin\right.\right.}{\sin(dx+}}{6d\sin(dx+c)\cos(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(-3*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-3
*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*
x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+4*cos(d*x+c)^2-8*cos(
d*x+c)+4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 2.66, size = 316, normalized size = 3.04

$$\left[\frac{3\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(\cos(dx+c)-1\right)\sin(dx+c)}{6\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}, -\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(\cos(dx+c)-1\right)\sin(dx+c)+\frac{3\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{3\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*log(-(2*sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x
 + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*s
in(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/3*(2*sqrt((a*cos(d
```

```
*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sin(d*x + c) + 3*sqrt(2)*(a*c
os(d*x + c)^2 + a*cos(d*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)
^2 + a*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 1.13, size = 113, normalized size = 1.09

$$-\frac{\sqrt{2}\left(\frac{4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\frac{3\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}}\right)}{3\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(2)*(4*a*tan(1/2*d*x + 1/2*c)^3/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) + 3*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*
c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(-a))/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^3\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)), x)
```

3.121 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=73

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)+2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3883, 3880, 209}

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
]])/(Sqrt[a]*d)) + (2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx\\
&= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 83, normalized size = 1.14

$$
-\frac{\left(\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-2\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(((Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - 2*Sqrt[1 - Sec[c + d*
x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

Maple [A]
time = 0.09, size = 121, normalized size = 1.66

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+2\cos(dx+c)-2\right)}{d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)+2*cos(d*x+c)-2)/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 2.70, size = 262, normalized size = 3.59

$$\left[\frac{\sqrt{2}\,(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)},\ \frac{\frac{\sqrt{2}\,(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{ad\cos(dx+c)+ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x
+ c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
, (sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a\,(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 1.12, size = 108, normalized size = 1.48

$$\frac{\sqrt{2}\left(\frac{\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}}-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\right)}{d\,\mathrm{sgn}(\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)))/sqrt(-a) - 2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x +
 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a))/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos \left(c+d\,x\right)^2\,\sqrt{a+\frac{a}{\cos \left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)), x)
```

3.122 $\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3880, 209}

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [**A**]
time = 0.06, size = 64, normalized size = 1.39

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(37) = 74.
time = 0.09, size = 95, normalized size = 2.07

method	result	size
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)}{da}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 3.87, size = 158, normalized size = 3.43

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x +
c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, -sqrt(2)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
/(sqrt(a)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 0.78, size = 59, normalized size = 1.28

$$-\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,d\,\mathrm{sgn}(\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)))/(sqrt(-a)*d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.123 $\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*a^
(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3861, 3859, 209, 3880}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\int \sqrt{a+a\sec(c+dx)}\,dx}{a} - \int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\cdots\right.}{d}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.03, size = 5402, normalized size = 63.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 0.09, size = 141, normalized size = 1.66

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)+\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos}{1+\cos}}}{\cdots}\right.\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
```

```
(d*x+c))+2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2)))/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 3.76, size = 294, normalized size = 3.46

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\cos\left(dx+c\right)\sin\left(dx+c\right)+3\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)-2\sqrt{-a}\log\left(\frac{2a\cos\left(dx+c\right)^{2}+2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)}{2ad},\ \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)-2\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/2*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x
+ c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 2*sqrt(-a)*log((2*a*cos(
d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d), (sqrt(2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a*sec(c + d*x) + a), x)
```

Giac [A]
time = 0.87, size = 69, normalized size = 0.81

$$\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}}{2\sqrt{a}}\right)}{\sqrt{a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(2)*arctan(sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqrt(a))/sqrt(a) - 2*ar
ctan(1/2*sqrt(2)*sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqrt(a))/sqrt(a))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + a/cos(c + d*x))^(1/2), x)
```

3.124 $\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=108

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+arctan(1/2*a^(
1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+sin(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3908, 3989, 3972, 492, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/(Sqrt[a]*d)) + (S
qrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
Sqrt[a]*d) + Sin[c + d*x]/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
```

```
b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a
+ b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{a-a\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a} \\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}a\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx \\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{a\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{d} \\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2\text{S}}{} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} +
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 105, normalized size = 0.97

$$-\frac{\left(\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-\cos(c+dx)\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(((ArcTanh[Sqrt[1 - Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]
]/Sqrt[2]] - Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 -
 Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(91) = 182$.
time = 0.12, size = 201, normalized size = 1.86

method	result
default	$\frac{\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)+2\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+c}{\sin(dx+c)}\right.\right.}{2d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*ln(-(
-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*cos(d*x+c)^2+2*cos(d*x+c)
)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 3.44, size = 417, normalized size = 3.86

$$\left[\frac{\sqrt{2}\,(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{-a}\,(\cos(dx+c)+1)\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)},\ \frac{\sqrt{a}\,(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-\frac{\sqrt{2}\,(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{ad\cos(dx+c)+ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x
 + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt
(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(
cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), (sqrt(a)*(cos(d*x + c) + 1)*arctan
(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)
)) + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - sq
rt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*
d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(91) = 182.
time = 1.08, size = 326, normalized size = 3.02

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{2\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}}-\frac{8\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}-\sqrt{-a}\,a\right)}{\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2}\right)}{4d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/abs(a) - 2*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) - 8*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a) - sqrt(-a)*a)/((sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))
/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/2), x)
```

3.125 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=147

$$\frac{7\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{\cos}{2d}$$

```
[Out] 7/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*
a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-1/4*si
n(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+
c))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3908, 4107, 4005, 3859, 209, 3880}

$$\frac{7\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (7*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sqrt[a]*d) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(Sqrt[a]*d) - Sin[c + d*x]/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (Cos[c + d*x]
*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a
+ b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)(a-3a\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{-\frac{7a^2}{2}+\frac{1}{2}a^2\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,d}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{7\int \sqrt{a+a\sec(c+dx)}}{8a}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{7\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\right.}{4a}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} -$$

Mathematica [A]
time = 0.25, size = 118, normalized size = 0.80

$$\frac{\left(7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+\cos(c+dx)(-1+2\cos(c+dx))\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((7*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*
x]]/Sqrt[2]] + Cos[c + d*x]*(-1 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])*T
an[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(122) = 244$.
time = 0.15, size = 380, normalized size = 2.59

method	result
default	$-\frac{\left(-7\sin(dx+c)\cos(dx+c)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-8\cos(dx+c)\sin(dx+c)\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/d*(-7*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)-8*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-
7*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*ln(-(-(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+8*cos(d*x+c)^4-12*cos(d*x+c)^3+4
*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 3.61, size = 446, normalized size = 3.03

$$\left[\frac{4\sqrt{2}\left(a\cos\left(dx+c\right)+a\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-7\sqrt{-a}\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\cos\left(dx+c\right)^2-\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{8\left(ad\cos\left(dx+c\right)+ad\right)}, -\frac{7\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\cos\left(dx+c\right)^2-\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)-\frac{4\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{4\left(ad\cos\left(dx+c\right)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*
x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 7*s
qrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a
)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), -1/4*(7*sq
rt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*(a*cos(d*x + c)
 + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(122) = 244.
time = 1.15, size = 423, normalized size = 2.88

$$-\frac{\sqrt{2}\left(\frac{7\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{8\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}}-\frac{8\left(17\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6\sqrt{-a}-57\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4\sqrt{-a}a+19\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\sqrt{-a}a^2-3\sqrt{-a}a^3\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)^2}\right)}{16d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(7*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*s
qrt(2)*abs(a) - 6*a))/abs(a) - 8*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) - 8*(17*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a) - 57*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a + 19
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sq
rt(-a)*a^2 - 3*sqrt(-a)*a^3)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/(d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

3.126 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{15\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}}+\frac{9\sec^2(c+dx}{10ad\sqrt{a+a}}$$

```
[Out] -15/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)
/d*2^(1/2)-1/2*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+31/5*tan(d*
x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+9/10*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d
*x+c))^(1/2)-13/10*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^2/d
```

Rubi [A]

time = 0.30, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3901, 4106, 4095, 4086, 3880, 209}

$$-\frac{15\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{13\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{10a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{9\tan(c+dx)\sec^2(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}+\frac{31\tan(c+dx)}{5ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x]
)^(3/2)) + (31*Tan[c + d*x])/(5*a*d*Sqrt[a + a*Sec[c + d*x]]) + (9*Sec[c +
d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) - (13*Sqrt[a + a*Sec
[c + d*x]]*Tan[c + d*x])/(10*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
```

```
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^3(c+dx)\left(3a-\frac{9}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\sec^2(c+dx)\left(-9a^2+\frac{39}{4}\right.}{\sqrt{a+a\sec(c}}}{5a^3}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{a+a\sec(c+}}{10a}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{9\sec^2(c+dx)\tan(c}{10ad\sqrt{a+a\sec(c+}}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{9\sec^2(c+dx)\tan(c}{10ad\sqrt{a+a\sec(c+}}$$

$$= -\frac{15\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31}{5ad\sqrt{a}}$$

Mathematica [A]
time = 0.46, size = 124, normalized size = 0.68

$$\frac{\left(-75\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))+2\sqrt{1-\sec(c+dx)}\left(49+36\sec(c+dx)-4\sec^2(c+dx)+4\sec^3(c+dx)\right)\right)\tan(c+dx)}{20d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-75*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) +
2*Sqrt[1 - Sec[c + d*x]]*(49 + 36*Sec[c + d*x] - 4*Sec[c + d*x]^2 + 4*Sec[c
 + d*x]^3))*Tan[c + d*x])/(20*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]
))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(156) = 312$.
time = 0.14, size = 417, normalized size = 2.28

method	result

default	$\frac{\left(75(\cos^4(dx+c))\sin(dx+c)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+150(\cos^3(dx+c))\sin(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/80/d*(75*cos(d*x+c)^4*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)+150*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-150
*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(-(-(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-75*ln(-(-(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)+392*cos(d*x+c)^5-496*cos(d*x+
c)^4-216*cos(d*x+c)^3+384*cos(d*x+c)^2-96*cos(d*x+c)+32)*(a*(1+cos(d*x+c))/
cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^3/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 3.92, size = 414, normalized size = 2.26

$$\left[-\frac{75\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(49\cos(dx+c)^3+36\cos(dx+c)^2-4\cos(dx+c)+4\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{40\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3+a^2d\cos(dx+c)^2\right)}, \frac{75\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left(49\cos(dx+c)^3+36\cos(dx+c)^2-4\cos(dx+c)+4\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{20\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3+a^2d\cos(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/40*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqr
t(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^3 + 36*cos(d*x + c)^2 -
 4*cos(d*x + c) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2), 1/2
```

```
0*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*
arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) + 2*(49*cos(d*x + c)^3 + 36*cos(d*x + c)^2 - 4*cos(d*x + c
) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
 + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.21, size = 210, normalized size = 1.15

$$-\frac{\frac{\left(\left(\left(\frac{5\sqrt{2}\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{\mathrm{sgn}(\cos(dx+c))}-\frac{127\sqrt{2}\,a}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{175\sqrt{2}\,a}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{85\sqrt{2}\,a}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}-\frac{75\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/20*((((5*sqrt(2)*a*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 127*sqrt(2
)*a/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 + 175*sqrt(2)*a/sgn(cos(d*x +
 c)))*tan(1/2*d*x + 1/2*c)^2 - 85*sqrt(2)*a/sgn(cos(d*x + c)))*tan(1/2*d*x
+ 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a)) - 75*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^5\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(3/2)), x)
```

3.127 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{11\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}}+\frac{7\sqrt{a+a\sec(}{}$$

```
[Out] 11/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/
d*2^(1/2)-1/2*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-13/3*tan(d*x
+c)/a/d/(a+a*sec(d*x+c))^(1/2)+7/6*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3901, 4095, 4086, 3880, 209}

$$\frac{11\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{7\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{13\tan(c+dx)}{3ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (11*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2)) - (13*Tan[c + d*x])/(3*a*d*Sqrt[a + a*Sec[c + d*x]]) + (7*Sqrt[a +
a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
```

```
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^2(c+dx)\left(2a-\frac{7}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{6a^2d} - \frac{\int \frac{\sec(c+dx)(\ldots}{\sqrt{a+\ldots}}}{\ldots}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{a+a\sec(c+dx)}}{6a^2d}\ldots$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{a+a\sec(c+dx)}}{6a^2d}\ldots$$

$$= \frac{11\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan\ldots}{3ad\sqrt{a+\ldots}}$$

Mathematica [A]

time = 0.31, size = 114, normalized size = 0.79

$$\frac{\left(33\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))+2\sqrt{1-\sec(c+dx)}\,(-19-12\sec(c+dx)+4\sec^2(c+dx))\right)\tan(c+dx)}{12d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((33*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) + 2
*Sqrt[1 - Sec[c + d*x]]*(-19 - 12*Sec[c + d*x] + 4*Sec[c + d*x]^2))*Tan[c +
 d*x])/(12*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(122) = 244.
time = 0.12, size = 322, normalized size = 2.22

method	result
default	$\frac{(-1+\cos(dx+c))\left(33\sin(dx+c)\left(\cos^2(dx+c)\right)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+66\cos(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))*(33*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)+66*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)+33*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-76*cos(d*
x+c)^3+28*cos(d*x+c)^2+64*cos(d*x+c)-16)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2
)/sin(d*x+c)^3/cos(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 4.31, size = 387, normalized size = 2.67

$$\left[-\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^{3}+2\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+3a\cos\left(dx+c\right)^{2}+2a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)+4\left(19\cos\left(dx+c\right)^{2}+12\cos\left(dx+c\right)-4\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{24\left(a^{2}d\cos\left(dx+c\right)^{3}+2a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx+c\right)\right)},\ -\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^{3}+2\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)+2\left(19\cos\left(dx+c\right)^{2}+12\cos\left(dx+c\right)-4\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{12\left(a^{2}d\cos\left(dx+c\right)^{3}+2a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(33*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(
-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)) + 4*(19*cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3
+ 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/12*(33*sqrt(2)*(cos(d*x
+ c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*(19*
cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos
(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.26, size = 179, normalized size = 1.23

$$\frac{\frac{\left(\left(\frac{3\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{46\sqrt{2}}{\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+\frac{27\sqrt{2}}{\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}-\frac{33\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*(((3*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 46*sqrt(2)/sgn
(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 + 27*sqrt(2)/sgn(cos(d*x + c)))*tan(
```

```
1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)) - 33*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)), x)
```

3.128 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{7\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -7/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/
d*2^(1/2)+1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+2*tan(d*x+c)/a/d/(a+a*sec
(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3884, 4086, 3880, 209}

$$-\frac{7\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (2*Tan[
c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
```

```
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)\left(-\frac{3a}{2}+2a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{7\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx}}}{4a}$$

$$= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{7\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x,\right.}{2}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2\,\text{t}}{ad\sqrt{a}}$$

Mathematica [A]
time = 0.32, size = 104, normalized size = 0.99

$$\frac{\left(-7\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))+2\sqrt{1-\sec(c+dx)}(5+4\sec(c+dx))\right)\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
[Out] ((-7*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) + 2
*Sqrt[1 - Sec[c + d*x]]*(5 + 4*Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - S
ec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(88) = 176$.
time = 0.11, size = 225, normalized size = 2.14

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(7\sin(dx+c)\left(\cos^2(dx+c)\right)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,-7\ln\left(-\right.\right.}{4d\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(7*sin(d*x+c)*cos(d*x+c)^2*ln(-(-
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-7*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)+10*cos(d*x+c)^3-12*cos(d*x+c)^2-6*cos(d*x+c)+8)/sin(d*x+c)^
3/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 5.59, size = 336, normalized size = 3.20

$$\left[-\frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\,\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(5\cos(dx+c)+4\right)\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)},\ \frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(5\cos(dx+c)+4\right)\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x + c) +
 4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/
```

```
4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x + c) + 4)*sin(d*x +
 c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.23, size = 157, normalized size = 1.50

$$\frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a\mathrm{sgn}(\cos(dx+c))}-\frac{9\sqrt{2}}{a\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}+\frac{7\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a
*sgn(cos(d*x + c))) - 9*sqrt(2)/(a*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)
/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 7*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x +
 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c
))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)), x)
```

3.129 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 3/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d
*2^(1/2)-1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3882, 3880, 209}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sq
rt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{3\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\ &= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{3\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}\\ &= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.20, size = 94, normalized size = 1.22

$$\frac{\left(-2\sqrt{1-\sec(c+dx)}+3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))\right)\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-2*Sqrt[1 - Sec[c + d*x]] + 3*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt
[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 +
Sec[c + d*x]))^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(62) = 124$.
time = 0.10, size = 222, normalized size = 2.88

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\cos(dx+c)+3\ln\left(-\right.\right.}{4d(1+\cos(dx+c))\sin(dx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*sin(d*x+c)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos
```

```
(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)+3*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)+2*cos(d*x+c)^2-2*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.
time = 3.51, size = 329, normalized size = 4.27

$$\left[-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^{2}+2a\cos(dx+c)-a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}, -\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(3*sqr
t(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 1.18, size = 108, normalized size = 1.40

$$-\frac{\frac{3\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,a\text{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\text{sgn}(\cos(dx+c))}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))) + sqrt(2)*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)), x)
```

3.130 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 1/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d
*2^(1/2)+1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3881, 3880, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2
]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$
$$= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [**A**]
time = 0.12, size = 93, normalized size = 1.21

$$\frac{\left(2\sqrt{1-\sec(c+dx)} + \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))\right)\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]
]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec
[c + d*x]))^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(62) = 124$.
time = 0.08, size = 220, normalized size = 2.86

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\cos(dx+c)+\ln\left(-\frac{-\sqrt{}}{}\right.\right.}{4d(1+\cos(dx+c))\sin(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(sin(d*x+c)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d
```

```
*x+c)-1)/sin(d*x+c))*cos(d*x+c)+ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)-2*cos(d*x+c)^2+2*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(62) = 124$.
time = 4.06, size = 327, normalized size = 4.25

$$\left[-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\log\left(\frac{2\,\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)\sin(dx+c)+3\,a\cos(dx+c)^{2}+2\,a\cos(dx+c)-a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)-4\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)\sin\left(dx+c\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)},\,-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)}{\sqrt{a}\,\sin(dx+c)}\right)-2\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)\sin\left(dx+c\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)
*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 0.91, size = 108, normalized size = 1.40

$$-\frac{\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}-\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{2}\mathrm{sgn}(\cos(dx+c))}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))) - sqrt(2)*sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.131 $\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctan(1/
2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*
tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]

time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3862, 4005, 3859, 209, 3880}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
5*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqr
t[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
```

```
gerQ[2*n]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-2a+\frac{1}{2}a\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{5\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c}}}{4a}$$

$$= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.15, size = 5524, normalized size = 48.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(93) = 186$.
time = 0.09, size = 370, normalized size = 3.25

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(4\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\ \cos(dx+c)+5\sin(d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*sin(d*x+c)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+5*sin(d*x+c)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)+4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^
(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c
)*2^(1/2))*sin(d*x+c)+5*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)-2*cos(d*x+c)^2+2*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(93) = 186$.
time = 3.91, size = 491, normalized size = 4.31

$$\left[-\frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+8\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, \frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-8\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
```

```
+ c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) + 8*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 8*(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec{(c+dx)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(-3/2), x)
```

Giac [**A**]
time = 0.70, size = 47, normalized size = 0.41

$$-\frac{\sqrt{2}\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{4\,a^{2}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*
d*sgn(cos(d*x + c)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a/cos(c + d*x))^(3/2), x)
```

3.132 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{9\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{}{2ad\sqrt{}}$$

```
[Out] -3*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*sin(d*x+
c)/d/(a+a*sec(d*x+c))^(3/2)+9/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*
sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+3/2*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1
/2)
```

Rubi [A]
time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3902, 4107, 4005, 3859, 209, 3880}

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{3\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(9*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sq
rt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*Sin[c
 + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(-3a+\frac{3}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{3a^2-\frac{3}{2}a^2\sec(c+dx)}{\sqrt{a+a\sec(c+dx}}}{2a^3}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{3\int \sqrt{a+a\sec(c+a}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{3\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x\right.}{}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{}{2}$$

Mathematica [A]
time = 0.86, size = 129, normalized size = 0.90

$$\frac{\left(2(3+2\cos(c+dx))\sqrt{1-\sec(c+dx)} - 12\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)(1+\sec(c+dx)) + 9\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))\right)\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*(3 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 12*ArcTanh[Sqrt[1 - Sec[c
 + d*x]]]*(1 + Sec[c + d*x]) + 9*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqr
t[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 +
 Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(119) = 238$.
time = 0.12, size = 384, normalized size = 2.67

method	result
default	$-\frac{\left(6\sin(dx+c)(\cos^2(dx+c))\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+9\sin(dx+c)(\cos^2(dx+c))\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(6*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))+9*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)-6*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-4*cos(d*x+
c)^4-9*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^3+
8*cos(d*x+c)^2-6*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)
^3/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 2.62, size = 518, normalized size = 3.60

$$\left[-\frac{9\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+12\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-4\left(2\cos(dx+c)^2+3\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, -\frac{9\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-12\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left(2\cos(dx+c)^2+3\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*(2*cos(d*x
+ c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*sqrt(2)*
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 12*(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos
```

```
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*cos(d*x + c)^2 + 3*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*c
os(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(119) = 238.
time = 1.24, size = 387, normalized size = 2.69

$$\frac{\frac{16\sqrt{2}\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}-\frac{9\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}+\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\,\mathrm{sgn}(\cos(dx+c))}+\frac{12\log\left(\frac{\left|-2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|+6a\right|}{\left|-2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|+6a\right|}\right)}{\sqrt{-a}\,|a|\,\mathrm{sgn}(\cos(dx+c))}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] 1/8*(16*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)*sgn(cos(d*x + c))) - 9*sqrt(2)*log((
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sq
rt(-a)*a*sgn(cos(d*x + c))) + 2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))) + 12*log(abs(-2*(sqrt(-a)*tan
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(
a) + 6*a)/abs(-2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 + 4*sqrt(2)*abs(a) + 6*a))/(sqrt(-a)*abs(a)*sgn(cos(d*x + c))
))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(3/2), x)
```

3.133 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{19\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d}-\frac{13\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{}{4ad}$$

```
[Out] 19/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*cos(d*
x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-13/4*arctan(1/2*a^(1/2)*tan(d*x+c)
*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-7/4*sin(d*x+c)/a/d/(a+a*
sec(d*x+c))^(1/2)+cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3902, 4107, 4005, 3859, 209, 3880}

$$\frac{19\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{13\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{7\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)\cos(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (19*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*a^(3/2)*d)
- (13*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2)) - (7*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Sec[c + d*x]]) + (Cos[c + d*x]
*Sin[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos^2(c+dx)\left(-4a+\frac{5}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2} \\
&= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)(7a^2-6a^2\sec(}{\sqrt{a+a\sec(c+}}}{4a^3} \\
&= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+}{ad\sqrt{a+a\sec(c+}} \\
&= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+}{ad\sqrt{a+a\sec(c+}} \\
&= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+}{ad\sqrt{a+a\sec(c+}} \\
&= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{13\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} -
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.03, size = 197, normalized size = 1.06

$$-\frac{\sin(2(c+dx)) - \frac{(1+\cos(c+dx))\left(13\left(7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-\cos(c+dx)\sqrt{1-\sec(c+dx)}+2\cos^2(c+dx)\sqrt{1-\sec(c+dx)}\right)-40\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}\right)\sec(c+dx)\tan(c+dx)}{4\sqrt{1-\sec(c+dx)}}}{4d(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/4*(Sin[2*(c + d*x)] - ((1 + Cos[c + d*x])*(13*(7*ArcTanh[Sqrt[1 - Sec[c
+ d*x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - Cos[c + d*x]
*Sqrt[1 - Sec[c + d*x]] + 2*Cos[c + d*x]^2*Sqrt[1 - Sec[c + d*x]]) - 40*Hyp
ergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sec[c
 + d*x]*Tan[c + d*x])/(4*Sqrt[1 - Sec[c + d*x]]))/(d*(a*(1 + Sec[c + d*x]))
^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(156) = 312$.
time = 0.15, size = 560, normalized size = 3.03

method	result
default	$-\frac{(-1+\cos(dx+c))\left(19\sin(dx+c)(\cos^2(dx+c))\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{2}+26\sin(dx+ \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/16/d*(-1+cos(d*x+c))*(19*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)+26*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+38*sin(d*x+c)*cos(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+52*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+19*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+26*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*cos(d*x+c)^5+20*cos(d*x+c)^4+16*cos(d*x+c)^3-28*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)/a^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)

Fricas [A]
time = 3.06, size = 536, normalized size = 2.90

$$\left[-\frac{13\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+19\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(2\cos(dx+c)^3-3\cos(dx+c)^2-7\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, \frac{13\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-19\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(2\cos(dx+c)^3-3\cos(dx+c)^2-7\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/8*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x

```
 + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*cos(d*
x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2
*d), 1/4*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c))) - 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (2*c
os(d*x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
+ a^2*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(156) = 312.
time = 1.68, size = 473, normalized size = 2.56

$$\frac{\frac{13\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}-\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\mathrm{sgn}(\cos(dx+c))}+\frac{19\log\left(\frac{\left|8\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-16\sqrt{2}|a|-24a\right|}{\left|8\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+16\sqrt{2}|a|-24a\right|}\right)}{\sqrt{-a}\,|a|\,\mathrm{sgn}(\cos(dx+c))}-\frac{4\sqrt{2}\left(29\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6-133\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4a+55\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a^2-7a^3\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)^2\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(13*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(cos(d*x + c))) - 2*sqrt(2)*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))) + 19*log
(abs(8*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 - 16*sqrt(2)*abs(a) - 24*a)/abs(8*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 16*sqrt(2)*abs(a) - 24*a))/(sqrt(-a)*a
bs(a)*sgn(cos(d*x + c))) - 4*sqrt(2)*(29*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6 - 133*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a + 55*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^2 - 7*a^3)/(((sqrt(-a)*tan(1
```

```
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*sqrt(-
a)*sgn(cos(d*x + c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

3.134 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{163\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}-\frac{197}{24a^2d\sqrt{\cdots}}$$

```
[Out] 163/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2
)/d*2^(1/2)-1/4*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-17/16*sec(
d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-197/24*tan(d*x+c)/a^2/d/(a+a
*sec(d*x+c))^(1/2)+95/48*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^3/d
```

Rubi [A]
time = 0.30, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3901, 4104, 4095, 4086, 3880, 209}

$$\frac{163\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{95\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{197\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{17\tan(c+dx)\sec^2(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x
])^(5/2)) - (17*Sec[c + d*x]^2*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(
3/2)) - (197*Tan[c + d*x])/(24*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + (95*Sqrt[a
 + a*Sec[c + d*x]]*Tan[c + d*x])/(48*a^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
```

```
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^3(c+dx)\left(3a-\frac{11}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^2(c+dx)(17a^2-}{\sqrt{a+a\sec}}}{8a}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{95\sqrt{a+a\sec(c}}{4}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{197\tan(c}{24a^2d\sqrt{a+a\sec}}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{197\tan(c}{24a^2d\sqrt{a+a\sec}}$$

$$= \frac{163\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2}{16ad(}$$

Mathematica [A]
time = 1.32, size = 135, normalized size = 0.74

$$\frac{\left(978\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c+dx)}\left(-299-503\sec(c+dx)-160\sec^2(c+dx)+32\sec^3(c+dx)\right)\right)\tan(c+dx)}{48d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((978*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Se
c[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-299 - 503*Sec[c + d*x] - 160*Sec[c
+ d*x]^2 + 32*Sec[c + d*x]^3))*Tan[c + d*x])/(48*d*Sqrt[1 - Sec[c + d*x]]*(
a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(156) = 312$.
time = 0.14, size = 417, normalized size = 2.28

method	result

default	$-\frac{(-1+\cos(dx+c))^2\left(489\sin(dx+c)\left(\cos^3(dx+c)\right)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+1467\sin$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/d*(-1+cos(d*x+c))^2*(489*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)+1467*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)+1467*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)+489*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-1196*
cos(d*x+c)^4-816*cos(d*x+c)^3+1372*cos(d*x+c)^2+768*cos(d*x+c)-128)*(a*(1+c
os(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 3.07, size = 455, normalized size = 2.49

$$\left[-\frac{489\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left(299\cos(dx+c)^3+503\cos(dx+c)^2+160\cos(dx+c)-32\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{192\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)\right)}, -\frac{489\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left(299\cos(dx+c)^3+503\cos(dx+c)^2+160\cos(dx+c)-32\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{96\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(489*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x
+ c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(299*cos(d*x + c)^3 +
503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a
```

```
^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), -1/96*(489*sqrt(2)*(cos(d*x + c)
^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) + 2*(299*cos(d*x + c)^3 + 503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)
^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.22, size = 219, normalized size = 1.20

$$\frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a\operatorname{sgn}(\cos(dx+c))}+\frac{23\sqrt{2}}{a\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{668\sqrt{2}}{a\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{465\sqrt{2}}{a\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{489\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,a^2\operatorname{sgn}(\cos(dx+c))}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/96*(((3*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a*sgn(cos(d*x + c))) + 23*sqrt
(2)/(a*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 668*sqrt(2)/(a*sgn(cos(
d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 465*sqrt(2)/(a*sgn(cos(d*x + c))))*tan
(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)) - 489*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^5\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(5/2)), x)
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{75\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}+\frac{9\tan(c+dx)}{4a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -75/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2
)/d*2^(1/2)-1/4*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+13/16*tan(
d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+9/4*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/
2)
```

Rubi [A]
time = 0.20, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3901, 4093, 4086, 3880, 209}

$$-\frac{75\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{9\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-75*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x
])^(5/2)) + (13*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (9*Tan[
c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
```

```
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^2(c+dx)\left(2a-\frac{9}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)\left(-\frac{39a^2}{4}+\right.}{\sqrt{a+a\sec}}}{8a}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{9\tan(c+}{4a^2d\sqrt{a+a\sec(}}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{9\tan(c+}{4a^2d\sqrt{a+a\sec(}}$$

$$= -\frac{75\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{}{16ad(}$$

Mathematica [A]
time = 0.73, size = 125, normalized size = 0.86

$$\frac{\left(-150\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c+dx)}\,(49+85\sec(c+dx)+32\sec^2(c+dx))\right)\tan(c+dx)}{16d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-150*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*S
ec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(49 + 85*Sec[c + d*x] + 32*Sec[c + d
*x]^2))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(
5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(122) = 244$.
time = 0.12, size = 316, normalized size = 2.18

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(75\sin(dx+c)\left(\cos^2(dx+c)\right)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\sqrt{-\frac{2}{1+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(75*sin(d*x+c
)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+150*sin(d*x+c)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)+75*ln(-(-(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)+98*cos(d*x+c)^3+72*cos(d*x+c)^2-106*cos(d*x+
c)-64)/sin(d*x+c)^5/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 2.38, size = 404, normalized size = 2.79

$$\left[-\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)-3a\cos\left(dx+c\right)^{2}-2a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)-4\left(49\cos\left(dx+c\right)^{2}+85\cos\left(dx+c\right)+32\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)},\ \frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)+2\left(49\cos\left(dx+c\right)^{2}+85\cos\left(dx+c\right)+32\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^2 + 85*cos(d*x + c)
 + 32)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
 + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(75*
sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) + 2*(49*cos(d*x + c)^2 + 85*cos(d*x + c) + 32)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*co
s(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.22, size = 188, normalized size = 1.30

$$\frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\left(\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}+\frac{17\sqrt{2}}{a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\frac{83\sqrt{2}}{a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}+\frac{75\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{-a}\,a^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^2*sgn(cos(d*x + c))) + 17*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c)^2 - 83*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/(a*t
an(1/2*d*x + 1/2*c)^2 - a) + 75*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2
```

```
*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))
))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+dx)^4 \left(a+\frac{a}{\cos(c+dx)}\right)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)), x)
```

3.136 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{19\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 19/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)
/d*2^(1/2)+1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-13/16*tan(d*x+c)/a/d/(a+
a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3884, 4085, 3880, 209}

$$\frac{19\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) + Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (13*Ta
n[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
```

```
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec(c+dx)\left(-\frac{5a}{2}+4a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{19\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+}}}{32a^2}$$

$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{19\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx\right.}{}$$

$$= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\mathrm{t}}{16ad(a+}$$

Mathematica [A]
time = 0.66, size = 116, normalized size = 1.08

$$\frac{\left(76\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx) - 2\sqrt{1-\sec(c+dx)}\,(9+13\sec(c+dx))\right)\tan(c+dx)}{32d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((76*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Sec
[c + d*x]^2 - 2*Sqrt[1 - Sec[c + d*x]]*(9 + 13*Sec[c + d*x]))*Tan[c + d*x])
/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(88) = 176.
time = 0.11, size = 323, normalized size = 3.02

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(19\sin(dx+c)\left(\cos^2(dx+c)\right)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\sqrt{-\frac{}{1}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(19*sin(d*x+c)*
cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+38*sin(d*x+c)*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)+19*ln(-(-(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)+18*cos(d*x+c)^3+8*cos(d*x+c)^2-26*cos(d*x+c))/(
1+cos(d*x+c))/sin(d*x+c)^3/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(88) = 176.
time = 2.55, size = 403, normalized size = 3.77

$$\left[-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left(9\cos(dx+c)^2+13\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)},\ -\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left(9\cos(dx+c)^2+13\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*c
os(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(9*cos(d*x + c)^2 + 13*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(19*sqrt(2
```

```
)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c))) + 2*(9*cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.28, size = 139, normalized size = 1.30

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}+\frac{11\,\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{19\,\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^3*sgn(cos(d*x + c))) + 11*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c) + 19*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)), x)
```

3.137 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 5/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/
d*2^(1/2)-1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+5/16*tan(d*x+c)/a/d/(a+a*
sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3882, 3881, 3880, 209}

$$\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{5\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (5*Tan[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

```
&& IntegerQ[2*m]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{5\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c-}}}{32a^2}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{5\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx\right.}{}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\tan}{16ad(a+a}$$

Mathematica [A]
time = 0.65, size = 115, normalized size = 1.07

$$\frac{\left(10\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c+dx)}\,(1+5\sec(c+dx))\right)\tan(c+dx)}{16d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((10*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Sec
[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(1 + 5*Sec[c + d*x]))*Tan[c + d*x])/(1
6*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(88) = 176.
time = 0.09, size = 315, normalized size = 2.94

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(5\sin(dx+c)\left(\cos^2(dx+c)\right)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}+10\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*sin(d*x+c)*cos(d*x+c)^2*ln(-(
-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+10*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*cos(d*x+c)+5*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-2*cos(d*x+c)^3-8*cos(d*x+c)^2+10*cos(d*x+c))/(1+cos(d*x+c))^2/sin(
d*x+c)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 2.40, size = 399, normalized size = 3.73

$$\left[-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(\cos(dx+c)^2+5\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, -\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left(\cos(dx+c)^2+5\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3
*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(5*sqrt(2)*(co
s(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
```

```
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c
))) - 2*(cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.26, size = 139, normalized size = 1.30

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{3}\mathrm{sgn}(\cos(dx+c))}+\frac{3\sqrt{2}}{a^{3}\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{5\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}\,a^{2}\mathrm{sgn}(\cos(dx+c))}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^3*sgn(cos(d*x + c))) + 3*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x +
 1/2*c) - 5*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)), x)
```

3.138 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 3/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/
d*2^(1/2)+1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+3/16*tan(d*x+c)/a/d/(a+a*
sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3881, 3880, 209}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{3\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) + Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*Tan[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

```
&& IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{3\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+}}}{32a^2}$$

$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{3\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx,\right.}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan}{16ad(a+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 52, normalized size = 0.49

$$\frac{{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)\tan(c+dx)}{4a^2d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
[Out] (Hypergeometric2F1[1/2, 3, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*x])/(4*a^2*
d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(88) = 176.
time = 0.08, size = 315, normalized size = 2.94

method	result
default	$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\sin(dx+c)\left(\cos^2(dx+c)\right)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}+6\sin(d\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)+3*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-14*cos(d*x+c)^3+8*cos(d*x+c)^2+6*cos(d*x+c))/(1+cos(d*x+c))^2/sin(d*x+c)/a^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(88) = 176.
time = 2.47, size = 403, normalized size = 3.77

$$\left[-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+3a\cos\left(dx+c\right)^2+2a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1}\right)-4\left(7\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{64\left(a^3d\cos\left(dx+c\right)^3+3a^3d\cos\left(dx+c\right)^2+3a^3d\cos\left(dx+c\right)+a^3d\right)}, -\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)-2\left(7\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{32\left(a^3d\cos\left(dx+c\right)^3+3a^3d\cos\left(dx+c\right)^2+3a^3d\cos\left(dx+c\right)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(7*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(7*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 1.01, size = 139, normalized size = 1.30

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\,\sqrt{2}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{3}\mathrm{sgn}(\cos(dx+c))}-\frac{5\,\sqrt{2}}{a^{3}\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\,\sqrt{2}\,\log\left(\left|-\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}\,a^{2}\mathrm{sgn}(\cos(dx+c))}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^3*sgn(cos(d*x + c))) - 5*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x
+ 1/2*c) + 3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.139 $\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{43\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-16a\ldots$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-43/32*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/
4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-11/16*tan(d*x+c)/a/d/(a+a*sec(d*x+c))
^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3862, 4007, 4005, 3859, 209, 3880}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{11\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
43*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (11*Tan
[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
```

```
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-4a+\frac{3}{2}a\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{8a^2-\frac{11}{4}a^2\sec(c+d}{\sqrt{a+a\sec(c+}}}{8a^4}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\sec(c+}}{a^3}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx\right.}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{}{4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 24.19, size = 5564, normalized size = 38.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(119) = 238$.
time = 0.10, size = 550, normalized size = 3.82

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(32\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(}{2\cos(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(32*sin(d*x+c)*c
os(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+43*sin(d*x+c)
*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x
+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+64*sin(d*x+c)*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*cos(d*x+c)+86*sin(d*x+c)*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)+32*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+43*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)-30*cos(d*x+c)^3+8*cos(d*x+c)^2+22*cos(d*x+c))/(1+cos(d*x+
c))/sin(d*x+c)^3/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^(-5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(119) = 238$.
time = 2.51, size = 585, normalized size = 4.06

$$\left[-\frac{43\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\log\left(-\frac{2\,\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)-3\,a\cos(dx+c)^{2}-2\,a\cos(dx+c)+a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+64\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^{2}+2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+4\left(15\,\cos\left(dx+c\right)^{2}+11\,\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)},\ \frac{43\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-64\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left(15\,\cos\left(dx+c\right)^{2}+11\,\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \sec\left(c + dx\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(d*x+c))**(5/2),x)

[Out] Integral((a*sec(c + d*x) + a)**(-5/2), x)

Giac [A]
time = 0.80, size = 78, normalized size = 0.54

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\,\sqrt{2}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{13\,\sqrt{2}}{a^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/
(a^3*sgn(cos(d*x + c))) - 13*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x +
 1/2*c)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + a/cos(c + d*x))^(5/2), x)
```

3.140 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{115\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{}{16}$$

```
[Out] -5*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*sin(d*x+
c)/d/(a+a*sec(d*x+c))^(5/2)-15/16*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+115
/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d
*2^(1/2)+35/16*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3902, 4105, 4107, 4005, 3859, 209, 3880}

$$-\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{35\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{15\sin(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(115*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (15*S
in[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (35*Sin[c + d*x])/(16*a^
2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\cos(c+dx)\left(-5a+\frac{5}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(-\frac{35a^2}{2}+\frac{45}{4}\right.}{\sqrt{a+a\sec(c}}}{8a^4}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx}{16a^2d\sqrt{a+a\sec(c}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx}{16a^2d\sqrt{a+a\sec(c}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx}{16a^2d\sqrt{a+a\sec(c}}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]
time = 1.90, size = 169, normalized size = 0.97

$$\frac{460\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right) - 80\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)(1+\sec(c+dx))^2\tan(c+dx) + \sqrt{1-\sec(c+dx)}\,(16\sin(c+dx)+5(11+7\sec(c+dx))\tan(c+dx))}{16d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (460*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^5*Sec
[c + d*x]^3*Sin[(c + d*x)/2] - 80*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[
c + d*x])^2*Tan[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(16*Sin[c + d*x] + 5*(11
+ 7*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c
 + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(145) = 290.
time = 0.14, size = 552, normalized size = 3.17

method	result

default	$\frac{(-1+\cos(dx+c))^2\left(80\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+115\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d*(-1+cos(d*x+c))^2*(80*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)/cos(d*x+c)*2^(1/2))+115*sin(d*x+c)*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)+160*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1
/2))*2^(1/2)*cos(d*x+c)+230*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*
x+c))*cos(d*x+c)+80*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctanh(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d
*x+c)-32*cos(d*x+c)^4+115*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)-78*cos(d*x+c)^3+40*cos(d*x+c)^2+70*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*
x+c))^(1/2)/sin(d*x+c)^5/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 4.90, size = 606, normalized size = 3.48

$$\left[-\frac{115\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+160\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-4\left(16\cos(dx+c)^3+55\cos(dx+c)^2+35\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)},\ -\frac{115\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-160\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left(16\cos(dx+c)^3+55\cos(dx+c)^2+35\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos
```

```
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2
 + 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) - 4*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 35*
cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*
cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1
/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)/(sqrt(a)*sin(d*x + c))) - 2*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 3
5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (c+dx)}{(a (\sec (c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(145) = 290.
time = 1.38, size = 424, normalized size = 2.44

$$-\frac{2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{21\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{128\sqrt{2}\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}+\frac{115\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}-\frac{160\log\left(\left|\frac{-2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|+6a}{-2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|+6a}\right|\right)}{\sqrt{-a}\,a|a|\mathrm{sgn}(\cos(dx+c))}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c
)^2/(a^3*sgn(cos(d*x + c))) - 21*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d
*x + 1/2*c) - 128*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)*a*sgn(cos(d*x + c))) + 115
*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^2)/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 160*log(abs(-2*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a
) + 6*a)/abs(-2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
```

```
c)^2 + a))^2 + 4*sqrt(2)*abs(a) + 6*a))/(sqrt(-a)*a*abs(a)*sgn(cos(d*x + c)
)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)}{\left(a+\frac{a}{\cos(c+dx)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/2), x)
```

3.141 $\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{3880, 209\}$

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])
])/(Sqrt[a]*d))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.41, size = 94, normalized size = 1.96

$$\frac{i\sqrt{2}\left(-1+e^{i(c+dx)}\right)\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*(-1 + E^(I*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*
Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(39) = 78$.
time = 0.11, size = 83, normalized size = 1.73

method	result	size
default	$-\frac{2(-1+\cos(dx+c))\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)}{d\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(-1+cos(d*x+c))*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(a*(-1+
cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(-a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 2.66, size = 161, normalized size = 3.35

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2d}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqr
t((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*sin(
d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))/d, sqrt(2)*arctan(sqrt(2)*sqrt
((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/(s
qrt(a)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{-a(\sec(c+dx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(-a*(sec(c + d*x) - 1)), x)
```

Giac [A]
time = 0.53, size = 67, normalized size = 1.40

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}\,d\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*d*sgn(t
an(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x + c)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2)), x)
```

3.142 $\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*a^
(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3861, 3859, 209, 3880}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/
(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{\int \sqrt{a-a\sec(c+dx)}\,dx}{a} + \int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d} - \frac{2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{a}{\sqrt{a-}}\right.}{d}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.49, size = 127, normalized size = 1.46

$$-\frac{i\left(-1+e^{i(c+dx)}\right)\left(\sinh^{-1}\left(e^{i(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sec[c + d*x]],x]
[Out] ((-I)*(-1 + E^(I*(c + d*x)))*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1
 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt
[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Se
c[c + d*x]])
```

Maple [A]
time = 0.10, size = 119, normalized size = 1.37

method	result	size
default	$-\frac{\left(\sqrt{2}\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)+2\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\right)(-1+\cos(dx+c))\sqrt{2}}{d\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*arctan(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))*(-1+cos(d*x+c))/(a*(-1+cos(d*
x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(
1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 2.43, size = 301, normalized size = 3.46

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)-2\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)}{2ad}, \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*
sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*s
in(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))) - 2*sqrt(-a)*log((2*(cos(d*
x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) -
 (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)))/(a*d), (sqrt(2)*sqrt(a
)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sec{(c+dx)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sec(c + d*x) + a), x)
```

Giac [A]
time = 0.77, size = 69, normalized size = 0.79

$$\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}}{2\sqrt{a}}\right)}{\sqrt{a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a) - 2*arc
tan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x))^(1/2), x)
```

3.143 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=383

$$-\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d}+\frac{57(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{80d(1+\sec(c+dx))}+\frac{3(a+a\sec(c+dx))^{5/3}\tan(c+}{8ad}$$

```
[Out] -9/40*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+57/80*(a+a*sec(d*x+c))^(2/3)*tan(
d*x+c)/d/(1+sec(d*x+c))+3/8*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/a/d-19/160*3^
(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c)
)^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/
3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c
))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3885, 4086, 3913, 3912, 52, 65, 231}

$$-\frac{19\,3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/3}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{80\sqrt[3]{2}\,d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}-\frac{9\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{40d}+\frac{57\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{80d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-9*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(40*d) + (57*(a + a*Sec[c + d*
x])^(2/3)*Tan[c + d*x])/(80*d*(1 + Sec[c + d*x])) + (3*(a + a*Sec[c + d*x])
^(5/3)*Tan[c + d*x])/(8*a*d) - (19*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 -
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + S
ec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 +
 Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]
*Tan[c + d*x])/(80*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-((
(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (
1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8ad} + \frac{3\int \sec(c+dx)\left(\frac{5a}{3}-a\sec(\right.}{}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan}{8ad}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan}{8ad}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan}{8ad}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\mathrm{t}}{80d(1+\sec(c+d}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\mathrm{t}}{80d(1+\sec(c+d}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\mathrm{t}}{80d(1+\sec(c+d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.29, size = 105, normalized size = 0.27

$$\frac{(a(1+\sec(c+dx)))^{2/3}\left(38\sqrt[6]{2}\ {}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{1+\sec(c+dx)}\,(2+7\sec(c+dx)+5\sec^2(c+dx))\right)\tan(c+dx)}{40d(1+\sec(c+dx))^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(38*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2,
 (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(1/6)*(2 + 7*Sec[c + d*x] + 5
*Sec[c + d*x]^2))*Tan[c + d*x])/(40*d*(1 + Sec[c + d*x])^(7/6))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^3(dx+c)\right)(a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{2}{3}}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

3.144 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=353

$$\frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d}+\frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d(1+\sec(c+dx))}-\frac{3^{3/4}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\cdots}\right.\right.\cdots}{\cdots}$$

```
[Out] 3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*x+
c)/d/(1+sec(d*x+c))-1/10*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2))
)^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x
+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*Elliptic
F((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(
1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*
(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+se
c(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*
x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3
)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3883, 3913, 3912, 52, 65, 231}

$$-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (3*(a + a*Sec[c + d*x])
^(2/3)*Tan[c + d*x])/(5*d*(1 + Sec[c + d*x])) - (3^(3/4)*EllipticF[ArcCos[(
2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^
(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x]
)^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d
*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))
/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{2}{5}\int \sec(c+dx)(a+a\sec$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{(2(a+a\sec(c+dx))^{2/3})}{5(1+}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{(2(a+a\sec(c+dx))^{2/3}\text{ta}}{5d\sqrt{1-}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan}{5d(1+\sec(c+dx)}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan}{5d(1+\sec(c+dx)}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan}{5d(1+\sec(c+dx)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.14, size = 85, normalized size = 0.24

$$\frac{(a(1+\sec(c+dx)))^{2/3}\left(4\sqrt[6]{2}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3(1+\sec(c+dx))^{7/6}\right)\tan(c+dx)}{5d(1+\sec(c+dx))^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(4*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2,
(1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(7/6))*Tan[c + d*x])/(5*d*(1 +
 Sec[c + d*x])^(7/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)\left(a+a\sec(dx+c)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(\sec(c+dx)+1)\right)^{\frac{2}{3}}\sec^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(2/3),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
```

```
[Out] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

3.145 $\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=326

$$\frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))}-\frac{3^{3/4}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a}{2\sqrt[3]{2}\,d(1-\sec(c+dx))(}$$

[Out] 3/2*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))-1/4*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.21, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3913, 3912, 52, 65, 231}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{2d(\sec(c+dx)+1)}-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/3}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[3]{2}\,d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]

[Out] (3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])) - (3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \sec(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}}\,dx, x, \sec(\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{((a+a\sec(c+dx))^{2/3}\tan(c}{2d\sqrt{1-}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{(3(a+a\sec(c+dx))^{2/3}\tan(}{d\sqrt{1-}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right.}{\sqrt[3]{2}-\left(1+\sqrt{3}\right.}\right.\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 66, normalized size = 0.20

$$\frac{2\sqrt[6]{2}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)(a(1+\sec(c+dx)))^{2/3}\tan(c+dx)}{d(1+\sec(c+dx))^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
Sec[c + d*x]))^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\left(a+a\sec(dx+c)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (c + dx) + 1))^{\frac{2}{3}} \sec (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{2/3}}{\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x),x)
[Out] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

3.146 $\int (a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$\frac{3\sqrt{2}\,F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{1-\sec(c+dx)}}$$

```
[Out] 3/7*AppellF1(7/6,1,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c
))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3864, 3863, 141}

$$\frac{3\sqrt{2}\,\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*Sqrt[2]*AppellF1[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt[6]{1+x}}{\sqrt{1-x}\,x}\,dx,x,\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$

$$= \frac{3\sqrt{2}\,F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}}{7d\sqrt{1-\sec(c+dx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 691 vs. 2(77) = 154.
time = 5.07, size = 691, normalized size = 8.97

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (45*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(a*
(1 + Sec[c + d*x]))^(5/3)*Sin[c + d*x]*(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(a*d*(40*(3*AppellF1
[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/
2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Sec[c + d*x]*Si
n[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 6*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[c + d*x]^2*Sin[(c + d*x)/2]^2*(15*Appe
llF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-7 + 16*Co
s[c + d*x] - 3*Cos[2*(c + d*x)]) + 10*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*(7 - 16*Cos[c + d*x] + 3*Cos[2*(c + d*x)]) -
 24*(9*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
- 6*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 5
*AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c
 + d*x]*Tan[(c + d*x)/2]^2) + 135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]^2*(3 + 3*Cos[c + d*x] + 2*Tan[c + d*x]^2)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(c+dx\right)+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3),x)
[Out] int((a + a/cos(c + d*x))^(2/3), x)
```

3.147 $\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$-\frac{3\sqrt{2}\,F_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{1-\sec(c+dx)}}$$

```
[Out] -3/7*AppellF1(7/6,2,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+
c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3913, 3912, 141}

$$-\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Sqrt[2]*AppellF1[7/6, 1/2, 2, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx &= \frac{(a+a\sec(c+dx))^{2/3}\int \cos(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}\\ &= -\frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}\,x^2}\,dx, x, s\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}\\ &= -\frac{3\sqrt{2}\,F_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\,s}{7d\sqrt{1-\sec(c+dx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2700 vs. $2(77) = 154$.
time = 16.25, size = 2700, normalized size = 35.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(Sin[
c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(2/3)) - (2^(2/3)*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*((Sec[(c +
d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/6 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1
 + Sec[c + d*x])^(2/3))/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/
3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(9*d*(1 + Sec[c +
 d*x])^(2/3)*(-1/9*(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2
/3)*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(C
os[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2
, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)
/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[
(c + d*x)/2]^2)))/2^(1/3) - (2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3
```

```
)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan
[(c + d*x)/2] + (Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*
((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)
+ (2*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Ta
n[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3))
- (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]*Sin[(c + d*x)/2])/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (81*Cos[(c + d*x)/2]^2*(
-1/3*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/
(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2
*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2
*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(
c + d*x)/2]^2) - (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-1/3
*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*T
an[(c + d*x)/2]^2*(3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2
, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/5) - 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/
2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2]))))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/9 - (2*2^(2/3)*Tan[(c + d*x)/2]*(A
ppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c +
 d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2, 2/3,
 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*A
ppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*A
ppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*Appe
llF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d
*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*
x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(27*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(
```

```
1/3))))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(a+a\sec\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
[Out] int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{2}{3}}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(2/3), x)
```

3.148 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=413

$$\frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d}+\frac{1029a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{880d(1+\sec(c+dx))}-\frac{9(a+a\sec(c+dx))^{5/3}\,\mathrm{t}}{88d}$$

```
[Out] 147/440*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+1029/880*a*(a+a*sec(d*x+c))^(
2/3)*tan(d*x+c)/d/(1+sec(d*x+c))-9/88*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/d+3
/11*(a+a*sec(d*x+c))^(8/3)*tan(d*x+c)/a/d-343/1760*3^(3/4)*a*((2^(1/3)-(1+s
ec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^
2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))
^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1
/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3
)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*
(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-
(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3885, 4086, 3913, 3912, 52, 65, 231}

$$-\frac{343\, 3^{3/4} a \tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/3} F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{880\sqrt[3]{2}\, d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}-\frac{9\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{88d}+\frac{147a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{440d}+\frac{1029a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{880d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (147*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(440*d) + (1029*a*(a + a*Se
c[c + d*x])^(2/3)*Tan[c + d*x])/(880*d*(1 + Sec[c + d*x])) - (9*(a + a*Sec[
c + d*x])^(5/3)*Tan[c + d*x])/(88*d) + (3*(a + a*Sec[c + d*x])^(8/3)*Tan[c
+ d*x])/(11*a*d) - (343*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])
*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3
))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c +
d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c +
```

```
d*x])/(880*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec
[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+dx)}{11ad} + \frac{3\int \sec(c+dx)\left(\frac{8a}{3}-a\sec(\right.}{}$$

$$= -\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\text{ta}}{11ad}$$

$$= -\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\text{ta}}{11ad}$$

$$= -\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\text{ta}}{11ad}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} - \frac{9(a+a\sec(c+dx))^{5/3}\text{t}}{88d}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx)}{880d(1+\sec(c}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx)}{880d(1+\sec(c}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx)}{880d(1+\sec(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.34, size = 96, normalized size = 0.23

$$\frac{a(a(1+\sec(c+dx)))^{2/3}\left(196\sqrt[6]{2}\ {}_2F_1\left(-\frac{7}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3(1+\sec(c+dx))^{13/6}(5+8\sec(c+dx))\right)\tan(c+dx)}{88d(1+\sec(c+dx))^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (a*(a*(1 + Sec[c + d*x]))^(2/3)*(196*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3
/2, (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(13/6)*(5 + 8*Sec[c + d*x]
))*Tan[c + d*x])/(88*d*(1 + Sec[c + d*x])^(7/6))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(dx+c\right)\right)\left(a+a\sec\left(dx+c\right)\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(a*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^3, x)
```

3.149 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=383

$$\frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d}+\frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{16d(1+\sec(c+dx))}+\frac{3(a+a\sec(c+dx))^{5/3}\tan(c}{8d}$$

```
[Out] 3/8*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+21/16*a*(a+a*sec(d*x+c))^(2/3)*ta
n(d*x+c)/d/(1+sec(d*x+c))+3/8*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/d-7/32*3^(3
/4)*a*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c)
)^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/
3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c
))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3883, 3913, 3912, 52, 65, 231}

$$-\frac{7\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}+\frac{3a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{8d}+\frac{21a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{16d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*d) + (21*a*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(16*d*(1 + Sec[c + d*x])) + (3*(a + a*Sec[c + d*x
])^(5/3)*Tan[c + d*x])/(8*d) - (7*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3) - (1
- Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 +
Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1
+ Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2
]*Tan[c + d*x])/(16*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(
((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) -
(1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{5}{8}\int \sec(c+dx)(a+a\sec$$

$$= \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{\left(5a(a+a\sec(c+dx))^{2/3}\right)}{8(1+}$$

$$= \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} - \frac{\left(5a(a+a\sec(c+dx))^{2/3}\right.}{8d\sqrt{1-}}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+a\sec(c+dx))^{5/3}\mathrm{ta}}{8d}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}}{16d(1+\sec(c+}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}}{16d(1+\sec(c+}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}}{16d(1+\sec(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.46, size = 106, normalized size = 0.28

$$\frac{a(a(1+\sec(c+dx)))^{2/3}\left(5\sqrt[6]{2}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt[6]{1+\sec(c+dx)}\right)\tan(c+dx)}{2d(1+\sec(c+dx))^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (a*(a*(1 + Sec[c + d*x]))^(2/3)*(5*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2
, (1 - Sec[c + d*x])/2] + 3*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(1 + Sec[c +
d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(7/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\sec^{2}\left(dx+c\right)\right) \left(a+a\sec\left(dx+c\right)\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(a*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{3}}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/3)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^2,x)
[Out] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^2, x)
```

3.150 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=356

$$\frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d}+\frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{10d(1+\sec(c+dx))}-\frac{7\,3^{3/4}aF\left(\operatorname{ArcCos}\left(\frac{\sqrt[3]{2}-(1-\cdots}{\sqrt[3]{2}-(1+\cdots}\right.\right.}{\cdots}$$

```
[Out] 3/5*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+21/10*a*(a+a*sec(d*x+c))^(2/3)*ta
n(d*x+c)/d/(1+sec(d*x+c))-7/20*3^(3/4)*a*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-
3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1
+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*
EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c)
)^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/
3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)
*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3913, 3912, 52, 65, 231}

$$-\frac{7\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}F\left(\operatorname{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{3a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}+\frac{21a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{10d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (21*a*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(10*d*(1 + Sec[c + d*x])) - (7*3^(3/4)*a*Elliptic
F[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])
^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Se
c[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)*d*(1 - Sec[c + d*x])*(1
+ Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{\left(a(a+a\sec(c+dx))^{2/3}\right)\int \sec(c+dx)(1+\sec(c+dx))^{5/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1+x)^{7/6}}{\sqrt{1-x}}\,dx, x, \sec\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{\left(7a(a+a\sec(c+dx))^{2/3}\text{ta}\right.}{5d\sqrt{1-\text{s}}}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\text{ta}}{10d(1+\sec(c+dx}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\text{ta}}{10d(1+\sec(c+dx}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\text{ta}}{10d(1+\sec(c+dx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 66, normalized size = 0.19

$$\frac{4\sqrt[6]{2}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)(a(1+\sec(c+dx)))^{5/3}\tan(c+dx)}{d(1+\sec(c+dx))^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (4*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
Sec[c + d*x]))^(5/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(13/6))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\left(a+a\sec(dx+c)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^2 + a*sec(d*x + c))*(a*sec(d*x + c) + a)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec{(c+dx)}+1))^{\frac{5}{3}}\sec{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x),x)
[Out] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x), x)
```

3.151 $\int (a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=86

$$\frac{3\sqrt{2}\,aF_1\left(\frac{13}{6};\frac{1}{2},1;\frac{19}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{13d\sqrt{1-\sec(c+dx)}}$$

```
[Out] 3/13*a*AppellF1(13/6,1,1/2,19/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(1+sec(d*x
+c))*(a+a*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3864, 3863, 141}

$$\frac{3\sqrt{2}\,a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},1;\frac{19}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{13d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*Sqrt[2]*a*AppellF1[13/6, 1/2, 1, 19/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
 d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*Sq
rt[1 - Sec[c + d*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/3}\,dx = \frac{(a(a+a\sec(c+dx))^{2/3})\int (1+\sec(c+dx))^{5/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$
$$= -\frac{(a(a+a\sec(c+dx))^{2/3}\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{(1+x)^{7/6}}{\sqrt{1-x}\,x}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$
$$= \frac{3\sqrt{2}\,aF_1\left(\frac{13}{6};\frac{1}{2},1;\frac{19}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec(c+dx))(a}{13d\sqrt{1-\sec(c+dx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2694 vs. $2(86) = 172$.
time = 16.00, size = 2694, normalized size = 31.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*Tan
[(c + d*x)/2])/(2*d*(1 + Sec[c + d*x])^(5/3)) + ((Cos[(c + d*x)/2]^2*Sec[c
+ d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((3*Sec[(c + d*x)/2]^2*(1 + Sec[
c + d*x])^(2/3))/4 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2
/3))/2)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]
^2 + (135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*2^(1/3)*d*(1 + Sec[c + d*x])^(5/
3)*((Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(AppellF1[3
/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec
[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (135*AppellF1[1/2, 2/3, 1, 3/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1
/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[
3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2
, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2
)))/(6*2^(1/3)) + ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]
*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
```

```
c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2] + (C
os[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2,
 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + (2*AppellF1[3/2,
 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(
-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) - (135*AppellF1[1/
2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*S
in[(c + d*x)/2])/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (135*Cos[(c + d*x)/2]^2*(-1/3*(AppellF1[3
/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2,
 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2
, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) -
 (135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-1/3*(AppellF1[3/2
, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)/2
]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 2, 7/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8/3, 1, 7/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2]))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2])*Tan[(c + d*x)/2]^2)^2))/(3*2^(1/3)) + (2^(2/3)*Tan[(c + d*x)/2]*(Appell
F1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]
*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (135*AppellF1[1/2, 2/3, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*Appell
F1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*Appel
lF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1
[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/
2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2
]^2*Sec[c + d*x]*Tan[c + d*x]))/(9*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3))
))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (a + a\sec(dx + c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(c + dx) + a)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/3), x)
```

3.152 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=86

$$-\frac{3\sqrt{2}\,aF_1\left(\frac{13}{6};\frac{1}{2},2;\frac{19}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}\tan(c+d}{13d\sqrt{1-\sec(c+dx)}}$$

```
[Out] -3/13*a*AppellF1(13/6,2,1/2,19/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(1+sec(d*
x+c))*(a+a*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3913, 3912, 141}

$$-\frac{3\sqrt{2}\,a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},2;\frac{19}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{13d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[2]*a*AppellF1[13/6, 1/2, 2, 19/6, (1 + Sec[c + d*x])/2, 1 + Sec[c
 + d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*S
qrt[1 - Sec[c + d*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{\left(a(a+a\sec(c+dx))^{2/3}\right)\int \cos(c+dx)(1+\sec(c+dx))^{5/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1+x)^{7/6}}{\sqrt{1-x}\,x^2}\,dx, x\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$

$$= -\frac{3\sqrt{2}\,aF_1\left(\frac{13}{6};\frac{1}{2},2;\frac{19}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+}{13d\sqrt{1-\sec(c+d}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2700 vs. 2(86) = 172.
time = 16.14, size = 2700, normalized size = 31.40

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*(Sin[
c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(5/3)) - (2^(2/3)*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((2*Sec[(c
+ d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/3 + (5*Cos[c + d*x]*Sec[(c + d*x)/2]^
2*(1 + Sec[c + d*x])^(2/3))/6)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(2/3)*Tan[(c + d*x)/2]^2 + (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(9*d*(1 + Se
c[c + d*x])^(5/3)*(-1/9*(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(2/3)*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (243*Appell
F1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)
/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
```

```
)*Tan[(c + d*x)/2]^2)))/2^(1/3) - (2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/
3)*Tan[(c + d*x)/2] + (Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)
/2]^2*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
])/5) + (2*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(
1/3)) - (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2])/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (243*Cos[(c + d*x
)/2]^2*(-1/3*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/9))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Tan[(c + d*x)/2]^2) - (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(3*AppellF1[3/2, 2/3, 2, 5/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
- 9*(-1/3*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
/9) + 2*Tan[(c + d*x)/2]^2*(3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*App
ellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/5) - 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + Ap
pellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/9 - (2*2^(2/3)*Tan[(c + d
*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (243*AppellF1
[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2
]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*
Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + C
```

```
os[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(27*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^(1/3))))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cos \left(d x+c\right) \left(a+a \sec \left(d x+c\right)\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
[Out] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3),x)
```

```
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3), x)
```

3.153 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=371

$$\frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}}+\frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}}-\frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad}+\frac{37\ 3^{3/4}F\Big(\mathrm{Arc}\cdots}{\cdots}$$

```
[Out] 99/80*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+3/8*sec(d*x+c)^2*tan(d*x+c)/d/(a+
a*sec(d*x+c))^(1/3)-3/40*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/a/d+37/160*3^(3/
4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1
/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/
3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(
1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4
*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+s
ec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(
1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/
(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c
))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3909, 4095, 4086, 3913, 3912, 65, 231}

$$\frac{37\ 3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\mathrm{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{80\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{40ad}+\frac{99\tan(c+dx)}{80d\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (99*Tan[c + d*x])/(80*d*(a + a*Sec[c + d*x])^(1/3)) + (3*Sec[c + d*x]^2*Tan
[c + d*x])/(8*d*(a + a*Sec[c + d*x])^(1/3)) - (3*(a + a*Sec[c + d*x])^(2/3)
*Tan[c + d*x])/(40*a*d) + (37*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3
) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) -
 (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(80*2^(1/3)*d*(1
```

```
- Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)
*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3909

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(m + n - 1))), x] + Dist[d^2/(b*(m + n - 1)), Int
[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
 2] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\int \frac{\sec^2(c+dx)\left(2a-\frac{1}{3}a\sec(c+dx)\right)}{\sqrt[3]{a+a\sec(c+dx)}}dx}{8a}$$

$$= \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad} + \frac{9\int \frac{\sec(c+dx)}{\sqrt[3]{a}}}{}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/}}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/}}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/}}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/}}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/}}{40ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.51, size = 108, normalized size = 0.29

$$\frac{(a(1+\sec(c+dx)))^{2/3}\left(129\tan\left(\frac{1}{2}(c+dx)\right)-37\ 2^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{2/3}\tan\left(\frac{1}{2}(c+dx)\right)+6(-6+5\sec(c+dx))\tan(c+dx)\right)}{80ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(129*Tan[(c + d*x)/2] - 37*2^(2/3)*Hypergeome
tric2F1[1/2, 2/3, 3/2, Tan[(c + d*x)/2]^2]*((1 + Sec[c + d*x])^(-1))^(2/3)*
Tan[(c + d*x)/2] + 6*(-6 + 5*Sec[c + d*x])*Tan[c + d*x]))/(80*a*d)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/3)), x)
```

3.154 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=336

$$-\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}}+\frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad}-\frac{7\,3^{3/4}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\text{se}}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\text{se}}}\right.\right.}{10\sqrt[3]{2}\,d}$$

```
[Out] -9/10*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*
x+c)/a/d-7/20*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*
(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/
3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(
1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2
^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a
*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2
^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3885, 4086, 3913, 3912, 65, 231}

$$-\frac{7\,3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}-\frac{9\tan(c+dx)}{10d\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-9*Tan[c + d*x])/(10*d*(a + a*Sec[c + d*x])^(1/3)) + (3*(a + a*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(5*a*d) - (7*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1
- Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(
2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)
*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])
^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 +
Sec[c + d*x])^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
```

```
1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} + \frac{3\int \frac{\sec(c+dx)\left(\frac{2a}{3}-a\sec(c+dx)\right)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} + \frac{7}{10}\int \frac{}{\sqrt[3]{a}}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} + \frac{\left(7\sqrt[3]{1+s}\right.}{}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{(7\tan(c}{10d\sqrt{1-}}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{(21\tan(c}{5d\sqrt{1-}}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{7\,3^{3/4}F\Bigg(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.21, size = 95, normalized size = 0.28

$$\frac{\left(7\sqrt[6]{2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{1+\sec(c+dx)}\,(-1+2\sec(c+dx))\right)\tan(c+dx)}{10d\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((7*2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2] + 3*(1 +
 Sec[c + d*x])^(1/6)*(-1 + 2*Sec[c + d*x]))*Tan[c + d*x])/(10*d*(1 + Sec[c
+ d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(dx+c\right)}{\left(a+a\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(c+dx\right)}{\sqrt[3]{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/3)), x)
```

3.155 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=306

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}}+\frac{3^{3/4}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\right.}{2\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-}}$$

```
[Out] 3/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+1/4*3^(3/4)*((2^(1/3)-(1+sec(d*x+c)
)^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/
(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1
+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/
3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(2^(
1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*
x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)
*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^
(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3883, 3913, 3912, 65, 231}

$$\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}+\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*EllipticF[ArcC
os[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[
3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c +
d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c +
d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1
 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1
+ Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} - \frac{1}{2}\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$$

$$= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} - \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{\sec(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{2\sqrt[3]{a+a\sec(c+dx)}}$$

$$= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{\tan(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/6}}\,dx, x, \sec(c+\right.}{2d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(}$$

$$= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{(3\tan(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{2-x^6}}\,dx, x, \sqrt[6]{1+\sec(c\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(}$$

$$= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)\right|}{2\sqrt[3]{2}\,d(1-\sec(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.12, size = 85, normalized size = 0.28

$$\frac{\left(-\sqrt[6]{2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{1+\sec(c+dx)}\right)\tan(c+dx)}{2d\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
[Out] ((-(2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]) + 3*(1
+ Sec[c + d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/6)*(a*(1 +
Sec[c + d*x]))^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/3)), x)
```

3.156 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=276

$$-\frac{3^{3/4}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{\frac{2^{2/3}}{}}}{\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{}\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\right.}}}$$

```
[Out] -1/2*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(
d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2
)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^
(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1
/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+
c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1
+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3913, 3912, 65, 231}

$$-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] -((3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*
x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
 d*x])^(1/3))^2]*Tan[c + d*x])/(2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{\sec(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}$$
$$= -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$
$$= -\frac{(6\tan(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{2-x^6}}\,dx, x, \sqrt[6]{1+\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$
$$= -\frac{3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\right.}{\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 65, normalized size = 0.24

$$\frac{\sqrt[6]{2}\ {}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)\tan(c+dx)}{d\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
[Out] (2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*x
])/(d*(1 + Sec[c + d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/3)), x)
```

3.157 $\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3\sqrt{2}\,F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$

```
[Out] 3*AppellF1(1/6,1,1/2,7/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d*x+c
)/d/(a+a*sec(d*x+c))^(1/3)/(1-sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3864, 3863, 141}

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x
]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{1}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}$$
$$= -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$
$$= \frac{3\sqrt{2}\,F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 718 vs. 2(75) = 150.
time = 4.81, size = 718, normalized size = 9.57

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-1/3),x]
```

```
[Out] (45*AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Co
s[c + d*x]*(1 + Sec[c + d*x])^2*Tan[(c + d*x)/2]*(9*AppellF1[1/2, -1/3, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(3*AppellF1[3/2, -1/3, 2,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(d*(a*(1 +
Sec[c + d*x]))^(1/3)*(40*(3*AppellF1[3/2, -1/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 6*
AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[c
+ d*x]^2*Sin[(c + d*x)/2]^2*(-15*AppellF1[3/2, -1/3, 2, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*(1 - 10*Cos[c + d*x] + 3*Cos[2*(c + d*x)]) - 5*A
ppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 - 10*
Cos[c + d*x] + 3*Cos[2*(c + d*x)]) - 24*(9*AppellF1[5/2, -1/3, 3, 7/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2) + 135*Appell
F1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*(3 + 3*Sec
[c + d*x] - 3*Sin[c + d*x]*Tan[c + d*x] - Tan[c + d*x]^2)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + a \sec (dx + c))^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(1/3),x)
[Out] int(1/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a \sec (c + dx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral((a*sec(c + d*x) + a)**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(1/3),x)
```

```
[Out] int(1/(a + a/cos(c + d*x))^(1/3), x)
```

3.158 $\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{3\sqrt{2}\,F_1\left(\frac{1}{6};\frac{1}{2},2;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$

```
[Out] -3*AppellF1(1/6,2,1/2,7/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/3)/(1-sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3913, 3912, 141}

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},2;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Sqrt[2]*AppellF1[1/6, 1/2, 2, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*
x]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{\cos(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}} \\
&= -\frac{\tan(c+dx)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}} \\
&= -\frac{3\sqrt{2}\,F_1\left(\frac{1}{6};\frac{1}{2},2;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(75) = 150.
time = 2.10, size = 240, normalized size = 3.20

$$\frac{(a(1+\sec(c+dx)))^{2/3}\left(\frac{20F_1\left(\frac{3}{2};\frac{2}{3},1;\frac{5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sin^3\left(\frac{1}{2}(c+dx)\right)}{6\left(3F_1\left(\frac{5}{2};\frac{2}{3},2;\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2F_1\left(\frac{5}{2};\frac{5}{3},1;\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+45F_1\left(\frac{3}{2};\frac{2}{3},1;\frac{5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))}+\sin(c+dx)-\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*((20*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]^3)/(6*(3*Appe
llF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF
1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c +
 d*x]) + 45*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*(1 + Cos[c + d*x])) + Sin[c + d*x] - Tan[(c + d*x)/2]))/(a*d)
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(c + dx \right)}}{\sqrt[3]{a \left(\sec{\left(c + dx \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/3), x)
```

3.159 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=766

$$-\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}}+\frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}}+\frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2/3}}+\frac{375\left(1+\sqrt{3}\right.}{28a^2d(1+\sec(c+dx)}$$

```
[Out] -33/28*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)+3/4*sec(d*x+c)^2*tan(d*x+c)/d/(a
+a*sec(d*x+c))^(5/3)+135/14*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(2/3)+375/28*(a
+a*sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a^2/d/(1+sec(d*x+c))^(2/3)/(2^(
1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))-375/28*3^(1/4)*((2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2
)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*
(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(
1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a
+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+se
c(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)
/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-125/56*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/
3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1
/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1
+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(
d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*((2^(2/3)+2^(1/3)*
(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1
+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+sec(d*x+c))^
(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.79, antiderivative size = 766, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3909, 4093, 4085, 3913, 3912, 65, 314, 231, 1895}

$$-\frac{125\ 3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{28\ 2^{2/3}a^2d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}-\frac{375\sqrt[4]{3}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{14\ 2^{2/3}a^2d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{375\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{28a^2d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}+\frac{135\tan(c+dx)}{14ad(a\sec(c+dx)+a)^{2/3}}-\frac{33\tan(c+dx)}{28d(a\sec(c+dx)+a)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-33*Tan[c + d*x])/(28*d*(a + a*Sec[c + d*x])^(5/3)) + (3*Sec[c + d*x]^2*Ta
n[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/3)) + (135*Tan[c + d*x])/(14*a*d*(
a + a*Sec[c + d*x])^(2/3)) + (375*(1 + Sqrt[3])*(a + a*Sec[c + d*x])^(1/3)*
Tan[c + d*x])/(28*a^2*d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))) - (375*3^(1/4)*EllipticE[ArcCos[(2^(1/3) - (1 - S
qrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec
[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + S
ec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*T
an[c + d*x])/(14*2^(2/3)*a^2*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*
Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (125*3^(3/4)*(1 - Sqr
t[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a
*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2
^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(28*2^(2/3)*a^2*d*(1 -
 Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2
^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3909

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(m + n - 1))), x] + Dist[d^2/(b*(m + n - 1)), Int
[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
 2] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{3\int \frac{\sec^2(c+dx)\left(2a-\frac{5}{3}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{5/3}}\,dx}{4a}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} - \frac{9\int \frac{\sec(c+dx)\left(-\frac{55a}{9}\right.}{(a+a\sec}}{2}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+}{14ad(a+a\sec(c}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+}{14ad(a+a\sec(c}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+}{14ad(a+a\sec(c}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+}{14ad(a+a\sec(c}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+}{14ad(a+a\sec(c}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+}{14ad(a+a\sec(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.60, size = 111, normalized size = 0.14

$$\frac{\left(-2502^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)\sec(c+dx)\sqrt[6]{1+\sec(c+dx)}\ +3(79+90\sec(c+dx)+7\sec^2(c+dx))\right)\tan(c+dx)}{28d(a(1+\sec(c+dx)))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] ((-250*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Sec
[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6) + 3*(79 + 90*Sec[c + d*
x] + 7*Sec[c + d*x]^2))*Tan[c + d*x])/(28*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/3)), x)
```

3.160 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=731

$$\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}}-\frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}}-\frac{57\left(1+\sqrt{3}\right)\sqrt[3]{a+a\sec(c+dx)}\,\tan(c+dx}{7a^2d(1+\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec}\right.}$$

```
[Out] 3/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)-36/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c)
)^(2/3)-57/7*(a+a*sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a^2/d/(1+sec(d*x
+c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))+57/7*3^(1/4)*((2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(
1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+
2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^
(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+sec(
d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)+19/14*3^(3/4)*((2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)
^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(
1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2
/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2
))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*((2^(2
/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+
sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.61, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3884, 4085, 3913, 3912, 65, 314, 231, 1895}

$$\frac{19\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\,2^{2/3}a^2d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{57\sqrt[3]{2}\sqrt[4]{3}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^2d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}-\frac{57\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{7a^2d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{36\tan(c+dx)}{7ad(a\sec(c+dx)+a)^{2/3}}+\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (36*Tan[c + d*x])/(7*a*
d*(a + a*Sec[c + d*x])^(2/3)) - (57*(1 + Sqrt[3])*(a + a*Sec[c + d*x])^(1/3
)*Tan[c + d*x])/(7*a^2*d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))) + (57*2^(1/3)*3^(1/4)*EllipticE[ArcCos[(2^(1/3)
- (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) -
(1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3)
+ (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))^2]*Tan[c + d*x])/(7*a^2*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*S
qrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (19*3^(3/4)*(1 - Sqrt[
3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(
1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a^2*d*(1 - Se
c[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1
/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x]
)^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
```

```
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{3\int \frac{\sec(c+dx)\left(-\frac{5a}{3}+\frac{7}{3}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{2/3}}\,dx}{7a^2}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{19\int \sec(c+dx)\sqrt[3]{a \cdots}}{7 \cdots}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(19\sqrt[3]{a+a\sec(c+ \cdots}\right.}{\cdots}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{\left(19\sqrt[3]{a+a\sec(c+ \cdots}\right.}{\cdots}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{\left(114\sqrt[3]{a+a\sec(c+ \cdots}\right.}{\cdots}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(57\sqrt[3]{a+a\sec(c+ \cdots}\right.}{\cdots}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{57\left(1+\sqrt{3} \cdots\right.}{7a^2d(1+\sec(c+dx \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.29, size = 98, normalized size = 0.13

$$\frac{\left(-33-36\sec(c+dx)+38\ 2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)\sec(c+dx)\sqrt[6]{1+\sec(c+dx)}\right)\tan(c+dx)}{7d(a(1+\sec(c+dx)))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] ((-33 - 36*Sec[c + d*x] + 38*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1
/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6))*
Tan[c + d*x])/(7*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/3),x)
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/3)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/3)), x)
```

3.161 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=731

$$-\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}}+\frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}}+\frac{15\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}\,\tan(c+a}{7ad(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+s}\right.}$$

```
[Out] -3/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)+15/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c
))^(2/3)+15/7*(1+sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))-15/7*2^(1/3)*3^(1/4)*(
(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*
(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)
*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(
2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x
+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/a/d/(1-sec(d*x+c))/(a+a*sec(d*x
+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/14*3^(3/4)*((2^(1/3)-(1+sec(d*x
+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/
2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)
*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*((2^(2/3)+2^
(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1
/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a/d/(1-sec(d*x+c))/(a+a*sec(d*
x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-
(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.51, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3882, 3913, 3912, 53, 65, 314, 231, 1895}

$$-\frac{5\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\,2^{2/3}ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/3}}-\frac{15\sqrt[3]{2}\sqrt[4]{3}\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/3}}+\frac{15\tan(c+dx)}{7ad(a\sec(c+dx)+a)^{2/3}}+\frac{15\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}}{7ad\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) + (15*Tan[c + d*x])/(7*a
*d*(a + a*Sec[c + d*x])^(2/3)) + (15*(1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3)
*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))) - (15*2^(1/3)*3^(1/4)*EllipticE[ArcCos[(2^(1/3) -
 (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[
c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (
1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
^2]*Tan[c + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt
[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (5*3^(3/4)*(1 - Sqrt[3])*
EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c +
 d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(
1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a*d*(1 - Sec[c + d*
x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) -
(1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3
))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx}{7a}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{\left(5(1+\sec(c+dx))^{2/3}\right)\int \frac{\sec(c+dx)}{(1+\sec(c+dx))^{2/3}}\,dx}{7a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{\left(5\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\text{Subst}\left(\int -\right.}{7ad\sqrt{1-\sec(c+dx)}\,(a+a}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(5\sqrt[6]{1+\sec(c+d\right.}{}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(30\sqrt[6]{1+\sec(c+\right.}{}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{\left(15\sqrt[6]{1+\sec(c+\right.}{}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{15\left(1+\right.}{7ad(a+a\sec(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.35, size = 90, normalized size = 0.12

$$\frac{\left(-3+5\ 2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\ {}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)\sec(c+dx)\sqrt[6]{1+\sec(c+dx)}\right)\tan(c+dx)}{7d(a(1+\sec(c+dx)))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] ((-3 + 5*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - S
ec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(7*d*(
a*(1 + Sec[c + d*x]))^(5/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/3),x)
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/3)),x)
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/3)), x)
```

3.162 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=744

$$\frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}}+\frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}+\frac{6\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c}{7ad(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1\right.\right.}$$

```
[Out] 6/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(2/3)+3/7*tan(d*x+c)/a/d/(1+sec(d*x+c))
/(a+a*sec(d*x+c))^(2/3)+6/7*(1+sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a/d
/(a+a*sec(d*x+c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))-6/7*2^(1
/3)*3^(1/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)
))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(
d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))
^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/
3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/a/d/(1-sec(d*x+c))
/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3
))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/7*3^(3/4)*((2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(
1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*
((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+se
c(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a/d/(1-sec(d*x+c))
/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3
))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 744, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3913, 3912, 53, 65, 314, 231, 1895}

$$-\frac{\sqrt[3]{2}\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/3}}-\frac{6\sqrt[3]{2}\sqrt[4]{3}\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/3}}+\frac{6\tan(c+dx)}{7ad(a\sec(c+dx)+a)^{2/3}}+\frac{3\tan(c+dx)}{7ad(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}+\frac{6\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}}{7ad\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)(a\sec(c+dx)+a)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (6*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)) + (3*Tan[c + d*x])/(7*a
*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)) + (6*(1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3)*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (6*2^(1/3)*3^(1/4)*EllipticE[
ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/
3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
+ d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (2^(1/3)
*3^(3/4)*(1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + S
qrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sq
rt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*
(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{(1+\sec(c+dx))^{2/3}\int \frac{\sec(c+dx)}{(1+\sec(c+dx))^{5/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(1+x)^{13/6}}\,dx, x, \sec(c+\right.}{ad\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+dx))^{2/3}}$$

$$= \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} - \frac{\left(2\sqrt[6]{1+\sec(c+dx)}\,\tan(c+\right.}{7ad\sqrt{1-\mathrm{se}}}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{(}{}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{(}{}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} - \frac{(}{}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{}{7a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 68, normalized size = 0.09

$$\frac{{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)(1+\sec(c+dx))^{7/6}\tan(c+dx)}{2\sqrt[6]{2}\,d(a(1+\sec(c+dx)))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (Hypergeometric2F1[1/2, 13/6, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])
^(7/6)*Tan[c + d*x])/(2*2^(1/6)*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx + c)}{(a + a\sec (dx + c))^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(a^2*sec(d*x + c)^2 + 2*a^
2*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (c + dx)}{(a (\sec (c + dx) + 1))^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3)), x)
```

3.163 $\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=90

$$-\frac{3\sqrt{2}\,F_1\left(-\frac{7}{6};\frac{1}{2},1;-\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

```
[Out] -3/7*AppellF1(-7/6,1,1/2,-1/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(
d*x+c)/a/d/(1+sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(1-sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3864, 3863, 141}

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},1;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}\,(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-5/3),x]
```

```
[Out] (-3*Sqrt[2]*AppellF1[-7/6, 1/2, 1, -1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*Tan[c + d*x])/(7*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a
*Sec[c + d*x])^(2/3))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
```

```
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{(1+\sec(c+dx))^{2/3}\int \frac{1}{(1+\sec(c+dx))^{5/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$
$$= -\frac{\left(\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x(1+x)^{13/6}}\,dx, x, \sec(c+$$
$$= -\frac{3\sqrt{2}\,F_1\left(-\frac{7}{6};\frac{1}{2},1;-\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3007 vs. $2(90) = 180$.
time = 16.42, size = 3007, normalized size = 33.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-5/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((27*Sin[
c + d*x])/7 - (30*Tan[(c + d*x)/2])/7 + (3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) + (2^(1/3)*(1 + Sec[c + d*x])^(5
/3)*((16*(1 + Sec[c + d*x])^(1/3))/7 - (27*Cos[c + d*x]*(1 + Sec[c + d*x])^
(1/3))/7)*Tan[(c + d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2
, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3
/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9
)))))/(7*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(
5/3)*((Sec[(c + d*x)/2]^2*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2,
 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/
2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/
```

```
3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)
))))/(7*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) + (2^(1/3)*Tan[(c
 + d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(2/3) - (3*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (A
ppellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) +
(2*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[
(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/3) - Cos[
(c + d*x)/2]*Sin[(c + d*x)/2]*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2,
 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/
2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/
3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)
)) + Cos[(c + d*x)/2]^2*((5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-1 + Tan[(c + d
*x)/2]^2)^2*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2])*Tan[(c + d*x)/2]^2)/9)) - (5*(-1/3*(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/9))/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2, 1
/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2,
 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/3,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9))
+ (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-
1/3*(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9 + (2*
(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + A
ppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/9 + (2*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2,
4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2])/5 + (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - 3*((-6*AppellF1[5/
2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)))/9))/((-1 + Tan[(
c + d*x)/2]^2)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2])*Tan[(c + d*x)/2]^2)/9)^2))))/(7*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2
```

```
/3)) - (2*2^(1/3)*Tan[(c + d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c +
 d*x)/2]^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1...
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+a\sec\left(dx+c\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/3),x)
[Out] int(1/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(c+dx\right)+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/3),x)
[Out] Integral((a*sec(c + d*x) + a)**(-5/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(5/3),x)
```

```
[Out] int(1/(a + a/cos(c + d*x))^(5/3), x)
```

3.164 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt{2}\,F_1\left(-\frac{7}{6};\frac{1}{2},2;-\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

```
[Out] 3/7*AppellF1(-7/6,2,1/2,-1/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d
*x+c)/a/d/(1+sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(1-sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3913, 3912, 141}

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},2;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}\,(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*Sqrt[2]*AppellF1[-7/6, 1/2, 2, -1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*Tan[c + d*x])/(7*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a*
Sec[c + d*x])^(2/3))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{(1+\sec(c+dx))^{2/3}\int \frac{\cos(c+dx)}{(1+\sec(c+dx))^{5/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2(1+x)^{13/6}}\,dx, x, \sec(c+\right.}{ad\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+dx))^{2/3}}$$

$$= \frac{3\sqrt{2}\,F_1\left(-\frac{7}{6};\frac{1}{2},2;-\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3011 vs. $2(90) = 180$.
time = 16.11, size = 3011, normalized size = 33.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((-48*Sin
[c + d*x])/7 + (51*Tan[(c + d*x)/2])/7 - (3*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) - (5*2^(1/3)*(1 + Sec[c + d*x])
^(5/3)*((-30*(1 + Sec[c + d*x])^(1/3))/7 + (55*Cos[c + d*x]*(1 + Sec[c + d*
x])^(1/3))/7)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*
x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[
1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF
1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2
, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2
)/9)))))/(63*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]
))^(5/3)*((-5*Sec[(c + d*x)/2]^2*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/3, 1, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*(AppellF
1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*Appel
```

```
lF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3
/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]
^2)/9)))))/(63*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) - (5*2^(1/3
)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^(2/3) - (11*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/5 + (AppellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(2/3) + (22*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*
x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(5/3)) - 9*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*(-11 - AppellF1[1/2, 1/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*
(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(
-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2)/9))) + 9*Cos[(c + d*x)/2]^2*((AppellF1[1/2, 1/3, 1, 3/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-
1 + Tan[(c + d*x)/2]^2)^2*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)) - (-1/3*(AppellF1[3/2, 1/3, 2, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2]) + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9)/((-1 + Tan[(c + d*x)/2]^2)*(Appel
lF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*App
ellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1
[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/
2]^2)/9)) + (AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(-1/3*(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (AppellF1[3/2, 4/3, 1, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
/9 + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9 + (2*Tan[(c + d*x)/2]^2*((-3*Appell
F1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/5 + (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - 3*((-6*App
ellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)))/9))/((-1
 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
```

```
 + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)^2))))/(63*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^(2/3)) + (10*2^(1/3)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*
x]*Sec[(c + d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/
3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2...
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cos (dx+c)}{(a+a\sec (dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (c+dx)}{(a(\sec (c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/3),x)
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/3), x)
```

3.165 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$-\frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+6a\surd$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/5
*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3853, 3856, 2720, 2719}

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec^{\frac{5}{2}}(c+dx)\,dx + a\int \sec^{\frac{7}{2}}(c+dx)\,dx\\ &= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int\\ &= \frac{6a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\,\mathrm{s}}{}\\ &= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6a\sqrt{\sec(c+d}}{5}\\ &= -\frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+}}{}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 115, normalized size = 0.76

$$\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))\left(-9\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+9\sin(c+dx)+5\tan(c+dx)+3\sec(c+dx)\tan(c+dx)\right)}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(-9*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*Sin[c +
 d*x] + 5*Tan[c + d*x] + 3*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[Sec[c + d
*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(179) = 358$.
time = 0.12, size = 384, normalized size = 2.54

method	result
default	$-\frac{a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.}}{10\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/10*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d
*x+1/2*c)^2-1/2)^3-12/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1
/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+28/15*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6/5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))-1/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 188, normalized size = 1.25

$$\frac{-5i\sqrt{2}\,a\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,a\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-9i\sqrt{2}\,a\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+9i\sqrt{2}\,a\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(9a\cos(dx+c)^2+5a\cos(dx+c)+3a\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a*cos(d*x + c)^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*
```

```
I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*a*cos(d*x + c)^2 + 5*a*cos(d*x + c
) + 3*a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

3.166 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=123

$$-\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a\sqrt{}}{}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3853, 3856, 2719, 2720}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\int \sec^{\frac{3}{2}}(c+dx)\,dx + a\int \sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}a$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\Big(a$$

$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+}}{}$$

Mathematica [A]
time = 0.21, size = 83, normalized size = 0.67

$$\frac{a\sec^{\frac{3}{2}}(c+dx)\left(-6\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)+3\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^(3/2)*(-6*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*Sin[c + d*x] + 3*Sin[2*(c
+ d*x)]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(159) = 318$.
time = 0.08, size = 368, normalized size = 2.99

method	result

default	$-\frac{2a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2
*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 167, normalized size = 1.36

$$\frac{-i\sqrt{2}\,a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,a\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,a\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2(3a\cos(dx+c)+a)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a
*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sec^{\frac{3}{2}}\left(c+dx\right)dx+\int \sec^{\frac{5}{2}}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(5/2), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

3.167 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{}}{}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3856, 2720, 3853, 2719}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))\,dx = a\int \sqrt{\sec(c+dx)}\,dx + a\int \sec^{\frac{3}{2}}(c+dx)\,dx$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \Big(a\sqrt{\mathrm{co}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\sec(c+}}{}$$

$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c}}{}$$

Mathematica [A]
time = 0.15, size = 68, normalized size = 0.70

$$\frac{2a\sqrt{\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]))/d
```

Maple [A]
time = 0.06, size = 148, normalized size = 1.53

method	result
default	$\frac{2a\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\sqrt{2\,(}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 124, normalized size = 1.28

$$\frac{-i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2a\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sq
rt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*a*sin(d*x + c)/sqrt(cos(d*x + c)))/
d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{\sec(c+dx)}\,dx+\int\sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

3.168 $\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 3856, 2719, 2720}

$$\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a\int \sqrt{\sec(c+dx)}\,dx$$
$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{d}$$

Mathematica [A]
time = 0.09, size = 49, normalized size = 0.65

$$\frac{2a\sqrt{\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|2\right)+F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
[Out] (2*a*Sqrt[Cos[c + d*x]]*(EllipticE[(c + d*x)/2, 2] + EllipticF[(c + d*x)/2,
 2])*Sqrt[Sec[c + d*x]])/d
```

Maple [A]
time = 0.11, size = 150, normalized size = 2.00

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{Elliptic}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}}$
risch	$-\frac{ia\sqrt{2}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\left(\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,\text{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+\right.\right.}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 107, normalized size = 1.43

$$\frac{-i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sq
rt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\sqrt{\sec(c+dx)}}\,dx+\int\sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

3.169 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a\sin\cdots}{3d\sqrt{\sec\cdots}}$$

```
[Out] 2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3854, 3856, 2720, 2719}

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\\
&= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\\
&= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\Big(a\sqrt{c}\\
&= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{3d}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 73, normalized size = 0.72

$$\frac{a\sqrt{\sec(c+dx)}\left(6\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)]))/(3*d)
```

Maple [A]
time = 0.05, size = 225, normalized size = 2.23

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.32, size = 125, normalized size = 1.24

$$\frac{2\,a\sqrt{\cos(dx+c)}\,\sin(dx+c)-i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*a*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)**(-3/2), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

3.170 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a\sin}{5d\sec}$$

```
[Out] 2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5
*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3854, 3856, 2719, 2720}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}(3a)\int \frac{}{\sqrt{\sec}}\\ &= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{}}\\ &= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{3d}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 93, normalized size = 0.73

$$\frac{a\sqrt{\sec(c+dx)}\left(36\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+3\sin(c+dx)+10\sin(2(c+dx))+3\sin(3(c+dx))\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(36*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*Sin[c + d*x] + 10*Sin[2*(
c + d*x)] + 3*Sin[3*(c + d*x)]))/(30*d)
```

Maple [A]
time = 0.06, size = 219, normalized size = 1.72

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-28\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{}\right.}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 145, normalized size = 1.14

$$\frac{-5i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+9i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-9i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3a\cos(dx+c)^2+5a\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + 5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
)) + 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c)^2 +
5*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)**(-5/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

3.171 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}+\frac{2a\,\mathrm{si}}{7d\,\mathrm{sec}}$$

```
[Out] 2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10/
21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3854, 3856, 2720, 2719}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}(5a)\int \frac{}{\sec}$$
$$= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(5a)\int \sqrt{\sec(c+d}$$
$$= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c}{5d\sec^{\frac{3}{2}}(c}$$
$$= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{21d}$$

Mathematica [A]
time = 0.28, size = 103, normalized size = 0.68

$$\frac{a\sqrt{\sec(c+dx)}\left(504\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+200\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+42\sin(c+dx)+130\sin(2(c+dx))+42\sin(3(c+dx))+15\sin(4(c+dx))\right)}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(504*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2
00*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 130*Sin
[2*(c + d*x)] + 42*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(420*d)
```

Maple [A]
time = 0.05, size = 270, normalized size = 1.79

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-528\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+25*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.93, size = 156, normalized size = 1.03

$$\frac{-25i\sqrt{2}\,a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+25i\sqrt{2}\,a\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+63i\sqrt{2}\,a\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-63i\sqrt{2}\,a\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\left(15\,a\cos(dx+c)^3+21\,a\cos(dx+c)^2+25\,a\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/105*(-25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a*cos(d*x + c
)^3 + 21*a*cos(d*x + c)^2 + 25*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(7/2),x)
[Out] a*(Integral(sec(c + d*x)**(-7/2), x) + Integral(sec(c + d*x)**(-5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(7/2), x)
```

3.172 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=187

$$-\frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{7d}+\frac{1\!\!\!\!\!\!\quad}{}$$

```
[Out] 8/7*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+2/7*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d+12/5*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)
/d-12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/7*a^2*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3853, 3856, 2719, 4131, 2720}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{7d}+\frac{12a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}-\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(7*d) + (12*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(7*d) + (4*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \sec^{\frac{7}{2}}(c+dx)\,dx+\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right) \\
&= \frac{4a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{1}{5}(\\
&= \frac{12a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{8a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d}+ \\
&= \frac{12a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{8a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d}+ \\
&= -\frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{8a^2\sqrt{\cos}}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.17, size = 285, normalized size = 1.52

$$
\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-\frac{2i\sqrt{2}\,e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\cos^2(c+dx)\left(21e^{ic}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{idx}\left(10\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)+7e^{i(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\right)}{-1+e^{2ic}}+\frac{42\cos(dx)\csc(c)+(15+14\cos(c+dx)+10\cos(2(c+dx)))\sec^2(c+dx)\tan(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\right)}{70d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*
x]^2*(21*E^(I*c)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] +
E^(I*d*x)*(10*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)
*(c + d*x))] + 7*E^(I*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)
*(c + d*x))])))/(E^(I*d*x)*(-1 + E^((2*I)*c))) + (42*Cos[d*x]*Csc[c] + (15
+ 14*Cos[c + d*x] + 10*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c + d*x])/Sec[c
 + d*x]^(3/2)))/(70*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(211) = 422$.
time = 0.10, size = 439, normalized size = 2.35

method	result
default	$-\frac{a^2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.}}{28\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/28*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^4-4/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+124/35*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/5*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x
+1/2*c)^2-1/2)^3-24/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2
*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-12/5*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 215, normalized size = 1.15

$$-\frac{2\left(10i\sqrt{2}\,a^2\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-10i\sqrt{2}\,a^2\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+21i\sqrt{2}\,a^2\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-21i\sqrt{2}\,a^2\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{\left(42a^2\cos(dx+c)^3+20a^2\cos(dx+c)^2+14a^2\cos(dx+c)+5a^2\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}\right)}{35\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/35*(10*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2)*a^2*cos(d*x + c)^3*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 21*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (42*a^2*cos(d*x + c)^3 + 20
*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 5*a^2)*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2), x)
```

3.173 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=161

$$-\frac{16a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{1\ldots}{}$$

```
[Out] 4/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+16/5*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3853, 3856, 2720, 4131, 2719}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{16a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (16*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \int \sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right) \\ &= \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(\\ &= \frac{16a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \\ &= \frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{16a^2\sqrt{\sec(c} \\ &= -\frac{16a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos}}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.52, size = 269, normalized size = 1.67

$$\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(12\left(1+e^{2i(c+dx)}\right)+12\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}+\frac{24\cos(dx)\csc(c)+(10+3\sec(c+dx))\tan(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(12*(1 + E^((2*I)*(c + d*
x))) + 12*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*
c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I
)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (24*Cos[d*x]*Csc[c]
+ (10 + 3*Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(3/2)))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(189) = 378$.
time = 0.09, size = 386, normalized size = 2.40

method	result
default	$-\frac{a^2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.}}{10\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/10*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-32/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos
(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+68/15*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-16/5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 202, normalized size = 1.25

$$-\frac{2\left(5i\sqrt{2}\,a^2\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-5i\sqrt{2}\,a^2\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+12i\sqrt{2}\,a^2\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-12i\sqrt{2}\,a^2\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{\left(24\,a^2\cos(dx+c)^2+10\,a^2\cos(dx+c)+3\,a^2\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}\right)}{15\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*I*sqrt(2)*a^2*cos(d*x + c)^2*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) - 12*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (24*a^2*cos(d*x + c)^2 + 10*a
^2*cos(d*x + c) + 3*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

3.174 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=131

$$-\frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{4a}{}$$

```
[Out] 2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3853, 3856, 2719, 4131, 2720}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (4*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\sec(c+dx)}\,\left(a^2+a^2\sec^2(c+d$$

$$= \frac{4a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} +$$

$$= \frac{4a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} +$$

$$= -\frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.23, size = 264, normalized size = 2.02

$$\frac{1}{3}a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-\frac{i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+2e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{d\left(-1+e^{2ic}\right)}+\frac{6\cos(dx)\csc(c)+\tan(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-I)*Sqrt[2]*Sqrt[E^(I*(c +
d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(3*(1 + E^((2*I)*(c + d*x))
```

```
) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1
/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c
 + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (6*Cos[d*x]*Csc[c] + T
an[c + d*x])/(2*d*Sec[c + d*x]^(3/2))))/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(167) = 334$.
time = 0.09, size = 371, normalized size = 2.83

method	result
default	$-\frac{4a^2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*
c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-7*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 179, normalized size = 1.37

$$-\frac{2\left(2i\sqrt{2}a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-2i\sqrt{2}a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}a^2\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}a^2\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{(6a^2\cos(dx+c)+a^2)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I
*sqrt(2)*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sqrt{\sec\left(c+dx\right)}\,dx+\int 2\sec^{\frac{3}{2}}\left(c+dx\right)dx+\int\sec^{\frac{5}{2}}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sqrt(sec(c + d*x)), x) + Integral(2*sec(c + d*x)**(3/2), x)
+ Integral(sec(c + d*x)**(5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

3.175 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=64

$$\frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}$$

```
[Out] 2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3873, 3856, 2720, 4128}

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = (2a^2)\int \sqrt{\sec(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \left(2a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos}}$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c-}{d}$$

Mathematica [A]
time = 0.15, size = 48, normalized size = 0.75

$$\frac{2a^2\sqrt{\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a^2*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 Sin[c + d*x]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(84) = 168$.
time = 0.07, size = 185, normalized size = 2.89

method	result
default	$-\frac{4a^2\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{d}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*a^2*(-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
```

```
4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 77, normalized size = 1.20

$$-\frac{2\left(i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-\frac{a^2\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*(I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
 - I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
- a^2*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{1}{\sqrt{\sec(c+dx)}}\,dx+\int 2\sqrt{\sec(c+dx)}\,dx+\int\sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
[Out] a**2*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(2*sqrt(sec(c + d*x)), x)
 + Integral(sec(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

3.176 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a^2}{3d\sqrt{}}$$

```
[Out] 2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3873, 3856, 2719, 4130, 2720}

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(2a^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(4a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \left(2a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\Big(4$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.81, size = 156, normalized size = 1.46

$$\frac{a^2\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(-i\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(12-\frac{24\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+8\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+2i\sin(c+dx)\right)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(Cos[c/2] - I*Sin[c/2])*((-I)*Cos[c/2] + Sin[c/2])*(12 - (24*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x)
)] + 8*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]*Sec[c + d*x] + (2*I)*Sin[c + d*x]))/(3*d*Sqrt[Sec[c + d*x]
])
```

Maple [A]
time = 0.05, size = 228, normalized size = 2.13

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{\ }\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 134, normalized size = 1.25

$$\frac{2\left(a^2\sqrt{\cos(dx+c)}\,\sin(dx+c)-2i\sqrt{2}\,a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+2i\sqrt{2}\,a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3i\sqrt{2}\,a^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3i\sqrt{2}\,a^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*sqrt(cos(d*x + c))*sin(d*x + c) - 2*I*sqrt(2)*a^2*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a^2*weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*s
qrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{2}{\sqrt{\sec(c+dx)}}\,dx+\int\sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**(-3/2), x) + Integral(2/sqrt(sec(c + d*x)), x)
 + Integral(sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

3.177 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{16a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a^2}{5d\,s}$$

```
[Out] 2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3854, 3856, 2720, 4130, 2719}

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= (2a^2)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}(2a^2)\int \sqrt{\sec(c+dx)}\,dx+\frac{1}{5}(8a^2)\\ &= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\\ &= \frac{16a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.09, size = 136, normalized size = 1.01

$$\frac{a^2\left(-96i+\frac{192i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-40i\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+40\sin(c+dx)+6\sin(2(c+dx))\right)}{30d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

[Out] (a^2*(-96*I + ((192*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (40*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 40*Sin
[c + d*x] + 6*Sin[2*(c + d*x)]))/(30*d*Sqrt[Sec[c + d*x]])

Maple [A]
time = 0.06, size = 250, normalized size = 1.85

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^{2}\left(-12\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+32\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}{15\sqrt{-2\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-13*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.55, size = 157, normalized size = 1.16

$$-\frac{2\left(5i\sqrt{2}\,a^{2}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,a^{2}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-12i\sqrt{2}\,a^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+12i\sqrt{2}\,a^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(3a^{2}\cos(dx+c)^{2}+10a^{2}\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] -2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,

```
 cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^2*cos(d*x
 + c)^2 + 10*a^2*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int\frac{2}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
[Out] a**2*(Integral(sec(c + d*x)**(-5/2), x) + Integral(2/sec(c + d*x)**(3/2), x
) + Integral(1/sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

3.178 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{7d}+\frac{2a^2}{7d\,\mathrm{s}}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+8/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3854, 3856, 2719, 4130, 2720}

$$\frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]
])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \left(2a^2\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(6a^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}\left(12a^2\right) \\
&= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(4a^2\right)\int \sqrt{\sec(c+} \\
&= \frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\,\mathrm{s}}{5d\,\mathrm{se}} \\
&= \frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.31, size = 149, normalized size = 0.93

$$
\frac{a^2\left(\frac{672i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2\left(-168i - 80i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 85\sin(c+dx) + 28\sin(2(c+dx)) + 5\sin(3(c+dx))\right)\right)}{140d\sqrt{\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqr
t[1 + E^((2*I)*(c + d*x))] + 2*(-168*I - (80*I)*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 85
*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)])))/(140*d*Sqrt[Sec
[c + d*x]])
```

Maple [A]
time = 0.06, size = 272, normalized size = 1.69

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2 \left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-39*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 170, normalized size = 1.06

$$-\frac{2\left(10i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}\,a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\,a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(5a^2\cos(dx+c)^3+14a^2\cos(dx+c)^2+20a^2\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/35*(10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) - 10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*a^2*cos(d
*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 20*a^2*cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\int \frac{2}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**(-7/2), x) + Integral(2/sec(c + d*x)**(5/2), x
) + Integral(sec(c + d*x)**(-3/2), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2), x)
```

3.179 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=187

$$-\frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{52a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}+$$

```
[Out] 52/21*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)
/d+2/7*a^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+28/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3853, 3856, 2719, 2720}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{28a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+dx)+a^3\sec\right. \\
&= a^3\int \sec^{\frac{3}{2}}(c+dx)\,dx+a^3\int \sec^{\frac{9}{2}}(c+dx)\,dx+\left(3a^3\right)\int \sec^{\frac{5}{2}}(c+ \\
&= \frac{2a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}+\frac{6}{} \\
&= \frac{28a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{52a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+ \\
&= -\frac{2a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sqrt{\cos(c}}{} \\
&= -\frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{52a^3\sqrt{\mathrm{co}}}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.13, size = 287, normalized size = 1.53

$$
\frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3\left(-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^3(c+dx)\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+65e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}+\frac{294\cos(dx)\csc(c)+(80+63\cos(c+dx)+65\cos(2(c+dx)))\sec^2(c+dx)\tan(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\right)}{420d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(147*(1 + E^((2*I)*(c + d
```

```
*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (294*Cos[d*x]*Csc
[c] + (80 + 63*Cos[c + d*x] + 65*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c + d
*x])/Sec[c + d*x]^(5/2)))/(420*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(211) = 422$.
time = 0.11, size = 439, normalized size = 2.35

method	result
default	$-\frac{a^3\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{28\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/28*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^4-26/21*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+848/105*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-56/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos
(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-28/5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 215, normalized size = 1.15

$$-\frac{2\left(65i\sqrt{2}\,a^3\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-65i\sqrt{2}\,a^3\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+147i\sqrt{2}\,a^3\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-147i\sqrt{2}\,a^3\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{\left(294\,a^3\cos(dx+c)^3+130\,a^3\cos(dx+c)^2+63\,a^3\cos(dx+c)+15\,a^3\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}\right)}{105\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/105*(65*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 65*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a^3*cos(d*x + c)^3*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 147*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (294*a^3*cos(d*x + c)^3
+ 130*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 15*a^3)*sin(d*x + c)/sqrt(
cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

3.180 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=157

$$-\frac{36a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+3$$

```
[Out] 2*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d+3
6/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3856, 2720, 3853, 2719}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{36a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\sqrt{\sec(c+dx)}+3a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+a\right.\\
&= a^3\int \sqrt{\sec(c+dx)}\,dx+a^3\int \sec^{\frac{7}{2}}(c+dx)\,dx+\left(3a^3\right)\int \sec^{\frac{3}{2}}(\\
&= \frac{6a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}+\\
&= \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{36a^3\sqrt{\sec}}{}\\
&= -\frac{6a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4a^3\sqrt{\cos}}{}\\
&= -\frac{36a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{c}}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.80, size = 267, normalized size = 1.70

$$\frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3\left(-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^3(c+dx)\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}+\frac{18\cos(dx)\csc(c)+(5+\sec(c+dx))\tan(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\right)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(9*(1 + E^((2*I)*(c + d*x
))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[
-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c)
```

```
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*
(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (18*Cos[d*x]*Csc[c] +
(5 + Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(5/2)))/(20*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(189) = 378$.
time = 0.10, size = 386, normalized size = 2.46

method	result
default	$-\frac{a^3 \sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} \left(\frac{56\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{E}}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(56/5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
10*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^3-72/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(
-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-36/5*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 200, normalized size = 1.27

$$-\frac{2\left(5i\sqrt{2}\,a^3\cos\left(dx+c\right)^2 \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-5i\sqrt{2}\,a^3\cos\left(dx+c\right)^2 \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+9i\sqrt{2}\,a^3\cos\left(dx+c\right)^2 \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-9i\sqrt{2}\,a^3\cos\left(dx+c\right)^2 \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{\left(18\,a^3\cos\left(dx+c\right)^2+5\,a^3\cos\left(dx+c\right)+a^3\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}\right)}{5\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a^3*cos(d*x + c)^2*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 - 9*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (18*a^3*cos(d*x + c)^2 + 5*a^3*c
os(d*x + c) + a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^3\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

3.181 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=131

$$-\frac{4a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+6\ldots$$

```
[Out] 2/3*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3856, 2719, 2720, 3853}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \left(\frac{a^3}{\sqrt{\sec(c+dx)}} + 3a^3\sqrt{\sec(c+dx)} + 3a^3\sec^{\frac{3}{2}}(c+dx) + a^3\sec^{\frac{5}{2}}(c+dx\right.$$

$$= a^3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^3\int \sec^{\frac{5}{2}}(c+dx)\,dx + \left(3a^3\right)\int \sqrt{\sec(c+dx)}\,dx$$

$$= \frac{6a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}a^3\int \sqrt{\sec(}$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{}$$

$$= -\frac{4a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.36, size = 187, normalized size = 1.43

$$\frac{a^3e^{-2i(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(-6-6\cos(2(c+dx))+6e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+20\sqrt{1+e^{2i(c+dx)}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)+2i\sin(c+dx)+9i\sin(2(c+dx))\right)(-i\cos(2(c+dx))+\sin(2(c+dx)))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(-6 - 6*Cos[2*(c + d*x)] + (6*(1 + E^((2*I)*(c + d*
x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/E^((2*I
)*(c + d*x)) + 20*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]*Hypergeometric
2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + (2*I)*Sin[c + d*x] + (9*I)*Sin[2
*(c + d*x)])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*d*E^((2*I)*(c +
 d*x)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(167) = 334$.
time = 0.09, size = 371, normalized size = 2.83

method	result
default	$-\frac{4a^3\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(18\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-10\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2
*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-10*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.34, size = 179, normalized size = 1.37

$$-\frac{2\left(5i\sqrt{2}\,a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{(9a^3\cos(dx+c)+a^3)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0,
```

```
 cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I
*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (9*a^3*cos(d*x + c) + a^3)*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{1}{\sqrt{\sec{(c+dx)}}}\,dx + \int 3\sqrt{\sec{(c+dx)}}\,dx + \int 3\sec^{\frac{3}{2}}{(c+dx)}\,dx + \int \sec^{\frac{5}{2}}{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
[Out] a**3*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(3*sqrt(sec(c + d*x)), x)
 + Integral(3*sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

3.182 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{4a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a}{3d}$$

```
[Out] 2/3*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d+4
*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3876, 3854, 3856, 2720, 2719, 3853}

$$\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}+3a^3\sqrt{\sec(c+dx)}+a^3\sec^{\frac{3}{2}}(c+dx)\right) \\ &= a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+a^3\int \sec^{\frac{3}{2}}(c+dx)\,dx+\left(3a^3\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+ \\ &= \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{1}{3}a^3\int \sqrt{\sec(c+dx)}\,dx \\ &= \frac{6a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{} \\ &= \frac{4a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.07, size = 169, normalized size = 1.29

$$\frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{24i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-6i-10i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+\sin(c+dx)+3\tan(c+dx)\right)\right)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((24*I)*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] +
 2*(-6*I - (10*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2,
 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c + d*x] + 3*Tan[c + d*x]))
/(3*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.06, size = 172, normalized size = 1.31

method	result
default	$-\frac{4a^3\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{Ellip}}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.16, size = 148, normalized size = 1.13

$$-\frac{2\left(5i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-5i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3i\sqrt{2}\,a^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3i\sqrt{2}\,a^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{\left(a^3\cos\left(dx+c\right)+3\,a^3\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
```

```
+ c)) - 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c)
 + 3*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\, dx + \int \frac{3}{\sqrt{\sec(c+dx)}}\, dx + \int 3\sqrt{\sec(c+dx)}\, dx + \int \sec^{\frac{3}{2}}(c+dx)\, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**(-3/2), x) + Integral(3/sqrt(sec(c + d*x)), x)
 + Integral(3*sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

3.183 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{36a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3}{5d\,s}$$

```
[Out] 2/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+3
6/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3854, 3856, 2719, 2720}

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

```
[Out] (36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])
/(d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \int \left(\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}+a^3\sqrt{\sec(c+dx)}\right)dx\\ &= a^3\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+a^3\int \sqrt{\sec(c+dx)}\,dx+\left(3a^3\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\\ &= \frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{1}{5}\left(3a^3\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+a^3\int \sqrt{\text{s}}\\ &= \frac{6a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{d}\\ &= \frac{36a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.02, size = 171, normalized size = 1.31

$$\frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{144i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-36i-20i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+10\sin(c+dx)+\sin(2(c+dx))\right)\right)}{10d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((144*I)*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))]
+ 2*(-36*I - (20*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/
2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 10*Sin[c + d*x] + Sin[2*(c + d
*x)])))/(10*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.06, size = 250, normalized size = 1.91

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(-4\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{5\sqrt{-2\,(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.90, size = 156, normalized size = 1.19

$$-\frac{2\left(5i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-9i\sqrt{2}\,a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+9i\sqrt{2}\,a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(a^3\cos(dx+c)^2+5a^3\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c)
^2 + 5*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{1}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx+\int \frac{3}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{3}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \sqrt{\sec\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**(-5/2), x) + Integral(3/sec(c + d*x)**(3/2), x
) + Integral(3/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2), x)
```

3.184 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{52a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}+\frac{2a}{7a}$$

```
[Out] 2/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)+6/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+52/21*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A]
time = 0.14, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3854, 3856, 2720, 2719}

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$= a^3\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a^3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(3a^3\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx +$$

$$= \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^3\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin}{5d\sec}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{52a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.47, size = 146, normalized size = 0.91

$$\frac{a^3\left(-2352i+\frac{4704i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-1040i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+1070\sin(c+dx)+252\sin(2(c+dx))+30\sin(3(c+dx))\right)}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^3*(-2352*I + ((4704*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (1040*I)*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1
070*Sin[c + d*x] + 252*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt
[Sec[c + d*x]])
```

Maple [A]
time = 0.06, size = 272, normalized size = 1.69

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^{3}\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)+65*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.95, size = 170, normalized size = 1.06

$$-\frac{2\left(65i\sqrt{2}\,a^{3}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65i\sqrt{2}\,a^{3}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-147i\sqrt{2}\,a^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+147i\sqrt{2}\,a^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(15a^{3}\cos(dx+c)^{3}+63a^{3}\cos(dx+c)^{2}+130a^{3}\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/105*(65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) - 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*I*sqrt(2)*a^3*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*a^3*c
os(d*x + c)^3 + 63*a^3*cos(d*x + c)^2 + 130*a^3*cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\int \frac{3}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{3}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**(-7/2), x) + Integral(3/sec(c + d*x)**(5/2), x
) + Integral(3/sec(c + d*x)**(3/2), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2), x)
```

3.185 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{68a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{44a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}+\frac{2a}{9a}$$

```
[Out] 2/9*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)+6/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+68/45*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+44/21*a^3*sin(d*x+c)/d/sec(d*x+c)^
(1/2)+68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+44/21*a^3*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3854, 3856, 2719, 2720}

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{68a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
5*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]
^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}\right)dx$$
$$= a^3\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + (3$$
$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}$$
$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} +$$
$$= \frac{18a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}$$
$$= \frac{68a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{44a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.87, size = 156, normalized size = 0.83

$$\frac{a^3\left(-11424i+\frac{22848i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-5280i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+5820\sin(c+dx)+2044\sin(2(c+dx))+540\sin(3(c+dx))+70\sin(4(c+dx))\right)}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^3*(-11424*I + ((22848*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (5280*I)*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
 5820*Sin[c + d*x] + 2044*Sin[2*(c + d*x)] + 540*Sin[3*(c + d*x)] + 70*Sin[
4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.06, size = 260, normalized size = 1.39

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^{3}\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-600\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+212\left(\cos^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos
(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos
(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 183, normalized size = 0.98

$$-\frac{2\left(165i\sqrt{2}\,a^{3}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-165i\sqrt{2}\,a^{3}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-357i\sqrt{2}\,a^{3}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+357i\sqrt{2}\,a^{3}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(35a^{3}\cos(dx+c)^{4}+135a^{3}\cos(dx+c)^{3}+238a^{3}\cos(dx+c)^{2}+330a^{3}\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*a^3
*cos(d*x + c)^4 + 135*a^3*cos(d*x + c)^3 + 238*a^3*cos(d*x + c)^2 + 330*a^3
*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2), x)
```

3.186 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=213

$$-\frac{152a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{7d}+$$

```
[Out] 32/7*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+122/45*a^4*sec(d*x+c)^(5/2)*sin(d*x+
c)/d+8/7*a^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/9*a^4*sec(d*x+c)^(9/2)*sin(d*x
+c)/d+152/15*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-152/15*a^4*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d+32/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3853, 3856, 2719, 2720}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{122a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{32a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{7d}+\frac{152a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}-\frac{152a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(7*d) + (152*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (32*a^4*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(7*d) + (122*a^4*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(45*d) + (8*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + (2*a^4*Sec[
c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec^{\frac{3}{2}}(c+dx)+4a^4\sec^{\frac{5}{2}}(c+dx)+6a^4\sec^{\frac{7}{2}}(c+dx)+4a^4\,\mathrm{s}\right.$$

$$= a^4\int \sec^{\frac{3}{2}}(c+dx)\,dx + a^4\int \sec^{\frac{11}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sec^{\frac{5}{2}}(c+$$

$$= \frac{2a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{}$$

$$= \frac{46a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{32a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} +$$

$$= -\frac{2a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(}}{}$$

$$= -\frac{46a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\mathrm{co}}}{}$$

$$= -\frac{152a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{32a^4\sqrt{\mathrm{c}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.84, size = 287, normalized size = 1.35

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-\frac{4i\sqrt{2}\,e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^4(c+dx)\left(399e^{ic}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{idx}\left(180\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)+133e^{i(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\right)}{-1+e^{2ic}}+\frac{1596\cos(dx)\csc(c)+\left(720+427\sec(c+dx)+180\sec^2(c+dx)+35\sec^3(c+dx)\right)\tan(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\right)}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*
x]^4*(399*E^(I*c)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] +
 E^(I*d*x)*(180*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*
I)*(c + d*x))] + 133*E^(I*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))])))/(E^(I*d*x)*(-1 + E^((2*I)*c))) + (1596*Cos[d*x]*Csc[c]
+ (720 + 427*Sec[c + d*x] + 180*Sec[c + d*x]^2 + 35*Sec[c + d*x]^3)*Tan[c +
 d*x])/Sec[c + d*x]^(7/2)))/(2520*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(233) = 466$.
time = 0.12, size = 492, normalized size = 2.31

method	result
default	$-\frac{a^4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{7\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/7*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^4-16/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1544/105*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/72*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^5-61/90*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-304/15*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)
^(1/2)-152/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 228, normalized size = 1.07

$$-\frac{2 \left(360 i \sqrt{2}\, a^{4} \cos \left(d x+c\right)^{4} \mathrm{weierstrassPInverse}\left(-4, 0, \cos \left(d x+c\right)+i \sin \left(d x+c\right)\right)-360 i \sqrt{2}\, a^{4} \cos \left(d x+c\right)^{4} \mathrm{weierstrassPInverse}\left(-4, 0, \cos \left(d x+c\right)-i \sin \left(d x+c\right)\right)+798 i \sqrt{2}\, a^{4} \cos \left(d x+c\right)^{4} \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos \left(d x+c\right)+i \sin \left(d x+c\right)\right)\right)-798 i \sqrt{2}\, a^{4} \cos \left(d x+c\right)^{4} \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos \left(d x+c\right)-i \sin \left(d x+c\right)\right)\right)-\frac{\left(1596 a^{4} \cos(dx+c)^{4}+720 a^{4} \cos(dx+c)^{3}+427 a^{4} \cos(dx+c)^{2}+180 a^{4} \cos(dx+c)+35 a^{4}\right) \sin(dx+c)}{\sqrt{\cos \left(d x+c\right)}}\right)}{315 d \cos \left(d x+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/315*(360*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 360*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 798*I*sqrt(2)*a^4*cos(d*x + c)^
4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 798*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (1596*a^4*cos(d*x + c)
^4 + 720*a^4*cos(d*x + c)^3 + 427*a^4*cos(d*x + c)^2 + 180*a^4*cos(d*x + c)
 + 35*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^4\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2), x)
```

3.187 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=187

$$-\frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{136a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}$$

```
[Out] 94/21*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+8/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)
/d+2/7*a^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+64/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+136/21*a^4*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3856, 2720, 3853, 2719}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{136a^4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{64a^4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^4\,dx &= \int \left(a^4\sqrt{\sec(c+dx)}+4a^4\sec^{\frac{3}{2}}(c+dx)+6a^4\sec^{\frac{5}{2}}(c+dx)+4a \right.\\
&= a^4\int\sqrt{\sec(c+dx)}\,dx+a^4\int\sec^{\frac{9}{2}}(c+dx)\,dx+\left(4a^4\right)\int\sec^{\frac{3}{2}}(c\\
&= \frac{8a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{4a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}+\\
&= \frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{64a^4\sqrt{\sec(c}{}\\
&= -\frac{8a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{6a^4\sqrt{\cos(}{}\\
&= -\frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{136a^4\sqrt{c}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.63, size = 279, normalized size = 1.49

$$
\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-\frac{4i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^4(c+dx)\left(168\left(1+e^{2i(c+dx)}\right)+168\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+85e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}+\frac{672\cos(dx)\csc(c)+\left(235+84\sec(c+dx)+15\sec^2(c+dx)\right)\tan(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\right)}{840d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(168*(1 + E^((2*I)*(c + d
```

```
*x))) + 168*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 85*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (672*Cos[d*x]*Csc
[c] + (235 + 84*Sec[c + d*x] + 15*Sec[c + d*x]^2)*Tan[c + d*x])/Sec[c + d*x
]^(7/2)))/(840*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(211) = 422$.
time = 0.11, size = 439, normalized size = 2.35

method	result
default	$-\frac{a^4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2024\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{105\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2024/105*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/28*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-47/21*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-2/5*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^3-128/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos
(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-64/5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.15, size = 215, normalized size = 1.15

$$-\frac{2\left(170i\sqrt{2}\,a^4\cos\left(dx+c\right)^3 \text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-170i\sqrt{2}\,a^4\cos\left(dx+c\right)^3 \text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+336i\sqrt{2}\,a^4\cos\left(dx+c\right)^3 \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-336i\sqrt{2}\,a^4\cos\left(dx+c\right)^3 \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-\frac{\left(672\,a^4\cos(dx+c)^3+235\,a^4\cos(dx+c)^2+84\,a^4\cos(dx+c)+15\,a^4\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}\right)}{105\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/105*(170*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 170*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 336*I*sqrt(2)*a^4*cos(d*x + c)^
3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 336*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (672*a^4*cos(d*x + c)^
3 + 235*a^4*cos(d*x + c)^2 + 84*a^4*cos(d*x + c) + 15*a^4)*sin(d*x + c)/sqr
t(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^4\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2), x)
```

3.188 $\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=161

$$-\frac{56a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}-$$

```
[Out] 8/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+66/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

Rubi [A]
time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3856, 2719, 2720, 3853}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{56a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx &= \int \left(\frac{a^4}{\sqrt{\sec(c+dx)}} + 4a^4\sqrt{\sec(c+dx)} + 6a^4\sec^{\frac{3}{2}}(c+dx) + 4a^4\sec^{\frac{5}{2}}(c+dx)\right. \\
&= a^4\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^4\int \sec^{\frac{7}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sqrt{\sec(c+dx)}\,dx + \\
&= \frac{12a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^4\sec^{\frac{5}{2}}(c+}{5} \\
&= \frac{2a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c-\right.}{d} \\
&= -\frac{10a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\right.}{} \\
&= -\frac{56a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\right.}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.73, size = 286, normalized size = 1.78

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-\frac{8i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^4(c+dx)\left(21\left(1+e^{2i(c+dx)}\right)+21\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+20e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}+\frac{-3(-61+5\cos(2c))\cos(dx)\csc(c)+30\cos(c)\sin(dx)+2(20+3\sec(c+dx))\tan(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-8*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(21*(1 + E^((2*I)*(c + d*
x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 20*E^(I*(c + d*x))*(-1 + E^((2*I)
*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*
I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (-3*(-61 + 5*Cos[2*
c])*Cos[d*x]*Csc[c] + 30*Cos[c]*Sin[d*x] + 2*(20 + 3*Sec[c + d*x])*Tan[c +
d*x])/Sec[c + d*x]^(7/2)))/(240*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(189) = 378$.
time = 0.10, size = 386, normalized size = 2.40

method	result
default	$-\frac{a^4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{56\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-56/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+328/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/10*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1
/2)^3-132/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c
)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 202, normalized size = 1.25

$$-\frac{2\left(40i\,\sqrt{2}\,a^{4}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-40i\,\sqrt{2}\,a^{4}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+42i\,\sqrt{2}\,a^{4}\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-42i\,\sqrt{2}\,a^{4}\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{\left(99\,a^{4}\cos\left(dx+c\right)^{2}+20\,a^{4}\cos\left(dx+c\right)+3\,a^{4}\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}\right)}{15\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 42*I*sqrt(2)*a^4*cos(d*x + c)^2*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 42*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (99*a^4*cos(d*x + c)^2 + 20
*a^4*cos(d*x + c) + 3*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)
^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^4}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

3.189 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{40a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2a$$

```
[Out] 2/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+8*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+40/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3876, 3854, 3856, 2720, 2719, 3853}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{40a^4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \int \left(\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}+6a^4\sqrt{\sec(c+dx)}+4a^4\sec^{\frac{3}{2}}(c+dx)+
\\
&= a^4\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+a^4\int \sec^{\frac{5}{2}}(c+dx)\,dx+\left(4a^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\left(
\\
&= \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3d}
\\
&= \frac{8a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{12a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c}{d}
\\
&= \frac{40a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^4\sqrt{}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.32, size = 70, normalized size = 0.59

$$
\frac{a^4\sec^{\frac{3}{2}}(c+dx)\left(80\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sin(c+dx)+24\sin(2(c+dx))+\sin(3(c+dx))\right)}{6d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^4*Sec[c + d*x]^(3/2)*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[c + d*x] + 24*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(128) = 256$.
time = 0.10, size = 292, normalized size = 2.47

method	result
default	$\frac{8a^4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/3*a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+7*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 121, normalized size = 1.03

$$-\frac{2\left(10i\sqrt{2}\,a^4\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-10i\sqrt{2}\,a^4\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-\frac{\left(a^4\cos(dx+c)^2+12\,a^4\cos(dx+c)+a^4\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}\right)}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4,
```

```
0, cos(d*x + c) - I*sin(d*x + c)) - (a^4*cos(d*x + c)^2 + 12*a^4*cos(d*x +
c) + a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{4}{\sqrt{\sec(c+dx)}}\,dx+\int 6\sqrt{\sec(c+dx)}\,dx+\int 4\sec^{\frac{3}{2}}(c+dx)\,dx+\int\sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)
[Out] a**4*(Integral(sec(c + d*x)**(-3/2), x) + Integral(4/sqrt(sec(c + d*x)), x)
 + Integral(6*sqrt(sec(c + d*x)), x) + Integral(4*sec(c + d*x)**(3/2), x) +
 Integral(sec(c + d*x)**(5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2), x)
```

3.190 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{56a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2}{5}$$

```
[Out] 2/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3876, 3854, 3856, 2719, 2720, 3853}

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{56a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

```
[Out] (56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c +
d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^4}{\sqrt{\sec(c+dx)}}+4a^4\sqrt{\sec(c+dx)}+a^4\,\mathrm{s}$$

$$= a^4\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+a^4\int \sec^{\frac{3}{2}}(c+dx)\,dx+\left(4a^4\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\left(4a\right.$$

$$= \frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{1}{5}\left(3a^4\right)$$

$$= \frac{12a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{d}$$

$$= \frac{56a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.21, size = 184, normalized size = 1.16

$$\frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(-336i+\frac{672i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-320i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+80\sin(c+dx)+3\sec(c+dx)\sin(3(c+dx))+63\tan(c+dx)\right)}{30d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-336*I + ((672*I)*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c
+ d*x))] - (320*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2
, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 80*Sin[c + d*x] + 3*Sec[c + d*x
]*Sin[3*(c + d*x)] + 63*Tan[c + d*x]))/(30*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.06, size = 194, normalized size = 1.22

method	result
default	$\frac{8a^4\left(6\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-26\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+19\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-20\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/15*a^4*(6*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-26*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-20*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+21*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.24, size = 162, normalized size = 1.02

$$-\frac{2\left(40i\sqrt{2}a^4\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-40i\sqrt{2}a^4\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-42i\sqrt{2}a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+42i\sqrt{2}a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{(3a^4\cos(dx+c)^2+20a^4\cos(dx+c)+15a^4)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) - 40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
```

```
*x + c)) - 42*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 42*I*sqrt(2)*a^4*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^4*cos(d
*x + c)^2 + 20*a^4*cos(d*x + c) + 15*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/
d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{4}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{6}{\sqrt{\sec(c+dx)}}\,dx+\int 4\sqrt{\sec(c+dx)}\,dx+\int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**(-5/2), x) + Integral(4/sec(c + d*x)**(3/2), x
) + Integral(6/sqrt(sec(c + d*x)), x) + Integral(4*sqrt(sec(c + d*x)), x) +
 Integral(sec(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^4}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2), x)
```

3.191 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{136a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}+$$

```
[Out] 2/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+8/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+94/21*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)+64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/d
```

Rubi [A]

time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3854, 3856, 2720, 2719}

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

```
[Out] (64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{7}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{6a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}+a^4\sqrt{\sec(\right.$$

$$= a^4\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a^4\int \sqrt{\sec(c+dx)}\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + ($$

$$= \frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^4\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{8a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c-\right.}{d}$$

$$= \frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{6a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{d}$$

$$= \frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.27, size = 180, normalized size = 1.12

$$\frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(-5376i+\frac{10752i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-2720i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+1910\sin(c+dx)+336\sin(2(c+dx))+30\sin(3(c+dx))\right)}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

[Out] (a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-5376*I + ((10752*I)*
Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*
(c + d*x))] - (2720*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4,
 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1910*Sin[c + d*x] + 336*Sin
[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt[Sec[c + d*x]])

Maple [A]
time = 0.06, size = 272, normalized size = 1.69

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^4\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-258\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] -8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/
2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-167*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)+85*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 170, normalized size = 1.06

$$-\frac{2\left(170i\sqrt{2}\,a^4\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-170i\sqrt{2}\,a^4\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-336i\sqrt{2}\,a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+336i\sqrt{2}\,a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(15a^4\cos(dx+c)^3+84a^4\cos(dx+c)^2+235a^4\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")

```
[Out] -2/105*(170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 336*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 336*I*sqrt(2)*a^4*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*a^4
*cos(d*x + c)^3 + 84*a^4*cos(d*x + c)^2 + 235*a^4*cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\int \frac{4}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{6}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{4}{\sqrt{\sec(c+dx)}}\,dx+\int \sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**(-7/2), x) + Integral(4/sec(c + d*x)**(5/2), x
) + Integral(6/sec(c + d*x)**(3/2), x) + Integral(4/sqrt(sec(c + d*x)), x)
+ Integral(sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2), x)
```

3.192 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{152a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{7d}+$$

```
[Out] 2/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/2)+8/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+122/45*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+32/7*a^4*sin(d*x+c)/d/sec(d*x+c)^
(1/2)+152/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+32/7*a^4*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3854, 3856, 2719, 2720}

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{152a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x
]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{9}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{7}{2}}(c+dx)}+\frac{6a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^4}{\sqrt{\sec(c+\right.$$

$$= a^4\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^4\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + ($$

$$= \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{12a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{9}$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin}{7d\sec^{\frac{5}{2}}}$$

$$= \frac{46a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{3}$$

$$= \frac{152a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.80, size = 156, normalized size = 0.83

$$\frac{a^4\left(-25536i+\frac{51072i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-11520i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+12240\sin(c+dx)+3556\sin(2(c+dx))+720\sin(3(c+dx))+70\sin(4(c+dx))\right)}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^4*(-25536*I + ((51072*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (11520*I)*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
+ 12240*Sin[c + d*x] + 3556*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 70*Si
n[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.06, size = 260, normalized size = 1.39

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^4\left(280\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+34\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*cos
(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*cos(
1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-399
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 183, normalized size = 0.98

$$-\frac{2\left(360i\sqrt{2}\,a^4\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-360i\sqrt{2}\,a^4\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-798i\sqrt{2}\,a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+798i\sqrt{2}\,a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{\left(35\,a^4\cos(dx+c)^4+180\,a^4\cos(dx+c)^3+427\,a^4\cos(dx+c)^2+720\,a^4\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/315*(360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 798*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 798*I*sqrt(2)*a^4*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*a^4
*cos(d*x + c)^4 + 180*a^4*cos(d*x + c)^3 + 427*a^4*cos(d*x + c)^2 + 720*a^4
*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2),x)
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2), x)
```

3.193 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{128a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{904a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{231d}$$

```
[Out] 2/11*a^4*sin(d*x+c)/d/sec(d*x+c)^(9/2)+8/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/2
)+150/77*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+128/45*a^4*sin(d*x+c)/d/sec(d*x+
c)^(3/2)+904/231*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)+128/15*a^4*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+904/231*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.21, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3876, 3854, 3856, 2720, 2719}

$$\frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{904a^4\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{904a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{128a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] (128*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (904*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(231*d) + (2*a^4*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (8*a^4*Si
n[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (150*a^4*Sin[c + d*x])/(77*d*Sec[c +
 d*x]^(5/2)) + (128*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (904*a^4*
Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \int \left(\frac{a^4}{\sec^{\frac{11}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{9}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{a^4}{\sec^{\frac{3}{2}}(c+\right. \\
&= a^4 \int \frac{1}{\sec^{\frac{11}{2}}(c+dx)}\,dx + a^4 \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(4a^4\right) \int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + (4 \\
&= \frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{12a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{3d} \\
&= \frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \\
&= \frac{24a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{3} \\
&= \frac{128a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{74a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{} \\
&= \frac{128a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{904a^4\sqrt{\cos(c+dx)}\,F\left(\right.}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.70, size = 293, normalized size = 1.38

$$-\frac{a^4\csc(c)\sqrt{\sec(c+dx)}\left(427504\cos(dx)+518672\cos(2c+dx)-137055\cos(c+2dx)+137055\cos(3c+2dx)-48664\cos(2c+3dx)+48664\cos(4c+3dx)-14760\cos(3c+4dx)+14760\cos(5c+4dx)-3080\cos(4c+5dx)+3080\cos(6c+5dx)-315\cos(5c+6dx)+315\cos(7c+6dx)-473088e^{-idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-157696e^{idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+433920i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sin(c)\right)}{110880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] -1/110880*(a^4*Csc[c]*Sqrt[Sec[c + d*x]]*(427504*Cos[d*x] + 518672*Cos[2*c
+ d*x] - 137055*Cos[c + 2*d*x] + 137055*Cos[3*c + 2*d*x] - 48664*Cos[2*c +
3*d*x] + 48664*Cos[4*c + 3*d*x] - 14760*Cos[3*c + 4*d*x] + 14760*Cos[5*c +
4*d*x] - 3080*Cos[4*c + 5*d*x] + 3080*Cos[6*c + 5*d*x] - 315*Cos[5*c + 6*d*
x] + 315*Cos[7*c + 6*d*x] - (473088*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/E^(I*d*x) - 157696*E^(I*d*x
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))] + (433920*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/
4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sin[c]))/d
```

Maple [A]
time = 0.07, size = 273, normalized size = 1.28

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^4 \left(5040\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5320\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1740\left(\cos^{9}\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*c
os(1/2*d*x+1/2*c)^13-5320*cos(1/2*d*x+1/2*c)^11+1740*cos(1/2*d*x+1/2*c)^9+3
26*cos(1/2*d*x+1/2*c)^7+678*cos(1/2*d*x+1/2*c)^5-4465*cos(1/2*d*x+1/2*c)^3+
1695*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3696*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2001*cos(1/2*
d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.89, size = 196, normalized size = 0.92

$$-\frac{2\left(3390i\sqrt{2}\,a^4\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-3390i\sqrt{2}\,a^4\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-7392i\sqrt{2}\,a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+7392i\sqrt{2}\,a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(315a^4\cos(dx+c)^5+1540a^4\cos(dx+c)^4+3375a^4\cos(dx+c)^3+4928a^4\cos(dx+c)^2+6780a^4\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] -2/3465*(3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) - 3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) - 7392*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 7392*I*sqrt(2)*a^4*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
15*a^4*cos(d*x + c)^5 + 1540*a^4*cos(d*x + c)^4 + 3375*a^4*cos(d*x + c)^3 +
 4928*a^4*cos(d*x + c)^2 + 6780*a^4*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2), x)
```

3.194 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=164

$$\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad}-\frac{3\sqrt{\sec(}}{}$$

```
[Out] 5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(
d*x+c))-3*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]

time = 0.11, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3903, 3872, 3853, 3856, 2719, 2720}

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d
) - (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (5*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(3*a*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{5}{2}a\sec(c+dx)\right)\,dx}{a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{5\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a}$$

$$= -\frac{3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+}{d(a+a\sec(c+dx}$$

$$= -\frac{3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+}{d(a+a\sec(c+dx}$$

$$= \frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right.}{3ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.08, size = 291, normalized size = 1.77

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}-\sqrt{\sec(c+dx)}\left(18\cos(dx)\csc(c)+\sec(c+dx)\left(-5\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{3}{2}(c+dx)\right)+\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{3ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*S
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x
))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + Sec[c + d
*x]*(-5*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + Tan[(c + d*x)/2]))))/(3*a*d
*(1 + Sec[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(198) = 396$.
time = 0.08, size = 413, normalized size = 2.52

method	result
default	$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(10\cos\left(\frac{dx}{2}+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/sin(1/2*d*x
+1/2*c)^3/cos(1/2*d*x+1/2*c)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*cos(1/2*d*x+1/
2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-18*cos(1/2*d*x+1/2*c)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*cos(1
/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 248, normalized size = 1.51

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+9\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+9\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+\frac{2\left(9\cos(dx+c)^{2}+4\cos(dx+c)-2\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{6\left(ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(5*(I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^2 -
 I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9
*(I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*cos(d*x
+ c)^2 + 4*cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x
+ c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x)), x)
```

3.195 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{3\sqrt{\sec}}{}$$

```
[Out] -sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))+3*sin(d*x+c)*sec(d*x+c)^(1/
2)/a/d-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]

time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3903, 3872, 3856, 2720, 3853, 2719}

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
- (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{3}{2}a\sec(c+dx)\right)\,dx}{a^2} \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a} \\
&= \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} - \\
&= -\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \\
&= -\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{ad}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.68, size = 262, normalized size = 1.93

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{d\left(-1+e^{2ic}\right)}+\frac{\sqrt{\sec(c+dx)}\left(6\cos(dx)\csc(c)-2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\right)}{a(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(
c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*
x))*(-1 + E^((2*I)*c))) + (Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] - 2*Tan[(c
 + d*x)/2]))/d))/(a*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.06, size = 253, normalized size = 1.86

method	result
default	$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\text{Ellip}\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/cos
(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.32, size = 196, normalized size = 1.44

$$\frac{\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2(3\cos(dx+c)+2)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)
 + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(
3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*
d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(5/2)/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x)), x)
```

3.196 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\sec(c+}}{d(a+}$$

```
[Out] -sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3903, 3872, 3856, 2719, 2720}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} \\
&= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} \\
&= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}}{2a} \\
&= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.57, size = 201, normalized size = 1.83

$$
-\frac{2ie^{-i(c+dx)}\cos^2\left(\frac{1}{2}(c+dx)\right)\left(1+e^{2i(c+dx)}-\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^{\frac{3}{2}}(c+dx)}{ad\left(1+e^{i(c+dx)}\right)(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((-2*I)*Cos[(c + d*x)/2]^2*(1 + E^((2*I)*(c + d*x)) - (1 + E^(I*(c + d*x)))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*
(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]
^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.06, size = 200, normalized size = 1.82

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 184, normalized size = 1.67

$$\frac{\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\sqrt{\cos(dx+c)}\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) +
 I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*sqrt(
cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{a+\frac{a}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x)), x)
```

3.197 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{\sqrt{\sec(c}}{d(a\,-}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3905, 3872, 3856, 2719, 2720}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d))
+ (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Dist[d*((n - 1)/(a*b)), Int[(d*Csc[e + f*x])^(
n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[
a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx &= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{2a^2} \\
&= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} \\
&= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} \\
&= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.53, size = 202, normalized size = 1.84

$$
-\frac{2ie^{-i(c+dx)}\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-1-e^{2i(c+dx)}+\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^{\frac{3}{2}}(c+dx)}{ad\left(1+e^{i(c+dx)}\right)(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((-2*I)*Cos[(c + d*x)/2]^2*(-1 - E^((2*I)*(c + d*x)) + (1 + E^(I*(c + d*x))
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)
*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x
]^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.06, size = 198, normalized size = 1.80

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{si}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.83, size = 184, normalized size = 1.67

$$\frac{\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(ad\cos(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c)
- I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(
cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x)), x)
```

3.198 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=112

$$\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\sec(c+}}{d(a+}$$

```
[Out] -sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3904, 3872, 3856, 2719, 2720}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx &= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} \\
&= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{3\int \frac{}{\sqrt{\sec(}}}{2} \\
&= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{2a} \\
&= \frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.48, size = 317, normalized size = 2.83

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{d\left(-1+e^{2ic}\right)} - \frac{\left(\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+2\cos\left(\frac{1}{2}(c+3dx)\right)+\cos\left(\frac{1}{2}(5c+3dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{2d}\right)\sec(c+dx)}{a(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E
^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))) - ((Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 2*Cos[(c + 3*d*x)/2
] + Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d
*x]])/(2*d))*Sec[c + d*x])/(a*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.07, size = 199, normalized size = 1.78

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\mathrm{E}\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/
2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*s
in(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 186, normalized size = 1.66

$$\frac{\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(ad\cos(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x + c
) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*s
qrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(1/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

3.199 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=140

$$-\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad}+\frac{5\,\mathrm{s}}{3ad\sqrt{}}$$

```
[Out] 5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-sin(d*x+c)/d/(a+a*sec(d*x+c))/sec(d*x+c
)^(1/2)-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3904, 3872, 3854, 3856, 2720, 2719}

$$\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
+ (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*
d) + (5*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Sec
[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx &= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2} \\
&= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{5 \ldots}{\ldots} \\
&= \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} + \frac{5\int \sqrt{\mathrm{se}\ldots}}{\ldots} \\
&= -\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+\ldots}} \\
&= -\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}\ldots}{\ldots}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.03, size = 318, normalized size = 2.27

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}+2\sqrt{\sec(c+dx)}\left(3(2+\cos(2c))\cos(dx)\csc(c)+\cos(2dx)\sin(2c)-3\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)-6\cos(c)\sin(dx)+\cos(2c)\sin(2dx)-3\tan\left(\frac{c}{2}\right)\right)\right)}{3ad(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]

[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + 2*Sqrt[Sec[c + d*x]]*(3*(2 + Cos[2*c])*Cos[d*x]*Csc[c] + Cos[2*d*x]*Sin[2*c] - 3*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 6*Cos[c]*Sin[d*x] + Cos[2*c]*Sin[2*d*x] - 3*Tan[c/2])))/(3*a*d*(1 + Sec[c + d*x]))

Maple [A]
time = 0.07, size = 215, normalized size = 1.54

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/3/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.84, size = 207, normalized size = 1.48

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+9\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+9\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))-\frac{2\left(2\cos(dx+c)^2+5\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(5*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(d*x
 + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
- 2*(2*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

3.200 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=168

$$\frac{21\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad}-\frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad}+\frac{7\sin}{5ad\,\mathrm{se}}$$

```
[Out] 7/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(
d*x+c))-5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+21/5*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3904, 3872, 3854, 3856, 2719, 2720}

$$-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{21\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (21*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a*d) + (7*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d
*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx &= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}\\
&= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{7\int \frac{1}{\sec^{\frac{5}{2}}(c+}}{2a}\\
&= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c}\\
&= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c}\\
&= \frac{21\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{5\sqrt{\cos(c+dx)}}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.42, size = 347, normalized size = 2.07

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{8i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(63\left(1+e^{2i(c+dx)}\right)+63\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+25e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}} - \sqrt{\sec(c+dx)}\left(18(17+11\cos(2c))\cos(dx)\csc(c)+4\left(10\cos(2dx)\sin(2c)-3\cos(3dx)\sin(3c)-30\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)-99\cos(c)\sin(dx)+10\cos(2c)\sin(2dx)-3\cos(3c)\sin(3dx)-30\tan\left(\frac{c}{2}\right)\right)\right)\right)}{60ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1 + E^((2*I)*c))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*
(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c +
d*x))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*(17 + 11*Cos[2*c])*Cos[d
*x]*Csc[c] + 4*(10*Cos[2*d*x]*Sin[2*c] - 3*Cos[3*d*x]*Sin[3*c] - 30*Sec[c/2
]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 99*Cos[c]*Sin[d*x] + 10*Cos[2*c]*Sin[2*d*
x] - 3*Cos[3*c]*Sin[3*d*x] - 30*Tan[c/2]))))/(60*a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.08, size = 229, normalized size = 1.36

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d
*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)
^4+23*sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.89, size = 217, normalized size = 1.29

$$-\frac{25\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+25\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+63\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+63\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(6\cos(dx+c)^3-4\cos(dx+c)^2-25\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/30*(25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(6*cos(d*x + c)^3 - 4*cos(d*x + c)^2 - 25*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)

[Out] Integral(1/(sec(c + d*x)**(7/2) + sec(c + d*x)**(5/2)), x)/a

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)),x)

[Out] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)), x)

3.201 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=202

$$\frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}-\frac{7\sqrt{\sec}}{}$$

```
[Out] 10/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d-7/3*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/
d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-7*sin
(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3901, 4104, 3872, 3853, 3856, 2719, 2720}

$$-\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) - (7*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (10*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*a^2*d) - (7*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(
1 + Sec[c + d*x])) - (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{21a}{2}\right.}{}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{7\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a^2}$$

$$= -\frac{7\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)}{3a^2d(1+\sec}$$

$$= -\frac{7\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)}{3a^2d(1+\sec}$$

$$= \frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.49, size = 287, normalized size = 1.42

$$-\frac{e^{-\frac{1}{2}i(4c+3dx)}(-1+e^{ic})\cos\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)\left(-10-37e^{i(c+dx)}-65e^{2i(c+dx)}-82e^{3i(c+dx)}-68e^{4i(c+dx)}-53e^{5i(c+dx)}-21e^{6i(c+dx)}+10i\left(1+e^{i(c+dx)}\right)^3\left(1+e^{2i(c+dx)}\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+7e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec^{\frac{5}{2}}(c+dx)}{12a^2d\left(1+e^{2i(c+dx)}\right)(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/12*((-1 + E^(I*c))*Cos[(c + d*x)/2]*Csc[c/2]*(-10 - 37*E^(I*(c + d*x)) -
 65*E^((2*I)*(c + d*x)) - 82*E^((3*I)*(c + d*x)) - 68*E^((4*I)*(c + d*x)) -
 53*E^((5*I)*(c + d*x)) - 21*E^((6*I)*(c + d*x)) + (10*I)*(1 + E^(I*(c + d*
x)))^3*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] + 7*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*(1 + E^((2*I)*(c + d*x)))^(3
/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^(5
/2))/(a^2*d*E^((I/2)*(4*c + 3*d*x))*(1 + E^((2*I)*(c + d*x)))*(1 + Sec[c +
d*x])^2)
```

Maple [A]
time = 0.09, size = 413, normalized size = 2.04

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}+14\sqrt{\frac{1}{2}\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(6*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)+14*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3-22/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2
)-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 328, normalized size = 1.62

$$-\frac{10\left(i\sqrt{2}\cos(dx+c)^3+2i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10\left(-i\sqrt{2}\cos(dx+c)^3-2i\sqrt{2}\cos(dx+c)^2-i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21\left(-i\sqrt{2}\cos(dx+c)^3-2i\sqrt{2}\cos(dx+c)^2-i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21\left(i\sqrt{2}\cos(dx+c)^3+2i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2(21\cos(dx+c)^3+32\cos(dx+c)^2+8\cos(dx+c)-2)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(10*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)
*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
10*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(
d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*(-
```

```
I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x +
 c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I
*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^3 + 32*cos(d*x + c)^2 +
8*cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3
+ 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2, x)
```

3.202 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$-\frac{4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}+\frac{4\sqrt{\sec}}{}$$

```
[Out] -5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(5/2)*
sin(d*x+c)/d/(a+a*sec(d*x+c))^2+4*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d-4*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3901, 4104, 3872, 3856, 2720, 3853, 2719}

$$-\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - (5*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{5a}{2}\right.}{}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int \sqrt{\sec(c+dx)}\,d}{6a^2}$$

$$= \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c}{3d(a+a\sec(c+d}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{4\sqrt{\sec(c+dx)}\sin(c+dx}{a^2d}$$

$$= -\frac{4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{3a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.31, size = 252, normalized size = 1.43

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-4ie^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+40\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+i(29+50\cos(c+dx)+17\cos(2(c+dx))+12i\sin(c+dx)+7i\sin(2(c+dx)))\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{6a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6*(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-4*I)*(1 + E^(I*(c + d*x)))^3*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])/E^(I*(c + d*x)) + 40*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(29 + 50*C
os[c + d*x] + 17*Cos[2*(c + d*x)] + (12*I)*Sin[c + d*x] + (7*I)*Sin[2*(c +
d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^2*d*E^(I*d*x)*(1 +
Sec[c + d*x])^2)
```

Maple [A]
time = 0.07, size = 405, normalized size = 2.30

method	result

default	$-\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{c}{\ldots}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(
1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 278, normalized size = 1.58

$$-\frac{5\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+12\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+12\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(12\cos(dx+c)^2+19\cos(dx+c)+6\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(5*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*co
s(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2*
```

```
I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(12*cos(d*x + c)^2 + 19*cos(
d*x + c) + 6)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^
2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2, x)
```

3.203 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}-\frac{\sqrt{\sec(c+}}{a^2d(1}$$

```
[Out] -1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-sin(d*x+c)*sec(d*x+c)
^(1/2)/a^2/d/(1+sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3901, 4104, 3872, 3856, 2719, 2720}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) +
 (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2
*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{5}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^4}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{s}\ldots}\right.}{}$$

$$= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\ldots\right.}{3a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 1.18, size = 242, normalized size = 1.62

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-ie^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+16\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+i(5+14\cos(c+dx)+5\cos(2(c+dx))+i\sin(2(c+dx)))\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{6a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])/E^(I*(c + d*x)) + 16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(5 + 14*Cos[c + d
*x] + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin
[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.06, size = 257, normalized size = 1.72

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 277, normalized size = 1.86

$$-\frac{2\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+2\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3\cos(dx+c)^2+4\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*co
s(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sq
rt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*
sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 4*cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(5/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2, x)
```

3.204 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=77

$$\frac{\sqrt{\cos(c+dx)}\, F\left(\frac{1}{2}(c+dx)\middle|2\right) \sqrt{\sec(c+dx)}}{3a^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] 1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3900, 21, 3856, 2720}

$$\frac{\sqrt{\cos(c+dx)}\, \sqrt{\sec(c+dx)}\, F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d)
 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3900

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}\\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \sqrt{\sec(c+dx)}\,dx}{6a^2}\\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{6a^2}\\
&= \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 98, normalized size = 1.27

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(4\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(4*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] - Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3
*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(93) = 186.
time = 0.06, size = 188, normalized size = 2.44

method	result

default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticF}\cdots\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 150, normalized size = 1.95

$$\frac{\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{\cos(dx+c)}\sin(dx+c)}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*((-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x
 + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2, x)
```

3.205 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}+\frac{\sqrt{\sec(}}{a^2d(}$$

```
[Out] -1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+sin(d*x+c)*sec(d*x+c)
^(1/2)/a^2/d/(1+sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3902, 4104, 3872, 3856, 2719, 2720}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{5a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^4}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a^2} -$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{3a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 1.37, size = 239, normalized size = 1.60

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(16\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+i\left(-7-10\cos(c+dx)-7\cos(2(c+dx))+e^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+i\sin(2(c+dx))\right)\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{6a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(
-7 - 10*Cos[c + d*x] - 7*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^3*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])/E^(I*(c + d*x)) + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c
+ 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 257, normalized size = 1.72

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}\right.}{6a^2\sqrt{-2\left(\sin\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/
2*d*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x
+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.17, size = 277, normalized size = 1.86

$$-\frac{2\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+2\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(3\cos(dx+c)^2+2\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*co
s(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqr
t(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*
sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*cos(d*x + c)^2 + 2*cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2, x)
```

3.206 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$\frac{4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}-\frac{5\sqrt{\sec(}}{3a^2d}$$

```
[Out] -5/3*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*x+c)*sec(d*
x+c)^(1/2)/d/(a+a*sec(d*x+c))^2+4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3902, 4105, 3872, 3856, 2719, 2720}

$$-\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a
^2*d) - (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{7a}{2}+\frac{3}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \cdots$$

$$= -\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \cdots$$

$$= -\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \cdots$$

$$= \frac{4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+d\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.92, size = 260, normalized size = 1.71

$$-\frac{i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-3-16e^{i(c+dx)}-23e^{2i(c+dx)}-25e^{3i(c+dx)}-20e^{4i(c+dx)}-9e^{5i(c+dx)}-5ie^{i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+4e^{2i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{3a^2d\left(1+e^{i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((-1/3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-3 - 16*
E^(I*(c + d*x)) - 23*E^((2*I)*(c + d*x)) - 25*E^((3*I)*(c + d*x)) - 20*E^((
4*I)*(c + d*x)) - 9*E^((5*I)*(c + d*x)) - (5*I)*E^(I*(c + d*x))*(1 + E^(I*(
c + d*x)))^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 4*E^((2*I)*(c +
 d*x))*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric
2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(a^2*d*E^(I*(c + d*x))*(1 + E^(I
*(c + d*x)))^3)
```

Maple [A]
time = 0.09, size = 257, normalized size = 1.69

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{6a^2\sqrt{-2\left(\sin\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2
*d*x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d
*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.94, size = 277, normalized size = 1.82

$$-\frac{5\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+12\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+12\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+\frac{2\left(6\cos\left(dx+c\right)^{2}+5\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(5*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*co
s(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*s
qrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*
I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(6*cos(d*x + c)^2 + 5*cos(d*
x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*co
s(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{5}{2}}{\left(c+dx\right)}+2\sec^{\frac{3}{2}}{\left(c+dx\right)}+\sqrt{\sec{\left(c+dx\right)}}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x)
)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)
```

3.207 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=178

$$-\frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}+\frac{10\,\mathrm{s}}{3a^2d\,\sqrt{}}$$

```
[Out] 10/3*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)-7/3*sin(d*x+c)/a^2/d/(1+sec(d*x+c))/
sec(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-7*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3902, 4105, 3872, 3854, 3856, 2720, 2719}

$$\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(\sec(c+dx)+1)}+\frac{10\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (7*Sin[c + d*x])
/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[S
ec[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{9a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,d}{3a^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\,s}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\,s}$$

$$= \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{}{3d\sqrt{}}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sin(c+}{3a^2d\sqrt{\sec(c+}}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.69, size = 257, normalized size = 1.44

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-84i\cos\left(\frac{1}{2}(c+dx)\right)-63i\cos\left(\frac{3}{2}(c+dx)\right)-21i\cos\left(\frac{5}{2}(c+dx)\right)+80\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+7ie^{-\frac{1}{2}i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+3\sin\left(\frac{1}{2}(c+dx)\right)+10\sin\left(\frac{3}{2}(c+dx)\right)+12\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)}{6a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((-84*I)*Cos[(
c + d*x)/2] - (63*I)*Cos[(3*(c + d*x))/2] - (21*I)*Cos[(5*(c + d*x))/2] + 8
0*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((7*I)*
(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + 3*Sin[(c + d*x)/2]
 + 10*Sin[(3*(c + d*x))/2] + 12*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]
))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.09, size = 270, normalized size = 1.52

method	result

default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(16\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}\right.}{6a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/
2*d*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2
*d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*
d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 287, normalized size = 1.61

$$-\frac{10\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(2\cos(dx+c)^3+13\cos(dx+c)^2+10\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(10*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 10*(-I*sqrt(2)*
cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*
sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(-I*sqrt(2)*cos(d*x + c)^2 -
2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*cos(d*x + c)^3 + 13*cos
(d*x + c)^2 + 10*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+2\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(7/2) + 2*sec(c + d*x)**(5/2) + sec(c + d*x)**(3/
2)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)
```

3.208 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$\frac{56\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d}-\frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{56\,\mathrm{s}}{15a^2d}$$

```
[Out] 56/15*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)-3*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)
/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-5*sin(
d*x+c)/a^2/d/sec(d*x+c)^(1/2)+56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.19, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3902, 4105, 3872, 3854, 3856, 2719, 2720}

$$-\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{56\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (56*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^2
*d) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + (56*Sin[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])
/(a^2*d*Sqrt[Sec[c + d*x]]) - (3*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{11a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,d}{3a^2}$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(}$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(}$$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+d}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+}$$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+d}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+}$$

$$= \frac{56\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5\sqrt{\cos(c+d}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.77, size = 271, normalized size = 1.36

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(1344i\cos\left(\frac{1}{2}(c+dx)\right)+1008i\cos\left(\frac{3}{2}(c+dx)\right)+336i\cos\left(\frac{5}{2}(c+dx)\right)-1200\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)-112ie^{-\frac{1}{2}i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-34\sin\left(\frac{1}{2}(c+dx)\right)-148\sin\left(\frac{3}{2}(c+dx)\right)-168\sin\left(\frac{5}{2}(c+dx)\right)-11\sin\left(\frac{7}{2}(c+dx)\right)+3\sin\left(\frac{9}{2}(c+dx)\right)\right)}{60a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((1344*I)*Cos[
(c + d*x)/2] + (1008*I)*Cos[(3*(c + d*x))/2] + (336*I)*Cos[(5*(c + d*x))/2]
 - 1200*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (
(112*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) - 34*Sin[(c
+ d*x)/2] - 148*Sin[(3*(c + d*x))/2] - 168*Sin[(5*(c + d*x))/2] - 11*Sin[(7
*(c + d*x))/2] + 3*Sin[(9*(c + d*x))/2]))/(60*a^2*d*E^(I*d*x)*(1 + Sec[c +
d*x])^2)
```

Maple [A]
time = 0.10, size = 283, normalized size = 1.42

method	result

default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(96\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-150\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/30/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1
/2*d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 297, normalized size = 1.48

$$-\frac{75\left(-i\sqrt{2}\cos\left(dx+c\right)^2-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+75\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+168\left(-i\sqrt{2}\cos\left(dx+c\right)^2-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+168\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))-\frac{2\left(6\cos(dx+c)^4-8\cos(dx+c)^3-94\cos(dx+c)^2-75\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{30\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2)
)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 75*(I*sqrt(2)
*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 168*(-I*sqrt(2)*cos(d*x + c)^2 - 2
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*(I*sqrt(2)*cos(d*x + c)^2
 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(6*cos(d*x + c)^4 - 8*c
os(d*x + c)^3 - 94*cos(d*x + c)^2 - 75*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)+2\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(9/2) + 2*sec(c + d*x)**(7/2) + sec(c + d*x)**(5/
2)), x)/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)), x)
```

3.209 $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=247

$$\frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{11\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{2a^3d}-\frac{119\sqrt{}}{}$$

```
[Out] 11/2*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/d-1/5*sec(d*x+c)^(9/2)*sin(d*x+c)/d/(a
+a*sec(d*x+c))^3-2/3*sec(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-119
/30*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))-119/10*sin(d*x+c)*se
c(d*x+c)^(1/2)/a^3/d+119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.25, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3901, 4104, 3872, 3853, 3856, 2719, 2720}

$$-\frac{119\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2a^3d}-\frac{119\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{119\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(2*a^3*d) - (119*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) + (11*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(2*a^3*d) - (Sec[c + d*x]^(9/2)*Sin[c + d*x])/(5
*d*(a + a*Sec[c + d*x])^3) - (2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*a*d*(a
+ a*Sec[c + d*x])^2) - (119*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(30*d*(a^3 + a
^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(\frac{7a}{2}-\frac{13}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(25a^2-\frac{69}{2}a^2}{a+a\sec(c+dx}}{15a^4}$$

$$= -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sec^{\frac{5}{2}}(c+dx)\sin(}{30d\,(a^3+a^3\sec(c-}$$

$$= -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sec^{\frac{5}{2}}(c+dx)\sin(}{30d\,(a^3+a^3\sec(c-}$$

$$= -\frac{119\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} + \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d} - \frac{\sec^{\frac{9}{2}}(c+dx)\,s}{5d(a+a\sec(}$$

$$= -\frac{119\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} + \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d} - \frac{\sec^{\frac{9}{2}}(c+dx)\,s}{5d(a+a\sec(}$$

$$= \frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{2a^3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.97, size = 378, normalized size = 1.53

$$\frac{e^{-idx}\csc\left(\frac{c}{2}\right)\left(-119\sqrt{2}\,e^{2idx}(-1+e^{2ic})\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^6\left(\frac{1}{2}(c+dx)\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\sec\left(\frac{c}{2}\right)\sec^3(c+dx)+\frac{e^{-\frac{3}{2}i(2c+dx)}(-1+e^{ic})\cos\left(\frac{1}{2}(c+dx)\right)\left(165+944e^{i(c+dx)}+2476e^{2i(c+dx)}+4148e^{3i(c+dx)}+5134e^{4i(c+dx)}+4664e^{5i(c+dx)}+3340e^{6i(c+dx)}+1620e^{7i(c+dx)}+357e^{8i(c+dx)}-165i(1+e^{i(c+dx)})^5(1+e^{2i(c+dx)})\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\sec^{\frac{7}{2}}(c+dx)}{16(1+e^{2i(c+dx)})}\right)}{15a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*(-119*Sqrt[2]*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x
))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2
]^6*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c +
 d*x]^3 + ((-1 + E^(I*c))*Cos[(c + d*x)/2]*(165 + 944*E^(I*(c + d*x)) + 247
6*E^((2*I)*(c + d*x)) + 4148*E^((3*I)*(c + d*x)) + 5134*E^((4*I)*(c + d*x))
 + 4664*E^((5*I)*(c + d*x)) + 3340*E^((6*I)*(c + d*x)) + 1620*E^((7*I)*(c +
 d*x)) + 357*E^((8*I)*(c + d*x)) - (165*I)*(1 + E^(I*(c + d*x)))^5*(1 + E^(
(2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sec[c + d*x
]^(7/2))/(16*E^(((3*I)/2)*(2*c + d*x))*(1 + E^((2*I)*(c + d*x))))))/(15*a^3
*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.10, size = 453, normalized size = 1.83

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{32\sqrt{-}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(1/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+32/15*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+118/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 404, normalized size = 1.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] -1/60*(165*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, co

```
s(d*x + c) + I*sin(d*x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt(2)
*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 357*(-I*sqrt(2)*cos
(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*s
qrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 357*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*c
os(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 + 2*(357*cos(d*x + c)^4 + 906*cos(d*x + c)^3 + 695*cos(d*x + c)^2 + 120*co
s(d*x + c) - 20)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^4 + 3
*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(11/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(11/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(11/2)/(a + a/cos(c + d*x))^3, x)
```

3.210 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{49\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}+49$$

```
[Out] -1/5*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*sec(d*x+c)^(5/2)
*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-13/6*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+
a^3*sec(d*x+c))+49/10*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d-49/10*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3901, 4104, 3872, 3856, 2720, 3853, 2719}

$$-\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{8\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(6*a^3*d) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) - (Sec[c + d*
x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (8*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (13*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{11}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(12a^2-\frac{41}{2}a \ldots\right.}{a+a\sec(c+\ldots}}{15a^4}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin\ldots}{6d\left(a^3+a^3\sec(c\ldots\right.}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin\ldots}{6d\left(a^3+a^3\sec(c\ldots\right.}$$

$$= \frac{49\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin\ldots}{15ad(a+a\sec(c\ldots}$$

$$= -\frac{13\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} + \frac{49\sqrt{\sec(c+dx)}\,\sin(c\ldots}{10a^3d}$$

$$= -\frac{49\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\ldots\right.}{6\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.95, size = 371, normalized size = 1.68

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-65e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}+\frac{1}{32}\left(1284\cos\left(\frac{1}{2}(c-dx)\right)+921\cos\left(\frac{1}{2}(3c+dx)\right)+1243\cos\left(\frac{1}{2}(c+3dx)\right)+374\cos\left(\frac{1}{2}(5c+3dx)\right)+670\cos\left(\frac{1}{2}(3c+5dx)\right)+65\cos\left(\frac{1}{2}(7c+5dx)\right)+147\cos\left(\frac{1}{2}(5c+7dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^3(c+dx)}{15a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-
1 + E^((2*I)*c))) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 1243
*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2] +
 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[
(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c
 + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(245) = 490$.
time = 0.08, size = 555, normalized size = 2.51

method	result
default	$-\frac{-2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(65\,\mathrm{EllipticF}\left(\cos\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.84, size = 354, normalized size = 1.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqr
t(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 +
 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 147*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x
 + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(-I*sqrt(2)*co
s(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sq
rt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) - 2*(147*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 295*cos(d*x +
c) + 60)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^3, x)
```

3.211 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{9\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{2a^3d}-\frac{\sec^{\frac{5}{2}}(c+d}{5d(a+a}$$

```
[Out] -1/5*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-2/5*sec(d*x+c)^(3/2)*
sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-9/10*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a
^3*sec(d*x+c))+9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/2*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]

time = 0.23, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3901, 4104, 3872, 3856, 2719, 2720}

$$-\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3
*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*
a^3*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (
2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (9*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\,(3a^2}{a+a\sec(c+}}{15a^4}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\,\sin(}{10d\,(a^3+a^3\sec(c+}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\,\sin(}{10d\,(a^3+a^3\sec(c+}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\,\sin(}{10d\,(a^3+a^3\sec(c+}$$

$$= \frac{9\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{2a^3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.70, size = 274, normalized size = 1.41

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(-3ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+160\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+2i(34+69\cos(c+dx)+34\cos(2(c+dx))+7\cos(3(c+dx))+2i\sin(c+dx)+6i\sin(2(c+dx))+2i\sin(3(c+dx)))\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{40a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(((-3*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^((2*I)*(c + d*x)) + 160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*(34 +
69*Cos[c + d*x] + 34*Cos[2*(c + d*x)] + 7*Cos[3*(c + d*x)] + (2*I)*Sin[c +
d*x] + (6*I)*Sin[2*(c + d*x)] + (2*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2
] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.06, size = 268, normalized size = 1.37

method	result

default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{20a^3\cos\left(\frac{d}{}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 353, normalized size = 1.81

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+9\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+9\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+\frac{2\left(9\cos(dx+c)^3+22\cos(dx+c)^2+15\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{20\left(a^3d\cos\left(dx+c\right)^3+3a^3d\cos\left(dx+c\right)^2+3a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/20*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(
2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x +
c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(I*sqrt(2)*cos(d*x
+ c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
```

```
 + c))) + 2*(9*cos(d*x + c)^3 + 22*cos(d*x + c)^2 + 15*cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^3, x)
```

3.212 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}-\frac{\sec^{\frac{3}{2}}(c+\cdots}{5d(a+a\cdots}$$

```
[Out] -1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/15*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a
^3*sec(d*x+c))+1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.23, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3901, 4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d
) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^
3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (4*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{-2a^2-\frac{9}{2}a^2\,\mathrm{se}}{\sqrt{\sec(c+dx)}\,(}}{15a}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin}{6d\,(a^3+a^3\sec(c}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin}{6d\,(a^3+a^3\sec(c}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin}{6d\,(a^3+a^3\sec(c}$$

$$= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.}{6a^3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.77, size = 371, normalized size = 1.90

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}-\frac{1}{32}\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+7\cos\left(\frac{1}{2}(c+3dx)\right)+26\cos\left(\frac{1}{2}(5c+3dx)\right)+10\cos\left(\frac{1}{2}(3c+5dx)\right)+5\cos\left(\frac{1}{2}(7c+5dx)\right)+3\cos\left(\frac{1}{2}(5c+7dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^3(c+dx)}{15a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] -
 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^
((2*I)*c))) - ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 7*Cos[(c + 3*d
*x)/2] + 26*Cos[(5*c + 3*d*x)/2] + 10*Cos[(3*c + 5*d*x)/2] + 5*Cos[(7*c + 5
*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqr
t[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.07, size = 270, normalized size = 1.38

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{60a^3\cos\left(\frac{dx}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 353, normalized size = 1.81

$$-\frac{5\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3\cos(dx+c)^3+4\cos(dx+c)^2-5\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(
2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x +
c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x
+ c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))
```

```
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + 2*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 5*cos(d*x + c))*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3
*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**(5/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^3, x)
```

3.213 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$-\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}+\frac{\sec^{\frac{3}{2}}(c+}{5d(a+a}$$

```
[Out] 1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*sin(d*x+c)*sec(d*
x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^
3*sec(d*x+c))-1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.23, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3900, 4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/10*(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^
3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6
*a^3*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3900

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{a^2}{2}+3a^2\sec(c+dx}{\sqrt{\sec(c+dx)}\,(a+a\,\mathrm{s}}}{15a^4}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+}{6d\left(a^3+a^3\sec(c+d\right.}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+}{6d\left(a^3+a^3\sec(c+d\right.}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+}{6d\left(a^3+a^3\sec(c+d\right.}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.}{6a^3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.69, size = 371, normalized size = 1.90

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}+\frac{1}{32}\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+17\cos\left(\frac{1}{2}(c+3dx)\right)+16\cos\left(\frac{1}{2}(5c+3dx)\right)+20\cos\left(\frac{1}{2}(3c+5dx)\right)-5\cos\left(\frac{1}{2}(7c+5dx)\right)+3\cos\left(\frac{1}{2}(5c+7dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^3(c+dx)}{15a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
+ 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E
^((2*I)*c))) + ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 17*Cos[(c + 3
*d*x)/2] + 16*Cos[(5*c + 3*d*x)/2] + 20*Cos[(3*c + 5*d*x)/2] - 5*Cos[(7*c +
 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*S
qrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.08, size = 270, normalized size = 1.38

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}}{60a^3 c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 353, normalized size = 1.81

$$-\frac{5\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(3\cos(dx+c)^3+14\cos(dx+c)^2+5\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(
2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) + 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c
)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x
+ c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))
```

```
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) - 2*(3*cos(d*x + c)^3 + 14*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^3, x)
```

3.214 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$-\frac{9\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{2a^3d}-\frac{\sec^{\frac{3}{2}}(c}{5d(a}$$

```
[Out] -1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+2/5*sin(d*x+c)*sec(d*
x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^
3*sec(d*x+c))-9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/2*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.23, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3902, 4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{5ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2
*a^3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{9a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{3a^2-\frac{9}{2}a^2\sec}{\sqrt{\sec(c+dx)}\,(}}{15a}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin}{2d\,(a^3+a^3\sec(c}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin}{2d\,(a^3+a^3\sec(c}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin}{2d\,(a^3+a^3\sec(c}$$

$$= -\frac{9\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{2a^3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.11, size = 272, normalized size = 1.39

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(160\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+i\left(-68-128\cos(c+dx)-68\cos(2(c+dx))-24\cos(3(c+dx))+3e^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+6i\sin(c+dx)+8i\sin(2(c+dx))+6i\sin(3(c+dx))\right)\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{40a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*
(-68 - 128*Cos[c + d*x] - 68*Cos[2*(c + d*x)] - 24*Cos[3*(c + d*x)] + (3*(1
 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + (6*I)*Sin[c + d*x] +
 (8*I)*Sin[2*(c + d*x)] + (6*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*
Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.10, size = 270, normalized size = 1.38

method	result

default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{20a^{3}\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1
/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)
^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2
*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 353, normalized size = 1.81

$$-\frac{5\left(i\sqrt{2}\cos(dx+c)^{3}+3i\sqrt{2}\cos(dx+c)^{2}+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(-i\sqrt{2}\cos(dx+c)^{3}-3i\sqrt{2}\cos(dx+c)^{2}-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+9\left(i\sqrt{2}\cos(dx+c)^{3}+3i\sqrt{2}\cos(dx+c)^{2}+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+9\left(-i\sqrt{2}\cos(dx+c)^{3}-3i\sqrt{2}\cos(dx+c)^{2}-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(9\cos(dx+c)^{3}+12\cos(dx+c)^{2}+5\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{20\left(a^{3}d\cos(dx+c)^{3}+3a^{3}d\cos(dx+c)^{2}+3a^{3}d\cos(dx+c)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/20*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(
2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c
)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*cos(d*x
+ c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
```

```
 + c))) - 2*(9*cos(d*x + c)^3 + 12*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^3, x)
```

3.215 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{49\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}-\frac{\sqrt{\sec(}{5d(a}$$

```
[Out] -1/5*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-13/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+
a^3*sec(d*x+c))+49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13
/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3902, 4105, 3872, 3856, 2719, 2720}

$$-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{13\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{8\sin(c+dx)\sqrt{\sec(c+dx)}}{15ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(6*a^3*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3)
 - (8*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{11a}{2}+\frac{5}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \int$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - 13$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - 13$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - 13$$

$$= \frac{49\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - 13\sqrt{\cos(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.91, size = 386, normalized size = 1.98

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+65e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}-\frac{1}{32}\left(1134\cos\left(\frac{1}{2}(c-dx)\right)+1071\cos\left(\frac{1}{2}(3c+dx)\right)+923\cos\left(\frac{1}{2}(c+3dx)\right)+694\cos\left(\frac{1}{2}(5c+3dx)\right)+470\cos\left(\frac{1}{2}(3c+5dx)\right)+265\cos\left(\frac{1}{2}(7c+5dx)\right)+117\cos\left(\frac{1}{2}(5c+7dx)\right)+30\cos\left(\frac{1}{2}(9c+7dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^3(c+dx)}{15a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1
 + E^((2*I)*c))) - ((1134*Cos[(c - d*x)/2] + 1071*Cos[(3*c + d*x)/2] + 923*
Cos[(c + 3*d*x)/2] + 694*Cos[(5*c + 3*d*x)/2] + 470*Cos[(3*c + 5*d*x)/2] +
265*Cos[(7*c + 5*d*x)/2] + 117*Cos[(5*c + 7*d*x)/2] + 30*Cos[(9*c + 7*d*x)/
2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*
x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.10, size = 270, normalized size = 1.38

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(348\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}\right.}{60a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1
/2*d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*
c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos
(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.97, size = 353, normalized size = 1.81

$$-\frac{65\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+65\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+147\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+147\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(87\cos(dx+c)^3+146\cos(dx+c)^2+65\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqr
t(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 +
 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*
x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(I*sqrt(2)*co
s(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sq
```

```
rt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(87*cos(d*x + c)^3 + 146*cos(d*x + c)^2 + 65*cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+3\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(5/2) + 3*sec(c + d*x)**(
3/2) + sqrt(sec(c + d*x))), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x)
```

3.216 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{11\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{2a^3d}+\frac{}{2a}$$

```
[Out] 11/2*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3/
sec(d*x+c)^(1/2)-2/3*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-119
/30*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/sec(d*x+c)^(1/2)-119/10*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^3/d
```

Rubi [A]

time = 0.24, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3902, 4105, 3872, 3854, 3856, 2720, 2719}

$$\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\,(a^3\sec(c+dx)+a^3)}+\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (-119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*
a^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x
]/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d
*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Sqrt
[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{13a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{2\sin(c+}{3ad\sqrt{\sec(c+dx)}\,(a+}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{2\sin(c+}{3ad\sqrt{\sec(c+dx)}\,(a+}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{2\sin(c+}{3ad\sqrt{\sec(c+dx)}\,(a+}$$

$$= \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{}{3a}$$

$$= -\frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sin(}{2a^3d\sqrt{\sec}}$$

$$= -\frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sqrt{\cos(c}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.33, size = 285, normalized size = 1.29

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-5355i\cos\left(\frac{1}{2}(c+dx)\right)-3927i\cos\left(\frac{3}{2}(c+dx)\right)-1785i\cos\left(\frac{5}{2}(c+dx)\right)-357i\cos\left(\frac{7}{2}(c+dx)\right)+5280\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+119ie^{-\frac{3}{2}i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+193\sin\left(\frac{1}{2}(c+dx)\right)+579\sin\left(\frac{3}{2}(c+dx)\right)+555\sin\left(\frac{5}{2}(c+dx)\right)+227\sin\left(\frac{7}{2}(c+dx)\right)+10\sin\left(\frac{9}{2}(c+dx)\right)\right)}{120a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-5355*I)*Cos
[(c + d*x)/2] - (3927*I)*Cos[(3*(c + d*x))/2] - (1785*I)*Cos[(5*(c + d*x))/
2] - (357*I)*Cos[(7*(c + d*x))/2] + 5280*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + ((119*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
/E^(((3*I)/2)*(c + d*x)) + 193*Sin[(c + d*x)/2] + 579*Sin[(3*(c + d*x))/2]
+ 555*Sin[(5*(c + d*x))/2] + 227*Sin[(7*(c + d*x))/2] + 10*Sin[(9*(c + d*x)
)/2]))/(120*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.09, size = 283, normalized size = 1.28

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(
1/2*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*co
s(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*
cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 363, normalized size = 1.64

$$-\frac{165\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+165\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+357\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+357\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(20\cos(dx+c)^4+237\cos(dx+c)^3+376\cos(dx+c)^2+165\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqr
t(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 357*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d
*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*(-I*sqrt(2)*
```

```
cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) - 2*(20*cos(d*x + c)^4 + 237*cos(d*x + c)^3 + 376*cos(d*x +
 c)^2 + 165*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)+3\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(9/2) + 3*sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(
5/2) + sec(c + d*x)**(3/2)), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)
```

3.217 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=247

$$\frac{231\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{21\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{2a^3d}+\frac{77}{10a^3d}$$

```
[Out] 77/10*sin(d*x+c)/a^3/d/sec(d*x+c)^(3/2)-1/5*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(
a+a*sec(d*x+c))^3-4/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-63
/10*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a^3+a^3*sec(d*x+c))-21/2*sin(d*x+c)/a^3/
d/sec(d*x+c)^(1/2)+231/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/
d-21/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.26, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3902, 4105, 3872, 3854, 3856, 2719, 2720}

$$-\frac{63\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{21\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{231\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (231*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (21*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(2*a^3*d) + (77*Sin[c + d*x])/(10*a^3*d*Sec[c + d*x]^(3/2)) - (21*Sin[c +
 d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(5*d*Sec[c + d*x]^(3/2)*
(a + a*Sec[c + d*x])^3) - (4*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)*(a + a
*Sec[c + d*x])^2) - (63*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3 + a^3*S
ec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{15a}{2}+\frac{9}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,d}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= \frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+d}{5d\sec^{\frac{3}{2}}(c+dx)(a+a}$$

$$= \frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+d}{5d\sec^{\frac{3}{2}}(c+dx)(a+a}$$

$$= \frac{231\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{21\sqrt{\cos(c+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.72, size = 297, normalized size = 1.20

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-3465i\cos\left(\frac{1}{2}(c+dx)\right)-2541i\cos\left(\frac{3}{2}(c+dx)\right)-1155i\cos\left(\frac{5}{2}(c+dx)\right)-231i\cos\left(\frac{7}{2}(c+dx)\right)+3360\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+77ie^{-\frac{3}{2}i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+125\sin\left(\frac{1}{2}(c+dx)\right)+359\sin\left(\frac{3}{2}(c+dx)\right)+350\sin\left(\frac{5}{2}(c+dx)\right)+138\sin\left(\frac{7}{2}(c+dx)\right)+5\sin\left(\frac{9}{2}(c+dx)\right)-\sin\left(\frac{11}{2}(c+dx)\right)\right)}{40a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] -1/40*(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-3465*
I)*Cos[(c + d*x)/2] - (2541*I)*Cos[(3*(c + d*x))/2] - (1155*I)*Cos[(5*(c +
d*x))/2] - (231*I)*Cos[(7*(c + d*x))/2] + 3360*Cos[(c + d*x)/2]^5*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((77*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(((3*I)/2)*(c + d*x)) + 125*Sin[(c + d*x)/2] + 359*Sin[(3*(c + d*x)
)/2] + 350*Sin[(5*(c + d*x))/2] + 138*Sin[(7*(c + d*x))/2] + 5*Sin[(9*(c +
d*x))/2] - Sin[(11*(c + d*x))/2]))/(a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.10, size = 296, normalized size = 1.20

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(64\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-288\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-76\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-210\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1
/2*d*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*cos
(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.97, size = 373, normalized size = 1.51

$$-\frac{105\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+105\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+231\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+231\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(4\cos(dx+c)^5-8\cos(dx+c)^4-147\cos(dx+c)^3-238\cos(dx+c)^2-105\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{20\left(a^3d\cos\left(dx+c\right)^3+3a^3d\cos\left(dx+c\right)^2+3a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/20*(105*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sq
rt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 105*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2
 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 231*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(
d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 231*(I*sqrt(2)*
cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*
```

```
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) - 2*(4*cos(d*x + c)^5 - 8*cos(d*x + c)^4 - 147*cos(d*x + c)
^3 - 238*cos(d*x + c)^2 - 105*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{11}{2}}(c+dx)+3\sec^{\frac{9}{2}}(c+dx)+3\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
[Out] Integral(1/(sec(c + d*x)**(11/2) + 3*sec(c + d*x)**(9/2) + 3*sec(c + d*x)**
(7/2) + sec(c + d*x)**(5/2)), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)), x)
```

3.218 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{3\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 3/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+3/4*a*sec(
d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*sec(d*x+c)^(5/2)*sin
(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3888, 3886, 221}

$$\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{3a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (3*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
```

```
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}$$

$$= \frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{3}{8}\int \sqrt{}$$

3Subst

$$= \frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}-$$

$$= \frac{3\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{4d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 0.54, size = 100, normalized size = 0.86

$$\frac{2a\left(\frac{1}{8}\cos(c+dx)(2+3\cos(c+dx))+\frac{3\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)}{8\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\right)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*((Cos[c + d*x]*(2 + 3*Cos[c + d*x]))/8 + (3*ArcSin[Sqrt[1 - Sec[c + d*
x]]])/(8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)))*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(96) = 192.
time = 0.36, size = 221, normalized size = 1.91

method	result

default	$$\frac{\left(3(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-3(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(d}}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(3*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(
d*x+c)+sin(d*x+c))*2^(1/2))-3*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*
x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+6*cos(d*x+c)*sin(d*x+c)*(-2
/(1+cos(d*x+c)))^(1/2)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*(
1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(96) = 192$.
time = 0.60, size = 1264, normalized size = 10.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(
2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1
)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
```

```
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*
c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 2.86, size = 356, normalized size = 3.07

$$\left[\frac{3\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(3\cos(dx+c)+2)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{16\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)},\ \frac{3\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)+\frac{2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(3\cos(dx+c)+2)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{8\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*c
os(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos
(d*x + c)), 1/8*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2), x)
```

3.219 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*sec(d*x+c)^(
3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3888, 3886, 221}

$$\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+$$

$$= \frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a}{\sqrt{a+}}\right.}{d}$$

$$= \frac{\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [**A**]
time = 0.25, size = 75, normalized size = 1.04

$$\frac{a\left(\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (a*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x
])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(62) = 124$.
time = 0.15, size = 186, normalized size = 2.58

method	result
default	$-\frac{(-1+\cos(dx+c))\left(\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{}{1+c}}}{}\right.\right.}{2d\sqrt{-\frac{2}{1+\cos(}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+
cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)-cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*
```

```
x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)+2*sin(d*x+c)*(-2/(1
+cos(d*x+c)))^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c))/cos(
d*x+c))^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(62) = 124$.
time = 0.58, size = 662, normalized size = 9.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)
/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(62) = 124$.
time = 2.18, size = 302, normalized size = 4.19

$$\left[\frac{\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+\frac{4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{4\left(d\cos\left(dx+c\right)+d\right)}, \frac{\sqrt{-a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)+\frac{2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{2\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(co
s(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(sqrt(-a)*(cos(d*x + c) + 1)*arctan(
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sec^{\frac{3}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)
```

3.220 $\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3886, 221}

$$\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]

[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

Mathematica [**A**]
time = 0.12, size = 54, normalized size = 1.46

$$-\frac{2\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])
/(d*Sqrt[1 - Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(31) = 62$.
time = 0.14, size = 150, normalized size = 4.05

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{\frac{1}{\cos(dx+c)}}\cos(dx+c)\left(\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-\arctan\left(\frac{\sqrt{-\frac{}{1-}}}{}\right.\right.}{2d\sin(dx+c)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*cos(d*x+c)*(
arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-arc
tan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2)))*(-2/(
1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*2^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(31) = 62$.
time = 0.57, size = 241, normalized size = 6.51

$$\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
```

```
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(31) = 62$.
time = 2.28, size = 189, normalized size = 5.11

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2d}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

3.221 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=36

$$\frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3889}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 0.10, size = 39, normalized size = 1.08

$$\frac{2\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.13, size = 52, normalized size = 1.44

method	result	size
default	$-\frac{2(-1+\cos(dx+c))\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{d\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}}$	52
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{a\left(\mathrm{e}^{i(dx+c)}+1\right)^{2}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(\mathrm{e}^{i(dx+c)}-1\right)}{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(\mathrm{e}^{i(dx+c)}+1\right)d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(-1+cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(1/cos(
d*x+c))^(1/2)
```

Maxima [A]
time = 0.53, size = 20, normalized size = 0.56

$$\frac{2\sqrt{2}\,\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Fricas [A]
time = 2.76, size = 49, normalized size = 1.36

$$\frac{2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [B]
time = 0.54, size = 53, normalized size = 1.47

$$\frac{\sin\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] (sin(2*c + 2*d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2))/(d*(cos(c + d*x) + 1))
```

3.222 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{4a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+4/3*a*sin(d*x+c)
*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3890, 3889}

$$\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{2}{3}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{4a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 49, normalized size = 0.64

$$\frac{2(2+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*Sqr
t[Sec[c + d*x]])
```

Maple [A]
time = 0.15, size = 68, normalized size = 0.88

method	result	size
default	$-\frac{2\left(\cos^{2}(dx+c)+\cos(dx+c)-2\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{2}(dx+c)\right)}{3d\sin(dx+c)}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)^2+cos(d*x+c)-2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/c
os(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)
```

Maxima [A]
time = 0.54, size = 113, normalized size = 1.47

$$\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\right)\sqrt{a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 3.88, size = 66, normalized size = 0.86

$$\frac{2\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(d*x + c)^2 + 2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.30, size = 69, normalized size = 0.90

$$\frac{\cos\left(c+d\,x\right)\,\left(4\,\sin\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}}{3\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)*(4*sin(c + d*x) + sin(2*c + 2*d*x))*(1/cos(c + d*x))^(1/2)*((
a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(3*d*(cos(c + d*x) + 1))
```

3.223 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{16a\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+8/15*a*sin(d*x+c
)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/15*a*sin(d*x+c)*sec(d*x+c)^(
1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3890, 3889}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a
*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{4}{5}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 0.20, size = 61, normalized size = 0.53

$$\frac{(19+8\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[
(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.14, size = 80, normalized size = 0.70

method	result	size
default	$-\frac{2(3(\cos^3(dx+c))+\cos^2(dx+c)+4\cos(dx+c)-8)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^3(dx+c))}{15d\sin(dx+c)}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(3*cos(d*x+c)^3+cos(d*x+c)^2+4*cos(d*x+c)-8)*(a*(1+cos(d*x+c))/cos(
d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^3/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(97) = 194.
time = 0.55, size = 203, normalized size = 1.77

$$\frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-30\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-5\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+6\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+30\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)\sqrt{a}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x +
 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 3.19, size = 78, normalized size = 0.68

$$\frac{2\left(3\cos\left(dx+c\right)^{3}+4\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + 8*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x +
c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.67, size = 82, normalized size = 0.71

$$\frac{\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(35\,\sin\left(c+d\,x\right)+8\,\sin\left(2\,c+2\,d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)\right)}{30\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^
(1/2)*(35*sin(c + d*x) + 8*sin(2*c + 2*d*x) + 3*sin(3*c + 3*d*x)))/(30*d*(c
os(c + d*x) + 1))
```

3.224 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{16a\sin(c+dx)}{35d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+12/35*a*sin(d*x+
c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+16/35*a*sin(d*x+c)/d/sec(d*x+c
)^(1/2)/(a+a*sec(d*x+c))^(1/2)+32/35*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*s
ec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3890, 3889}

$$\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(c+dx)\sqrt{\sec(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)}{35d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (12*
a*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (16*a*
Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (32*a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{6}{7}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} \\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} \\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 71, normalized size = 0.46

$$\frac{(76+47\cos(c+dx)+12\cos(2(c+dx))+5\cos(3(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{70d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((76 + 47*Cos[c + d*x] + 12*Cos[2*(c + d*x)] + 5*Cos[3*(c + d*x)])*Sqrt[a*(
1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(70*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.15, size = 90, normalized size = 0.59

method	result	size
default	$-\frac{2\left(5\left(\cos^{4}(dx+c)\right)+\cos^{3}(dx+c)+2\left(\cos^{2}(dx+c)\right)+8\cos(dx+c)-16\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(\cos^{4}(dx+c)\right)}{35d\sin(dx+c)}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/d*(5*cos(d*x+c)^4+cos(d*x+c)^3+2*cos(d*x+c)^2+8*cos(d*x+c)-16)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^4/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(129) = 258.
time = 0.56, size = 293, normalized size = 1.92

$$\frac{\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\cos\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)-105\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-35\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-7\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+10\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sin\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+35\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+105\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\right)\sqrt{a}}{280d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d

Fricas [A]
time = 3.09, size = 88, normalized size = 0.58

$$\frac{2\left(5\cos\left(dx+c\right)^{4}+6\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}+16\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 2/35*(5*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 16*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Mupad [B]
time = 2.20, size = 93, normalized size = 0.61

$$\frac{\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(140\,\sin(c+d\,x)+42\,\sin(2\,c+2\,d\,x)+12\,\sin(3\,c+3\,d\,x)+5\,\sin(4\,c+4\,d\,x))}{140\,d\,(\cos(c+d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] (cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^
(1/2)*(140*sin(c + d*x) + 42*sin(2*c + 2*d*x) + 12*sin(3*c + 3*d*x) + 5*sin
(4*c + 4*d*x)))/(140*d*(cos(c + d*x) + 1))
```

3.225 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=160

$$\frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec}{3d}$$

```
[Out] 11/8*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+11/8*a^2*
sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+11/12*a^2*sec(d*x+c)^(
5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^(7/2)*sin(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3899, 21, 3888, 3886, 221}

$$\frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
 + (11*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ (11*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{3}a\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{11a}{2}+\frac{11}{2}a\sec(\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}(11a)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}$$

$$= \frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}$$

$$= \frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c}{8d\sqrt{a+a\sec(c+a}}$$

Mathematica [A]

time = 0.58, size = 112, normalized size = 0.70

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}\left(66\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)+54\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])]*(66*Sqrt[
2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] +
 11*(Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d)
```

Maple [A]
time = 0.19, size = 246, normalized size = 1.54

method	result
default	$\left(33\left(\cos^3(dx+c)\right)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-33\left(\cos^3(dx+c)\right)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}}{\ldots}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96/d*(33*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+
sin(d*x+c))*2^(1/2))*2^(1/2)-33*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))
^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)+66*sin(d*x+c)*cos(d*x+c)^
2*(-2/(1+cos(d*x+c)))^(1/2)+44*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2)+16*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^
(1/2)*(1/cos(d*x+c))^(5/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+
c)^2-1)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2361 vs. 2(134) = 268.
time = 0.66, size = 2361, normalized size = 14.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
```

qrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2
*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*
a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos
(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d
*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x
 + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c)
+ a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a
*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*c
os(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin
(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x +
2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*
c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) +
 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 +
 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 1
8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos
(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d
*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4
*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqr
t(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x +

```
4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*
cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) +
3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*si
n(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*
x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sq
rt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2
)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x
 + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sqrt(a)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*d)
```

Fricas [A]
time = 2.59, size = 400, normalized size = 2.50

$$\left[\frac{33\,\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\,\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+\frac{4\,\left(33\,a\cos(dx+c)^{2}+22\,a\cos(dx+c)+8\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{96\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)},\ \frac{33\,\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)+\frac{2\,\left(33\,a\cos(dx+c)^{2}+22\,a\cos(dx+c)+8\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{48\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(33*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x +
 c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(33*(a*cos(d*x + c)^3
+ a*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
 c) - 2*a)) + 2*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2), x)
```

3.226 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=120

$$\frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 7/4*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+7/4*a^2*se
c(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)^(5/2)
*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3899, 21, 3888, 3886, 221}

$$\frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (7*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
(a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}a\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{7a}{2}+\frac{7}{2}a\sec(c\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$=\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{4}(7a)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(}$$

$$=\frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{8}($$

$$(7$$

$$=\frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}-$$

$$=\frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+}{4d\sqrt{a+a\sec(c+dx}}$$

Mathematica [A]

time = 0.43, size = 99, normalized size = 0.82

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}\left(7\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^2(c+dx)-3\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])]*(7*Sqrt[2
]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7
*Sin[(3*(c + d*x))/2]))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(100) = 200$.
time = 0.16, size = 214, normalized size = 1.78

method	result
default	$-\frac{(-1+\cos(dx+c))\left(7(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-7(\cos^2(dx+c))\sqrt{2}\,\mathrm{arctan}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(7*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-7*cos(d*x+c)^2*2^(1/2)*arctan(1
/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+14*cos(d*x+
c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(-2/(1+cos(d*
x+c)))^(1/2)/sin(d*x+c)^2*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. $2(100) = 200$.
time = 0.64, size = 2244, normalized size = 18.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*sqr
t(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin(3/
2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(7/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(5/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x + 3/2
```

```
*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a*co
s(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a
)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(
8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*
log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
```

```
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)*a*
cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(si...
```

Fricas [A]
time = 2.49, size = 370, normalized size = 3.08

$$\left[\frac{7\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+\frac{4\left(7\,a\cos(dx+c)+2\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{16\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)},\ \frac{7\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)+\frac{2\left(7\,a\cos(dx+c)+2\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{8\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3
- 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos
(d*x + c)^3 + cos(d*x + c)^2)) + 4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2
 + d*cos(d*x + c)), 1/8*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-a)*arc
tan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(7*a*cos(d*x + c
) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

3.227 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 3*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*sec(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3899, 21, 3886, 221}

$$\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + a\int \frac{\sqrt{\sec(c+dx)}\left(\frac{3a}{2}+\frac{3}{2}a\sec\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}(3a)\int \sqrt{\sec(c+dx)}\,\sqrt{a+a}$$

$$= \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(3a)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \right.}{d}$$

$$= \frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c-}{d\sqrt{a+a\sec(c+d}}$$

Mathematica [A]
time = 0.29, size = 75, normalized size = 1.00

$$\frac{a^2\left(-3\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-3*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*
x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(65) = 130$.
time = 0.14, size = 184, normalized size = 2.45

method	result

default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-3\cos(dx+c)\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c
)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2
^(1/2)-3*cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(
d*x+c))*2^(1/2))*2^(1/2)+2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos
(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1143 vs. $2(65) = 130$.
time = 0.57, size = 1143, normalized size = 15.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x + 3/
2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
```

```
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 4
*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*
x + 2*c))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(65) = 130$.
time = 2.41, size = 310, normalized size = 4.13

$$\left[\frac{3\,(a\cos(dx+c)+a)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\,(\cos(dx+c)^2-2\,\cos(dx+c))\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\,(d\cos(dx+c)+d)},\ \frac{3\,(a\cos(dx+c)+a)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)+\frac{2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2\,(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(3*(a*cos(d*x + c) + a)*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*a*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c) + d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(3/2),x)
```

[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)

[Out] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)

3.228 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*sin(d*
x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3898, 21, 3886, 221}

$$\frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(2a)\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d\\
&= \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+a\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx\\
&= \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(2a)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,-\frac{a\tan(c}{\sqrt{a+a\sec}}\right.}{d}\\
&= \frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 86, normalized size = 1.13

$$\frac{2a^2\left(\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,\sin(c+dx)+\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a^2*(Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + ArcSin[Sqr
t[1 - Sec[c + d*x]]]*Tan[c + d*x]))/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + S
ec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(66) = 132.
time = 0.13, size = 174, normalized size = 2.29

method	result
default	$-\frac{\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cdots}}}{}\right.\cdots\right.}{2d\sin(dx+c)\sqrt{\frac{\cdots}{\cos\cdots}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+
c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2^(1/2)*arctan(1/4*(-2/(1
+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)+4*cos(d*x+c)-4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*
x+c)/(1/cos(d*x+c))^(1/2)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(66) = 132$.
time = 0.56, size = 274, normalized size = 3.61

$$\frac{\sqrt{2}\left(\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+8\,a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*sqrt(
a)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(66) = 132$.
time = 3.71, size = 307, normalized size = 4.04

$$\left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\,\sin(dx+c)+(a\cos(dx+c)+a)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2\,(d\cos(dx+c)+d)},\ \frac{2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\,\sin(dx+c)+(a\cos(dx+c)+a)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\,\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)}{d\cos(dx+c)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + c
os(d*x + c)^2)))/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) +
d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c+dx)+1))^{\frac{3}{2}}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

3.229 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{8a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] 8/3*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+
c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3894, 3889}

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(4a)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{8a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Mathematica [A]
time = 0.23, size = 50, normalized size = 0.63

$$\frac{2a(5+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
[Out] (2*a*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*S
qrt[Sec[c + d*x]])
```

Maple [A]
time = 0.14, size = 71, normalized size = 0.90

method	result	size
default	$-\frac{2\left(\cos^2(dx+c)+4\cos(dx+c)-5\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{3d\sin(dx+c)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/d*(cos(d*x+c)^2+4*cos(d*x+c)-5)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*co
s(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)*a
```

Maxima [A]
time = 0.54, size = 38, normalized size = 0.48

$$\frac{\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqr
t(a)/d
```

Fricas [A]
time = 3.83, size = 69, normalized size = 0.87

$$\frac{2\left(a\cos\left(dx+c\right)^{2}+5\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(d*x + c)^2 + 5*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a(\sec\left(c+dx\right)+1)\right)^{\frac{3}{2}}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.41, size = 70, normalized size = 0.89

$$\frac{a\,\cos\left(c+d\,x\right)\,\left(10\,\sin\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}}{3\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] (a*cos(c + d*x)*(10*sin(c + d*x) + sin(2*c + 2*d*x))*(1/cos(c + d*x))^(1/2)
*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(3*d*(cos(c + d*x) + 1))
```

3.230 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{8a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}+\frac{2(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/5*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/5*a^2*sin(d*x+c)
*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)*(a+a*sec(d*x+c)
)^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3897, 3894, 3889}

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{5d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]) + (2*(a
+ a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 3897

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(m + 1))), x] + Dist[a*(m/(b*d*(m + 1))), Int[(a + b*Csc[e
+ f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{3}{5}\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \ldots$$

$$= \frac{8a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2(a+\ldots}{\ldots}$$

Mathematica [A]
time = 0.28, size = 60, normalized size = 0.52

$$\frac{a(13+6\cos(c+dx)+\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{5d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[
(c + d*x)/2])/(5*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.14, size = 81, normalized size = 0.70

method	result	size
default	$-\frac{2\left(\cos^3(dx+c)+2\left(\cos^2(dx+c)\right)+3\cos(dx+c)-6\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\cos^3(dx+c)\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}a}{5d\sin(dx+c)}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(cos(d*x+c)^3+2*cos(d*x+c)^2+3*cos(d*x+c)-6)*(a*(1+cos(d*x+c))/cos(d
*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(98) = 196.
time = 0.56, size = 210, normalized size = 1.81

$$\frac{\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\,a\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-20\,a\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-5\,a\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+2\,a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\,a\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+20\,a\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)\sqrt{a}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/
2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin
(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)))*sqrt(a)/d
```

Fricas [A]
time = 3.35, size = 80, normalized size = 0.69

$$\frac{2\left(a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)^{2}+6\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 + 6*a*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x
 + c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a(\sec\left(c+dx\right)+1)\right)^{\frac{3}{2}}}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.77, size = 81, normalized size = 0.70

$$\frac{a \cos(c+d x) \sqrt{\frac{1}{\cos(c+d x)}} (25 \sin(c+d x)+6 \sin(2 c+2 d x)+\sin(3 c+3 d x)) \sqrt{\frac{a (\cos(c+d x)+1)}{\cos(c+d x)}}}{10 d (\cos(c+d x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*(25*sin(c + d*x) + 6*sin(2*c + 2*d*x
) + sin(3*c + 3*d*x))*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(10*d*(c
os(c + d*x) + 1))
```

3.231 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)} dx$

Optimal. Leaf size=161

$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{104a^2\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+26/35*a^2*sin(
d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+104/105*a^2*sin(d*x+c)/d/s
ec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)*sec(d*x+c)^(1
/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3898, 21, 3890, 3889}

$$\frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{104a^2\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
6*a^2*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (1
04*a^2*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(208*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
```

```
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}(2a)\int \frac{\frac{13a}{2}+\frac{13}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}(13a)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 0.36, size = 72, normalized size = 0.45

$$\frac{a(494+253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{210d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqr
t[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.15, size = 93, normalized size = 0.58

method	result	s
default	$-\frac{2\left(15\left(\cos^4(dx+c)\right)+24\left(\cos^3(dx+c)\right)+13\left(\cos^2(dx+c)\right)+52\cos(dx+c)-104\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(\cos^4(dx+c)\right)a}{105d\sin(dx+c)}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/d*(15*cos(d*x+c)^4+24*cos(d*x+c)^3+13*cos(d*x+c)^2+52*cos(d*x+c)-104
)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^4/sin
(d*x+c)*a
```

<u>Maxima</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(137) = 274$.
time = 0.56, size = 303, normalized size = 1.88

$$\frac{\sqrt{2}\left(735\,a\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+175\,a\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+63\,a\cos\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)-735\,a\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-175\,a\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-63\,a\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+30\,a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+63\,a\sin\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+175\,a\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+735\,a\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\right)\sqrt{a}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*
x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 1
75*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

<u>Fricas</u> [A]
time = 2.85, size = 92, normalized size = 0.57

$$\frac{2\left(15\,a\cos\left(dx+c\right)^{4}+39\,a\cos\left(dx+c\right)^{3}+52\,a\cos\left(dx+c\right)^{2}+104\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 10
4*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*
cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

Mupad [B]
time = 2.33, size = 94, normalized size = 0.58

$$\frac{a\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(910\,\sin\left(c+d\,x\right)+238\,\sin\left(2\,c+2\,d\,x\right)+78\,\sin\left(3\,c+3\,d\,x\right)+15\,\sin\left(4\,c+4\,d\,x\right)\right)}{420\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x)
)^(1/2)*(910*sin(c + d*x) + 238*sin(2*c + 2*d*x) + 78*sin(3*c + 3*d*x) + 15
*sin(4*c + 4*d*x)))/(420*d*(cos(c + d*x) + 1))
```

3.232 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{68a^2\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/9*a^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+34/63*a^2*sin(
d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+68/105*a^2*sin(d*x+c)/d/se
c(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+272/315*a^2*sin(d*x+c)/d/sec(d*x+c)^(
1/2)/(a+a*sec(d*x+c))^(1/2)+544/315*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*
sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3898, 21, 3890, 3889}

$$\frac{68a^2\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{544a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{272a^2\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (3
4*a^2*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (6
8*a^2*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (
272*a^2*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
 (544*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
```

```
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(2a)\int \frac{\frac{17a}{2}+\frac{17}{2}a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(17a)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 0.55, size = 80, normalized size = 0.40

$$\frac{2a^2(35+85\sec(c+dx)+102\sec^2(c+dx)+136\sec^3(c+dx)+272\sec^4(c+dx))\sin(c+dx)}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(35 + 85*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 136*Sec[c + d*x]^3 + 27
2*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c
 + d*x])])
```

Maple [A]
time = 0.15, size = 103, normalized size = 0.51

method	result
default	$-\frac{2\left(35\left(\cos^{5}\left(dx+c\right)\right)+50\left(\cos^{4}\left(dx+c\right)\right)+17\left(\cos^{3}\left(dx+c\right)\right)+34\left(\cos^{2}\left(dx+c\right)\right)+136\cos\left(dx+c\right)-272\right)\sqrt{\frac{a\left(1+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\,\left(\frac{1}{\cos\left(dx+c\right)}\right)^{\frac{9}{2}}}{315d\sin\left(dx+c\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*(35*cos(d*x+c)^5+50*cos(d*x+c)^4+17*cos(d*x+c)^3+34*cos(d*x+c)^2+1
36*cos(d*x+c)-272)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)
*cos(d*x+c)^5/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(171) = 342$.
time = 0.58, size = 396, normalized size = 1.97

$$\frac{\sqrt{2}\left(3780\,a\cos\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+1050\,a\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+378\,a\cos\left(\frac{4}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+135\,a\cos\left(\frac{2}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)-3780\,a\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\sin\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)-\ldots\right)\sqrt{a}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5040*sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c)
, cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(9
/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2/
9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c)
 - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/
2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x
+ 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arcta
n2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*c
)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(9
/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) +
 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*sqrt(
a)/d
```

Fricas [A]
time = 2.27, size = 103, normalized size = 0.51

$$\frac{2\left(35\,a\cos\left(dx+c\right)^{5}+85\,a\cos\left(dx+c\right)^{4}+102\,a\cos\left(dx+c\right)^{3}+136\,a\cos\left(dx+c\right)^{2}+272\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a*cos(d*x + c)^5 + 85*a*cos(d*x + c)^4 + 102*a*cos(d*x + c)^3 + 1
36*a*cos(d*x + c)^2 + 272*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2), x)
```

Mupad [**B**]
time = 3.02, size = 105, normalized size = 0.52

$$\frac{a\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(4830\,\sin\left(c+d\,x\right)+1428\,\sin\left(2\,c+2\,d\,x\right)+513\,\sin\left(3\,c+3\,d\,x\right)+170\,\sin\left(4\,c+4\,d\,x\right)+35\,\sin\left(5\,c+5\,d\,x\right)\right)}{2520\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x)
)^(1/2)*(4830*sin(c + d*x) + 1428*sin(2*c + 2*d*x) + 513*sin(3*c + 3*d*x) +
 170*sin(4*c + 4*d*x) + 35*sin(5*c + 5*d*x)))/(2520*d*(cos(c + d*x) + 1))
```

3.233 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=200

$$\frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}}+17$$

```
[Out] 163/64*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+163/64*
a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+163/96*a^3*sec(d*x
+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+17/24*a^3*sec(d*x+c)^(7/2)*si
n(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*
sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.24, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3899, 4101, 3888, 3886, 221}

$$\frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{17a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*
d) + (163*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x
]]) + (163*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*
x]]) + (17*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*
x]]) + (a^2*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
```

```
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{1}{4}a\int \sec^{\frac{5}{2}}(c+$$

$$= \frac{17a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+d}}{4d}$$

$$= \frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(}{64d\sqrt{a+a\sec(c+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.33, size = 582, normalized size = 2.91

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/6144*(a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*((7824*I)*ArcTan[(
Cos[(c + d*x)/4] - (-1 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c +
 d*x)/4] - Sin[(c + d*x)/4])] + (7824*I)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sq
rt[2])*Sin[(c + d*x)/4])/((-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4
])] + Sec[c + d*x]^4*(-2934*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] + 1467*Log[2
- Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] - 1956*Cos[2*(c + d*
x)]*(2*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] - Log[2 - Sqrt[2]*Cos[(c + d*x)/2]
 - Sqrt[2]*Sin[(c + d*x)/2]] - Log[2 + Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*S
in[(c + d*x)/2]]) - 489*Cos[4*(c + d*x)]*(2*Log[Sqrt[2] + 2*Sin[(c + d*x)/2
]] - Log[2 - Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] - Log[2 +
 Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]]) + 1467*Log[2 + Sqrt[
2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] + 2060*Sqrt[2]*Sin[(c + d*x
)/2] - 6204*Sqrt[2]*Sin[(3*(c + d*x))/2] - 652*Sqrt[2]*Sin[(5*(c + d*x))/2]
 - 1956*Sqrt[2]*Sin[(7*(c + d*x))/2])))/(Sqrt[2]*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.15, size = 286, normalized size = 1.43

method	result
default	$\frac{\left(489\left(\cos^4(dx+c)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-489\left(\cos^4(dx+c)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{}{1+c}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768/d*(489*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+c
os(d*x+c)+sin(d*x+c))*2^(1/2))-489*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+c
os(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+978*sin(d*x+c)*cos(d*x
+c)^3*(-2/(1+cos(d*x+c)))^(1/2)+652*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+
c)))^(1/2)+368*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+96*sin(d*x+c
)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x
+c))^(5/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)^2-
1)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3860 vs. 2(168) = 336.
time = 0.85, size = 3860, normalized size = 19.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) +
 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x
+ 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*si
n(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x +
 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2
*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*
```

c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*
sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(sqrt(
2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x
 + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 4
8*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2
*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c
) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4
*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*
x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x +
 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos
(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a
^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^
2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x +
2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*
x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2
*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*
x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*
d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2
+ 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16
*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x
 + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x
+ 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*
x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*si
n(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x
 + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x +
6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*
cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2

```
+ a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4
*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^
2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*
d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(
4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*c
os(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2
*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin...
```

Fricas [**A**]
time = 2.63, size = 446, normalized size = 2.23

$$\left[\frac{489\left(a^2\cos(dx+c)^4+a^2\cos(dx+c)^3\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(489a^2\cos(dx+c)^3+326a^2\cos(dx+c)^2+184a^2\cos(dx+c)+48a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{768\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}, \frac{489\left(a^2\cos(dx+c)^4+a^2\cos(dx+c)^3\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2\left(489a^2\cos(dx+c)^3+326a^2\cos(dx+c)^2+184a^2\cos(dx+c)+48a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{384\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*
x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(489*a^2*cos(d*x + c)^3 + 326*a
^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(
d*x + c)^3), 1/384*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(489*a^2*cos(
d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2), x)
```

3.234 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=160

$$\frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\,\mathrm{se}}{}$$

```
[Out] 25/8*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+25/8*a^3*
sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+13/12*a^3*sec(d*x+c)^(
5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^(5/2)*sin(d*x+c
)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3899, 4101, 3888, 3886, 221}

$$\frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{13a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{25a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (25*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
 + (25*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ (13*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
```

```
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}a\int \sec^{\frac{3}{2}}(c+$$

$$= \frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+d}{3d}$$

$$= \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c-}{8d\sqrt{a+a\sec(c+dx}$$

Mathematica [C] Result contains complex when optimal does not.

time = 7.86, size = 458, normalized size = 2.86

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(-600i\text{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)-600i\text{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)+\sec^3(c+dx)\left(225\cos(c+dx)\left(2\log\left(\sqrt{2}+2\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(2-\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(2+\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+75\cos(3(c+dx))\left(2\log\left(\sqrt{2}+2\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(2-\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(2+\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+4\sqrt{2}\left(114\sin\left(\frac{1}{2}(c+dx)\right)-7\sin\left(\frac{3}{2}(c+dx)\right)+75\sin\left(\frac{5}{2}(c+dx)\right)\right)\right)\right)}{384\sqrt{2}\,d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*((-600*I)*ArcTan[(Cos[(c +
 d*x)/4] - (-1 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c + d*x)/4]
 - Sin[(c + d*x)/4])] - (600*I)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sqrt[2])*Si
n[(c + d*x)/4])/((-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4])] + Sec
[c + d*x]^3*(225*Cos[c + d*x]*(2*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] - Log[2
- Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] - Log[2 + Sqrt[2]*Co
s[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]]) + 75*Cos[3*(c + d*x)]*(2*Log[Sq
rt[2] + 2*Sin[(c + d*x)/2]] - Log[2 - Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Si
n[(c + d*x)/2]] - Log[2 + Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/
2]]) + 4*Sqrt[2]*(114*Sin[(c + d*x)/2] - 7*Sin[(3*(c + d*x))/2] + 75*Sin[(5
*(c + d*x))/2]))))/(384*Sqrt[2]*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.17, size = 254, normalized size = 1.59

method	result
default	$-\frac{(-1+\cos(dx+c))\left(75\left(\cos^3(dx+c)\right)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-75\left(\cos^3(dx+c)\right)\arctan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(75*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/
2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)-75*cos(d*x+c)^3*arctan(1/4*(-
2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)+150*sin(
d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+68*cos(d*x+c)*sin(d*x+c)*(-2/
(1+cos(d*x+c)))^(1/2)+16*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/cos(d*x+c)/sin(d*x+c)^2/(-2/(1
+cos(d*x+c)))^(1/2)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3469 vs. $2(134) = 268$.
time = 0.79, size = 3469, normalized size = 21.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^

```
2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2
+ 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*
a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) +
a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*
sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*l
og(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(
6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*...
```

Fricas [A]
time = 3.02, size = 420, normalized size = 2.62

$$\left[\frac{75\,\left(a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\,\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\,\left(75\,a^2\cos(dx+c)^2+34\,a^2\cos(dx+c)+8\,a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{96\,\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)},\ \frac{75\,\left(a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)+\frac{2\,\left(75\,a^2\cos(dx+c)^2+34\,a^2\cos(dx+c)+8\,a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{48\,\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(75*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(75*a^2*cos(d*x + c)^2 + 34*a^2*c
os(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(75*(a^2*cos
(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
```

```
 - a*cos(d*x + c) - 2*a)) + 2*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c)
+ 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)
```

3.235 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=120

$$\frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,s}{2d}$$

```
[Out] 19/4*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+9/4*a^3*s
ec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)^(3/2
)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3899, 4101, 3886, 221}

$$\frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
 + (9*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
 (a^2*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}a\int \sqrt{\sec(}$$

$$= \frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}{2d}$$

$$= \frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}{2d}$$

$$= \frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c}{4d\sqrt{a+a\sec(c+}}$$

Mathematica [A]
time = 0.47, size = 106, normalized size = 0.88

$$\frac{a^3\left(-19\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)+2\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+11\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^3*(-19*ArcSin[Sqrt[Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x
]^(3/2) + 11*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(4*d*
Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(100) = 200$.

time = 0.16, size = 226, normalized size = 1.88

method	result
default	$\frac{\left(19(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-19(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\sqrt{-\frac{}{1+\cos}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(19*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos
(d*x+c)+sin(d*x+c))*2^(1/2))-19*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(
d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+22*cos(d*x+c)*sin(d*x+c)*
(-2/(1+cos(d*x+c)))^(1/2)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos
(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/s
in(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)^2-1)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2826 vs. $2(100) = 200$.

time = 3.31, size = 2826, normalized size = 23.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a^
2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 - 7
6*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
```

+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)
^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)
- 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
- 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2
*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22
*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)

```
 + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*
(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) +
 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2...
```

Fricas [A]
time = 2.77, size = 386, normalized size = 3.22

$$\left[\frac{19 \left(a^2 \cos\left(dx+c\right)^2 + a^2 \cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a \cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - \frac{4\left(\cos(dx+c)^2 - 2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}} + 8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right) + \frac{4\left(11\,a^2\cos(dx+c)+2\,a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{16\left(d\cos\left(dx+c\right)^2 + d\cos\left(dx+c\right)\right)}, \frac{19\left(a^2\cos\left(dx+c\right)^2 + a^2\cos\left(dx+c\right)\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2 - a\cos(dx+c) - 2\,a}\right) + \frac{2\left(11\,a^2\cos(dx+c)+2\,a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{8\left(d\cos\left(dx+c\right)^2 + d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x +
c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c)^2 + d*cos(d*x + c)), 1/8*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c
))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(
11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

3.236 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=112

$$\frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] 5*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^3*sin(d*x+
c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+a^2*sin(d*x+c)*sec(d*x+c)^(1/2
)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3899, 4100, 3886, 221}

$$\frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (5*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqr
t[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
```

```
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^2\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} + a\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c}}$$

$$= \frac{a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+}{d}$$

$$= \frac{a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+}{d}$$

$$= \frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{s}}{}$$

Mathematica [A]
time = 0.71, size = 91, normalized size = 0.81

$$\frac{a^3\left(5\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)+(1+2\cos(c+dx))\sqrt{(-1+\cos(c+dx))\sec^2(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]] + (1 + 2*Cos[c + d*x])*Sqrt[(-1 + Co
s[c + d*x])*Sec[c + d*x]^2])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(98) = 196$.
time = 0.15, size = 199, normalized size = 1.78

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(5\cos(dx+c)\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*cos(d*x+c)*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d
*x+c))*2^(1/2))*2^(1/2)-5*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*a
rctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1
/2)+8*cos(d*x+c)^2-4*cos(d*x+c)-4)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 11494 vs. $2(98) = 196$.
time = 0.65, size = 11494, normalized size = 102.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^2*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c) + 16*a^2*cos(1/2*d*x
 + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^3 + 8*a^2*sin(1/2*d*x + 1/2*c)^5 + 5*(sqrt
(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^4 + 10*(sqrt(2)*a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s
```

in(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 5
*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)
*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*c)^4 + (8*a^2*sin
(1/2*d*x + 1/2*c)^3 + (5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) + 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a^2
*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 5*(sqrt(2)*a^2*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2 + (5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) + 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 8*a^2*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 5*(sqrt(2)
*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*c)^2 + 2*(8*a^2*cos(1/2*d*
x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*
sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si

```
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)...
```

Fricas [A]
time = 1.95, size = 346, normalized size = 3.09

$$\left[\frac{5\left(a^{2}\cos\left(dx+c\right)+a^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+\frac{4\left(2\,a^{2}\cos(dx+c)+a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{4\left(d\cos\left(dx+c\right)+d\right)}, \frac{5\left(a^{2}\cos\left(dx+c\right)+a^{2}\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)+\frac{2\left(2\,a^{2}\cos(dx+c)+a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{2\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] [1/4*(5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(
5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*co
s(d*x + c) - 2*a)) + 2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

3.237 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+14/3*a^3*sin
(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*sin(d*x+c)*(a+a*s
ec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3898, 4100, 3886, 221}

$$\frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
4*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
```

```
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}(2a)\int\frac{\sqrt{a+a\sec(c+dx)}\,\left(\frac{7a}{2}+\frac{3}{2}a\right.}{\sqrt{\sec(c+dx)}}$$

$$=\frac{14a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+a^2\int$$

$$(2a^2$$

$$=\frac{14a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-$$

$$=\frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{14a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2}{}$$

Mathematica [A]
time = 0.45, size = 103, normalized size = 0.87

$$\frac{2a^3\left(3\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sec^{\frac{3}{2}}(c+dx)+\sqrt{1-\sec(c+dx)}\,(1+8\sec(c+dx))\right)\sin(c+dx)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^3*(3*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2) + Sqrt[1 - Sec[
c + d*x]]*(1 + 8*Sec[c + d*x]))*Sin[c + d*x])/(3*d*Sqrt[-((-1 + Sec[c + d*x
])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.15, size = 195, normalized size = 1.65

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*2^(1/2)*arctan(1/4*(-2/(1+cos
(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-s
in(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2+28*
cos(d*x+c)-32)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(100) = 200$.
time = 0.59, size = 593, normalized size = 5.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*ar
```

```
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*x
 + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*sqrt(a)/d
```

Fricas [A]
time = 2.64, size = 364, normalized size = 3.08

$$\left[\frac{3\,{\left(a^{2} \cos\left(dx + c\right) + a^{2}\right)} \sqrt{a} \log\left(\frac{a \cos\left(dx + c\right)^{3} - 7\,a \cos\left(dx + c\right)^{2} - \frac{4\,{\left(\cos\left(dx + c\right)^{2} - 2\,\cos\left(dx + c\right)\right)} \sqrt{a} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \sin\left(dx + c\right)}{\sqrt{\cos\left(dx + c\right)}} + 8\,a}{\cos\left(dx + c\right)^{3} + \cos\left(dx + c\right)^{2}}\right) + \frac{4\,{\left(a^{2} \cos\left(dx + c\right)^{2} + 8\,a^{2} \cos\left(dx + c\right)\right)} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \sin\left(dx + c\right)}{\sqrt{\cos\left(dx + c\right)}}}{6\,{\left(d \cos\left(dx + c\right) + d\right)}}, \frac{3\,{\left(a^{2} \cos\left(dx + c\right) + a^{2}\right)} \sqrt{-a} \arctan\left(\frac{2\,\sqrt{-a} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \sqrt{\cos\left(dx + c\right)} \sin\left(dx + c\right)}{a \cos\left(dx + c\right)^{2} - a \cos\left(dx + c\right) - 2\,a}\right) + \frac{2\,{\left(a^{2} \cos\left(dx + c\right)^{2} + 8\,a^{2} \cos\left(dx + c\right)\right)} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \sin\left(dx + c\right)}{\sqrt{\cos\left(dx + c\right)}}}{3\,{\left(d \cos\left(dx + c\right) + d\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c) + d), 1/3*(3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c) + d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

3.238 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{64a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/5*a*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+64/15*a^3*sin(d*
x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+16/15*a^2*sin(d*x+c)*(a+a*se
c(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3894, 3889}

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (64*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) +
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(8a)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$
$$= \frac{64a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a}{}$$

Mathematica [A]
time = 0.33, size = 64, normalized size = 0.54

$$\frac{a^2(89+28\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
[Out] (a^2*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]
*Tan[(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.13, size = 85, normalized size = 0.71

method	result	size
default	$-\frac{2\left(3\left(\cos^3(dx+c)\right)+11\left(\cos^2(dx+c)\right)+29\cos(dx+c)-43\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\cos^3(dx+c)\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}a^2}{15d\sin(dx+c)}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/15/d*(3*cos(d*x+c)^3+11*cos(d*x+c)^2+29*cos(d*x+c)-43)*(a*(1+cos(d*x+c))
/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)*a^2
```

Maxima [A]
time = 0.54, size = 60, normalized size = 0.50

$$\frac{\left(3\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}\,a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A]
time = 2.46, size = 87, normalized size = 0.73

$$\frac{2\left(3a^{2}\cos\left(dx+c\right)^{3}+14a^{2}\cos\left(dx+c\right)^{2}+43a^{2}\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 43*a^2*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*s
qrt(cos(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.83, size = 85, normalized size = 0.71

$$\frac{a^2\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(175\,\sin\left(c+d\,x\right)+28\,\sin\left(2\,c+2\,d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)\right)}{30\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2)*(175*sin(c + d*x) + 28*sin(2*c + 2*d*x) + 3*sin(3*c + 3*d*x)))/(3
0*d*(cos(c + d*x) + 1))
```

3.239 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{64a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{21d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/7*a*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*(a+a*sec(d*x
+c))^(5/2)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+64/21*a^3*sin(d*x+c)*sec(d*x+c)^(1
/2)/d/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/
sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3897, 3894, 3889}

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (64*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) +
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(3/2)) + (2
*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 3897

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Dist[a*(m/(b*d*(m + 1))), Int[(a + b*Csc[e
+ f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{5}{7}\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + $$
$$= \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + $$
$$= \frac{64a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + $$

Mathematica [A]
time = 0.35, size = 74, normalized size = 0.47

$$\frac{a^2(208+101\cos(c+dx)+24\cos(2(c+dx))+3\cos(3(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{42d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
[Out] (a^2*(208 + 101*Cos[c + d*x] + 24*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)])*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(42*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 0.15, size = 95, normalized size = 0.61

method	result	siz
default	$-\frac{2\left(3\left(\cos^4(dx+c)\right)+9\left(\cos^3(dx+c)\right)+11\left(\cos^2(dx+c)\right)+23\cos(dx+c)-46\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(\cos^4(dx+c)\right)a^2}{21d\sin(dx+c)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(3*cos(d*x+c)^4+9*cos(d*x+c)^3+11*cos(d*x+c)^2+23*cos(d*x+c)-46)*(a
*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^4/sin(d*x
+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(132) = 264$.
time = 0.57, size = 323, normalized size = 2.07

$$\frac{\sqrt{2}\left(315\,a^2\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+77\,a^2\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21\,a^2\cos\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)-315\,a^2\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-77\,a^2\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-21\,a^2\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+6\,a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21\,a^2\sin\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+77\,a^2\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+315\,a^2\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\right)\sqrt{a}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*cos
(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*s
in(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 4.13, size = 100, normalized size = 0.64

$$\frac{2\left(3\,a^2\cos\left(dx+c\right)^4+12\,a^2\cos\left(dx+c\right)^3+23\,a^2\cos\left(dx+c\right)^2+46\,a^2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^4 + 12*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2
+ 46*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

Mupad [**B**]
time = 2.32, size = 96, normalized size = 0.62

$$\frac{a^2\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(392\,\sin\left(c+d\,x\right)+98\,\sin\left(2\,c+2\,d\,x\right)+24\,\sin\left(3\,c+3\,d\,x\right)+3\,\sin\left(4\,c+4\,d\,x\right)\right)}{84\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2)*(392*sin(c + d*x) + 98*sin(2*c + 2*d*x) + 24*sin(3*c + 3*d*x) + 3
*sin(4*c + 4*d*x)))/(84*d*(cos(c + d*x) + 1))
```

3.240 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{584a^3\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 38/63*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+146/105*a^3*
sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+584/315*a^3*sin(d*x+c)
/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+1168/315*a^3*sin(d*x+c)*sec(d*x+
c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)
/d/sec(d*x+c)^(7/2)
```

Rubi [A]
time = 0.25, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3898, 4100, 3890, 3889}

$$\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{1168a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{584a^3\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (38*a^3*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) +
(146*a^3*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
+ (584*a^3*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
) + (1168*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{1}{9}(2a)\int \frac{\sqrt{a+a\sec(c+dx)}\,\left(\frac{19a}{2}+\frac{15}{2}\right.}{\sec^{\frac{7}{2}}(c+dx)}$$
$$= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$
$$= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A]
time = 0.58, size = 80, normalized size = 0.40

$$\frac{2a^3(35+130\sec(c+dx)+219\sec^2(c+dx)+292\sec^3(c+dx)+584\sec^4(c+dx))\sin(c+dx)}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(35 + 130*Sec[c + d*x] + 219*Sec[c + d*x]^2 + 292*Sec[c + d*x]^3 + 5
84*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[
c + d*x])])
```

Maple [A]
time = 0.15, size = 105, normalized size = 0.52

method	result
default	$-\frac{2\left(35\left(\cos^{5}\left(dx+c\right)\right)+95\left(\cos^{4}\left(dx+c\right)\right)+89\left(\cos^{3}\left(dx+c\right)\right)+73\left(\cos^{2}\left(dx+c\right)\right)+292\cos\left(dx+c\right)-584\right)\sqrt{\frac{a\left(1+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\,\left(\frac{1}{\cos\left(dx+c\right)}\right)}{315d\sin\left(dx+c\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*(35*cos(d*x+c)^5+95*cos(d*x+c)^4+89*cos(d*x+c)^3+73*cos(d*x+c)^2+2
92*cos(d*x+c)-584)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)
*cos(d*x+c)^5/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(171) = 342.
time = 0.56, size = 422, normalized size = 2.10

$$\frac{\sqrt{2}\left(8190\,a^{2}\cos\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+2100\,a^{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+756\,a^{2}\cos\left(\frac{4}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+225\,a^{2}\cos\left(\frac{2}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)-8190\,a^{2}\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\sin\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)-2100\,a^{2}\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)-756\,a^{2}\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\sin\left(\frac{4}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)-225\,a^{2}\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\sin\left(\frac{2}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+70\,a^{2}\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+225\,a^{2}\sin\left(\frac{7}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+756\,a^{2}\sin\left(\frac{5}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+2100\,a^{2}\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+8190\,a^{2}\sin\left(\frac{1}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\right)\sqrt{a}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/
2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a^
2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
+ 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)
*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos
(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos
(9/2*d*x + 9/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 3.51, size = 113, normalized size = 0.56

$$\frac{2\left(35\,a^{2}\cos\left(dx+c\right)^{5}+130\,a^{2}\cos\left(dx+c\right)^{4}+219\,a^{2}\cos\left(dx+c\right)^{3}+292\,a^{2}\cos\left(dx+c\right)^{2}+584\,a^{2}\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^5 + 130*a^2*cos(d*x + c)^4 + 219*a^2*cos(d*x + c
)^3 + 292*a^2*cos(d*x + c)^2 + 584*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

Mupad [**B**]
time = 2.96, size = 107, normalized size = 0.53

$$\frac{a^2\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(10290\,\sin\left(c+d\,x\right)+2856\,\sin\left(2\,c+2\,d\,x\right)+981\,\sin\left(3\,c+3\,d\,x\right)+260\,\sin\left(4\,c+4\,d\,x\right)+35\,\sin\left(5\,c+5\,d\,x\right)\right)}{2520\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2)*(10290*sin(c + d*x) + 2856*sin(2*c + 2*d*x) + 981*sin(3*c + 3*d*x
) + 260*sin(4*c + 4*d*x) + 35*sin(5*c + 5*d*x)))/(2520*d*(cos(c + d*x) + 1)
)
```

3.241 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=241

$$\frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{284a^3\sin(c+dx)}{231d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 46/99*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+710/693*a^3*
sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+284/231*a^3*sin(d*x+c)
/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1136/693*a^3*sin(d*x+c)/d/sec(d*
x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2272/693*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/
d/(a+a*sec(d*x+c))^(1/2)+2/11*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d
*x+c)^(9/2)
```

Rubi [A]
time = 0.29, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3898, 4100, 3890, 3889}

$$\frac{284a^3\sin(c+dx)}{231d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2272a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{693d\sqrt{a\sec(c+dx)+a}}+\frac{1136a^3\sin(c+dx)}{693d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] (46*a^3*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) +
(710*a^3*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]])
+ (284*a^3*Sin[c + d*x])/(231*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
) + (1136*a^3*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*
x]]) + (2272*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(11*d*Sec[c + d*x]^
(9/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{1}{11}(2a)\int\frac{\sqrt{a+a\sec(c+dx)}\left(\frac{23a}{2}+\ldots\right.}{\sec^{\frac{9}{2}}(c+dx)}\\ &= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}\ldots\\ &= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\ldots\\ &= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\ldots\\ &= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\ldots\\ &= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\ldots\end{aligned}$$

Mathematica [A]
time = 0.37, size = 90, normalized size = 0.37

$$\frac{2a^3(63+224\sec(c+dx)+355\sec^2(c+dx)+426\sec^3(c+dx)+568\sec^4(c+dx)+1136\sec^5(c+dx))\sin(c+dx)}{693d\sec^{\frac{9}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]

[Out] (2*a^3*(63 + 224*Sec[c + d*x] + 355*Sec[c + d*x]^2 + 426*Sec[c + d*x]^3 + 568*Sec[c + d*x]^4 + 1136*Sec[c + d*x]^5)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(9/2)*Sqrt[a*(1 + Sec[c + d*x])])

Maple [A]
time = 0.16, size = 115, normalized size = 0.48

method	result
default	$-\frac{2\left(63\left(\cos^{6}\left(dx+c\right)\right)+161\left(\cos^{5}\left(dx+c\right)\right)+131\left(\cos^{4}\left(dx+c\right)\right)+71\left(\cos^{3}\left(dx+c\right)\right)+142\left(\cos^{2}\left(dx+c\right)\right)+568\cos\left(dx+c\right)-1136\right)\sqrt{\frac{a\left(1+\cos\right.}{\cos(d}}}{693d\sin\left(dx+c\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)

[Out] -2/693/d*(63*cos(d*x+c)^6+161*cos(d*x+c)^5+131*cos(d*x+c)^4+71*cos(d*x+c)^3+142*cos(d*x+c)^2+568*cos(d*x+c)-1136)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^6*(1/cos(d*x+c))^(11/2)/sin(d*x+c)*a^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(205) = 410$.
time = 0.57, size = 521, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")

[Out] 1/22176*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 3465*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 31878*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2*c)

```
+ 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))
 + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)
)) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
11/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 3.28, size = 126, normalized size = 0.52

$$\frac{2\,\left(63\,a^{2}\cos\left(dx+c\right)^{6}+224\,a^{2}\cos\left(dx+c\right)^{5}+355\,a^{2}\cos\left(dx+c\right)^{4}+426\,a^{2}\cos\left(dx+c\right)^{3}+568\,a^{2}\cos\left(dx+c\right)^{2}+1136\,a^{2}\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{693\,\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^2*cos(d*x + c)^6 + 224*a^2*cos(d*x + c)^5 + 355*a^2*cos(d*x + c
)^4 + 426*a^2*cos(d*x + c)^3 + 568*a^2*cos(d*x + c)^2 + 1136*a^2*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) +
 d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/2), x)
```

Mupad [B]
time = 6.93, size = 356, normalized size = 1.48

$$\frac{\sqrt{a-\frac{a}{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1}}\,\left(2\sin\left(\frac{11c}{4}+\frac{11dx}{4}\right)^2+\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)\,1\mathrm{i}-1\right)\left(\frac{23a^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-2\sin\left(\frac{11c}{4}+\frac{11dx}{4}\right)^2+\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)1\mathrm{i}+1\right)}{4d}+\frac{19a^2\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)\left(-2\sin\left(\frac{11c}{4}+\frac{11dx}{4}\right)^2+\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)1\mathrm{i}+1\right)}{12d}+\frac{5a^2\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)\left(-2\sin\left(\frac{11c}{4}+\frac{11dx}{4}\right)^2+\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)1\mathrm{i}+1\right)}{8d}+\frac{13a^2\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)\left(-2\sin\left(\frac{11c}{4}+\frac{11dx}{4}\right)^2+\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)1\mathrm{i}+1\right)}{56d}+\frac{5a^2\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)\left(-2\sin\left(\frac{11c}{4}+\frac{11dx}{4}\right)^2+\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)1\mathrm{i}+1\right)}{72d}+\frac{a^2\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)\left(-2\sin\left(\frac{11c}{4}+\frac{11dx}{4}\right)^2+\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)1\mathrm{i}+1\right)}{88d}\right)}{2\sqrt{-\frac{1}{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1}}\,\left(2\sin\left(\frac{c}{4}+\frac{dx}{4}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(11/2),x)
```

```
[Out] ((a - a/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(sin((11*c)/2 + (11*d*x)/2)*1i
+ 2*sin((11*c)/4 + (11*d*x)/4)^2 - 1)*((23*a^2*sin(c/2 + (d*x)/2)*(sin((11*
c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(4*d) + (19*a^
2*sin((3*c)/2 + (3*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4
+ (11*d*x)/4)^2 + 1))/(12*d) + (5*a^2*sin((5*c)/2 + (5*d*x)/2)*(sin((11*c)/
2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(8*d) + (13*a^2*s
in((7*c)/2 + (7*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (
11*d*x)/4)^2 + 1))/(56*d) + (5*a^2*sin((9*c)/2 + (9*d*x)/2)*(sin((11*c)/2 +
 (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(72*d) + (a^2*sin((1
1*c)/2 + (11*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*
d*x)/4)^2 + 1))/(88*d)))/(2*(-1/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2*sin(
c/4 + (d*x)/4)^2 - 1))
```

3.242 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=38

$$\frac{4a^2\sec^{\frac{3}{4}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 4*a^2*sec(d*x+c)^(3/4)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3899, 8}

$$\frac{4a^2\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
```

```
[Out] (4*a^2*Sec[c + d*x]^(3/4)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx &= \frac{4a^2\sec^{\frac{3}{4}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(4a)\int 0\,dx\\ &= \frac{4a^2\sec^{\frac{3}{4}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 1.18

$$\frac{4\sec^{\frac{3}{4}}(c+dx)(a(1+\sec(c+dx)))^{3/2}\sin(c+dx)}{d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
[Out] (4*Sec[c + d*x]^(3/4)*(a*(1 + Sec[c + d*x]))^(3/2)*Sin[c + d*x])/(d*(1 + Se
c[c + d*x])^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
[Out] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(34) = 68.
time = 0.51, size = 121, normalized size = 3.18

$$\frac{4\left(\frac{\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="maxima")
[Out] 4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^(1/4))
```

Fricas [A]
time = 2.62, size = 50, normalized size = 1.32

$$\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)^{\frac{1}{4}}\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)^(1/4)*sin(d*x + c)
/(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/4),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/4), x)
```

Mupad [**B**]
time = 0.75, size = 55, normalized size = 1.45

$$\frac{2\,a\,\sin\left(2\,c+2\,d\,x\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/4}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/4),x)
```

```
[Out] (2*a*sin(2*c + 2*d*x)*(1/cos(c + d*x))^(3/4)*((a*(cos(c + d*x) + 1))/cos(c
+ d*x))^(1/2))/(d*(cos(c + d*x) + 1))
```

3.243 $\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3886, 221}

$$\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 54, normalized size = 1.46

$$-\frac{2\mathrm{ArcSin}\left(\sqrt{\sec(e+fx)}\right)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (-2*ArcSin[Sqrt[Sec[e + f*x]]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])
/(f*Sqrt[1 - Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(31) = 62.
time = 0.58, size = 147, normalized size = 3.97

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\sqrt{\frac{1}{\cos(fx+e)}}\cos(fx+e)(-1+\cos(fx+e))\left(\arctan\left(\frac{\sqrt{-\frac{2}{\cos(fx+e)+1}}(1+\cos(fx+e)-\sin(fx+e))\sqrt{2}}{4}\right)-a\right.}{f\sqrt{-\frac{2}{\cos(fx+e)+1}}\sin(fx+e)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(1/cos(f*x+e))^(1/2)*cos(f*x+e)*(-1
+cos(f*x+e))*(arctan(1/4*(-2/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)-sin(f*x+e)
)*2^(1/2))-arctan(1/4*(-2/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)+sin(f*x+e))*2
^(1/2)))/(-2/(cos(f*x+e)+1))^(1/2)/sin(f*x+e)^2*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(33) = 66.
time = 0.56, size = 257, normalized size = 6.95

$$\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\sqrt{2}\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\sqrt{2}\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right)+\log\left(2\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\sqrt{2}\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\sqrt{2}\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*sq
rt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) - log(2*co
s(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*sqrt(2)*cos(1/2*f*x + 1
/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) + log(2*cos(1/2*f*x + 1/2*e)^2
```

```
+ 2*sin(1/2*f*x + 1/2*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2)*sin
(1/2*f*x + 1/2*e) + 2) - log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2
*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2
))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(33) = 66$.
time = 2.91, size = 205, normalized size = 5.54

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2-\frac{4\left(\cos(fx+e)^2-2\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{\sqrt{\cos(fx+e)}}+8a}{\cos(fx+e)^3+\cos(fx+e)^2}\right)}{2f}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\cos(fx+e)}\sin(fx+e)}{a\cos(fx+e)^2-a\cos(fx+e)-2a}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2
 - 2*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e)/sqrt(cos(f*x + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))/f, sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x +
 e))*sin(f*x + e)/(a*cos(f*x + e)^2 - a*cos(f*x + e) - 2*a))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sqrt(sec(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{1}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^(1/2), x)
```

3.244 $\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

[Out] 2*arcsinh(a^(1/2)*tan(f*x+e)/(a-a*sec(f*x+e))^(1/2))*a^(1/2)/f

Rubi [A]

time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3886, 221}

$$\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]

[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a - a*Sec[e + f*x]]])/f

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \frac{a\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.93, size = 299, normalized size = 7.87

$$\frac{\csc\left(\frac{1}{2}(e+fx)\right)\left(-2i\mathrm{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(e+fx)\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{1}{4}(e+fx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e+fx)\right)}\right)+2i\mathrm{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(e+fx)\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{1}{4}(e+fx)\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e+fx)\right)}\right)-4\tanh^{-1}\left(\sqrt{2}\cos\left(\frac{1}{4}(2e+fx)\right)\sec\left(\frac{fx}{4}\right)+\tan\left(\frac{fx}{4}\right)\right)+\log\left(2-\sqrt{2}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(2+\sqrt{2}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sqrt{a-a\sec(e+fx)}}{2\sqrt{2}f\sqrt{-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]*((-2*I)*ArcTan[(Cos[(e + f*x)/4] - (-1 + Sqrt[2])*Sin[(e
+ f*x)/4])/((1 + Sqrt[2])*Cos[(e + f*x)/4] - Sin[(e + f*x)/4])] + (2*I)*Arc
Tan[(Cos[(e + f*x)/4] - (1 + Sqrt[2])*Sin[(e + f*x)/4])/((-1 + Sqrt[2])*Cos
[(e + f*x)/4] - Sin[(e + f*x)/4])] - 4*ArcTanh[Sqrt[2]*Cos[(2*e + f*x)/4]*S
ec[(f*x)/4] + Tan[(f*x)/4]] + Log[2 - Sqrt[2]*Cos[(e + f*x)/2] - Sqrt[2]*Si
n[(e + f*x)/2]] - Log[2 + Sqrt[2]*Cos[(e + f*x)/2] - Sqrt[2]*Sin[(e + f*x)/
2]])*Sqrt[a - a*Sec[e + f*x]])/(2*Sqrt[2]*f*Sqrt[-Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(32) = 64$.
time = 0.21, size = 124, normalized size = 3.26

method	result
default	$\frac{\left(\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{\cos(fx+e)+1}}\,(1+\cos(fx+e)-\sin(fx+e))}{2}\right)+\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{\cos(fx+e)+1}}\,(1+\cos(fx+e)+\sin(fx+e))}{2}\right)\right)\cos(fx+e)\sqrt{-}}{f\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(arctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)-sin(f*x+e)))+arctan
h(1/2*(1/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)+sin(f*x+e))))*cos(f*x+e)*(-1/c
os(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/(1/(cos(f*
x+e)+1))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(34) = 68$.
time = 0.59, size = 385, normalized size = 10.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*(log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(
```

```
f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2) + log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(34) = 68$.
time = 2.62, size = 233, normalized size = 6.13

$$\left[\frac{\sqrt{a}\log\left(\frac{4\left(\cos(fx+e)^3+3\cos(fx+e)^2+2\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)-a}{\cos(fx+e)}}\sqrt{-\frac{1}{\cos(fx+e)}}+\left(a\cos(fx+e)^2+8a\cos(fx+e)+8a\right)\sin(fx+e)}{\cos(fx+e)^2\sin(fx+e)}\right)}{2f}, -\frac{\sqrt{-a}\arctan\left(\frac{2\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)-a}{\cos(fx+e)}}\sqrt{-\frac{1}{\cos(fx+e)}}}{(a\cos(fx+e)+2a)\sin(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((4*(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 2*cos(f*x + e))*sq
rt(a)*sqrt((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e)) + (a*co
s(f*x + e)^2 + 8*a*cos(f*x + e) + 8*a)*sin(f*x + e))/(cos(f*x + e)^2*sin(f*
x + e)))/f, -sqrt(-a)*arctan(2*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-a)*sqr
t((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e))/((a*cos(f*x + e)
 + 2*a)*sin(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sec(e+fx)}\,\sqrt{-a(\sec(e+fx)-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**(1/2)*(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-sec(e + f*x))*sqrt(-a*(sec(e + f*x) - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(32) = 64$.

time = 0.80, size = 98, normalized size = 2.58

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)/
sqrt(-a))/sqrt(-a) - sqrt(2)*a^2*arctan(sqrt(a)/sqrt(-a))/sqrt(-a))*abs(a)*
sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(a^2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{-\frac{1}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^(1/2), x)
```

3.245 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/
d/a^(1/2)+sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3907, 4108, 3893, 212, 3886, 221}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/(Sqrt[a]*d)) + (
Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])]])/(Sqrt[a]*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[d^2/(b*(2*n - 3)), Int[(d
*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\,(a-a\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx}{2a} + \int \frac{\sqrt{}}{\sqrt{a}}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [**A**]
time = 0.26, size = 125, normalized size = 0.98

$$\frac{\left(\operatorname{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)+2\operatorname{ArcSin}\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*A
rcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + S
ec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(107) = 214.
time = 0.17, size = 222, normalized size = 1.73

method	result
default	$\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^2(dx+c))\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c}{4}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)
*(-1+cos(d*x+c))*(cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*
x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)-cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))
^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)-4*cos(d*x+c)*arctan(1/2*s
in(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)
)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/a
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 876 vs. 2(107) = 214.
time = 0.59, size = 876, normalized size = 6.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
```

```
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
+ 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)
```

Fricas [A]
time = 4.53, size = 481, normalized size = 3.76

$$\left[\frac{\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{a\cos\left(dx+c\right)^{3}-7\,a\cos\left(dx+c\right)^{2}+\frac{4\left(\cos\left(dx+c\right)^{2}-2\,\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)+\frac{2\sqrt{2}\left(a\cos\left(dx+c\right)+a\right)\log\left(-\frac{\cos\left(dx+c\right)^{2}-\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{\sqrt{a}}-2\,\cos\left(dx+c\right)-3}{\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1}\right)}{\sqrt{a}}+\frac{4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{4\left(ad\cos\left(dx+c\right)+ad\right)},-\frac{2\sqrt{2}\left(a\cos\left(dx+c\right)+a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\sqrt{\cos\left(dx+c\right)}}{\sin\left(dx+c\right)}\right)+\sqrt{-a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-2\,a}\right)-\frac{2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{2\left(ad\cos\left(dx+c\right)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/s
qrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a)
 + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
```

```
))/(a*d*cos(d*x + c) + a*d), -1/2*(2*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a
)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(co
s(d*x + c))/sin(d*x + c)) + sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*c
os(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.246 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)
/d/a^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3906, 3886, 221, 3893, 212}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3906

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e
 + f*x]], x], x] - Dist[a*(d/b), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx}{a} - \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx\right.}{}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.12, size = 89, normalized size = 0.94

$$\frac{\left(-2\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*
(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(78) = 156$.
time = 0.14, size = 184, normalized size = 1.94

method	result
default	$\frac{\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c))}{4}\right.\right.}{2d\,s}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c
))*2^(1/2))-2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(
d*x+c))*2^(1/2))-2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)*(-2/(1+cos(
d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(78) = 156$.
time = 0.58, size = 476, normalized size = 5.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

Fricas [A]
time = 3.06, size = 351, normalized size = 3.69

$$\left[\frac{\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\cos\left(dx+c\right)^{2}+\frac{2\,\sqrt{2}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{a}}-2\,\cos\left(dx+c\right)-3}{\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1}\right)+\sqrt{a}\,\log\left(\frac{a\cos\left(dx+c\right)^{3}-7\,a\cos\left(dx+c\right)^{2}-\frac{4\,\left(\cos\left(dx+c\right)^{2}-2\,\cos\left(dx+c\right)\right)\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)}{2\,ad},\ \frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{-\frac{1}{a}}\,\sqrt{\cos\left(dx+c\right)}}{\sin\left(dx+c\right)}\right)+\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-2\,a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c
) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arctan(sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/s
in(d*x + c)) + sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
 2*a)))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.247 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3893, 212}

$$\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.06, size = 75, normalized size = 1.34

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[
c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(45) = 90$.
time = 0.13, size = 99, normalized size = 1.77

method	result	s
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\cos(dx+c)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos^2(dx+c)-1\right)}{d\sin(dx+c)^2 a}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*cos(d*x+c)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*arc
tan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin
(d*x+c)^2*(cos(d*x+c)^2-1)/a
```

Maxima [A]
time = 0.54, size = 90, normalized size = 1.61

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{2\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Fricas [A]
time = 3.00, size = 160, normalized size = 2.86

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos\left(dx+c\right)^{2}-\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{\sqrt{a}}-2\cos\left(dx+c\right)-3}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)}{2\sqrt{a}\,d},\ -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\sqrt{\cos\left(dx+c\right)}}{\sin\left(dx+c\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x +
 c))/sin(d*x + c))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.248 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1
/2)
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3897, 3893, 212}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3897

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(m + 1))), x] + Dist[a*(m/(b*d*(m + 1))), Int[(a + b*Csc[e
+ f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c}}{\sqrt{a+}}\right.}{d}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sqrt{\sec(c+dx)}}{d\sqrt{a+a\sec(}}$$

Mathematica [A]
time = 0.20, size = 102, normalized size = 1.10

$$\frac{\left(\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\frac{2\sqrt{1-\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + (2*
Sqrt[1 - Sec[c + d*x]])/Sqrt[Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c
 + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.12, size = 102, normalized size = 1.10

method	result	size
default	$-\frac{\left(-\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)+2\cos(dx+c)-2\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{d\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}\,a}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+2*cos(d*x+c)-2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d
*x+c)/(1/cos(d*x+c))^(1/2)/a
```

Maxima [**A**]
time = 0.55, size = 104, normalized size = 1.12

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-4\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/(sq
rt(a)*d)
```

Fricas [**A**]
time = 3.19, size = 281, normalized size = 3.02

$$\left[\frac{\frac{\sqrt{2}\,(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^{2}+\frac{2\,\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\,\cos(dx+c)-3}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)}{\sqrt{a}}+4\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)},\ \frac{\sqrt{2}\,(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+2\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{ad\cos(dx+c)+ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) -
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*c
os(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(
d*x + c)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.249 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}-\frac{2\sqrt{\sec(c+dx)}\,\sin(c-}{3d\sqrt{a+a\sec(c+d}}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)+2/3*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(
1/2)-2/3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3908, 4098, 3893, 212}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]
*Sqrt[a + a*Sec[c + d*x]]) - (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[
a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
```

```
b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a
+ b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{a-2a\sec(c+dx}{\sqrt{\sec(c+dx)}\sqrt{a+a}}}{3a}$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+d}{3d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+d}{3d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c-}{3d\sqrt{\sec(c+dx)}\sqrt{a}}$$

Mathematica [A]
time = 0.26, size = 120, normalized size = 0.92

$$\frac{\left(2(-1+\cos(c+dx))\sqrt{1-\sec(c+dx)}-3\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*
Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x
])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])
])
```

Maple [A]
time = 0.15, size = 120, normalized size = 0.92

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\ \sin(dx+c)+2\left(\cos^{2}(dx+c)\right)-4\cos(dx+c)}{3d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2-4*c
os(d*x+c)+2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(108) = 216.
time = 0.57, size = 282, normalized size = 2.15

$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)-3\sqrt{2}\log\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^{2}+\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^{2}+2\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+1\right)+3\sqrt{2}\log\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^{2}+\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^{2}-2\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+1\right)-2\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sqrt{2}\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)}{6\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

Fricas [A]
time = 2.70, size = 318, normalized size = 2.43

$$\left[\frac{\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^{2}-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(\cos(dx+c)^{2}-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(ad\cos(dx+c)+ad\right)},\ -\frac{3\sqrt{2}\left(a\cos(dx+c)+a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\frac{2\left(\cos(dx+c)^{2}-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos(dx+c)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a)
- 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*(c
os(d*x + c)^2 - cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*(a*co
s(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(cos(d*x + c)^2 - co
s(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\ \sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.250 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=169

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}-\frac{2\sin(c+}{15d\sqrt{\sec(c+dx)}\sqrt{a}}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2/5*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^
(1/2)-2/15*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+26/15*sin(d
*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3908, 4107, 4098, 3893, 212}

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3
/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[
a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a
+ b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}}{5a}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a}}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a}}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{}}$$

Mathematica [A]
time = 1.13, size = 117, normalized size = 0.69

$$\frac{(29-2\cos(c+dx)+3\cos(2(c+dx)))\sqrt{\sec(c+dx)}\,\sin(c+dx)+\frac{15\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{\sqrt{1-\sec(c+dx)}}}{15d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((29 - 2*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
 + (15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*
Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.15, size = 130, normalized size = 0.77

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(6\left(\cos^3(dx+c)\right)-15\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-8\left(\cos^2(dx\right.\right.}{15d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(6*cos(d*x+c)^3-15*arctan(1/2*s
in(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8
*cos(d*x+c)^2+28*cos(d*x+c)-26)*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^3/sin(d*x+c
)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(140) = 280.
time = 0.57, size = 357, normalized size = 2.11

$$\frac{\sqrt{2}\left(60\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-60\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+5\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-30\log\left(\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)^2+\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)^2+2\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+1\right)+30\log\left(\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)^2+\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)^2-2\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+1\right)+6\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+60\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)}{60\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin
```

```
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

Fricas [A]
time = 2.71, size = 342, normalized size = 2.02

$$\left[\frac{\frac{15\sqrt{2}\left(a\cos(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(3\cos(dx+c)^3-\cos(dx+c)^2+13\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{30\left(ad\cos(dx+c)+ad\right)}, \frac{15\sqrt{2}\left(a\cos(dx+c)+a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\left(3\cos(dx+c)^3-\cos(dx+c)^2+13\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos(dx+c)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a
) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*
(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d
), 1/15*(15*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c))
 + 2*(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)
+ a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{\frac{5}{2}}\left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.251 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{3\,\mathrm{se}}{2a}$$

```
[Out] -3*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*sec(d*x
+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+9/4*arctanh(1/2*sin(d*x+c)*a^
(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+3/
2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3901, 4106, 4108, 3893, 212, 3886, 221}

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
 (9*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-3a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\,(-}{\sqrt{a+a\,\mathrm{se}}}}{2a}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{3\int \sqrt{\sec(c+dx)}}{}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{3}{}}}\right.}{}$$

$$= -\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}$$

Mathematica [A]
time = 0.60, size = 252, normalized size = 1.45

$$\frac{6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+4\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)-9\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)-9\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)+6\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\sec(c+dx))\tan(c+dx)+18\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\sec(c+dx))\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 4*Sqrt[1 - Sec[
c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[
Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqr
t[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x]
+ 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x] + 18*Arc
Sin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.15, size = 281, normalized size = 1.61

method	result

default	$\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}(\cos^3(dx+c))\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(3\cos(dx+c)\sin(dx+c)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cdots}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^3*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)
*(-1+cos(d*x+c))*(3*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x
+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-3*cos(d*x+c)*sin(d*x+c)*2^(1
/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))
+3*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-9*cos(d*x+c)*sin(d*x+c)*arctan(1/
2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2
)-2*(-2/(1+cos(d*x+c)))^(1/2))/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/a^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(143) = 286$.
time = 0.89, size = 4934, normalized size = 28.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(
2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqr
t(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sq
```

rt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*s
in(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqrt(
2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(
2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqr
t(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*
x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt
(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*
c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(
2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
 + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c)
+ 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +

```
 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +...
```

Fricas [A]
time = 2.57, size = 579, normalized size = 3.33

$$\left[\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+6\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(3\cos(dx+c)+2\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{8\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}, -\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+6\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)-\frac{2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(3\cos(dx+c)+2\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{4\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 6*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*sqrt(2)*(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 6*(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*
cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x
 + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d
*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.252 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*sec(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-5/4*arctanh(1/2*sin(d*x+c)*a^(
1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3901, 4108, 3893, 212, 3886, 221}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
(5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-2a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{5\int \cdots}{\cdots}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+d\cdots}}\right.}{a^2 d}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \cdots$$

Mathematica [A]
time = 0.54, size = 220, normalized size = 1.64

$$\frac{-2\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+5\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)+5\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)-2\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\sec(c+dx))\tan(c+dx)-10\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\sec(c+dx))\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 5*Sqrt[2]*ArcT
an[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 5*Sq
rt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d
*x]*Tan[c + d*x] - 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[
c + d*x] - 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(
4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(109) = 218$.
time = 0.15, size = 240, normalized size = 1.79

method	result
default	$\frac{\left(2\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sin(dx+c)-2\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x
+c))*2^(1/2))*sin(d*x+c)-2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+
cos(d*x+c)-sin(d*x+c))*2^(1/2))*sin(d*x+c)-5*arctan(1/2*sin(d*x+c)*(-2/(1+c
os(d*x+c)))^(1/2))*sin(d*x+c)+cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-(-2/(1+c
os(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)
*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. $2(109) = 218$.
time = 0.61, size = 2122, normalized size = 15.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*c
```

```
os(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
+ 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2
)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(
2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d
*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x +
2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)
```

```
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*s
in(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*s
in(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sq
rt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*
a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(109) = 218$.
time = 2.58, size = 559, normalized size = 4.17

$$\left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos\left(dx+c\right)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2a\cos\left(dx+c\right)-3a}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)+4\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^{3}-7a\cos\left(dx+c\right)^{2}-\frac{4\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}+8a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)-4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}, \frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin\left(dx+c\right)}\right)+4\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-2a}\right)-2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^
2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) -
 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.253 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+1/4*arctanh(1/2*si
n(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d
*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3895, 3893, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
```

```
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(97) = 194.
time = 0.53, size = 220, normalized size = 2.27

$$\frac{2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)+2\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\sec(c+dx))\tan(c+dx)+2\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\sec(c+dx))\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - Sqrt[2]
*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*T
an[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d
*x] + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sq
rt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.14, size = 146, normalized size = 1.51

method	result

default	$\frac{\left(\cos(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}-\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)-\sqrt{-\frac{2}{1+\cos(dx+c)}}\right)\sqrt{\frac{a(1+\cos(dx+c)}{\cos(dx+c)}}}{2d\sin(dx+c)^{3}\sqrt{-\frac{2}{1+\cos(dx+c)}}\, a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-arctan(1/2*sin(d*x+c)*(-2/(1+co
s(d*x+c)))^(1/2))*sin(d*x+c)-(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/c
os(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+
c)^3/(-2/(1+cos(d*x+c)))^(1/2)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15721 vs. $2(78) = 156$.
time = 1.19, size = 15721, normalized size = 162.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d
*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(
d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) +
3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3
/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3
*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1
)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*
x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(cos(3/2*d*x
+ 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + cos(d*x
 + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(3*d*x +
 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 9
6*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2*c)^2*sin(3/2*d
*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3
/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*
d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) +
 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3
```

```
/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*(2*(3*cos(d*x + c) + 1)*cos(2*
d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
 + 3*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 2*(3*sin(3/2*d*x + 3/2*c)*si
n(d*x + c) + cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) + (3*cos(d*x + c)^2 + 3
*sin(d*x + c)^2 + 2*cos(d*x + c))*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/
2*c)*sin(d*x + c))*cos(3*d*x + 3*c) - 4*(6*(sin(2*d*x + 2*c) + sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^3 + (2*(3*cos(2*d*x + 2*c) + 3*cos(
d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1
)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x
+ 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x
+ c) + 1)*sin(3*d*x + 3*c) + 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)
*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x +
 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin
(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 1
8*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*(2*(3*cos(2*
d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*
(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x +
c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3
*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x
 + c)^2 + 6*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*s
in(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*x + c)*sin(3/2*d*x + 3/2*c
) - 8*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*sin(2*d*x + 2*c)^2*sin(3
/2*d*x + 3/2*c) - 16*(3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 3*cos(3/2*d
*x + 3/2*c)*sin(d*x + c) - sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c) - 48*(cos
(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c
) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*
x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*
c)*sin(d*x + c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 16*(cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + 3*sin(2*d*x + 2*c)*sin(3/2
*d*x + 3/2*c) + 3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c))
*sin(3*d*x + 3*c) - 48*(3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x +
 3/2*c))*sin(2*d*x + 2*c) - 8*(9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 - 1)*sin
(3/2*d*x + 3/2*c) - 48*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 3*(2*(3*cos(2*d*
x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3
*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)
^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c
)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x +
 c)^2 + 6*cos(d*x + c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*sin
```

```
(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*...
```

Fricas [A]
time = 2.64, size = 338, normalized size = 3.48

$$\left[\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+4\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)},-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)-2\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/
4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*
x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.254 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] -1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+3/4*arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/
d*2^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3896, 3893, 212}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3896

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{3\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{2ad}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.19, size = 120, normalized size = 1.24

$$\frac{-2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx) - 3\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)(1+\sec(c+dx))\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 3*Sqrt[2]*ArcT
an[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*
Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.13, size = 146, normalized size = 1.51

method	result
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\cos(dx+c)\left(\cos(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}+3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx\right.}{4d\sin(dx+c)^3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(
cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x
+c)))^(1/2))*sin(d*x+c)-(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2
)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(78) = 156$.
time = 0.58, size = 1031, normalized size = 10.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*si
n(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
+ c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(
d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2*
sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c)
*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) +
 4*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(d
*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*sin
(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2*sq
rt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a)*d)
```

Fricas [A]

time = 3.49, size = 340, normalized size = 3.51

$$\left[\frac{3\,\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)-4\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{8\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)},-\frac{3\,\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+2\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -
1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*si
n(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.255 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -7/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+
c))^(3/2)+5/2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3902, 4098, 3893, 212}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
2*d*(a + a*Sec[c + d*x])^(3/2)) + (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{5a}{2}+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c}}}{2a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} -$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sqrt{\sec(c+dx)}\,\mathrm{si}}{2d(a+a\sec(c-}$$

Mathematica [A]
time = 0.47, size = 145, normalized size = 1.06

$$\frac{2\left(5\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+4\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sin(c+dx)+7\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)(1+\sec(c+dx))\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*(5*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*Sqrt[-((-1 + Sec[c + d*
x])*Sec[c + d*x])])*Sin[c + d*x] + 7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d
```

```
*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt[1
- Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.14, size = 175, normalized size = 1.28

method	result
default	$\frac{\left(-7\left(\cos^{2}(dx+c)\right)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}+8\left(\cos^{3}(dx+c)\right)+7\arctan\left(\frac{\sin(dx+c)\sqrt{\cdots}}{\cdots}\right.\right.}{4d\sin(dx+c)^{3}\sqrt{\frac{1}{\cos(dx+c}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(-7*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))
^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)+8*cos(d*x+c)^3+7*arctan(1/2*sin(d*x+c)*(-
2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-6*cos(d*x+c)^
2-12*cos(d*x+c)+10)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3/(1/cos
(d*x+c))^(1/2)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 7176 vs. 2(112) = 224.
time = 0.62, size = 7176, normalized size = 52.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)
^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) -
 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x
```

+ 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*
x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
c)^3 + 4(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/2
*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin(
1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/
2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*
cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(
1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2
+ 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*
cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*s
in(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/2
*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c
))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (
7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x

```
 + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x +
 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)
 - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + ...
```

Fricas [A]
time = 2.47, size = 378, normalized size = 2.76

$$\left[\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(4\cos(dx+c)^2+5\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{8\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}, \frac{7\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+\frac{2\left(4\cos(dx+c)^2+5\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{4\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*cos(d*x + c)^2 + 5*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.256 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}+\frac{7\sin(c+dx}{6ad\sqrt{\sec(c+dx)}\,\sqrt{a+}}$$

```
[Out] 11/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)/sec(d*x
+c)^(1/2)+7/6*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-19/6*s
in(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3902, 4107, 4098, 3893, 212}

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{19\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (11*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sqrt[Sec[c + d*x]
]*(a + a*Sec[c + d*x])^(3/2)) + (7*Sin[c + d*x])/(6*a*d*Sqrt[Sec[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) - (19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqr
t[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{7a}{2}+2a\sec(c+}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\text{se}}}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c-}{6ad\sqrt{\sec(c+dx)}\,\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c-}{6ad\sqrt{\sec(c+dx)}\,\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c-}{6ad\sqrt{\sec(c+dx)}\,\sqrt{}}$$

$$= \frac{11\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sin(c+}{2d\sqrt{\sec(c+dx)}\,(a+}$$

Mathematica [A]
time = 0.91, size = 150, normalized size = 0.85

$$\frac{-33\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+\sqrt{1-\sec(c+dx)}\,(4\sin(c+dx)-(12+19\sec(c+dx))\tan(c+dx))}{6d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
[Out] (-33*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(
4*Sin[c + d*x] - (12 + 19*Sec[c + d*x])*Tan[c + d*x]))/(6*d*Sqrt[-((-1 + Se
c[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.14, size = 193, normalized size = 1.09

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(33(\cos^2(dx+c))\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}+8(\cos^4(dx+c))-33\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/12/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(33*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)+8*cos(d*x+c)^4-33*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-40*cos(d*x+c)^3+18*cos(d*x+c)^2+52*cos(d*x+c)-38)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)^3/a^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33960 vs. $2(146) = 292$.
time = 0.97, size = 33960, normalized size = 191.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/12*(4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 4*sin(3/2*d*x + 3/2*c)^5 + 4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 4*(2*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 2*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 8*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sin(3/2*d*x + 3/2*c)^2 + 3)*sin(3*d*x + 3*c) + 20*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 7*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 18*(cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2 + cos

```
(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c))
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 32*(2*cos(3*d*x + 3*c)
^2*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/2*c)*sin(3
*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 2*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c
)*sin(3/2*d*x + 3/2*c) + 2*(sin(3/2*d*x + 3/2*c)^2 + 3)*sin(3*d*x + 3*c) +
20*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 7*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2
)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 18*(cos(3*
d*x + 3*c)^2*cos(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2
 + cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)*sin(3/2*d*x + 3
/2*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 4*(2*cos(3/2*d*x
 + 3/2*c)^2 + 7)*sin(3/2*d*x + 3/2*c)^3 + 4*((2*sin(3/2*d*x + 3/2*c)^2 + 1)
*cos(3*d*x + 3*c)^2 + (2*sin(3/2*d*x + 3/2*c)^2 + 1)*sin(3*d*x + 3*c)^2 + 2
*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 2*(sin(3/2*d*
x + 3/2*c)^2 + 3)*cos(3*d*x + 3*c) - 20*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3
*c)^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(co
s(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 8*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin
(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x +
3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 18*(cos(3*d*x + 3
*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + cos(
3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c))*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(7/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 32*((2*sin(3/2*d*x + 3/
2*c)^2 + 1)*cos(3*d*x + 3*c)^2 + (2*sin(3/2*d*x + 3/2*c)^2 + 1)*sin(3*d*x +
 3*c)^2 + 2*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 2*
(sin(3/2*d*x + 3/2*c)^2 + 3)*cos(3*d*x + 3*c) - 20*(cos(3*d*x + 3*c)^2 + si
n(3*d*x + 3*c)^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 7*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 18*(cos(3*d*x + 3*c)^2*sin(3/2*d*x +
3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin
(3*d*x + 3*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c))*sin(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^3 + 4*(4*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)^
2*sin(3/2*d*x + 3/2*c) + (sin(3/2*d*x + 3/2*c)^3 + (cos(3/2*d*x + 3/2*c)^2
+ 1)*sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c)^2 + 24*(cos(3*d*x + 3*c)^2*sin(
3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x +
 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + ...
```

Fricas [A]

time = 2.74, size = 398, normalized size = 2.25

$$\left[\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(4\cos(dx+c)^3-12\cos(dx+c)^2-19\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{24\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}, -\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)-\frac{2\left(4\cos(dx+c)^3-12\cos(dx+c)^2-19\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{12\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*sqrt(2)
*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) -
 2*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x +
c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.257 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}+\frac{9\sin(c+a}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a-}}$$

```
[Out] -1/2*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)-15/4*arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)
/d*2^(1/2)+9/10*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)-13/1
0*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+49/10*sin(d*x+c)*s
ec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3902, 4107, 4098, 3893, 212}

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{13\sin(c+dx)}{10ad\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sec[c + d*x]^(3/
2)*(a + a*Sec[c + d*x])^(3/2)) + (9*Sin[c + d*x])/(10*a*d*Sec[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]]) - (13*Sin[c + d*x])/(10*a*d*Sqrt[Sec[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{9a}{2}+3a\sec(c+}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sin(c}{2d\sec^{\frac{3}{2}}(c+dx)(a}$$

Mathematica [A]
time = 1.35, size = 163, normalized size = 0.75

$$\frac{75\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+(47+39\cos(c+dx)-2\cos(2(c+dx))+\cos(3(c+dx)))\sqrt{1-\sec(c+dx)}\sec(c+dx)\tan(c+dx)}{10d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos
[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + (47 + 39*Cos[c + d*x] - 2
*Cos[2*(c + d*x)] + Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*T
an[c + d*x])/(10*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c
+ d*x]))^(3/2))
```

Maple [A]
time = 0.16, size = 203, normalized size = 0.94

method	result

default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(8\left(\cos^5(dx+c)\right)-75\left(\cos^2(dx+c)\right)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}-24\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(8*cos(d*x+c)^5-75*cos(d*x+c)^2*
sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+
c)))^(1/2)-24*cos(d*x+c)^4+96*cos(d*x+c)^3+75*arctan(1/2*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-54*cos(d*x+c)^2-12
4*cos(d*x+c)+98)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^3/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.73, size = 418, normalized size = 1.93

$$\left[\frac{75\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(4\cos(dx+c)^4-4\cos(dx+c)^3+36\cos(dx+c)^2+49\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{40\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, \frac{75\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+\frac{2\left(4\cos(dx+c)^4-4\cos(dx+c)^3+36\cos(dx+c)^2+49\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{20\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos(d*x +
c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
, 1/20*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(
a*sin(d*x + c))) + 2*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos(d*x + c)
```

```
^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.258 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-$$

```
[Out] -5*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*sec(d*x
+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-15/16*sec(d*x+c)^(5/2)*sin(d*
x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+115/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d
*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+35/16*sec(d*x
+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3901, 4104, 4106, 4108, 3893, 212, 3886, 221}

$$\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{35\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{15\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
 (115*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a
*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (15*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (35*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
```

```
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-5a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{45a^2}{4}-\cdots\right.}{\sqrt{a+a\sec(\cdots}}}{8a^{\cdots}}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}\cdots}{16a^2d\sqrt{a+a\sec\cdots}}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}\cdots}{16a^2d\sqrt{a+a\sec\cdots}}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}\cdots}{16a^2d\sqrt{a+a\sec\cdots}}$$

$$= -\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+d\cdots}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right.}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]
time = 1.25, size = 340, normalized size = 1.59

$$\frac{70\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+110\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+32\sqrt{1-\sec(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)-115\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)-230\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)-115\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^2(c+dx)\tan(c+dx)+70\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\sec(c+dx))^2\tan(c+dx)+230\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\sec(c+dx))^2\tan(c+dx)}{32d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (70*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 110*Sqrt[1 - S
ec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 32*Sqrt[1 - Sec[c + d*x]]*Se
c[c + d*x]^(7/2)*Sin[c + d*x] - 115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*
x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 230*Sqrt[2]*ArcTan[(Sqrt[2]*Sqr
```

```
t[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 115*Sq
rt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d
*x]^2*Tan[c + d*x] + 70*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2
*Tan[c + d*x] + 230*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c +
 d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(177) = 354$.
time = 0.16, size = 454, normalized size = 2.12

method	result
default	$-\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{9}{2}}\left(\cos^4(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(40\left(\cos^2(dx+c)\right)\sin(dx+c)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/d*(1/cos(d*x+c))^(9/2)*cos(d*x+c)^4*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/
2)*(-1+cos(d*x+c))^2*(40*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+
cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-40*cos(d*x+c)^2*sin(d
*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c)
)*2^(1/2))+35*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)-115*cos(d*x+c)^2*sin(d
*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+40*cos(d*x+c)*sin(d*
x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))
*2^(1/2))-40*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(
1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+20*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))
^(1/2)-115*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^
(1/2))-39*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-16*(-2/(1+cos(d*x+c)))^(1/2)
)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 9048 vs. $2(177) = 354$.
time = 4.55, size = 9048, normalized size = 42.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

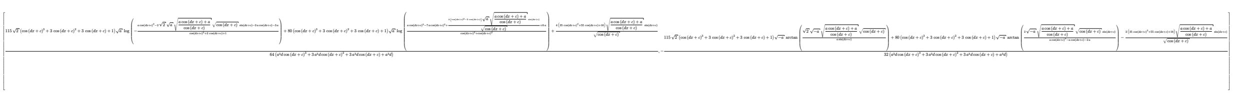
```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*
sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), co
```

s(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
6*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*c) + 7*si
n(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin(6*d*x + 6*c
) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*(24*sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 96*(
sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 300*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*
d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 140*(sin(6*d*x + 6*c) + 7*
sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x +
4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c
)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x +
 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*s
qrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2
)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c)
 + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*
cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c)
+ 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c)
+ 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt
(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqr
t(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*s
qrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d

```
*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(
sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x
 + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt(2)*cos
(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*
c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*
d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*...
```

Fricas [A]
time = 3.02, size = 667, normalized size = 3.12



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 80*(cos(d*x + c)^3 + 3*cos(d*x + c
)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(115*sq
rt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arc
tan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))/(a*sin(d*x + c))) + 80*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
+ c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
) - 2*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/
```

```
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.259 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{11}{16}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*sec(d*x+
c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-11/16*sec(d*x+c)^(3/2)*sin(d*x
+c)/a/d/(a+a*sec(d*x+c))^(3/2)-43/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x
+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3901, 4104, 4108, 3893, 212, 3886, 221}

$$-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
(43*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(4*d*(a + a*Sec[c + d*x])^(5/2)) - (11*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16
*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-4a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a}}}{}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{\sec(c+dx)}}{}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]
time = 0.70, size = 308, normalized size = 1.77

$$\frac{-22\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-30\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+43\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)+86\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)+43\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^2(c+dx)\tan(c+dx)-22\,\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\sec(c+dx))^2\tan(c+dx)-86\,\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\sec(c+dx))^2\tan(c+dx)}{32d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-22*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 30*Sqrt[1 - S
ec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*S
qrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 86*Sqrt[2]*ArcTan
[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c +
d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]
]]*Sec[c + d*x]^2*Tan[c + d*x] - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec
[c + d*x])^2*Tan[c + d*x] - 86*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x]
)^2*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(143) = 286$.
time = 0.15, size = 406, normalized size = 2.33

method	result
default	$\frac{(-1+\cos(dx+c))^{2}\left(16\cos(dx+c)\sin(dx+c)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-16\cos(dx+c)\sin(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(16*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(
1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-16*cos(d*x+c)*sin(d
*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c)
)*2^(1/2))-43*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)
))^(1/2))+16*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin
(d*x+c))*2^(1/2))*sin(d*x+c)-16*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2
)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*sin(d*x+c)+11*cos(d*x+c)^2*(-2/(1+cos(
d*x+c)))^(1/2)-43*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+
c)+4*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-15*(-2/(1+cos(d*x+c)))^(1/2))*(a*
(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+
c)^5/(-2/(1+cos(d*x+c)))^(1/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4988 vs. $2(143) = 286$.
time = 0.87, size = 4988, normalized size = 28.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(19*
sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2
)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
```

*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*s
qrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4
*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x +
4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 +
 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqr
t(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4
*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*s
in(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2
)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2
*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2)
*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqr
t(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16
*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6
*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*

```
c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x
 + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + ...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(143) = 286$.
time = 3.05, size = 665, normalized size = 3.82

$$\left[\frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+32\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)-\frac{4\left(11\cos(dx+c)^2+15\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, \frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+32\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)-\frac{2\left(11\cos(dx+c)^2+15\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) - 4*(11*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(c
os(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*
sin(d*x + c))) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*(1
1*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
 c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.260 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 1/4*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+3/16*sec(d*x+c)^(3
/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(1/
2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3895, 3893, 212}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
```

```
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}\\
&= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}+\frac{3\int \frac{\sqrt{\sec(c+d}}{\sqrt{a+a\sec(c+}}}{32a^2}\\
&= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}-\frac{3\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx\right.}{}\\
&= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\sec^{\frac{3}{2}}}{16ad(}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(137) = 274.
time = 0.68, size = 308, normalized size = 2.25

$$\frac{6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+14\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)-3\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)-6\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)-3\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^2(c+dx)\tan(c+dx)+6\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\sec(c+dx))^2\tan(c+dx)+6\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\sec(c+dx))^2\tan(c+dx)}{32d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 14*Sqrt[1 - Sec
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt
[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 6*Sqrt[2]*ArcTan[(Sq
rt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x]
 - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Se
c[c + d*x]^2*Tan[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d
*x])^2*Tan[c + d*x] + 6*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan
[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]
time = 0.15, size = 210, normalized size = 1.53

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(3\cos(dx+c)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)-3(\cos^2(dx+c))\sqrt{-\frac{2}{1+\cos(dx+c)}}+3\arctan\left(\right.\right.}{16d\sin(d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(3*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2
/(1+cos(d*x+c)))^(1/2))-3*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+3*arctan(1
/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-4*cos(d*x+c)*(-2/(1+cos
(d*x+c)))^(1/2)+7*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^
(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(1
/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. $2(112) = 224$.
time = 22.97, size = 84332, normalized size = 615.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5
/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c)
+ (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*
c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(5*d*
x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*
c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d
*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c)
 + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x
+ 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*si
n(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c)
 + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x
 + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) +
 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x +
5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(
5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) +
 cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x +
```

```
 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5
*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/
2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/
2*d*x + 5/2*c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))
)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + co
s(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*
c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*co
s(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c
) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d
*x + 5/2*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2
 - 512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*
d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) + (
2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)*
sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(5*d*x +
 5*c)^2 + 2560*cos(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 1024*(20*cos(2*d*x
 + 2*c) + 10*cos(d*x + c) + 1)*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 1024
0*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*sin(4*d*x + 4*c)^2*sin(5/2
*d*x + 5/2*c) + 10240*sin(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*(5*(2*
sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c
)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d
*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*s
in(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x +
 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(
8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*si
n(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*
sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x
 + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin
(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4
*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(6/
5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*si
n(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x +
 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5
/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c
)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2560*(5*(2*sin(2*d
*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5
*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/
2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*
d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*s
in(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(2/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 512*(5*cos(4*d*x + 4*
c)^2*sin(5/2*d*x + 5/2*c) + 4*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*co
s(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 20*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5
/2*c) + 5*sin(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 20*sin(3*d*x + 3*c)^2*s
```

```
in(5/2*d*x + 5/2*c) + 2*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2
*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c)
+ 2*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 2*(5*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*...
```

Fricas [A]
time = 3.02, size = 426, normalized size = 3.11

$$\left[\frac{3\,\sqrt{2}\,\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+\frac{4\left(3\,\cos(dx+c)^{2}+7\,\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)},\,-\frac{3\,\sqrt{2}\,\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)-\frac{2\left(3\,\cos(dx+c)^{2}+7\,\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x + c)^2 + 7*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/
(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3
*d), -1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(3*cos(d*x + c)^2 + 7*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.261 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

```
[Out] -1/4*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+5/16*sec(d*x+c)^(
3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+5/32*arctanh(1/2*sin(d*x+c)*a^(1
/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3896, 3895, 3893, 212}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (5*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
```

```
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3896

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{5\int \frac{\sqrt{\sec(c+}}{\sqrt{a+a\sec(c}}}{32a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{5\text{Subst}\left(\int \frac{1}{2a-x^2}\right.}{}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\sec^{\frac{3}{2}}}{16ad(}$$

Mathematica [A]
time = 1.85, size = 266, normalized size = 1.94

$$-\frac{8\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+10\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))\sin(c+dx)-10\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))^2\sin(c+dx)-10\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\sec(c+dx))^2\tan(c+dx)-10\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\sec(c+dx))^2\tan(c+dx)+5\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)(1+\sec(c+dx))^2\tan(c+dx)}{32d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/32*(8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 10*Sqrt[1
 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x])*Sin[c + d*x] - 10*Sq
rt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x] -
 10*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] - 10*A
rcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] + 5*Sqrt[2]*Arc
```

```
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])
^2*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]
time = 0.14, size = 210, normalized size = 1.53

method	result
default	$-\frac{(-1+\cos(dx+c))^2\left(5(\cos^2(dx+c))\sqrt{-\frac{2}{1+\cos(dx+c)}}-5\cos(dx+c)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)-4\cos(d\right.}{16d\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(5*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-5*cos(d
*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-4*cos(d*x
+c)*(-2/(1+cos(d*x+c)))^(1/2)-5*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2))*sin(d*x+c)-(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(
1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(1/
2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2875 vs. $2(112) = 224$.
time = 0.88, size = 2875, normalized size = 20.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*sin
(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x +
 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*c
) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x +
 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*c
```

os(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*d
*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 1
2*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*
(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c) +

```
48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*c
os(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2...
```

Fricas [A]
time = 3.93, size = 422, normalized size = 3.08

$$\left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+\frac{4\left(5\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)},-\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)-\frac{2\left(5\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(5*cos(d*x + c)^2 + cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
), -1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(5*cos(d*x + c)^2 + cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.262 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

```
[Out] -1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-9/16*sec(d*x+c)^(
3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+19/32*arctanh(1/2*sin(d*x+c)*a^(
1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3902, 4097, 3893, 212}

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(4*d*(a + a*Sec[c + d*x])^(5/2)) - (9*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*
a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4097

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n},
 x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m
, -1]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{7a}{2}+a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{19\int \frac{\sqrt{\sec(c-}}{\sqrt{a+a\sec}}}{32a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{19\text{Subst}\left(\int \frac{1}{2a-x^2}\right.}{}$$

$$= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec}{16ad}$$

Mathematica [A]
time = 0.94, size = 146, normalized size = 1.07

$$\frac{-76\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)-(9+13\cos(c+dx))\sqrt{1-\sec(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-76*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] - (9 + 13*Cos[c + d*x])*Sq
```

```
rt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c
+ d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]
time = 0.14, size = 208, normalized size = 1.52

method	result
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c)(-1+\cos(dx+c))^2\left(19\cos(dx+c)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\right.}{16d\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*
(-1+cos(d*x+c))^2*(19*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+co
s(d*x+c)))^(1/2))+13*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+19*arctan(1/2*s
in(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-4*cos(d*x+c)*(-2/(1+cos(d*x
+c)))^(1/2)-9*(-2/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(
1/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3049 vs. $2(112) = 224$.
time = 1.01, size = 3049, normalized size = 22.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3
*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(c
```

os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 +
 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(
1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x +
 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d
*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
+ 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4
c) + 104(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7/
2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10*
sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*si
n(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*s
in(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(1
/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*

```
c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) +
 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
- 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*co
s(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1...
```

Fricas [A]
time = 2.89, size = 426, normalized size = 3.11

$$\left[\frac{19 \sqrt{2} \left(\cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \sqrt{a} \log\left(-\frac{a \cos(dx+c)^{2}-2 \sqrt{2} \sqrt{a} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin(dx+c)-2 a \cos(dx+c)-3 a}{\cos(dx+c)^{2}+2 \cos(dx+c)+1}\right)-\frac{4 \left(13 \cos(dx+c)^{2}+9 \cos(dx+c)\right) \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{64 \left(a^{3} d \cos\left(dx+c\right)^{3}+3 a^{3} d \cos\left(dx+c\right)^{2}+3 a^{3} d \cos\left(dx+c\right)+a^{3} d\right)}, -\frac{19 \sqrt{2} \left(\cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \sqrt{-a} \arctan\left(\frac{\sqrt{2} \sqrt{-a} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)}}{a \sin(dx+c)}\right)+\frac{2 \left(13 \cos(dx+c)^{2}+9 \cos(dx+c)\right) \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{32 \left(a^{3} d \cos\left(dx+c\right)^{3}+3 a^{3} d \cos\left(dx+c\right)^{2}+3 a^{3} d \cos\left(dx+c\right)+a^{3} d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(13*cos(d*x + c)^2 + 9*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d), -1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c
) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(13*cos(d*x + c)^2 + 9*cos(d*
x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
 + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.263 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{13\sqrt{\sec(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}+\frac{49\sqrt{\ldots}}{16\ldots}$$

```
[Out] -75/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*
x+c))^(5/2)-13/16*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(3/2)+49
/16*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3902, 4105, 4098, 3893, 212}

$$-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (13*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
6*a*d*(a + a*Sec[c + d*x])^(3/2)) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
6*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-\frac{9a}{2}+2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} -$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} +$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} +$$

$$= -\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sqrt{\sec(c+dx)}\,\sin}{4d(a+a\sec(c+}$$

Mathematica [A]

time = 1.31, size = 186, normalized size = 1.05

$$\frac{300\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+\left(85\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+49\sqrt{1-\sec(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)+32\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sin(c+dx)}{16d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (300*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + (85*Sqrt[1 - Sec[c + d*x
]]*Sec[c + d*x]^(3/2) + 49*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 32*S
qrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]

time = 0.16, size = 236, normalized size = 1.33

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(75\left(\cos^2(dx+c)\right)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}+150\sin(dx+c)\cos(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/32/d*(-1+cos(d*x+c))^2*(75*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)+150*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)+75*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-64*cos(d*x+c)^3-106*cos(d*x+c)^2+72*cos(d*x+c)+98)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^5/(1/cos(d*x+c))^(1/2)/a^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 258456 vs. $2(146) = 292$.
time = 3.03, size = 258456, normalized size = 1460.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/32*(576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^6 + 14400*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^6 + 187500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^6 + 576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^6 + 5184*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^6 + 262500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c)^2 + 77700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^4 + 2700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^6 - 2304*sin(1/2*d*x + 1/2*c)^7 + 9

```
6*(86*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c
) + 10275*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8768*cos(1/2*d*x + 1/2*c
)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^5 + 88800*(75*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*cos(1/2*d*x + 1/2*c) - 64*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(
3/2*d*x + 3/2*c)^5 + 96*(62*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c
))*sin(3/2*d*x + 3/2*c) + 2625*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 224
0*sin(1/2*d*x + 1/2*c)^2 - 1996)*sin(5/2*d*x + 5/2*c)^5 + 864*(1275*(log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1))*sin(1/2*d*x + 1/2*c) - 1088*sin(1/2*d*x + 1/2*c)^2 - 920)*sin(3/
2*d*x + 3/2*c)^5 - 16*(4144*cos(1/2*d*x + 1/2*c)^2 + 675)*sin(1/2*d*x + 1/2
*c)^5 + 16*((75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x +
 5/2*c)^2 + 25*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*
x + 3/2*c)^2 + 7500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (75*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + 9*(75*log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 300*(log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 256*sin(1/2*d*x + 1/2*c)^3 + 10*((75*l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*
d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 150*(lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)...
```

Fricas [A]

time = 2.82, size = 446, normalized size = 2.52

$$\left[\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+\frac{4\left(32\cos(dx+c)^{3}+85\cos(dx+c)^{2}+49\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}, \frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+\frac{2\left(32\cos(dx+c)^{3}+85\cos(dx+c)^{2}+49\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 + 85*cos(d*x +
 c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x +
c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*cos(d
*x + c)^3 + 85*cos(d*x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a*(sec(c + d*x) + 1))**(5/2)*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.264 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}-\frac{17\sin(c}{16ad\sqrt{\sec(c+dx)}\,(a}$$

```
[Out] 163/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)/sec(d
*x+c)^(1/2)-17/16*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+95
/48*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-299/48*sin(d*x
+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3902, 4105, 4107, 4098, 3893, 212}

$$\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{299\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{95\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{17\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (163*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Sqrt[Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(5/2)) - (17*Sin[c + d*x])/(16*a*d*Sqrt[Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(3/2)) + (95*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (299*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48
*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-\frac{11a}{2}+3a\sec(c+\cdots}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+\cdots}}{4a^2}\cdots$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{17\sin\cdots}{16ad\sqrt{\sec(c+dx)}\cdots}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{17\sin\cdots}{16ad\sqrt{\sec(c+dx)}\cdots}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{17\sin\cdots}{16ad\sqrt{\sec(c+dx)}\cdots}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{17\sin\cdots}{16ad\sqrt{\sec(c+dx)}\cdots}$$

$$= \frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sin(\cdots}{4d\sqrt{\sec(c+dx)}\,(\cdots}$$

Mathematica [A]
time = 2.37, size = 165, normalized size = 0.76

$$-\frac{\sec(c+dx)\left(978\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+\sqrt{1-\sec(c+dx)}\,(487+379\sec(c+dx)+16\cos(2(c+dx))(-1+5\sec(c+dx)))\tan(c+dx)\right)}{48d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,(a(1+\sec(c+dx)))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -1/48*(Sec[c + d*x]*(978*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt
[1 - Sec[c + d*x]]*(487 + 379*Sec[c + d*x] + 16*Cos[2*(c + d*x)]*(-1 + 5*Se
c[c + d*x]))*Tan[c + d*x]))/(d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a
*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]
time = 0.15, size = 254, normalized size = 1.17

method	result

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(489(\cos^2(dx+c))\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(489*cos(d*x+
c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos
(d*x+c)))^(1/2)+64*cos(d*x+c)^4+978*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*
x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)-384*cos(d*x+c)^3+
489*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)-686*cos(d*x+c)^2+408*cos(d*x+c)+598)*cos(d*x+c)^2*(1/cos(d*x
+c))^(3/2)/sin(d*x+c)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 148823 vs. $2(180) = 360$.
time = 3.56, size = 148823, normalized size = 685.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(
3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*sin(3/2
*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3
*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 288*sin(3/2*d*x + 3/2*c)^5 + 32*(
cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x +
3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(11/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2
*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + si
n(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 8192*
(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x +
```

```
3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^4 + 4*(16*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*
c)*sin(3/2*d*x + 3/2*c) + 16*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/
2*d*x + 3/2*c) + 80*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2
*c) + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(
3/2*d*x + 3/2*c))*cos(9/2*d*x + 9/2*c) + 192*(cos(3*d*x + 3*c)^2*sin(3/2*d*
x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)
^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
128*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d
*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - (48*sin(3/2*d*x + 3/2*c)^2 - 751)*sin(3*d
*x + 3*c) + 33*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(10/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 409*(cos(3*d*x + 3*c)^2 + sin
(3*d*x + 3*c)^2)*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 519*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 88*(cos(3*d*x + 3*c)^2 + sin(3*d*x +
 3*c)^2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 240
*(cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(3*d*x
+ 3*c)^2 + 4*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*cos(9/2*d*x + 9/2*c)
 + 5*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) - 3*sin(3*d*x + 3*c)*sin(3/2*d*x
 + 3/2*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*co
s(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 864*(16*cos
(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 16*cos(3/2*d*x
+ 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 80*cos(3*d*x + 3*c)*cos(
3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d*x
+ 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c))*cos(9/2*d*x + 9/2*c) +
128*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d
*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - (48*sin(3/2*d*x + 3/2*c)^2 - 751)*sin(3*d
*x + 3*c) + 519*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 88*(cos(3*d*x + 3*c)^2 + sin(
3*d*x + 3*c)^2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - 240*(cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin
(3*d*x + 3*c)^2 + 4*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*cos(9/2*d*x +
 9/2*c) + 5*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) - 3*sin(3*d*x + 3*c)*sin(
3/2*d*x + 3/2*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 256*(
16*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 16*cos(3/
2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 80*cos(3*d*x + 3*c
)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 64*(cos(3*d*x + 3*c)^2*sin(3/
2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c))*cos(9/2*d*x + 9/2
```

```
*c) - (48*sin(3/2*d*x + 3/2*c)^2 - 751)*sin(3*d*x + 3*c) + 519*(cos(3*d*x +
 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 88*(cos(3*d*x + 3*c)^2 + sin(...
```

Fricas [A]
time = 2.79, size = 466, normalized size = 2.15

$$\left[\frac{489\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+\frac{4\left(32\,\cos(dx+c)^{4}-160\,\cos(dx+c)^{3}-503\,\cos(dx+c)^{2}-299\,\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{192\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)},\ -\frac{489\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)-\frac{2\left(32\,\cos(dx+c)^{4}-160\,\cos(dx+c)^{3}-503\,\cos(dx+c)^{2}-299\,\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{96\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^4 - 160*cos(d*
x + c)^3 - 503*cos(d*x + c)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt(2)*(co
s(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*s
in(d*x + c))) - 2*(32*cos(d*x + c)^4 - 160*cos(d*x + c)^3 - 503*cos(d*x + c
)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^
3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.265 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{7\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{4d}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}}$$

```
[Out] 7/4*arcsinh(tan(d*x+c)/(1+sec(d*x+c))^(1/2))/d-arcsinh(tan(d*x+c)/(1+sec(d*
x+c)))*2^(1/2)/d-1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)+1/2
*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3907, 4106, 4108, 3892, 221, 3886}

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{7\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (7*ArcSinh[Tan[c
+ d*x]/Sqrt[1 + Sec[c + d*x]]])/(4*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
4*d*Sqrt[1 + Sec[c + d*x]]) + (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[1
 + Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3892

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-Sqrt[2])*(Sqrt[a]/(b*f)), Subst[Int[1/Sqrt[1 +
```

```
x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[d^2/(b*(2*n - 3)), Int[(d
*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} + \frac{1}{4}\int \frac{(3-\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} + \frac{1}{4}\int \frac{\sqrt{\sec(c+dx)}\,(-}{\sqrt{1+\sec}}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} + \frac{7}{8}\int \sqrt{\sec(c+dx)}\,\sqrt{1}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} - \frac{7\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx\right.}{}$$

$$= -\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{7\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{4d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin}{4d\sqrt{1+\sec(c}}$$

Mathematica [A]
time = 0.43, size = 140, normalized size = 1.11

$$\frac{\cot(c+dx)\left(\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)+8\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)-4\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-2\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sqrt{-\tan^2(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 8*ArcSin[Sqrt[Sec[c + d*x]]
] - 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] -
 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Sqrt[-((-1 + Sec[c + d*x])*S
ec[c + d*x])])*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(108) = 216.
time = 0.20, size = 253, normalized size = 2.01

method	result
default	$-\frac{(-1+\cos(dx+c))\left(7(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-7(\cos^2(dx+c))\sqrt{2}\,\text{arctan}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(7*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-7*cos(d*x+c)^2*2^(1/2)*arctan(1
/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))-16*cos(d*x+
c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-2*cos(d*x+c)*sin(d*x+
c)*(-2/(1+cos(d*x+c)))^(1/2)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*((1+co
s(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^2/(-2/(1+cos(d*
x+c)))^(1/2)/sin(d*x+c)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1643 vs. $2(108) = 216$.
time = 0.60, size = 1643, normalized size = 13.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2
)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*co
s(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c)
```

```
, cos(d*x + c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
 + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(
sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d
*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) +
 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) +
2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(108) = 216$.

time = 2.38, size = 337, normalized size = 2.67

$$\frac{8\left(\sqrt{2}\cos\left(dx+c\right)^{2}+\sqrt{2}\cos\left(dx+c\right)\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)+\cos(dx+c)^{2}-2\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)-7\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(-\frac{\cos(dx+c)^{2}+2\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-\cos(dx+c)-2}{\cos(dx+c)+1}\right)+7\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(-\frac{\cos(dx+c)^{2}-2\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-\cos(dx+c)-2}{\cos(dx+c)+1}\right)-\frac{4\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{16\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*log(-(2*sqrt(2)*sqr
t((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*
x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 7*(
cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos
(d*x + c) + 1)) + 7*(cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c)^2 -
2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - c
os(d*x + c) - 2)/(cos(d*x + c) + 1)) - 4*sqrt((cos(d*x + c) + 1)/cos(d*x +
c))*(cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 +
 d*cos(d*x + c))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(sec(d*x + c) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

3.266 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}-\frac{\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}$$

```
[Out] -arcsinh(tan(d*x+c)/(1+sec(d*x+c))^(1/2))/d+arcsinh(tan(d*x+c)/(1+sec(d*x+c
)))*2^(1/2)/d+sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3907, 4108, 3892, 221, 3886}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d - ArcSinh[Tan[c + d*x]
/Sqrt[1 + Sec[c + d*x]]]/d + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 +
Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3892

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-Sqrt[2])*(Sqrt[a]/(b*f)), Subst[Int[1/Sqrt[1 +
 x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d,
```

```
 e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[d^2/(b*(2*n - 3)), Int[(d
*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx &= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}+\frac{1}{2}\int \frac{(1-\sec(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx \\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}-\frac{1}{2}\int \sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\,dx+\int \frac{\sqrt{}}{\sqrt{1}} \\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}- \\
&= \frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}-\frac{\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c}{d\sqrt{1+\sec(c+}}
\end{aligned}$$

Mathematica [A]
time = 0.31, size = 111, normalized size = 1.31

$$\frac{\left(\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)+2\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)}{d\sqrt{-\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

[Out] ((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[-Tan[c + d*x]^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(77) = 154$.
time = 0.14, size = 218, normalized size = 2.56

method	result
default	$-\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}} (\cos^2(dx+c)) \sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}\, (-1+\cos(dx+c)) \left(-\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\, (1+\cos(dx+c)+\sin(dx+c}{4}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*((1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)+cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)+2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+4*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(77) = 154$.
time = 0.58, size = 873, normalized size = 10.27

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan

```
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
+ 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(77) = 154$.
time = 3.37, size = 297, normalized size = 3.49

$$\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+(\cos(dx+c)+1)\log\left(-\frac{\cos(dx+c)^2+2\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)-2}{\cos(dx+c)+1}\right)-(\cos(dx+c)+1)\log\left(-\frac{\cos(dx+c)^2-2\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)-2}{\cos(dx+c)+1}\right)+\frac{4\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c) +
 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(
d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (cos(d*x + c) + 1)*l
og(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) - (cos(d*x + c)
+ 1)*log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) + 4*sqrt((
cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(sec(d*x + c) + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

3.267 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}$$

```
[Out] 2*arcsinh(tan(d*x+c)/(1+sec(d*x+c))^(1/2))/d-arcsinh(tan(d*x+c)/(1+sec(d*x+
c)))*2^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3906, 3886, 221, 3892}

$$\frac{2\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*ArcSinh[Tan[c
+ d*x]/Sqrt[1 + Sec[c + d*x]]])/d
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3892

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-Sqrt[2])*(Sqrt[a]/(b*f)), Subst[Int[1/Sqrt[1 +
 x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d,
 e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3906

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e
 + f*x]], x], x] - Dist[a*(d/b), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = -\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx + \int \sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\,dx$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d} + \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,d\right.}{d}$$

$$= -\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}$$

Mathematica [A]
time = 0.10, size = 76, normalized size = 1.41

$$\frac{\left(2\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right) - \sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\cot(c+dx)\sqrt{-\tan^2(c+dx)}}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] ((2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]]
)/Sqrt[1 - Sec[c + d*x]]])*Cot[c + d*x]*Sqrt[-Tan[c + d*x]^2])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(50) = 100$.
time = 0.13, size = 178, normalized size = 3.30

method	result
default	$-\frac{\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c}{4}\right.\right.}{d\sin(dx+c)^2\sqrt{-\frac{}{1+}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c)
)*2^(1/2))-2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d
*x+c))*2^(1/2))-2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*((1+cos
(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2
)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(50) = 100$.
time $= 0.57$, size $= 473$, normalized size $= 8.76$

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}-2\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)+1\right)-\log\left(2\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+2\sqrt{2}\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)+2\sqrt{2}\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)+2\right)+\log\left(2\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+2\sqrt{2}\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)-2\sqrt{2}\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}-2\sqrt{2}\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)+2\sqrt{2}\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)+2\right)+\log\left(2\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}-2\sqrt{2}\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)-2\sqrt{2}\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)+2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(50) = 100$.
time $= 2.46$, size $= 223$, normalized size $= 4.13$

$$\frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-3}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)-\log\left(-\frac{\cos\left(dx+c\right)^{2}+2\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\cos\left(dx+c\right)-2}{\cos\left(dx+c\right)+1}\right)+\log\left(-\frac{\cos\left(dx+c\right)^{2}-2\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\cos\left(dx+c\right)-2}{\cos\left(dx+c\right)+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)) - log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c) +
1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d
*x + c) + 1)) + log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1))
)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(sec(c + d*x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(sec(d*x + c) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

3.268 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}$$

```
[Out] arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3892, 221}

$$\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3892

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-Sqrt[2])*(Sqrt[a]/(b*f)), Subst[Int[1/Sqrt[1 +
 x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d,
 e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx &= -\frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}\\ &= \frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.48

$$\frac{2\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{1+\cos(c+dx)}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]*Sqrt[(1 + Cos[c + d*x])^(-1)]
)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(25) = 50.
time = 0.12, size = 95, normalized size = 3.52

method	result	size
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\cos(dx+c)\sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos^2(dx+c)-1\right)}{d\sin(dx+c)^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*cos(d*x+c)*((1+cos(d*x+c))/cos(d*x+c))^(1/2)*arcta
n(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d
*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(25) = 50.
time = 0.54, size = 87, normalized size = 3.22

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(25) = 50$.

time = 3.20, size = 88, normalized size = 3.26

$$\frac{\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(d*x + c) + 3)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1))/d
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(sec(c + d*x) + 1), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(sec(d*x + c) + 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(1/cos(c + d*x) + 1)^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

3.269 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}$$

```
[Out] -arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d+2*sin(d*x+c)*sec(d*x+c)^(1/2)
/d/(1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3897, 3892, 221}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3892

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-Sqrt[2])*(Sqrt[a]/(b*f)), Subst[Int[1/Sqrt[1 +
 x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d,
 e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3897

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Dist[a*(m/(b*d*(m + 1))), Int[(a + b*Csc[e
+ f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx &= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}} - \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx\\ &= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}} + \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}\\ &= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.18, size = 90, normalized size = 1.45

$$\frac{2\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sin(c+dx)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{-\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/(d*Sqrt[
-Tan[c + d*x]^2])
```

Maple [**A**]
time = 0.12, size = 96, normalized size = 1.55

method	result	size
default	$\frac{\left(\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\sin(dx+c)-2\cos(dx+c)+2\right)\sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}}{d\sqrt{\frac{1}{\cos(dx+c)}}\sin(dx+c)}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)-2*cos(d*x+c)+2)*((1+cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*
x+c))^(1/2)/sin(d*x+c)
```

Maxima [A]
time = 0.54, size = 101, normalized size = 1.63

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-4\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(56) = 112.
time = 2.45, size = 144, normalized size = 2.32

$$\frac{\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(-\frac{2\,\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)+\cos(dx+c)^{2}-2\,\cos(dx+c)-3}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+4\,\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x + c) +
1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos(d
*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\sec\left(c+dx\right)+1}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sec(c + d*x) + 1)*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(sec(d*x + c) + 1)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}+1}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.270 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}-\frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{1+\sec(c+dx)}}$$

```
[Out] arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d+2/3*sin(d*x+c)/d/sec(d*x+c)^(1
/2)/(1+sec(d*x+c))^(1/2)-2/3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(1+sec(d*x+c))^(
1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3908, 4098, 3892, 221}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\sec(c+dx)+1}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d + (2*Sin[c + d*x])/(3*
d*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) - (2*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(3*d*Sqrt[1 + Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3892

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-Sqrt[2])*(Sqrt[a]/(b*f)), Subst[Int[1/Sqrt[1 +
 x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d,
 e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a
+ b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx &= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}-\frac{1}{3}\int \frac{1-2\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}\\ &= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}-\frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{1+\sec(c+dx)}}\\ &= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}-\frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{1+\sec(c+dx)}}\\ &= \frac{\sqrt{2}\,\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}-\end{aligned}$$

Mathematica [A]
time = 0.26, size = 118, normalized size = 1.20

$$\frac{\left(2(-1+\cos(c+dx))\sqrt{1-\sec(c+dx)}-3\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,\sqrt{1+\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] ((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*
Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x
])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[1 + Sec[c + d*x]])
```

Maple [A]
time = 0.14, size = 116, normalized size = 1.18

method	result
default	$-\frac{\sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)+2\left(\cos^2(dx+c)\right)-4\cos(dx+c)+2}{3d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*((1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(1+co
s(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2-4*cos
(d*x+c)+2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(84) = 168$.
time = 0.55, size = 279, normalized size = 2.85

$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)-3\sqrt{2}\log\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^2+\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^2+2\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+1\right)+3\sqrt{2}\log\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^2+\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^2-2\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+1\right)-2\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sqrt{2}\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/d
```

Fricas [A]
time = 2.95, size = 163, normalized size = 1.66

$$\frac{3\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{6\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c) +
 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(
d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2 -
cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec{(c + dx)} + 1}\,\sec^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sec(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}+1}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.271 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=134

$$-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}+\frac{26}{}$$

[Out] -arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d+2/5*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2)-2/15*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2)+26/15*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(1+sec(d*x+c))^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3908, 4107, 4098, 3892, 221}

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{\sec(c+dx)+1}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]

[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[1 + Sec[c + d*x]])

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3892

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-Sqrt[2])*(Sqrt[a]/(b*f)), Subst[Int[1/Sqrt[1 + x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{1}{5}\int \frac{1-4\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c}$$

$$= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} -$$

Mathematica [A]
time = 0.31, size = 122, normalized size = 0.91

$$\frac{\left(15\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}\,(3-\sec(c+dx)+13\sec^2(c+dx))\right)\sin(c+dx)}{15d\sec^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] ((15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Se
c[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c + d*x] + 13*Sec[c +
d*x]^2))*Sin[c + d*x])/(15*d*Sec[c + d*x]^(3/2)*Sqrt[-Tan[c + d*x]^2])
```

Maple [A]
time = 0.14, size = 126, normalized size = 0.94

method	result
default	$-\frac{\sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}\left(6\left(\cos^3(dx+c)\right)-15\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-8\left(\cos^2(dx+c)\right.\right.}{15d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/d*((1+cos(d*x+c))/cos(d*x+c))^(1/2)*(6*cos(d*x+c)^3-15*arctan(1/2*sin
(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*c
os(d*x+c)^2+28*cos(d*x+c)-26)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(114) = 228.
time = 0.56, size = 354, normalized size = 2.64

$$\frac{\sqrt{2}\left(60\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-60\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+5\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-30\log\left(\cos\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)^2+\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)^2+2\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+1\right)+30\log\left(\cos\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)^2+\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)^2-2\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+1\right)+6\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+60\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))))/d
```

Fricas [A]
time = 2.86, size = 174, normalized size = 1.30

$$\frac{15\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)+\cos(dx+c)^{2}-2\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+\frac{4\left(3\cos(dx+c)^{3}-\cos(dx+c)^{2}+13\cos(dx+c)\right)\sqrt{\frac{\cos\left(dx+c\right)+1}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{30\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(15*(sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x + c
) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*c
os(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x + c)
^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec\left(c+dx\right)+1}\,\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sec(d*x + c) + 1)*sec(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}+1}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.272 $\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}+\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2eF\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-\right.}{5d(a-a\sec(c+dx))\sqrt{a+a}}$$

```
[Out] 6/5*a*e*(e*sec(d*x+c))^(1/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+4/5*3^(3/4
)*a^2*e*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+
c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3)
)*(1/2*6^(1/2)+1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d
*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+
c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c
))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3891, 52, 65, 224}

$$\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2e\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{5d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}+\frac{6ae\tan(c+dx)\sqrt[3]{e\sec(c+dx)}}{5d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (6*a*e*(e*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
+ (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*e*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3
) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3)
)], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1
/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3)
 - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*d*(a - a*Sec[c + d*x])*Sqrt[
a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 +
 Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{(a^2e\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{ex}}{\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{(2a^2e^2\tan(c+dx))\,\mathrm{Subst}}{5d\sqrt{a-a\sec(}}$$

$$= \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{(6a^2e\tan(c+dx))\,\mathrm{Subst}}{5d\sqrt{a-a\sec(}}$$

$$= \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2eF\Big(\mathrm{s}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.27, size = 71, normalized size = 0.22

$$\frac{2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};1-\sec(c+dx)\right)(e\sec(c+dx))^{4/3}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sec^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(4/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(4/3))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{\frac{4}{3}}\sqrt{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(4/3)*integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*e^(4/3)*sec(d*x + c)^(4/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(4/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(4/3),x)
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(4/3), x)
```

3.273 $\int \sqrt[3]{e\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=280

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\cdots}}{d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\,\mathrm{s}\cdots}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\cdots\right.}}}$$

```
[Out] 2*3^(3/4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*se
c(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))
^(1/3))*(1/2*6^(1/2)+1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e
*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*ta
n(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec
(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3891, 65, 224}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))],
-7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*
(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (
e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a + a*
Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[
3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{e\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{2/3}\sqrt{a-ax}}\,dx, x, \sec(c+dx\right.}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ax^3}{e}}}\,dx, x, \sqrt[3]{e\sec(c+d\right.}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right.\right.}{d(a-a\sec(c+dx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.15, size = 71, normalized size = 0.25

$$\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};1-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt[3]{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 2/3, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(1/3
)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(1/3))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{a+a\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(1/3)*integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*e^(1/3)*sec(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sqrt[3]{e\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3), x)
```

3.274 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=326

$$\frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,a^2F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\middle|-\right.}{2de(a-a\sec(c+dx))\sqrt{}}$$

[Out] 3/2*a*tan(d*x+c)/d/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2)+1/2*3^(3/4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*(1/2*6^(1/2)+1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.25, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3891, 53, 65, 224}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)}{2de(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}+\frac{3a\tan(c+dx)}{2d\sqrt{a\sec(c+dx)+a}\,(e\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]

[Out] (3*a*Tan[c + d*x])/(2*d*(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*d*e*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = -\frac{(a^2e\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{(ex)^{5/3}\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} - \frac{(a^2\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{(ex)^{2/3}\sqrt{a}\cdots}\right.}{4d\sqrt{a-a\sec(c+dx)}\sqrt{\cdots}}$$

$$= \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} - \frac{(3a^2\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\cdots}}\right.}{4de\sqrt{a-a\sec(c+dx)}\cdots}$$

$$= \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,a^2F\left(\sin^{-1}\left(\frac{(1-\sqrt{\cdots}}{(1+\sqrt{\cdots}}\right.\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.16, size = 71, normalized size = 0.22

$$\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{3}{2};1-\sec(c+dx)\right)\sec^{\frac{2}{3}}(c+dx)\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d(e\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 5/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(2/3)*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(2/3))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\sec(dx+c)}}{(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="maxima"
)
```

```
[Out] e^(-2/3)*integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*e^(-2/3)/sec(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\left(e\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(2/3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(2/3),x)
[Out] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(2/3), x)
```

3.275 $\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=716

$$\frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}}+\frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}}-\frac{240ae^3\tan(c}{91d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right.\right.}$$

```
[Out] 60/91*a*e^2*(e*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+6/13*a
*e*(e*sec(d*x+c))^(5/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-240/91*a*e^3*ta
n(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^(1/
2)-80/91*3^(3/4)*a^2*e^(7/3)*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^
(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)
-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*se
c(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d
*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*
x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)+120/91*3^
(1/4)*a^2*e^(7/3)*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(
e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x
+c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3
)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e
*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 716, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3891, 52, 65, 309, 224, 1891}

$$-\frac{80\sqrt{2}\,3^{3/4}a^2e^{7/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{91d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}+\frac{120\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2e^{7/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{91d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}-\frac{240ae^3\tan(c+dx)}{91d\sqrt{a\sec(c+dx)+a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}+\frac{60ae^2\tan(c+dx)(e\sec(c+dx))^{2/3}}{91d\sqrt{a\sec(c+dx)+a}}+\frac{6ae\tan(c+dx)(e\sec(c+dx))^{5/3}}{13d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (60*a*e^2*(e*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(91*d*Sqrt[a + a*Sec[c + d*x
]]) + (6*a*e*(e*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(13*d*Sqrt[a + a*Sec[c +
d*x]]) - (240*a*e^3*Tan[c + d*x])/(91*d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt
[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))) + (120*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^
2*e^(7/3)*EllipticE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))
/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3
```

```
) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3)
+ (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^
2]*Tan[c + d*x])/(91*d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(
e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))^2]) - (80*Sqrt[2]*3^(3/4)*a^2*e^(7/3)*EllipticF[ArcSin[((
1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*
Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*S
qrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1
 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(91*d*(a - a
*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c +
 d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{(a^2e\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{(ex)^{5/3}}{\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}} - \frac{(10a^2e^2\tan(c+dx))\,\mathrm{Subs}}{13d\sqrt{a-a\sec(c}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan}{13d\sqrt{a+a\sec(c+}}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan}{13d\sqrt{a+a\sec(c+}}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan}{13d\sqrt{a+a\sec(c+}}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan}{13d\sqrt{a+a\sec(c+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.25, size = 71, normalized size = 0.10

$$\frac{2\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};\frac{3}{2};1-\sec(c+dx)\right)(e\sec(c+dx))^{8/3}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sec^{\frac{8}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-5/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(8/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(8/3))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (e \sec (dx + c))^{\frac{8}{3}} \sqrt{a + a \sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(8/3)*integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(8/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*e^(8/3)*sec(d*x + c)^(8/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(8/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(8/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,\left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{8/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(8/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(8/3), x)
```

3.276 $\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=673

$$12\sqrt[4]{3}\sqrt{\ }$$

$$\frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}-\frac{24ae^2\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}+$$

```
[Out] 6/7*a*e*(e*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-24/7*a*e^2
*tan(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^
(1/2)-8/7*3^(3/4)*a^2*e^(4/3)*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3
^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3
)-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*s
ec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(
d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d
*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)+12/7*3^(
1/4)*a^2*e^(4/3)*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e
*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+
c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)
+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*
sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3891, 52, 65, 309, 224, 1891}

$$-\frac{8\sqrt{2}\,3^{3/4}a^2e^{4/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{7d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2e^{4/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{7d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}-\frac{24ae^2\tan(c+dx)}{7d\sqrt{a\sec(c+dx)+a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}+\frac{6ae\tan(c+dx)(e\sec(c+dx))^{2/3}}{7d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (6*a*e*(e*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
- (24*a*e^2*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1
/3) - (e*Sec[c + d*x])^(1/3))) + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*e^(4/3)*
EllipticE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec
[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c
 + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c +
```

```
 d*x])/(7*d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^
(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]) - (8*Sqrt[2]*3^(3/4)*a^2*e^(4/3)*EllipticF[ArcSin[((1 - Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x]
)^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3)
 + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*
e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*d*(a - a*Sec[c + d*x]
)*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{(a^2 e\tan(c+dx))\,\text{Subst}\left(\int \frac{(ex)^{2/3}}{\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{(4a^2e^2\tan(c+dx))\,\text{Subs}}{7d\sqrt{a-a\sec}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{(12a^2e\tan(c+dx))\,\text{Subs}}{7d\sqrt{a-a\,\text{se}}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{(12a^2e\tan(c+dx))\,\text{Subs}}{7d\sqrt{a-a}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{24}{7d\sqrt{a+a\sec(c+dx)}\,(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.23, size = 71, normalized size = 0.11

$$\frac{2\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};1-\sec(c+dx)\right)(e\sec(c+dx))^{5/3}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sec^{\frac{5}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-2/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(5/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(5/3))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(dx+c\right)\right)^{\frac{5}{3}}\sqrt{a+a\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(5/3)*integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*e^(5/3)*sec(d*x + c)^(5/3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/3)*(a+a*sec(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(5/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(5/3),x)
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(5/3), x)
```

3.277 $\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=624

$$-\frac{6ae\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\sqrt[3]{e}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{}$$

```
[Out] -6*a*e*tan(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*
x+c))^(1/2)-2*3^(3/4)*a^2*e^(1/3)*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*
(1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^
(1/3)-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+
(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*
tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*s
ec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)+3*3^
(1/4)*a^2*e^(1/3)*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(
e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x
+c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3
)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e
*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3891, 65, 309, 224, 1891}

$$-\frac{2\sqrt{2}\,3^{3/4}a^2\sqrt[3]{e}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\sqrt[3]{e}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}-\frac{6ae\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-6*a*e*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*e^(1/3)*Ellipti
cE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e
^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*
x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x]
)^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/
(d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (
```

```
e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]
) - (2*Sqrt[2]*3^(3/4)*a^2*e^(1/3)*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3)
- (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))]
, -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3
)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) -
 (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a +
a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
```

```
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{(a^2 e\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{ex}\,\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right.}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(3a^2\tan(c+dx))\,\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{ax^3}{e}}}\,dx, x, \sqrt[3]{e\sec(c+dx}\right.}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(3a^2\tan(c+dx))\,\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-x}{\sqrt{a-\frac{ax^3}{e}}}\,dx, x, \sqrt[3]{e\sec(c+}\right.}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{6ae\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\,\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.15, size = 71, normalized size = 0.11

$$\frac{2\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};1-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d\sec^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(2/3
)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(2/3))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{\frac{2}{3}} \sqrt{a+a\sec(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
[Out] int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
[Out] e^(2/3)*integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
[Out] integral(sqrt(a*sec(d*x + c) + a)*e^(2/3)*sec(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c+dx)+1)}\,(e\sec(c+dx))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2/3)*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3), x)
```

3.278 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=662

$$\frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{3a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}-\frac{3\sqrt[4]{3}}{}$$

[Out] 3*a*tan(d*x+c)/d/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2)+3*a*tan(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^(1/2)+3^(3/4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(2/3)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/2*3^(1/4)*a^2*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(2/3)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.41, antiderivative size = 662, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3891, 53, 65, 309, 224, 1891}

$$\frac{\sqrt{2}\,3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)}{de^{2/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)}{2de^{2/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}+\frac{3a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}\sqrt[3]{e\sec(c+dx)}}+\frac{3a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]

[Out] (3*a*Tan[c + d*x])/(d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]) + (3*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*EllipticE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sq

```
rt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1
+ Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*d*e^(2/3)*
(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*S
ec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]) +
 (Sqrt[2]*3^(3/4)*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c +
d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt
[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c +
d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(d*e^(2/3)*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec
[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx = -\frac{\left(a^2 e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{4/3}\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{ex}\sqrt{a-\cdots}}\right.}{2d\sqrt{a-a\sec(c+dx)}\sqrt{a+\cdots}}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{ax\cdots}{e}}}\right.}{2de\sqrt{a-a\sec(c+dx)}\sqrt{a\cdots}}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\cdots}{\sqrt{a-\cdots}}\right.}{2de\sqrt{a-a\sec(c+dx)}\cdots}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{3a\tan(c+dx)\cdots}{d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.16, size = 71, normalized size = 0.11

$$\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{3}{2};1-\sec(c+dx)\right)\sqrt[3]{\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt[3]{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(1/3)*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(1/3))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\sec(dx+c)}}{\left(e\sec(dx+c)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="maxima"
)
```

```
[Out] e^(-1/3)*integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*e^(-1/3)/sec(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sqrt[3]{e\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(1/3),x)
[Out] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(1/3), x)
```

3.279 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=715

$$\frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}}+\frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{}{8de\sqrt{a+a\sec(c+dx}}$$

```
[Out] 3/4*a*tan(d*x+c)/d/(e*sec(d*x+c))^(4/3)/(a+a*sec(d*x+c))^(1/2)+15/8*a*tan(d
*x+c)/d/e/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2)+15/8*a*tan(d*x+c)/d/e
/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^(1/2)+5/8*3^(
3/4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x
+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3
))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*
x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(5/3)/(a-a*sec(d*x
+c))*2^(1/2)/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))
/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)-15/16*3^(1/4)*a^2*Ell
ipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*(1/2*6^(1
/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3
))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(5/3
)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^
(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 715, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3891, 53, 65, 309, 224, 1891}

$$\frac{5\,3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,de^{5/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{16de^{5/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}+\frac{15a\tan(c+dx)}{8de\sqrt{a\sec(c+dx)+a}\sqrt[3]{e\sec(c+dx)}}+\frac{3a\tan(c+dx)}{4d\sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{4/3}}+\frac{15a\tan(c+dx)}{8de\sqrt{a\sec(c+dx)+a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*a*Tan[c + d*x])/(4*d*(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]]) +
(15*a*Tan[c + d*x])/(8*d*e*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]])
 + (15*a*Tan[c + d*x])/(8*d*e*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/
3) - (e*Sec[c + d*x])^(1/3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*EllipticE
```

```
[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(
1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x]
)^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^
(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(1
6*d*e^(5/3)*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^
(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]) + (5*3^(3/4)*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Se
c[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 -
4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Se
c[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(4*Sqrt[2]*d*e^(5/3)*(a - a*Sec[c + d*x]
)*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = -\frac{(a^2e\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{(ex)^{7/3}\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} - \frac{(5a^2\tan(c+dx))\,\text{Subst}\left(\int \frac{}{(ex)^{4/3}\sqrt{}}\right.}{8d\sqrt{a-a\sec(c+dx)}\,\sqrt{}}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\,\sqrt{a+a\sec(c+}}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\,\sqrt{a+a\sec(c+}}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\,\sqrt{a+a\sec(c+}}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\,\sqrt{a+a\sec(c+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.19, size = 71, normalized size = 0.10

$$\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{7}{3};\frac{3}{2};1-\sec(c+dx)\right)\sec^{\frac{4}{3}}(c+dx)\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d(e\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 7/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(4/3)*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(4/3))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \sec(dx + c)}}{(e \sec(dx + c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="maxima"
)
```

```
[Out] e^(-4/3)*integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*e^(-4/3)/sec(d*x + c)^(4/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a (\sec(c + dx) + 1)}}{(e \sec(c + dx))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(4/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(4/3), x)
```

3.280 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -3/2*AppellF1(2/3,1,1/2,5/3,-sec(d*x+c),sec(d*x+c))*(e*sec(d*x+c))^(2/3)*ta
n(d*x+c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3913, 3912, 129, 524}

$$-\frac{3\tan(c+dx)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-3*AppellF1[2/3, 1/2, 1, 5/3, Sec[c + d*x], -Sec[c + d*x]]*(e*Sec[c + d*x]
)^(2/3)*Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt{1+\sec(c+dx)}\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(e\tan(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt[3]{ex}\,(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(3\tan(c+dx))\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{3F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 760 vs. 2(78) = 156.
time = 7.15, size = 760, normalized size = 9.74

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (90*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]*Cos[c + d*x]^2*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c + d
*x])]*Sin[(c + d*x)/2]*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -T
```

```
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(a*d*(270*AppellF1[1/2, 1/6, 1/3,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^4*(1 + 2*C
os[c + d*x]) + 10*(-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - 3*App
ellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(10*Appe
llF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c +
d*x)/2]^2*(2 - 9*Cos[c + d*x] + Cos[2*(c + d*x)]) - 5*AppellF1[3/2, 7/6, 1/
3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2 - 9*
Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*(16*AppellF1[5/2, 1/6, 7/3, 7/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 4*AppellF1[5/2, 7/6, 4/3, 7/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 7*AppellF1[5/2, 13/6, 1/3, 7/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sin[(c + d*x)/2]^2)*Tan[(c
 + d*x)/2]^2))
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(e \sec (dx + c))^{\frac{2}{3}}}{\sqrt{a + a \sec (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(2/3)*integrate(sec(d*x + c)^(2/3)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sec{(c + dx)})^{\frac{2}{3}}}{\sqrt{a (\sec{(c + dx)} + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(2/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(2/3)/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{2/3}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(2/3)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(2/3)/(a + a/cos(c + d*x))^(1/2), x)
```

3.281 $\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$-\frac{3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\sec(c+dx),-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -3*AppellF1(1/3,1,1/2,4/3,-sec(d*x+c),sec(d*x+c))*(e*sec(d*x+c))^(1/3)*tan(
d*x+c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3913, 3912, 129, 440}

$$-\frac{3\tan(c+dx)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\sec(c+dx),-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-3*AppellF1[1/3, 1/2, 1, 4/3, Sec[c + d*x], -Sec[c + d*x]]*(e*Sec[c + d*x]
)^(1/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt{1+\sec(c+dx)}\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(e\tan(c+dx))\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(ex)^{2/3}(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(3\tan(c+dx))\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\sec(c+dx),-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 749 vs. 2(76) = 152.
time = 7.79, size = 749, normalized size = 9.86

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (720*e*AppellF1[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x])^2*Sin[(c + d*x)/2]*(9*AppellF1[1/2,
-1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - (4*AppellF1[3/2,
 -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 5
/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)
```

```
)/(d*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])]*(4320*AppellF1[1/2,
-1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]
^6*(-1 + 4*Cos[c + d*x]) + 160*(4*AppellF1[3/2, -1/6, 5/3, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^4 + 12*AppellF1
[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x
)/2]^2*(20*AppellF1[3/2, -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*(7 + 14*Cos[c + d*x] + 5*Cos[2*(c + d*x)] - 2*Cos[3*(c + d*x)]) + 5*
AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(7 +
14*Cos[c + d*x] + 5*Cos[2*(c + d*x)] - 2*Cos[3*(c + d*x)]) - 24*(40*AppellF
1[5/2, -1/6, 8/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 8*AppellF
1[5/2, 5/6, 5/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 5*AppellF1
[5/2, 11/6, 2/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x
]*Sin[(c + d*x)/2]^2)))
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(e \sec (dx+c))^{\frac{1}{3}}}{\sqrt{a+a \sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(1/3)*integrate(sec(d*x + c)^(1/3)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(1/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(1/3)/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/3)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(1/3)/(a + a/cos(c + d*x))^(1/2), x)
```

3.282 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{3F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 3*AppellF1(-1/3,1,1/2,2/3,-sec(d*x+c),sec(d*x+c))*tan(d*x+c)/d/(e*sec(d*x+c
))^(1/3)/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3913, 3912, 129, 524}

$$\frac{3\tan(c+dx)F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};\sec(c+dx),-\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}\sqrt[3]{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (3*AppellF1[-1/3, 1/2, 1, 2/3, Sec[c + d*x], -Sec[c + d*x]]*Tan[c + d*x])/(
d*Sqrt[1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{1+\sec(c+dx)}\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}$$
$$= -\frac{(e\tan(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(ex)^{4/3}(1+x)}\,dx, x, \sec(c+dx\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$
$$= -\frac{(3\tan(c+dx))\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c-\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{3F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3346 vs. $2(76) = 152$.
time = 20.44, size = 3346, normalized size = 44.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -(((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]*(-1 + Tan[(c +
d*x)/2]^2)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*
(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x
```

)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3)))/(d*(e*Sec[c + d*x])^(1/3)*Sqrt[a*(1 + Sec[c + d*x])]*(-(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]^2*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3))) - (Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*(-1 + Tan[(c + d*x)/2]^2)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3)))/2 - (Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (2*Tan[(c + d*x)/2]^2*(-1/5*(AppellF1[5/2, 1/6, 4/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (AppellF1[5/2, 7/6, 1/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/10))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) - (5*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(11/6)) - (Tan[(c + d*x)/2]*(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3) + (3*((-3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-1 + Tan[(c + d*x)/2]^2)^2*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) + (3*(-1/9*(AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/18))/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2

```
*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Ap
pellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
 + d*x)/2]^2)) - (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*((-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-1/9*(AppellF1[3/2, 1
/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]) + (AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/18) + Tan[(c + d*x)/2]^
2*(-1/5*(AppellF1[5/2, 7/6, 4/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (7*AppellF1[5/2, 13/6, 1/3, 7/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/10 - 2*((-4*AppellF1[5/2, 1/6, 7/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 7/6, 4/3
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/10))))/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2...
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \sec \left(dx+c\right)\right)^{\frac{1}{3}} \sqrt{a+a \sec \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] e^(-1/3)*integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sqrt[3]{e\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3)), x)
```

3.283 $\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{3F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}\,(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 3/2*AppellF1(-2/3,1,1/2,1/3,-sec(d*x+c),sec(d*x+c))*tan(d*x+c)/d/(e*sec(d*x
+c))^(2/3)/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3913, 3912, 129, 524}

$$\frac{3\tan(c+dx)F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};\sec(c+dx),-\sec(c+dx)\right)}{2d\sqrt{1-\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}\,(e\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (3*AppellF1[-2/3, 1/2, 1, 1/3, Sec[c + d*x], -Sec[c + d*x]]*Tan[c + d*x])/(
2*d*Sqrt[1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{1+\sec(c+dx)}\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(e\tan(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(ex)^{5/3}(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(3\tan(c+dx))\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}\,(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 585 vs. 2(78) = 156.
time = 7.35, size = 585, normalized size = 7.50

$$\frac{\sec^{\frac{7}{6}}(c+dx)\left(-\frac{3}{2}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{6}}(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)+\frac{5\sqrt{\frac{1}{1+\cos(c+dx)}}\,(-1+3\cos(c+dx))\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{5/6}\sin\left(\frac{1}{2}(c+dx)\right)\left(-3\cos^{\frac{5}{6}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\sqrt[3]{\sec^2\left(\frac{1}{2}(c+dx)\right)}+2F_1\left(\frac{3}{2};\frac{5}{6},\frac{2}{3};\frac{5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^{5/6}\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{-120F_1\left(\frac{3}{2};\frac{5}{6},\frac{2}{3};\frac{5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\frac{1}{1+\cos(c+dx)}\right)^{2/3}\left(\frac{\cos(c+dx)}{1+\cos(c+dx)}\right)^{5/6}\sin\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)+32F_1\left(\frac{5}{2};\frac{5}{6},\frac{5}{3};\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\frac{1}{1+\cos(c+dx)}\right)^{2/3}\left(\frac{\cos(c+dx)}{1+\cos(c+dx)}\right)^{5/6}\sin\left(\frac{1}{2}(c+dx)\right)\tan^3\left(\frac{1}{2}(c+dx)\right)+5\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)\left(3-4\sqrt{2}F_1\left(\frac{5}{2};\frac{11}{6},\frac{2}{3};\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\frac{1}{1+\cos(c+dx)}\right)^{2/3}\left(\frac{\cos(c+dx)}{1+\cos(c+dx)}\right)^{5/6}\tan^4\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{d(e\sec(c+dx))^{2/3}\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sec[c + d*x]^(7/6)*((-3*Cos[(c + d*x)/2]*Sec[c + d*x]^(5/6)*(Sin[(c + d*x)
/2] - Sin[(3*(c + d*x))/2]))/2 + (5*Sqrt[(1 + Cos[c + d*x])^(-1)]*(-1 + 3*C
os[c + d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/6)*Sin[(c + d*x)/2]*(-3*C
os[c + d*x]^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, 2*Sin[(c + d*x)/2]^2]*(S
ec[(c + d*x)/2]^2)^(1/3) + 2*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^
```

```
2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6)*Tan[(c + d*
x)/2]^2))/(-120*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(Cos[c + d*x]/(1 + Cos[c + d*x])
)^(5/6)*Sin[(c + d*x)/2]*Tan[(c + d*x)/2] + 32*AppellF1[5/2, 5/6, 5/3, 7/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(
Cos[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Sin[(c + d*x)/2]*Tan[(c + d*x)/2]^3
+ 5*Sqrt[2]*Cos[(c + d*x)/2]*(3 - 4*Sqrt[2]*AppellF1[5/2, 11/6, 2/3, 7/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(Co
s[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Tan[(c + d*x)/2]^4))))/(d*(e*Sec[c + d
*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \sec \left(dx+c\right)\right)^{\frac{2}{3}} \sqrt{a+a \sec \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] e^(-2/3)*integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \left(\sec \left(c+dx\right)+1\right)} \left(e \sec \left(c+dx\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3)), x)
```

3.284 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{2^{5/6}F_1\left(\frac{1}{2};-\frac{1}{3},\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{d(1+\sec(c+dx))^{5/6}}$$

```
[Out] 2^(5/6)*AppellF1(1/2,-1/3,1/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a*sec
(d*x+c))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(5/6)
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3910, 138}

$$\frac{2^{5/6}\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,F_1\left(\frac{1}{2};-\frac{1}{3},\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2^(5/6)*AppellF1[1/2, -1/3, 1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*x])
/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \frac{\sqrt[3]{a+a\sec(c+dx)}\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{1+\sec(c+dx)}\,dx}{\sqrt[3]{1+\sec(c+dx)}}$$

$$= \frac{\left(\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{\sqrt[6]{2-x}\sqrt{x}}\,dx, x,\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{5/6}}$$

$$= \frac{2^{5/6}F_1\left(\frac{1}{2}; -\frac{1}{3}, \frac{1}{6}; \frac{3}{2}; 1-\sec(c+dx), \frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\,\mathrm{se}}}{d(1+\sec(c+dx))^{5/6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1982 vs. $2(78) = 156$.
time = 23.25, size = 1982, normalized size = 25.41

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
[Out] (3*Sec[c + d*x]^(1/3)*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[c
 + d*x]))^(1/3)*Sin[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/3)) + (3*(a*(1 + S
ec[c + d*x]))^(1/3)*(-((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(2/3)) + (Sec[
c + d*x]^(1/3)*(1 + Sec[c + d*x])^(1/3))/2)*Tan[(c + d*x)/2]*(-1 + (3*Appel
lF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*Appel
lF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*Ap
pellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Hype
rgeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2)))
/(2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1
/3)*(1 + Sec[c + d*x])^(1/3)*((3*(Sec[(c + d*x)/2]^2)^(1/3)*(-1 + (3*Appell
F1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*Appell
F1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*App
ellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Hyper
geometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2)))/
(2*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)) - (2^(1/3)*Tan[(c + d*x
)/2]^2*(-1 + (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*
Tan[(c + d*x)/2]^2)))/((Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c
```

```
 + d*x])^(1/3)) + (3*Tan[(c + d*x)/2]*((3*((-2*AppellF1[3/2, -1/3, 5/3, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/9 - (HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, T
an[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2) - (3*AppellF1[1/2, -1/3, 2/3, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-2*(2*AppellF1[3/2, -1/3, 5/3, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4},
 {7/4}, Tan[(c + d*x)/2]^4])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*((-2*A
ppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2])/9 - (HypergeometricPFQ[{2/3, 3/4}, {7/4}, T
an[(c + d*x)/2]^4]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) - 2*Tan[(c + d*x
)/2]^2*(2*(-(AppellF1[5/2, -1/3, 8/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) - (AppellF1[5/2, 2/3, 5/3, 7/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2])/5) + (3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-HypergeometricPFQ[{2/3, 3
/4}, {7/4}, Tan[(c + d*x)/2]^4] + (1 - Tan[(c + d*x)/2]^4)^(-2/3)))/2)))/(9
*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2
*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2
]^2)^2))/(2^(2/3)*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*
x])^(1/3)) - (Tan[(c + d*x)/2]*(-1 + (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4
}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*
x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(2^(2
/3)*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(4/3)))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}\left(dx+c\right)\right)\left(a+a\sec\left(dx+c\right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{1/3}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^(4/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^(4/3), x)
```

3.285 $\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[6]{2}\,F_1\left(\frac{1}{2};-\frac{1}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{d(1+\sec(c+dx))^{7/6}}$$

```
[Out] 2*2^(1/6)*AppellF1(1/2,-1/3,-1/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a*
sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(7/6)
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3910, 138}

$$\frac{2\sqrt[6]{2}\,\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{1}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*AppellF1[1/2, -1/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*
x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6)
)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \sec^{\frac{4}{3}}(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= \frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-x}\,\sqrt[6]{2-x}}{\sqrt{x}}\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$

$$= \frac{2\sqrt[6]{2}\,F_1\left(\frac{1}{2};-\frac{1}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)(a+}{d(1+\sec(c+dx))^{7/6}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 25.47, size = 2618, normalized size = 33.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sec[c + d*x]^(1/3)*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c +
 d*x]))^(2/3)*Tan[(c + d*x)/2])/(d*(1 + Sec[c + d*x])^(2/3)) + (15*AppellF1
[1/2, 2/3, 1/3, 3/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*(a*(1 + Sec[c
 + d*x]))^(2/3)*(9*AppellF1[1/2, 2/3, 1/3, 3/2, Tan[(c + d*x)/4]^2, -Tan[(c
 + d*x)/4]^2] - 2*(AppellF1[3/2, 2/3, 4/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c
 + d*x)/4]^2] - 2*AppellF1[3/2, 5/3, 1/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c
+ d*x)/4]^2])*Tan[(c + d*x)/4]^2)*Tan[(c + d*x)/2])/(d*(Sec[c + d*x]/(1 + S
ec[c + d*x]))^(2/3)*(1 + Sec[c + d*x])^(2/3)*((135*AppellF1[1/2, 2/3, 1/3,
3/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]^2*(5 + Cos[c + d*x]))/2 + 20*
(AppellF1[3/2, 2/3, 4/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*
AppellF1[3/2, 5/3, 1/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])^2*Co
s[(c + d*x)/2]*Tan[(c + d*x)/4]^4 - 3*AppellF1[1/2, 2/3, 1/3, 3/2, Tan[(c +
 d*x)/4]^2, -Tan[(c + d*x)/4]^2]*Tan[(c + d*x)/4]^2*(5*AppellF1[3/2, 2/3, 4
/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*(5 - 12*Cos[(c + d*x)/2]
+ Cos[c + d*x]) - 10*AppellF1[3/2, 5/3, 1/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[
(c + d*x)/4]^2]*(5 - 12*Cos[(c + d*x)/2] + Cos[c + d*x]) + 24*(2*AppellF1[5
/2, 2/3, 7/3, 7/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*AppellF1[5/
2, 5/3, 4/3, 7/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + 5*AppellF1[5/2
, 8/3, 1/3, 7/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Cos[(c + d*x)/2]
*Tan[(c + d*x)/4]^2))) - ((Sec[(c + d*x)/2]^2)^(4/3)*Sec[c + d*x]^(1/3)*(a*
```

```
(1 + Sec[c + d*x]))^(2/3)*(AppellF1[-2/3, -1/3, -1/3, 1/3, (-1 - I)/(-1 + T
an[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x)/2
])/(-1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*
x)/2]))^(1/3)) - AppellF1[-2/3, -1/3, -1/3, 1/3, (1 - I)/(1 + Tan[(c + d*x)
/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(1 + Tan[(c
 + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3))))
/(2^(1/3)*d*(-(((Sec[(c + d*x)/2]^2)^(1/3)*Tan[(c + d*x)/2]*(AppellF1[-2/3,
 -1/3, -1/3, 1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c +
 d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)*((I + T
an[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)) - AppellF1[-2/3, -1/3, -1/
3, 1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]/(((
-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2]
)/(1 + Tan[(c + d*x)/2]))^(1/3))))/2^(1/3)) - (3*(Sec[(c + d*x)/2]^2)^(1/3)
*((((1/3 - I/3)*AppellF1[1/3, -1/3, 2/3, 4/3, (-1 - I)/(-1 + Tan[(c + d*x)/
2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1 + Tan[(c + d
*x)/2])^2 + ((1/3 + I/3)*AppellF1[1/3, 2/3, -1/3, 4/3, (-1 - I)/(-1 + Tan[(
c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1 + T
an[(c + d*x)/2])^2)/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3
)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)) - (AppellF1[-2/3,
 -1/3, -1/3, 1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c +
 d*x)/2])]*(Sec[(c + d*x)/2]^2/(2*(-1 + Tan[(c + d*x)/2])) - (Sec[(c + d*x)
/2]^2*(-I + Tan[(c + d*x)/2]))/(2*(-1 + Tan[(c + d*x)/2])^2)))/(3*((-I + Ta
n[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(4/3)*((I + Tan[(c + d*x)/2])/(-1
+ Tan[(c + d*x)/2]))^(1/3)) - (AppellF1[-2/3, -1/3, -1/3, 1/3, (-1 - I)/(-1
 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*(Sec[(c + d*x)/2]^2
/(2*(-1 + Tan[(c + d*x)/2])) - (Sec[(c + d*x)/2]^2*(I + Tan[(c + d*x)/2]))/
(2*(-1 + Tan[(c + d*x)/2])^2)))/(3*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c +
d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(4/3)) - (
((-1/3 - I/3)*AppellF1[1/3, -1/3, 2/3, 4/3, (1 - I)/(1 + Tan[(c + d*x)/2]),
 (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2])
^2 - ((1/3 - I/3)*AppellF1[1/3, 2/3, -1/3, 4/3, (1 - I)/(1 + Tan[(c + d*x)/
2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)
/2])^2)/(((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(
c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)) + (AppellF1[-2/3, -1/3, -1/3, 1
/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*(-1/2*(
Sec[(c + d*x)/2]^2*(-I + Tan[(c + d*x)/2]))/(1 + Tan[(c + d*x)/2])^2 + Sec[
(c + d*x)/2]^2/(2*(1 + Tan[(c + d*x)/2]))))/(3*((-I + Tan[(c + d*x)/2])/(1
+ Tan[(c + d*x)/2]))^(4/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^
(1/3)) + (AppellF1[-2/3, -1/3, -1/3, 1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (
1 + I)/(1 + Tan[(c + d*x)/2])]*(-1/2*(Sec[(c + d*x)/2]^2*(I + Tan[(c + d*x)
/2]))/(1 + Tan[(c + d*x)/2])^2 + Sec[(c + d*x)/2]^2/(2*(1 + Tan[(c + d*x)/2
]))))/(3*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(
c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(4/3))))/2^(1/3)))
```

Maple [F]

time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}(dx+c)\right)(a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(2/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{2/3}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(4/3),x)
[Out] int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(4/3), x)
```

3.286 $\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=327

$$-\frac{3a\sec^{\frac{5}{3}}(c+dx)\sin(c+dx)}{2d\sqrt[3]{a(1+\sec(c+dx))}}+\frac{9\sec^{\frac{2}{3}}(c+dx)(a(1+\sec(c+dx)))^{2/3}\sin(c+dx)}{4d}-\frac{9(a(1+\sec(c+dx)))^{2}}{4d\sqrt[3]{\frac{1}{1+\cos(c+dx)}}\,(1}$$

```
[Out] -3/2*a*sec(d*x+c)^(5/3)*sin(d*x+c)/d/(a*(1+sec(d*x+c)))^(1/3)+9/4*sec(d*x+c
)^(2/3)*(a*(1+sec(d*x+c)))^(2/3)*sin(d*x+c)/d-9/4*(a*(1+sec(d*x+c)))^(2/3)*
tan(d*x+c)/d/(1/(1+cos(d*x+c)))^(1/3)/(1+sec(d*x+c))^(7/3)+1/8*hypergeom([1
/4, 1/3],[5/4],tan(1/2*d*x+1/2*c)^4)*(cos(d*x+c)*sec(1/2*d*x+1/2*c)^4)^(1/3
)*(a*(1+sec(d*x+c)))^(2/3)*tan(d*x+c)/d/(1/(1+cos(d*x+c)))^(1/3)/(1+sec(d*x
+c))^(4/3)-5/8*hypergeom([1/3, 3/4],[7/4],tan(1/2*d*x+1/2*c)^4)*(cos(d*x+c)
*sec(1/2*d*x+1/2*c)^4)^(1/3)*(a*(1+sec(d*x+c)))^(2/3)*tan(d*x+c)^3/d/(1/(1+
cos(d*x+c)))^(1/3)/(1+sec(d*x+c))^(10/3)
```

Rubi [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 0.24, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3910, 138}

$$\frac{2\sqrt[6]{2}\,\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{2}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*AppellF1[1/2, -2/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*
x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6)
)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
```

```
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \sec^{\frac{5}{3}}(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= \frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1-x)^{2/3}\sqrt[6]{2-x}}{\sqrt{x}}\,dx,\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$

$$= \frac{2\sqrt[6]{2}\,F_1\left(\frac{1}{2};-\frac{2}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)(a+a}{d(1+\sec(c+dx))^{7/6}}$$

Mathematica [A]
time = 9.73, size = 274, normalized size = 0.84

$$\frac{(a(1+\sec(c+dx)))^{2/3}\left(-3\sec^3\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt[3]{1+\sec(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{3}{2}(c+dx)\right)\right)+\sqrt[3]{2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{5}{4};\tan^4\left(\frac{1}{2}(c+dx)\right)\right)\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}\sqrt[3]{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)-5\sqrt[3]{2}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{7}{4};\tan^4\left(\frac{1}{2}(c+dx)\right)\right)\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}\sqrt[3]{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\tan^3\left(\frac{1}{2}(c+dx)\right)\right)}{8d\sqrt[3]{\frac{1}{1+\cos(c+dx)}}(1+\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(-3*Sec[(c + d*x)/2]^3*Sec[c + d*x]*(1 + Sec[
c + d*x])^(1/3)*(Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2]) + 2^(1/3)*Hyper
geometric2F1[1/4, 1/3, 5/4, Tan[(c + d*x)/2]^4]*(Cos[c + d*x]*Sec[(c + d*x)
/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[(c + d*x)/2] - 5*2
^(1/3)*Hypergeometric2F1[1/3, 3/4, 7/4, Tan[(c + d*x)/2]^4]*(Cos[c + d*x]*S
ec[(c + d*x)/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[(c + d
*x)/2]^3))/(8*d*((1 + Cos[c + d*x])^(-1))^(1/3)*(1 + Sec[c + d*x])^(2/3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{5}{3}}\left(dx+c\right)\right)\left(a+a\sec\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos \left(c+d\,x\right)}\right)^{2/3}\,{\left(\frac{1}{\cos \left(c+d\,x\right)}\right)}^{5/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(5/3),x)
[Out] int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(5/3), x)
```

3.287 $\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2\ 2^{5/6} a F_1\left(\frac{1}{2};\frac{4}{3},-\frac{5}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\sec(c+dx)}\ \tan(c+dx)}{d(1+\sec(c+dx))^{5/6}}$$

```
[Out] 2*2^(5/6)*a*AppellF1(1/2,4/3,-5/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a
*sec(d*x+c))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(5/6)
```

Rubi [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3910, 138}

$$\frac{2\ 2^{5/6} a \tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\ F_1\left(\frac{1}{2};\frac{4}{3},-\frac{5}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

```
[Out] (2*2^(5/6)*a*AppellF1[1/2, 4/3, -5/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d
*x])/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6
))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \frac{\left(a\sqrt[3]{a+a\sec(c+dx)}\right)\int \frac{(1+\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx}{\sqrt[3]{1+\sec(c+dx)}}$$

$$= \frac{\left(a\sqrt[3]{a+a\sec(c+dx)}\,\tan(c+dx)\right)\text{Subst}\left(\int \frac{(2-x)^{5/6}}{(1-x)^{4/3}\sqrt{x}}\,dx,x,1-\sec(c+a}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{5/6}}$$

$$= \frac{2\,2^{5/6}aF_1\left(\frac{1}{2};\frac{4}{3},-\frac{5}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\sec(c+dx}{d(1+\sec(c+dx))^{5/6}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.33, size = 2325, normalized size = 29.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

```
[Out] (-3*(a*(1 + Sec[c + d*x]))^(4/3)*((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(1/
3) + Sec[c + d*x]^(2/3)*(1 + Sec[c + d*x])^(1/3))*(-8*Tan[(c + d*x)/2] + (A
ppellF1[-4/3, -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/
(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/(-1
+ Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))
^(2/3)) - AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)/2]),
(1 + I)/(1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)
/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[
(c + d*x)/2])^2))/(4*2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(1/3)*(1 + Sec[c + d*x]
)^(4/3)*((Tan[(c + d*x)/2]*(-8*Tan[(c + d*x)/2] + (AppellF1[-4/3, -2/3, -2/
3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])
]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2
/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)) - AppellF1[-4/3
, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d
*x)/2])]*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(
c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])^2))/(4*2^
(2/3)*(Sec[(c + d*x)/2]^2)^(1/3)) - (3*(-4*Sec[(c + d*x)/2]^2 + (Sec[(c + d
*x)/2]^2*(((-4/3 + (4*I)/3)*AppellF1[-1/3, -2/3, 1/3, 2/3, (-1 - I)/(-1 + T
```

```
an[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1
 + Tan[(c + d*x)/2])^2 - ((4/3 + (4*I)/3)*AppellF1[-1/3, 1/3, -2/3, 2/3, (-
1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c +
d*x)/2]^2)/(-1 + Tan[(c + d*x)/2])^2))/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[
(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)
) + (AppellF1[-4/3, -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1
 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(((-I +
 Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(
-1 + Tan[(c + d*x)/2]))^(2/3)) - AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(
1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2*(
(-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2
])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2]) - (2*AppellF1[-4/3,
 -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c
+ d*x)/2])]*Sec[(c + d*x)/2]^2*(Sec[(c + d*x)/2]^2/(2*(-1 + Tan[(c + d*x)/2
])) - (Sec[(c + d*x)/2]^2*(-I + Tan[(c + d*x)/2]))/(2*(-1 + Tan[(c + d*x)/2
])^2)))/(3*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(5/3)*((I + Ta
n[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)) - (2*AppellF1[-4/3, -2/3, -
2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2
])]*Sec[(c + d*x)/2]^2*(Sec[(c + d*x)/2]^2/(2*(-1 + Tan[(c + d*x)/2])) - (S
ec[(c + d*x)/2]^2*(I + Tan[(c + d*x)/2]))/(2*(-1 + Tan[(c + d*x)/2])^2)))/(
3*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*
x)/2])/(-1 + Tan[(c + d*x)/2]))^(5/3)) - ((-I + Tan[(c + d*x)/2])/(1 + Tan[
(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*
(1 + Tan[(c + d*x)/2])^2*(((4/3 + (4*I)/3)*AppellF1[-1/3, -2/3, 1/3, 2/3, (
1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x
)/2]^2)/(1 + Tan[(c + d*x)/2])^2 + ((4/3 - (4*I)/3)*AppellF1[-1/3, 1/3, -2/
3, 2/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec
[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2])^2) - (AppellF1[-4/3, -2/3, -2/3, -1
/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*((I + T
an[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])^2*(-1
/2*(Sec[(c + d*x)/2]^2*(-I + Tan[(c + d*x)/2]))/(1 + Tan[(c + d*x)/2])^2 +
Sec[(c + d*x)/2]^2/(2*(1 + Tan[(c + d*x)/2]))))/(3*((-I + Tan[(c + d*x)/2])
/(1 + Tan[(c + d*x)/2]))^(2/3)) - (AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)
/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*
x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])^2*(-1/2*(Sec[(c
 + d*x)/2]^2*(I + Tan[(c + d*x)/2]))/(1 + Tan[(c + d*x)/2])^2 + Sec[(c + d*
x)/2]^2/(2*(1 + Tan[(c + d*x)/2]))))/(3*((I + Tan[(c + d*x)/2])/(1 + Tan[(c
 + d*x)/2]))^(2/3))))/(4*2^(2/3)*(Sec[(c + d*x)/2]^2)^(1/3))))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sec (dx + c))^{\frac{4}{3}}}{\sec (dx + c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(4/3)/sec(d*x + c)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(4/3)/sec(d*x+c)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(4/3)/sec(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{4/3}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(4/3)/(1/cos(c + d*x))^(1/3),x)
[Out] int((a + a/cos(c + d*x))^(4/3)/(1/cos(c + d*x))^(1/3), x)
```

3.288 $\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$

Optimal. Leaf size=304

$$\frac{a^4(30+21n+4n^2)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)(3+n)}+\frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(e+fx)\right)^2\sin(e+fx)}{f(3+n)}+\frac{2(4+n)}{}$$

```
[Out] a^4*(4*n^2+21*n+30)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(n^3+6*n^2+11*n+6)+sec(f*
x+e)^(1+n)*(a^2+a^2*sec(f*x+e))^2*sin(f*x+e)/f/(3+n)+2*(4+n)*sec(f*x+e)^(1+
n)*(a^4+a^4*sec(f*x+e))*sin(f*x+e)/f/(2+n)/(3+n)-a^4*(8*n^2+24*n+3)*hyperge
om([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/
f/(3+n)/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+4*a^4*(3+2*n)*hypergeom([1/2, -1/2*n]
,[1-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(
1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3899, 4103, 4082, 3872, 3857, 2722}

$$-\frac{a^4(8n^2+24n+3)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)(n+3)\sqrt{\sin^2(e+fx)}}+\frac{4a^4(2n+3)\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn(n+2)\sqrt{\sin^2(e+fx)}}+\frac{a^4(4n^2+21n+30)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)(n+2)(n+3)}+\frac{2(n+4)\sin(e+fx)\left(a^4\sec(e+fx)+a^4\right)\sec^{n+1}(e+fx)}{f(n+2)(n+3)}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
```

```
[Out] (a^4*(30 + 21*n + 4*n^2)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)*(2 +
 n)*(3 + n)) + (Sec[e + f*x]^(1 + n)*(a^2 + a^2*Sec[e + f*x])^2*Sin[e + f*x
])/(f*(3 + n)) + (2*(4 + n)*Sec[e + f*x]^(1 + n)*(a^4 + a^4*Sec[e + f*x])*S
in[e + f*x])/(f*(2 + n)*(3 + n)) - (a^4*(3 + 24*n + 8*n^2)*Hypergeometric2F
1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e +
f*x])/(f*(1 - n)*(1 + n)*(3 + n)*Sqrt[Sin[e + f*x]^2]) + (4*a^4*(3 + 2*n)*H
ypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin
[e + f*x])/(f*n*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx = \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(e+fx)\right)^2\sin(e+fx)}{f(3+n)} + \frac{a\int \sec^n(e+f$$

$$= \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(e+fx)\right)^2\sin(e+fx)}{f(3+n)} + \frac{2(4+n)\sec^{1+}$$

$$= \frac{a^4(30+21n+4n^2)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\,(6+5n+n^2)} + \frac{\sec^{1+n}(e+fx)\,($$

$$= \frac{a^4(30+21n+4n^2)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\,(6+5n+n^2)} + \frac{\sec^{1+n}(e+fx)\,($$

$$= \frac{a^4(30+21n+4n^2)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\,(6+5n+n^2)} + \frac{\sec^{1+n}(e+fx)\,($$

$$= \frac{a^4(30+21n+4n^2)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\,(6+5n+n^2)} + \frac{\sec^{1+n}(e+fx)\,($$

Mathematica [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
[Out] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4, x]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(\sec^n(fx+e)\right)\left(a+a\sec(fx+e)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(f*x + e)^4 + 4*a^4*sec(f*x + e)^3 + 6*a^4*sec(f*x + e)^2
+ 4*a^4*sec(f*x + e) + a^4)*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{4}\left(\int 4\sec{\left(e+fx\right)}\sec^{n}{\left(e+fx\right)}\,dx+\int 6\sec^{2}{\left(e+fx\right)}\sec^{n}{\left(e+fx\right)}\,dx+\int 4\sec^{3}{\left(e+fx\right)}\sec^{n}{\left(e+fx\right)}\,dx+\int \sec^{4}{\left(e+fx\right)}\sec^{n}{\left(e+fx\right)}\,dx+\int \sec^{n}{\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**4,x)
```

```
[Out] a**4*(Integral(4*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(6*sec(e + f*x)
**2*sec(e + f*x)**n, x) + Integral(4*sec(e + f*x)**3*sec(e + f*x)**n, x) +
Integral(sec(e + f*x)**4*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^4\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^4*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^4*(1/cos(e + f*x))^n, x)
```

3.289 $\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$

Optimal. Leaf size=230

$$\frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)}+\frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin(e+fx)}{f(2+n)}-\frac{a^3(1+4n)\,{}_2F_1\left(\frac{1}{2},\right.}{}$$

```
[Out] a^3*(5+2*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+n)/(2+n)+sec(f*x+e)^(1+n)*(a^3
+a^3*sec(f*x+e))*sin(f*x+e)/f/(2+n)-a^3*(1+4*n)*hypergeom([1/2, 1/2-1/2*n],
[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+
e)^2)^(1/2)+a^3*(7+4*n)*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*sec
(f*x+e)^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3899, 4082, 3872, 3857, 2722}

$$-\frac{a^3(4n+1)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{a^3(4n+7)\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn(n+2)\sqrt{\sin^2(e+fx)}}+\frac{a^3(2n+5)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)(n+2)}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(5 + 2*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)*(2 + n)) + (Se
c[e + f*x]^(1 + n)*(a^3 + a^3*Sec[e + f*x])*Sin[e + f*x])/(f*(2 + n)) - (a^
3*(1 + 4*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Se
c[e + f*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[e + f*x]^2]) + (a^3
*(7 + 4*n)*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e
+ f*x]^n*Sin[e + f*x])/(f*n*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx &= \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin(e+fx)}{f(2+n)}+\frac{a\int \sec^n(e+
\\
&= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)}+\frac{\sec^{1+n}(e+fx)\left(a^3+a^3\right.}{f(2}
\\
&= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)}+\frac{\sec^{1+n}(e+fx)\left(a^3+a^3\right.}{f(2}
\\
&= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)}+\frac{\sec^{1+n}(e+fx)\left(a^3+a^3\right.}{f(2}
\\
&= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)}+\frac{\sec^{1+n}(e+fx)\left(a^3+a^3\right.}{f(2}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.52, size = 286, normalized size = 1.24

$$
-\frac{i2^{-3+n}a^3\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n(1+\cos(e+fx))^3\left(\frac{8e^{3i(e+fx)}\,{}_2F_1\left(1,\frac{1}{2}(-1-n);\frac{5+n}{2};-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2(3+n)}+\frac{6e^{i(e+fx)}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3+n}{2};-e^{2i(e+fx)}\right)}{1+n}+\frac{\left(1+e^{2i(e+fx)}\right){}_2F_1\left(1,1-\frac{n}{2};\frac{2+n}{2};-e^{2i(e+fx)}\right)}{n}+\frac{12e^{2i(e+fx)}\,{}_2F_1\left(1,-\frac{n}{2};\frac{4+n}{2};-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)(2+n)}\right)\sec^6\left(\frac{1}{2}(e+fx)\right)}{f}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((-I)*2^(-3 + n)*a^3*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + Cos
[e + f*x])^3*((8*E^((3*I)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (5 +
n)/2, -E^((2*I)*(e + f*x))])/((1 + E^((2*I)*(e + f*x)))^2*(3 + n)) + (6*E^(
I*(e + f*x))*Hypergeometric2F1[1, (1 - n)/2, (3 + n)/2, -E^((2*I)*(e + f*x)
)])/(1 + n) + ((1 + E^((2*I)*(e + f*x)))*Hypergeometric2F1[1, 1 - n/2, (2 +
 n)/2, -E^((2*I)*(e + f*x))])/n + (12*E^((2*I)*(e + f*x))*Hypergeometric2F1
[1, -1/2*n, (4 + n)/2, -E^((2*I)*(e + f*x))])/((1 + E^((2*I)*(e + f*x)))*(2
 + n)))*Sec[(e + f*x)/2]^6)/f
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (\sec^n (fx+e)) (a+a\sec (fx+e))^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int 3\sec (e+fx) \sec^n (e+fx) \,dx + \int 3\sec^2 (e+fx) \sec^n (e+fx) \,dx + \int \sec^3 (e+fx) \sec^n (e+fx) \,dx + \int \sec^n (e+fx) \,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(3*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(3*sec(e + f*x)
**2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**3*sec(e + f*x)**n, x) + In
tegral(sec(e + f*x)**n, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^3\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^3*(1/cos(e + f*x))^n, x)
```

3.290 $\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$

Optimal. Leaf size=172

$$\frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)}-\frac{a^2(1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\,2}{}$$

```
[Out] a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+n)-a^2*(1+2*n)*hypergeom([1/2, 1/2-1/2
*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(
f*x+e)^2)^(1/2)+2*a^2*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*sec(f
*x+e)^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3873, 3857, 2722, 4131}

$$-\frac{a^2(2n+1)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)) - (a^2*(1 + 2*n)*Hyperg
eometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n
)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[e + f*x]^2]) + (2*a^2*Hypergeometric2
F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x])/(f*
n*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = \left(2a^2\right)\int \sec^{1+n}(e+fx)\,dx + \int \sec^n(e+fx)\left(a^2+a^2\sec^2(e+fx\right.$$

$$= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)} + \frac{\left(a^2(1+2n)\right)\int \sec^n(e+fx)\,dx}{1+n} +$$

$$= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)} + \frac{2a^2\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right.}{fn\sqrt{\sin^2(e-}}$$

$$= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)} - \frac{a^2(1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(\right.}{f\left(1-n^2\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.10, size = 222, normalized size = 1.29

$$-\frac{i2^{-2+n}a^2\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n(1+\cos(e+fx))^2\left(\frac{4e^{i(e+fx)}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3+n}{2};-e^{2i(e+fx)}\right)}{1+n}+\frac{\left(1+e^{2i(e+fx)}\right){}_2F_1\left(1,1-\frac{n}{2};\frac{2+n}{2};-e^{2i(e+fx)}\right)}{n}+\frac{4e^{2i(e+fx)}\,{}_2F_1\left(1,-\frac{n}{2};\frac{4+n}{2};-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)(2+n)}\right)\sec^4\left(\frac{1}{2}(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((-I)*2^(-2 + n)*a^2*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + Cos
[e + f*x])^2*((4*E^(I*(e + f*x))*Hypergeometric2F1[1, (1 - n)/2, (3 + n)/2,
 -E^((2*I)*(e + f*x))])/(1 + n) + ((1 + E^((2*I)*(e + f*x)))*Hypergeometric
2F1[1, 1 - n/2, (2 + n)/2, -E^((2*I)*(e + f*x))])/n + (4*E^((2*I)*(e + f*x)
)*Hypergeometric2F1[1, -1/2*n, (4 + n)/2, -E^((2*I)*(e + f*x))])/((1 + E^((
2*I)*(e + f*x)))*(2 + n)))*Sec[(e + f*x)/2]^4)/f
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\sec^n(fx+e)\right)(a+a\sec(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\sec\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int\sec^{2}\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int\sec^{n}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(2*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**
2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos \left(e+f\,x\right)}\right)^2\,\left(\frac{1}{\cos \left(e+f\,x\right)}\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(1/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^2*(1/cos(e + f*x))^n, x)
```

3.291 $\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx$

Optimal. Leaf size=132

$$-\frac{a\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}+\frac{a\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}$$

```
[Out] -a*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*s
in(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)+a*hypergeom([1/2, -1/2*n],[1-1/2*n],
cos(f*x+e)^2)*sec(f*x+e)^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3872, 3857, 2722}

$$\frac{a\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] -((a*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f
*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeom
etric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x
])/(f*n*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = a\int \sec^n(e+fx)\,dx + a\int \sec^{1+n}(e+fx)\,dx$$
$$= (a\cos^n(e+fx)\sec^n(e+fx))\int \cos^{-1-n}(e+fx)\,dx + (a\cos^n(e+$$
$$= -\frac{a\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}} + $$

Mathematica [A]
time = 0.17, size = 106, normalized size = 0.80

$$\frac{a\csc(e+fx)\sec^{-1+n}(e+fx)\left((1+n)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\sec^2(e+fx)\right)+n\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sec^2(e+fx)\right)\sec(e+fx)\right)\sqrt{-\tan^2(e+fx)}}{fn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] (a*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*((1 + n)*Hypergeometric2F1[1/2, n/2,
(2 + n)/2, Sec[e + f*x]^2] + n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
 Sec[e + f*x]^2]*Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sec^n(fx+e)\right)\left(a+a\sec(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sec\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int \sec^{n}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e)),x)
```

```
[Out] a*(Integral(sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))*(1/cos(e + f*x))^n, x)
```

3.292 $\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=174

$$\frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))}+\frac{(1-n)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(e+fx)\right)\sec^{-2+n}(e+fx)\sin(e+fx)}{af(2-n)\sqrt{\sin^2(e+fx)}}-\frac{{}_2F_1\left(\frac{1}{2},\frac{1-n}{2}\right.}{}$$

```
[Out] sec(f*x+e)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))+(1-n)*hypergeom([1/2, 1-1/2*n],[
2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-2+n)*sin(f*x+e)/a/f/(2-n)/(sin(f*x+e)^2
)^(1/2)-hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1
+n)*sin(f*x+e)/a/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3905, 3872, 3857, 2722}

$$\frac{(1-n)\sin(e+fx)\sec^{n-2}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(e+fx)\right)}{af(2-n)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{af\sqrt{\sin^2(e+fx)}}+\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) + ((1 - n)*Hypergeom
etric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-2 + n)*S
in[e + f*x])/(a*f*(2 - n)*Sqrt[Sin[e + f*x]^2]) - (Hypergeometric2F1[1/2, (
1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e + f*x])/(a
*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Dist[d*((n - 1)/(a*b)), Int[(d*Csc[e + f*x])^(
n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[
a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx &= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{(1-n)\int \sec^{-1+n}(e+fx)(a-a\sec(e+fx))\,dx}{a^2}\\ &= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{(1-n)\int \sec^{-1+n}(e+fx)\,dx}{a} + \frac{(1-n)\int \sec^n(e+fx}{a}\\ &= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{((1-n)\cos^n(e+fx)\sec^n(e+fx))\int \cos^{1-n}(e+fx)}{a}\\ &= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{(1-n)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(e+fx)\right)\sec^{-2+n}(e+fx}{af(2-n)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [F]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x]), x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^n(fx+e)}{a+a\sec(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)
```

[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")

[Out] integral(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{n}(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e)),x)

[Out] Integral(sec(e + f*x)**n/(sec(e + f*x) + 1), x)/a

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="giac")

[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
[Out] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

3.293 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=217

$$-\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{(3-2n)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e}$$

```
[Out] -2/3*(2-n)*sec(f*x+e)^(1+n)*sin(f*x+e)/a^2/f/(1+sec(f*x+e))-1/3*sec(f*x+e)^
(1+n)*sin(f*x+e)/f/(a+a*sec(f*x+e))^2-1/3*(3-2*n)*hypergeom([1/2, 1/2-1/2*n
],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/a^2/f/(sin(f*x+e)^
2)^(1/2)+2/3*(2-n)*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*sec(f*x+
e)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3902, 4105, 3872, 3857, 2722}

$$-\frac{(3-2n)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{2(2-n)\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{2(2-n)\sin(e+fx)\sec^{n+1}(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-2*(2 - n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
 - (Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - ((3 -
 2*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e +
f*x]^(-1 + n)*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) + (2*(2 - n)*Hyp
ergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e
 + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx &= -\frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{\int \frac{\sec^n(e+fx)(a(-3+n)-a(-1+n)\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{3a^2}\\
&= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{\int \sec^n(e\ldots}{\ldots}\\
&= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{((3-2n\ldots}{\ldots}\\
&= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{((3-2n\ldots}{\ldots}\\
&= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{(3-2n\ldots}{\ldots}\end{aligned}$$

Mathematica [F]
time = 8.25, size = 0, normalized size = 0.00

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2, x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(fx+e\right)}{\left(a+a\sec\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{n}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

```
[Out] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

3.294 $\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=162

$$\frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}}+\frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+fx)}\,\sin(e+fx)}{f(3+2n)}+\frac{2(3+24n+16n^2}{f(1}$$

```
[Out] 2*(7+4*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(4*n^2+8*n+3)/(1+sec(f*x+e))^(1/2)+
2*sec(f*x+e)^(1+n)*sin(f*x+e)*(1+sec(f*x+e))^(1/2)/f/(3+2*n)+2*(16*n^2+24*n
+3)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(4*n^2+8*n+3)/(1+
sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3899, 4101, 3891, 67}

$$\frac{2(16n^2+24n+3)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\,\sec^{n+1}(e+fx)}{f(2n+3)}+\frac{2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)(2n+3)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt
[1 + Sec[e + f*x]]) + (2*Sec[e + f*x]^(1 + n)*Sqrt[1 + Sec[e + f*x]]*Sin[e
+ f*x])/(f*(3 + 2*n)) + (2*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n
, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[1 + Sec[
e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
```

```
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+fx)}\,\sin(e+fx)}{f(3+2n)} + \frac{2\int \sec^n(e+fx)}{}$$
$$= \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec}}{f(3+2}$$
$$= \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec}}{f(3+2}$$
$$= \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec}}{f(3+2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 62.75, size = 433, normalized size = 2.67

$$-\frac{i2^{-\frac{5}{2}+n}e^{-i\left(\frac{1}{2}+n\right)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\left(\frac{10e^{i(2+n)(e+fx)}\,{}_2F_1\left(1+\frac{n}{2},\frac{5}{2}+n;2+\frac{n}{2};-e^{2i(e+fx)}\right)}{2+n}+\frac{5e^{i(4+n)(e+fx)}\,{}_2F_1\left(2+\frac{n}{2},\frac{5}{2}+n;3+\frac{n}{2};-e^{2i(e+fx)}\right)}{4+n}+\frac{e^{in(e+fx)}\,{}_2F_1\left(\frac{n}{2},\frac{5}{2}+n;1+\frac{n}{2};-e^{2i(e+fx)}\right)}{n}+\frac{5e^{i(1+n)(e+fx)}\,{}_2F_1\left(\frac{1+n}{2},\frac{5}{2}+n;\frac{3+n}{2};-e^{2i(e+fx)}\right)}{1+n}+\frac{10e^{i(3+n)(e+fx)}\,{}_2F_1\left(\frac{5}{2}+n,\frac{3+n}{2};\frac{5+n}{2};-e^{2i(e+fx)}\right)}{3+n}+\frac{e^{i(5+n)(e+fx)}\,{}_2F_1\left(\frac{5}{2}+n,\frac{5+n}{2};\frac{7+n}{2};-e^{2i(e+fx)}\right)}{5+n}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^{5/2}}{f\sec^{\frac{5}{2}}(e+fx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1
 + E^((2*I)*(e + f*x)))^(1/2 + n)*((10*E^(I*(2 + n)*(e + f*x))*Hypergeometr
ic2F1[1 + n/2, 5/2 + n, 2 + n/2, -E^((2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(
4 + n)*(e + f*x))*Hypergeometric2F1[2 + n/2, 5/2 + n, 3 + n/2, -E^((2*I)*(e
 + f*x))])/(4 + n) + (E^(I*n*(e + f*x))*Hypergeometric2F1[n/2, 5/2 + n, 1 +
```

```
 n/2, -E^((2*I)*(e + f*x))])/n + (5*E^(I*(1 + n)*(e + f*x))*Hypergeometric2
F1[(1 + n)/2, 5/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n) + (10*E^(I
*(3 + n)*(e + f*x))*Hypergeometric2F1[5/2 + n, (3 + n)/2, (5 + n)/2, -E^((2
*I)*(e + f*x))])/(3 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[5/2 +
 n, (5 + n)/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n))*Sec[(e + f*x)/2]^
5*(1 + Sec[e + f*x])^(5/2))/(E^(I*(1/2 + n)*(e + f*x))*f*Sec[e + f*x]^(5/2)
)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(1+\sec\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x)
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] integral((sec(f*x + e)^2 + 2*sec(f*x + e) + 1)*sec(f*x + e)^n*sqrt(sec(f*x
+ e) + 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac {1}{\cos \left (e+f\,x\right)}+1\right)}^{5/2}\,{\left (\frac {1}{\cos \left (e+f\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(5/2)*(1/cos(e + f*x))^n,x)
```

```
[Out] int((1/cos(e + f*x) + 1)^(5/2)*(1/cos(e + f*x))^n, x)
```

3.295 $\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}+\frac{2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

```
[Out] 2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)+2*(1+4*n)*hype
rgeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1
/2)
```

Rubi [A]

time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3899, 21, 3891, 67}

$$\frac{2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
+ (2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e +
 f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\int \frac{\sec^n(e+fx)\left(\frac{1}{2}+2n+\left(\frac{1}{2}+2n\right)\sec(e+fx)\right)}{\sqrt{1+\sec(e+fx)}}}{1+2n}$$

$$= \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{(1+4n)\int \sec^n(e+fx)\sqrt{1+\sec(}{1+2n}$$

$$= \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{((1+4n)\tan(e+fx))\mathrm{Subst}\left(\int -\right.}{f(1+2n)\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(\right.}{f(1+2n)\sqrt{1+\sec(e}}$$

Mathematica [A]
time = 0.42, size = 83, normalized size = 0.85

$$\frac{\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+n;\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,\tan\left(\frac{1}{2}(e+fx)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/2
, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x
)/2])/(f*n)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sec^n(fx+e)\right)\left(1+\sec(fx+e)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\sec(e+fx)+1)^{\frac{3}{2}}\sec^n(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((sec(e + f*x) + 1)**(3/2)*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)}^{3/2}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(3/2)*(1/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^(3/2)*(1/cos(e + f*x))^n, x)
```

3.296 $\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{2\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

```
[Out] 2*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+sec(f*x+e))^(1/2
)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3891, 67}

$$\frac{2\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[1 + Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx &= -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{1-x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= \frac{2\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 1.00

$$\frac{2\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[1 + Sec[e + f*x]])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\sqrt{1+\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sec\left(e+fx\right)+1}\,\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(1/2)*(1/cos(e + f*x))^n,x)
```

```
[Out] int((1/cos(e + f*x) + 1)^(1/2)*(1/cos(e + f*x))^n, x)
```

3.297 $\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal. Leaf size=59

$$\frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

```
[Out] AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(1+sec
(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3910, 129, 440}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)\sqrt{x}}\,dx,x,1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{(1-x^2)^{-1+n}}{2-x^2}\,dx,x,\sqrt{1-\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2938 vs. $2(59) = 118$.
time = 15.95, size = 2938, normalized size = 49.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 + (-1 + 2*n)/2)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f*(3*AppellF1[1/2, -1/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(
1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]
*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*Appe
llF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(S
ec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f
*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Appel
lF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
```

```
[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e +
 f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + S
ec[e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF
1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*
(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(
Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e +
f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (
(-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2
*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2
+ n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*App
ellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n,
1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 +
n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(
e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2,
-1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n
)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e
```

```
 + f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]
) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*...
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(fx+e\right)}{\sqrt{1+\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x)
[Out] int(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(e+fx\right)}{\sqrt{\sec\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)**n/sqrt(sec(e + f*x) + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
[Out] int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

3.298 $\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2f\sqrt{1+\sec(e+fx)}}$$

```
[Out] 1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(1
+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3910, 129, 440}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*f*Sqrt[1 + Sec[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)^2\sqrt{x}}\,dx,x,1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{(1-x^2)^{-1+n}}{(2-x^2)^2}\,dx,x,\sqrt{1-\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2990 vs. $2(62) = 124$.
time = 16.46, size = 2990, normalized size = 48.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2
)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/
2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2))
/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*AppellF1[1/2
, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*
x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)
*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/
```

2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2

```
]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^
(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*(-(Cos[(e + f*x)/2]*
Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/...
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(fx+e\right)}{\left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(f*x
+ e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(e+fx\right)}{\left(\sec\left(e+fx\right)+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**n/(sec(e + f*x) + 1)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
[Out] int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

3.299 $\int (-\sec(e+fx))^n (1+\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}-\frac{(1+4n)\,{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] 2*(-sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)-(1+4*n)*hyperge
om([1/2, n],[1+n],sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/n/(1+2*n)/(1-sec
(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3899, 21, 3891, 66}

$$\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(-\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) - (
(1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*
Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\int \frac{(-\sec(e+fx))^n\left(\frac{1}{2}+2n+\left(\frac{1}{2}+2n\right)\right.}{\sqrt{1+\sec(e+fx}}}{1+2n}$$

$$= \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{(1+4n)\int(-\sec(e+fx))^n}{1+2n}$$

$$= \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{((1+4n)\tan(e+fx))\text{Subs}}{f(1+2n)\sqrt{1-\sec(e}}$$

$$= \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(1+4n)\,{}_2F_1\left(\frac{1}{2},n;1+n;\sec\right.}{fn(1+2n)\sqrt{1-\text{s}}}$$

Mathematica [A]
time = 0.42, size = 85, normalized size = 0.73

$$\frac{\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+n;\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,\tan\left(\frac{1}{2}(e+fx)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/2
, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e +
f*x)/2])/(f*n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
[Out] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sec(e+fx))^n (\sec(e+fx)+1)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
[Out] Integral((-sec(e + f*x))**n*(sec(e + f*x) + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2} \left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(3/2)*(-1/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^(3/2)*(-1/cos(e + f*x))^n, x)
```

3.300 $\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\, dx$

Optimal. Leaf size=64

$$-\frac{{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] -hypergeom([1/2, n],[1+n],sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/n/(1-sec
(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3891, 66}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*Tan[e +
 f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx &= \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{1-x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 67, normalized size = 1.05

$$\frac{2\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\sin(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*S
ec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\sqrt{1+\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
[Out] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(e+fx\right)\right)^{n}\sqrt{\sec\left(e+fx\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}\,{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(1/2)*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((1/cos(e + f*x) + 1)^(1/2)*(-1/cos(e + f*x))^n, x)
```

3.301 $\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal. Leaf size=73

$$-\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] -AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/
n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3911, 141}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[x^(m - 1/2)*(
(a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Rubi steps

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \frac{\tan(e+fx)\mathrm{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}\,x}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= -\frac{F_1\left(n; \frac{1}{2}, 1; 1+n; \sec(e+fx), -\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2951 vs. $2(73) = 146$.
time = 6.23, size = 2951, normalized size = 40.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2 -
n + (-1 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f*
(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -
1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*
(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + S
ec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]
^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2,
-1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n
)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(
e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]
*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 +
n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
```

```
) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[
e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1
[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sq
rt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/
2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Ap
pellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2 +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) +
 Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n,
 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 +
 2*n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*A
ppellF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2,
 -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n
)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2
*Sec[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f
*x]]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*
n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(
-1 + n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e +
 f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3
*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f...
```

Maple [F]

time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(fx+e))^n}{\sqrt{1+\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
[Out] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
[Out] Integral((-sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
[Out] int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

3.302 $\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] -AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/
n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3911, 141}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^nF_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[x^(m - 1/2)*(
(a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Rubi steps

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}\,x^2}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3003 vs. $2(73) = 146$.
time = 6.25, size = 3003, normalized size = 41.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(1/2 - n + (-3 +
 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 +
 Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
llF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^
(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1
+ Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*S
ec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
llF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e
 + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
```

```
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) +
 (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/
2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
+ n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)
/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1/3
*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/2
, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2,
 -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 +
n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)
/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 +
 Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) +
((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*
(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/
2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*
AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2
+ n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -
1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x
)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^
2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x...
```

Maple [**F**]

time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(fx+e))^n}{(1+\sec(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(f
*x + e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(e+fx))^n}{(\sec(e+fx)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
[Out] int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

3.303 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}-\frac{(1+4n)\,{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] 2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)-(1+4*n)*hyperg
eom([1/2, n],[1+n],sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f/n/(1+2*n)/(1-s
ec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3899, 21, 3891, 66}

$$\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) -
((1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
```

```
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\int \frac{(d\sec(e+fx))^n\left(\frac{1}{2}+2n+\left(\frac{1}{2}+2n\right)\sec(}{\sqrt{1+\sec(e+fx)}}}{1+2n}$$

$$= \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{(1+4n)\int(d\sec(e+fx))^n\sqrt{1}}{1+2n}$$

$$= \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(d(1+4n)\tan(e+fx))\text{Subst}(}{f(1+2n)\sqrt{1-\sec(e+}}$$

$$= \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(1+4n)\,{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e\right.}{fn(1+2n)\sqrt{1-\sec(}}$$

Mathematica [A]
time = 0.40, size = 85, normalized size = 0.73

$$\frac{\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+n;\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,\tan\left(\frac{1}{2}(e+fx)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/2
, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e +
 f*x)/2])/(f*n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n (\sec(e+fx)+1)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(sec(e + f*x) + 1)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos (e+f x)}+1\right)^{3/2} \left(\frac{d}{\cos (e+f x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(3/2)*(d/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^(3/2)*(d/cos(e + f*x))^n, x)
```

3.304 $\int (d\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\, dx$

Optimal. Leaf size=64

$$-\frac{{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] -hypergeom([1/2, n],[1+n],sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f/n/(1-se
c(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3891, 66}

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^n*Tan[e
+ f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx &= -\frac{(d\tan(e+fx))\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 1.05

$$\frac{2\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1 - n
)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\sqrt{1+\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
[Out] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^{n}\sqrt{\sec\left(e+fx\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(1/2)*(d/cos(e + f*x))^n,x)
```

```
[Out] int((1/cos(e + f*x) + 1)^(1/2)*(d/cos(e + f*x))^n, x)
```

3.305 $\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal. Leaf size=73

$$-\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] -AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f
/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3912, 138}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = -\frac{(d\tan(e+fx))\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}\,(1+x)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2951 vs. $2(73) = 146$.
time = 6.21, size = 2951, normalized size = 40.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 - n + (-1 + 2*n)/2)*
(d*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f
*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2,
-1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]
*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 +
Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2
 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2,
 -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 +
n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 +
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
```

```
2) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF
1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*S
qrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1
/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*A
ppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2
+ n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)
+ Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n
, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1
+ 2*n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*
AppellF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2
, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*
n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e +
f*x]]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]
*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^
(-1 + n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e
+ f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + ...
```

Maple [F]

time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (fx+e))^n}{\sqrt{1+\sec (fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
[Out] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (e+fx))^n}{\sqrt{\sec (e+fx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
[Out] Integral((d*sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
[Out] int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

3.306 $\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] -AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f
/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3912, 138}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = -\frac{(d\tan(e+fx))\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}\,(1+x)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= -\frac{F_1\left(n; \frac{1}{2}, 2; 1+n; \sec(e+fx), -\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3003 vs. $2(73) = 146$.
time = 6.24, size = 3003, normalized size = 41.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 - n + (-3 + 2*n)/2)*(d*Sec[e
+ f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1
+ Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)
^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1
 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/
2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/
2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*
Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(
e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
```

```
 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
+ (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1
/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x
)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1/
3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/
2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2
, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 +
 n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x
)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1
+ Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) +
 ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2
*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3
/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)
*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2
 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
-1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)
^2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*...
```

Maple [F]

time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^n}{(1+\sec(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(
f*x + e) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^n}{(\sec(e+fx)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
[Out] int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

3.307 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=177

$$\frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\sec(e+fx)}\,\sin(e+fx)}{f(3+2n)}+\frac{2a^3(3+24n-$$

```
[Out] 2*a^3*(7+4*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(4*n^2+8*n+3)/(a+a*sec(f*x+e))^
(1/2)+2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)*(a+a*sec(f*x+e))^(1/2)/f/(3+2*n)+2*
a^3*(16*n^2+24*n+3)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(
4*n^2+8*n+3)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3899, 4101, 3891, 67}

$$\frac{2a^3(16n^2+24n+3)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{a\sec(e+fx)+a}}+\frac{2a^3(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)(2n+3)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\,\sec^{n+1}(e+fx)}{f(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*a^3*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*
Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*Sec[e + f*x]^(1 + n)*Sqrt[a + a*Sec[e +
f*x]]*Sin[e + f*x])/(f*(3 + 2*n)) + (2*a^3*(3 + 24*n + 16*n^2)*Hypergeometr
ic2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*
n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
```

```
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\sec(e+fx)}\,\sin(e+fx)}{f(3+2n)} + \frac{(2a)\int \text{se}}{}$$

$$= \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{}}{}$$

$$= \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{}}{}$$

$$= \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.15, size = 435, normalized size = 2.46

$$-\frac{i2^{-\frac{5}{2}+n}e^{-i\left(\frac{1}{2}+n\right)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\left(\frac{10e^{i(2+n)(e+fx)}\,{}_2F_1\left(1+\frac{n}{2},\frac{5}{2}+n;2+\frac{n}{2};-e^{2i(e+fx)}\right)}{2+n}+\frac{5e^{i(4+n)(e+fx)}\,{}_2F_1\left(2+\frac{n}{2},\frac{5}{2}+n;3+\frac{n}{2};-e^{2i(e+fx)}\right)}{4+n}+\frac{e^{in(e+fx)}\,{}_2F_1\left(\frac{n}{2},\frac{5}{2}+n;1+\frac{n}{2};-e^{2i(e+fx)}\right)}{n}+\frac{5e^{i(1+n)(e+fx)}\,{}_2F_1\left(\frac{1+n}{2},\frac{5}{2}+n;\frac{3+n}{2};-e^{2i(e+fx)}\right)}{1+n}+\frac{10e^{i(3+n)(e+fx)}\,{}_2F_1\left(\frac{5}{2}+n,\frac{3+n}{2};\frac{5+n}{2};-e^{2i(e+fx)}\right)}{3+n}+\frac{e^{i(5+n)(e+fx)}\,{}_2F_1\left(\frac{5}{2}+n,\frac{5+n}{2};\frac{7+n}{2};-e^{2i(e+fx)}\right)}{5+n}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\left(a(1+\sec(e+fx))\right)^{5/2}}{f\sec^{\frac{5}{2}}(e+fx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1
 + E^((2*I)*(e + f*x)))^(1/2 + n)*((10*E^(I*(2 + n)*(e + f*x))*Hypergeometr
ic2F1[1 + n/2, 5/2 + n, 2 + n/2, -E^((2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(
4 + n)*(e + f*x))*Hypergeometric2F1[2 + n/2, 5/2 + n, 3 + n/2, -E^((2*I)*(e
 + f*x))])/(4 + n) + (E^(I*n*(e + f*x))*Hypergeometric2F1[n/2, 5/2 + n, 1 +
```

```
n/2, -E^((2*I)*(e + f*x))])/n + (5*E^(I*(1 + n)*(e + f*x))*Hypergeometric2
F1[(1 + n)/2, 5/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n) + (10*E^(I
*(3 + n)*(e + f*x))*Hypergeometric2F1[5/2 + n, (3 + n)/2, (5 + n)/2, -E^((2
*I)*(e + f*x))])/(3 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[5/2 +
 n, (5 + n)/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n))*Sec[(e + f*x)/2]^
5*(a*(1 + Sec[e + f*x]))^(5/2))/(E^(I*(1/2 + n)*(e + f*x))*f*Sec[e + f*x]^(
5/2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(5/2)*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*sec(f*x + e)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(5/2)*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(1/cos(e + f*x))^n, x)
```

3.308 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=108

$$\frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(1
+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+2*n)/(a+a*se
c(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3899, 21, 3891, 67}

$$\frac{2a^2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f
*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x
]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(2a)\int \frac{\sec^n(e+fx)\left(a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2\right.\right.}{\sqrt{a+a\sec(e+}}}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(a(1+4n))\int \sec^n(e+fx)\sqrt{}}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^3(1+4n)\tan(e+fx)\right)\text{Sub}}{f(1+2n)\sqrt{a-a\sec(e}}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1\right.}{f(1+2n)\sqrt{a+}}$$

Mathematica [A]
time = 0.44, size = 86, normalized size = 0.80

$$\frac{a\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+n;\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec^n(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3
/2, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e
 + f*x)/2])/(f*n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^n(fx+e)\right)\left(a+a\sec(fx+e)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^{\frac{3}{2}}\sec^{n}(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n, x)
```

3.309 $\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=48

$$\frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^
(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3891, 67}

$$\frac{2a\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*
Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 1.06

$$\frac{2\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sqrt[a*(1 + Sec[e +
 f*x])]*Tan[(e + f*x)/2])/f
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\sqrt{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n, x)
```

3.310 $\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=61

$$\frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(a+a*s
ec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3913, 3910, 129, 440}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx}{\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{(1-x^2)^{-1+n}}{2-x^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2964 vs. $2(61) = 122$.
time = 6.24, size = 2964, normalized size = 48.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 + (-1 + 2*n)/2)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2])/(f
*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2,
 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
```

```
e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appel
lF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e
+ f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqr
t[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF
1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1
 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2
)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x
)/2]*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*Ap
pellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1
/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*A
ppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Co
s[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((
2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*(
(1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2,
1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*Ap
pellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 +
n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n
, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*A
ppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
```

```
 + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)
/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])
/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n,
 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1
[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]
*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Se
c[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2...
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(fx+e\right)}{\sqrt{a+a\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

3.311 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/a/f/
(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3913, 3910, 129, 440}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx &= \frac{\sqrt{1+\sec(e+fx)}\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sec(e+fx)}}\\
&= \frac{\tan(e+fx)\mathrm{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(2\tan(e+fx))\mathrm{Subst}\left(\int \frac{(1-x^2)^{-1+n}}{(2-x^2)^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2992 vs. $2(67) = 134$.
time = 6.27, size = 2992, normalized size = 44.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2
)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF
1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
```

, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*

```
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/2 + n)*AppellF1[5/
2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*AppellF1[5/2, -1/2
 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 1 - n, 7/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
)/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (6*(3/
2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*(-(Cos[(e + f*x)
/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f...
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(fx+e\right)}{\left(a+a\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a^
2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

3.312 $\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(-\sec(e+}{f(1+2n)\sqrt{a}}$$

```
[Out] 2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec(
f*x+e)^(1-n)*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e)
)^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3899, 21, 3891, 69, 67}

$$\frac{2a^2(4n+1)\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec
[e + f*x])^n*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec
[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a +
a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
```

```
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(2a)\int\frac{(-\sec(e+fx))^n\left(a\left(\frac{1}{2}+2n\right.\right.}{\sqrt{a+a\sec}}}{1+2}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(a(1+4n))\int(-\sec(e+}{1}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(a^3(1+4n)\tan(e+fx))}{f(1+2n)\sqrt{a-a}}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{(a^3(1+4n)(-\sec(e+fx}{f(}$$

$$= \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)(-\sec(e+fx))}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 0.38, size = 88, normalized size = 0.68

$$\frac{a\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+n;\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(-\sec(e+fx))^n\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3
/2, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*Tan
[(e + f*x)/2])/(f*n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(f x +e \right)\right)^{n} \left(a +a \sec \left(f x +e \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- \sec{\left(e + f x \right)}\right)^{n} \left(a \left(\sec{\left(e + f x \right)} + 1\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n*(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n, x)
```

3.313 $\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec(f*x+e)^(1-
n)*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3891, 69, 67}

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n
*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\, dx = \frac{\left(a^2\tan(e+fx)\right) \text{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{a-ax}}\, dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2(-\sec(e+fx))^n \sec^{1-n}(e+fx)\sin(e+fx)\right) \text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+}}$$

$$= \frac{2a\, {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; 1-\sec(e+fx)\right)(-\sec(e+fx))^n \sec^{1-n}(e}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 0.13, size = 71, normalized size = 1.01

$$\frac{2\, {}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; 1-\sec(e+fx)\right)(-\sec(e+fx))^n \sec^{-n}(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*S
qrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (-\sec(fx+e))^n \sqrt{a+a\sec(fx+e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
[Out] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sec(e+fx))^n \sqrt{a(\sec(e+fx)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral((-sec(e + f*x))**n*sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n, x)
```

3.314 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/
n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3911, 141}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^nF_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[x^(m - 1/2)*(
(a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx}{\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}\,x}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2977 vs. $2(75) = 150$.
time = 6.22, size = 2977, normalized size = 39.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2 -
n + (-1 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Tan[(e + f*x)/2])/(f*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appell
F1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-
1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 +
 n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3
*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF
1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
```

[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(

```
e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e
+ f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e +
f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
 Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))...
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\sqrt{a+a\sec \left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
[Out] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec \left(e+fx\right)\right)^{n}}{\sqrt{a\left(\sec \left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

[Out] Integral((-sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(-\frac{1}{\cos(e+f\,x)}\right)}^n}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)

[Out] int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)

3.315 $\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/a/
f/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3911, 141}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^nF_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[x^(m - 1/2)*(
(a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx &= \frac{\sqrt{1+\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sec(e+fx)}}\\
&= \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}\,x^2}\,dx, x, 1+\sec(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3005 vs. $2(78) = 156$.
time = 6.22, size = 3005, normalized size = 38.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(1/2 - n + (-3 +
 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 +
 Tan[(e + f*x)/2]^2)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -3
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*
AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]
^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*
(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
-1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]
^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3
```

```
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*
AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Ta
n[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 +
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2) + (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e +
 f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF
1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*
(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e +
f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(
-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1
[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[
1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-
1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1
 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e +
f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(
-1 + Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 +
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2
]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*
(-3/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 -
 n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2,
1/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/
```

```
2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
+ f*x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]
^2)^2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e +...
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec \left(f x+e\right)\right)^{n}}{\left(a+a \sec \left(f x+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
[Out] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 + 2
*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \sec{\left(e + f x \right)}\right)^{n}}{\left(a \left(\sec{\left(e + f x \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)}^n}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

3.316 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d*
sec(f*x+e))^n*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(d*sec(f*x+
e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3899, 21, 3891, 69, 67}

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e
 + f*x]^(1 - n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Se
c[e + f*x]]) + (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a
+ a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
```

```
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{(2a)\int\frac{(d\sec(e+fx))^n\left(a\left(\frac{1}{2}+2n\right.\right.}{\sqrt{a+a\sec}}}{1+2}$$

$$=\frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{(a(1+4n))\int(d\sec(e+f}{1}$$

$$=\frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}-\frac{(a^3d(1+4n)\tan(e+fx)}{f(1+2n)\sqrt{a-a}}$$

$$=\frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}-\frac{(a^3(1+4n)\sec^{1-n}(e+f}{f(1}$$

$$=\frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{1-n}(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 0.37, size = 88, normalized size = 0.68

$$\frac{a\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+n;\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(d\sec(e+fx))^n\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3
/2, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*Ta
n[(e + f*x)/2])/(f*n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x +e \right)\right)^{n} \left(a +a \sec \left(f x +e \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec \left(e + f x \right) + 1\right)\right)^{\frac{3}{2}} \left(d \sec \left(e + f x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(d*sec(e + f*x))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

3.317 $\int (d\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d*sec(f*x+e)
)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3891, 69, 67}

$$\frac{2a\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1 -
 n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx = -\frac{(a^2 d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(a^2\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx))\,\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+}}$$

$$= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 0.13, size = 71, normalized size = 1.01

$$\frac{2\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{-n}(e+fx)(d\sec(e+fx))^n\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(d*Sec[e + f*x])^n*
Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\sqrt{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
[Out] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(d\sec\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(d*sec(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

3.318 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f
/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3912, 138}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx &= \frac{\sqrt{1+\sec(e+fx)}\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx}{\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{(d\tan(e+fx))\mathrm{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}\,(1+x)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2977 vs. $2(75) = 150$.
time = 6.21, size = 2977, normalized size = 39.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 - n + (-1 + 2*n)/2)*
(d*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x
]]*Tan[(e + f*x)/2])/(f*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appel
lF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1
+ n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*Appell
F1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec
[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x
]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF
```

1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e

```
+ f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e +
f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2])
+ Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x])...
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (fx + e))^n}{\sqrt{a + a \sec (fx + e)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
[Out] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (e + fx))^n}{\sqrt{a (\sec (e + fx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral((d*sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

3.319 $\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/a
/f/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3913, 3912, 138}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{afn\sqrt{1-\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{(d\tan(e+fx))\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}\,(1+x)^2}\,dx, x, \sec(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3005 vs. $2(78) = 156$.
time = 6.21, size = 3005, normalized size = 38.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 - n + (-3 + 2*n)/2)*(d*Sec[e
+ f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1
+ Tan[(e + f*x)/2]^2)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -
3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2
]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2
*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
 -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2
]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -
3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
```

2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(
e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + T
an[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]
^2) + (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e
+ f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*Appell
F1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2
*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e +
 f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*
(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF
1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1
[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(
-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n,
1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e +
 f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*
(-1 + Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
]) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/
2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3
*(-3/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1
- n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2,
 1/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3
/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
+ f*x)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +

```
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2
]^2)^2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e ...
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (fx+e))^n}{(a+a \sec (fx+e))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
[Out] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(a^2*sec(f*x + e)^2 +
2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (e+fx))^n}{(a (\sec (e+fx)+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral((d*sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

3.320 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=178

$$\frac{2a^3(3+24n+16n^2)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}}+\frac{2a^3(7+4n)(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a^3*(16*n^2+24*n+3)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f
/(4*n^2+8*n+3)/(a-a*sec(f*x+e))^(1/2)+2*a^3*(7+4*n)*(-sec(f*x+e))^n*tan(f*x
+e)/f/(4*n^2+8*n+3)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e))^n*(a-a*sec(f
*x+e))^(1/2)*tan(f*x+e)/f/(3+2*n)
```

Rubi [A]
time = 0.25, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3899, 4101, 3891, 67}

$$\frac{2a^3(16n^2+24n+3)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)(2n+3)\sqrt{a-a\sec(e+fx)}}+\frac{2a^3(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)(2n+3)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}\,(-\sec(e+fx))^n}{f(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*a^3*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f
*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^
3*(7 + 4*n)*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a -
 a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]]*Tan[e
 + f*x])/(f*(3 + 2*n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
```

```
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{5/2}\,dx = \frac{2a^2(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,\tan(e+fx)}{f(3+2n)} - \frac{(2a)}{}$$

$$= \frac{2a^3(7+4n)(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx}{}$$

$$= \frac{2a^3(7+4n)(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx}{}$$

$$= \frac{2a^3(3+24n+16n^2)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e}{f\left(3+8n+4n^2\right)\sqrt{a-a\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 26.26, size = 458, normalized size = 2.57

$$\frac{2^{-\frac{5}{2}+n}e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{-\frac{1}{2}+n}\csc^5\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{e^{ifnx}\,{}_2F_1\left(\frac{n}{2},\frac{5}{2}+n;\frac{2+n}{2};-e^{2i(e+fx)}\right)}{n}-\frac{5e^{i(e+f(1+n)x)}\,{}_2F_1\left(\frac{1+n}{2},\frac{5}{2}+n;\frac{3+n}{2};-e^{2i(e+fx)}\right)}{1+n}+\frac{10e^{i(2e+f(2+n)x)}\,{}_2F_1\left(\frac{2+n}{2},\frac{5}{2}+n;\frac{4+n}{2};-e^{2i(e+fx)}\right)}{2+n}-\frac{10e^{i(3e+f(3+n)x)}\,{}_2F_1\left(\frac{5}{2}+n,\frac{3+n}{2};\frac{5+n}{2};-e^{2i(e+fx)}\right)}{3+n}+\frac{5e^{i(4e+f(4+n)x)}\,{}_2F_1\left(\frac{5}{2}+n,\frac{4+n}{2};\frac{6+n}{2};-e^{2i(e+fx)}\right)}{4+n}-\frac{e^{i(5e+f(5+n)x)}\,{}_2F_1\left(\frac{5}{2}+n,\frac{5+n}{2};\frac{7+n}{2};-e^{2i(e+fx)}\right)}{5+n}\right)(-\sec(e+fx))^n\sec^{-\frac{5}{2}-n}(e+fx)(a-a\sec(e+fx))^{5/2}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2^(-5/2 + n)*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^(-1/2 + n)*(1 + E^((2*I)*(e + f*x)))^(-1/2 + n)*Csc[e/2 + (f*x
)/2]^5*((E^(I*f*n*x)*Hypergeometric2F1[n/2, 5/2 + n, (2 + n)/2, -E^((2*I)*(
e + f*x))])/n - (5*E^(I*(e + f*(1 + n)*x))*Hypergeometric2F1[(1 + n)/2, 5/2
 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n) + (10*E^(I*(2*e + f*(2 + n)
```

```
*x))*Hypergeometric2F1[(2 + n)/2, 5/2 + n, (4 + n)/2, -E^((2*I)*(e + f*x))]
)/(2 + n) - (10*E^(I*(3*e + f*(3 + n)*x))*Hypergeometric2F1[5/2 + n, (3 + n
)/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n) + (5*E^(I*(4*e + f*(4 + n)*x
))*Hypergeometric2F1[5/2 + n, (4 + n)/2, (6 + n)/2, -E^((2*I)*(e + f*x))])/
(4 + n) - (E^(I*(5*e + f*(5 + n)*x))*Hypergeometric2F1[5/2 + n, (5 + n)/2,
(7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n))*(-Sec[e + f*x])^n*Sec[e + f*x]^(
-5/2 - n)*(a - a*Sec[e + f*x])^(5/2))/f
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(a-a\sec \left(fx+e\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] integral((a^2*sec(f*x + e)^2 - 2*a^2*sec(f*x + e) + a^2)*sqrt(-a*sec(f*x +
e) + a)*(-sec(f*x + e))^n, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}\,{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(5/2)*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((a - a/cos(e + f*x))^(5/2)*(-1/cos(e + f*x))^n, x)
```

3.321 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=108

$$\frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f/(1+2*n)
/(a-a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a-a*sec
(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3899, 21, 3891, 67}

$$\frac{2a^2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e
 + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*
Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(2a)\int \frac{(-\sec(e+fx))^n\left(-a\left(\frac{1}{2}+2 \cdots\right.\right.}{\sqrt{a-a\sec \cdots}}}{1+2 \cdots}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a(1+4n))\int(-\sec(e+ \cdots}{1 \cdots}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a^3(1+4n)\tan(e+fx)) \cdots}{f(1+2n)\sqrt{a-a \cdots}}$$

$$= \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.70, size = 377, normalized size = 3.49

$$\frac{2^{-\frac{3}{2}+n}e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{-\frac{1}{2}+n}\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-e^{ifnx}(6+11n+6n^2+n^3)\,{}_2F_1\left(\frac{n}{2},\frac{3}{2}+n;\frac{2+n}{2};-e^{2i(e+fx)}\right)+3e^{i(e+f(1+n)x)}n(6+5n+n^2)\,{}_2F_1\left(\frac{1+n}{2},\frac{3}{2}+n;\frac{3+n}{2};-e^{2i(e+fx)}\right)+e^{2ie}n(1+n)\left(-3e^{if(2+n)x}(3+n)\,{}_2F_1\left(\frac{3}{2}+n,\frac{2+n}{2};\frac{4+n}{2};-e^{2i(e+fx)}\right)+e^{i(e+f(3+n)x)}(2+n)\,{}_2F_1\left(\frac{3}{2}+n,\frac{3+n}{2};\frac{5+n}{2};-e^{2i(e+fx)}\right)\right)\right)(-\sec(e+fx))^n\sec^{-\frac{3}{2}-n}(e+fx)(a-a\sec(e+fx))^{3/2}}{fn(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2^(-3/2 + n)*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^(-1/2 + n)*(1 + E^((2*I)*(e + f*x)))^(-1/2 + n)*Csc[(e + f*x)/
2]^3*(-(E^(I*f*n*x)*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1[n/2, 3/2 + n
, (2 + n)/2, -E^((2*I)*(e + f*x))]) + 3*E^(I*(e + f*(1 + n)*x))*n*(6 + 5*n
+ n^2)*Hypergeometric2F1[(1 + n)/2, 3/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x)
)] + E^((2*I)*e)*n*(1 + n)*(-3*E^(I*f*(2 + n)*x)*(3 + n)*Hypergeometric2F1[
3/2 + n, (2 + n)/2, (4 + n)/2, -E^((2*I)*(e + f*x))] + E^(I*(e + f*(3 + n)*
x))*(2 + n)*Hypergeometric2F1[3/2 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e +
 f*x))]))*(-Sec[e + f*x])^n*Sec[e + f*x]^(-3/2 - n)*(a - a*Sec[e + f*x])^(3
/2))/(f*n*(1 + n)*(2 + n)*(3 + n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(f x +e \right)\right)^{n} \left(a -a \sec \left(f x +e \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n,
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- \sec{\left(e + f x \right)}\right)^{n} \left(- a \left(\sec{\left(e + f x \right)} - 1\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
[Out] Integral((-sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((a - a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n, x)
```

3.322 $\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f/(a-a*sec(f*x+e))^
(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3891, 67}

$$\frac{2a\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e + f*x])/(f*
Sqrt[a - a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx &= \frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 74.20, size = 236, normalized size = 5.02

$$\frac{2^{-\frac{1}{2}+n}e^{-\frac{1}{2}i(e+f(1+2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\left(e^{ifnx}(1+n)\,{}_2F_1\left(\frac{n}{2},\frac{1}{2}+n;\frac{2+n}{2};-e^{2i(e+fx)}\right)-e^{i(e+f(1+n)x)}n\,{}_2F_1\left(\frac{1}{2}+n,\frac{1+n}{2};\frac{3+n}{2};-e^{2i(e+fx)}\right)\right)(-\sec(e+fx))^n\sec^{-\frac{1}{2}-n}(e+fx)\sqrt{a-a\sec(e+fx)}}{fn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2^(-1/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 + E^
((2*I)*(e + f*x)))^(1/2 + n)*Csc[e/2 + (f*x)/2]*(E^(I*f*n*x)*(1 + n)*Hyperg
eometric2F1[n/2, 1/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))] - E^(I*(e + f*(1
 + n)*x))*n*Hypergeometric2F1[1/2 + n, (1 + n)/2, (3 + n)/2, -E^((2*I)*(e +
 f*x))])*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2 - n)*Sqrt[a - a*Sec[e + f*x]]
)/(E^((I/2)*(e + f*(1 + 2*n)*x))*f*n*(1 + n))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\sqrt{a-a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sec(e+fx))^n \sqrt{-a(\sec(e+fx)-1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
[Out] Integral((-sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a-\frac{a}{\cos(e+f\,x)}}\,\left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n, x)
```

3.323 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] AppellF1(1/2,1-n,1,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f*x+e)/f/(a-a*s
ec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3913, 3910, 129, 440}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[a - a*Sec[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx &= \frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{\sqrt{1-\sec(e+fx)}}\,dx}{\sqrt{a-a\sec(e+fx)}}\\ &= \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)\sqrt{x}}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}\\ &= \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{(1-x^2)^{-1+n}}{2-x^2}\,dx, x, \sqrt{1+\sec(e+fx)}\right)}{f\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}\\ &= \frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}\end{aligned}$$

Mathematica [F]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
[Out] Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]], x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(fx+e))^n}{\sqrt{a-a\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a*sec(f*x + e) - a),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-\sec{(e+fx)})^n}{\sqrt{-a(\sec{(e+fx)}-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/sqrt(-a*(sec(e + f*x) - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(1/2),x)
[Out] int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(1/2), x)
```

3.324 $\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 1/2*AppellF1(1/2,1-n,2,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f*x+e)/a/f/
(a-a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3913, 3910, 129, 440}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{2af\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*a*f*Sqrt[a - a*Sec[e + f*x]])
```

Rule 129

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + b*(x^k/e))^m*(c + d*(x^k/e))^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx &= \frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{(1-\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a-a\sec(e+fx)}}\\
&= \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)^2\sqrt{x}}\,dx, x, 1+\sec(e+fx)\right)}{af\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}\\
&= \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{(1-x^2)^{-1+n}}{(2-x^2)^2}\,dx, x, \sqrt{1+\sec(e+fx)}\right)}{af\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}\\
&= \frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a-a\sec(e+fx)}}
\end{aligned}$$

Mathematica [F]
time = 74.99, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
[Out] Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2), x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\left(a-a\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/(-a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 -
2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(e+fx))^n}{(-a(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/(-a*(sec(e + f*x) - 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a-\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(3/2), x)
```

3.325 $\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}\sec^{1+n}(e+\ldots}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(1
+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*sec(f*x+e)^(1+n)*sin(f*x+e)/
f/(1+2*n)/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3899, 21, 3891, 69, 67}

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{n+1}(e+fx)(-\sec(e+fx))^{-n}\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f
*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x
]]*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a
 - a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
```

```
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(2a)\int \frac{\sec^n(e+fx)\left(-a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}-\right.\right.}{\sqrt{a-a\sec(e+}}}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a(1+4n))\int \sec^n(e+fx)\sqrt{}}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(a^3(1+4n)\tan(e+fx))\,\mathrm{Sub}}{f(1+2n)\sqrt{a-a\sec(}}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a^3(1+4n)(-\sec(e+fx))^{-n}}{f(1+}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.02, size = 363, normalized size = 2.79

$$\frac{2^{-\frac{3}{2}+n}e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{-\frac{1}{2}+n}\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-e^{ifnx}(6+11n+6n^2+n^3)\,{}_2F_1\left(\frac{n}{2},\frac{3}{2}+n;\frac{2+n}{2};-e^{2i(e+fx)}\right)+3e^{i(e+f(1+n)x)}n(6+5n+n^2)\,{}_2F_1\left(\frac{1+n}{2},\frac{3}{2}+n;\frac{3+n}{2};-e^{2i(e+fx)}\right)+e^{2ie}n(1+n)\left(-3e^{if(2+n)x}(3+n)\,{}_2F_1\left(\frac{3}{2}+n,\frac{2+n}{2};\frac{4+n}{2};-e^{2i(e+fx)}\right)+e^{i(e+f(3+n)x)}(2+n)\,{}_2F_1\left(\frac{3}{2}+n,\frac{3+n}{2};\frac{5+n}{2};-e^{2i(e+fx)}\right)\right)\right)(a-a\sec(e+fx))^{3/2}}{fn(1+n)(2+n)(3+n)\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2^(-3/2 + n)*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^(-1/2 + n)*(1 + E^((2*I)*(e + f*x)))^(-1/2 + n)*Csc[(e + f*x)/
2]^3*(-(E^(I*f*n*x)*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1[n/2, 3/2 + n
, (2 + n)/2, -E^((2*I)*(e + f*x))]) + 3*E^(I*(e + f*(1 + n)*x))*n*(6 + 5*n
+ n^2)*Hypergeometric2F1[(1 + n)/2, 3/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x)
)] + E^((2*I)*e)*n*(1 + n)*(-3*E^(I*f*(2 + n)*x)*(3 + n)*Hypergeometric2F1[
3/2 + n, (2 + n)/2, (4 + n)/2, -E^((2*I)*(e + f*x))] + E^(I*(e + f*(3 + n)*
x))*(2 + n)*Hypergeometric2F1[3/2 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e +
 f*x))]))*(a - a*Sec[e + f*x])^(3/2))/(f*n*(1 + n)*(2 + n)*(3 + n)*Sec[e +
f*x]^(3/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(a-a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-a*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a - a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n, x)
```

3.326 $\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*sec(f*x+e)^(1+n)*sin(f*x+e)/f/
((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3891, 69, 67}

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^{-n}\sec^{n+1}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Sec[e + f*x]^(1 +
 n)*Sin[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= \frac{(a^2(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin(e+fx))\,\text{Subst}\left(\int \frac{(-x)}{\sqrt{a+}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx}} \\
&= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}\sec^{1+n}(e+}{f\sqrt{a-a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.52, size = 222, normalized size = 3.22

$$
\frac{2^{-\frac{1}{2}+n}e^{-\frac{1}{2}i(e+f(1+2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\left(e^{ifnx}(1+n)\,{}_2F_1\left(\frac{n}{2},\frac{1}{2}+n;\frac{2+n}{2};-e^{2i(e+fx)}\right)-e^{i(e+f(1+n)x)}n\,{}_2F_1\left(\frac{1}{2}+n,\frac{1+n}{2};\frac{3+n}{2};-e^{2i(e+fx)}\right)\right)\sqrt{a-a\sec(e+fx)}}{fn(1+n)\sqrt{\sec(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2^(-1/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 + E^
((2*I)*(e + f*x)))^(1/2 + n)*Csc[e/2 + (f*x)/2]*(E^(I*f*n*x)*(1 + n)*Hyperg
eometric2F1[n/2, 1/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))] - E^(I*(e + f*(1
 + n)*x))*n*Hypergeometric2F1[1/2 + n, (1 + n)/2, (3 + n)/2, -E^((2*I)*(e +
 f*x))])*Sqrt[a - a*Sec[e + f*x]])/(E^((I/2)*(e + f*(1 + 2*n)*x))*f*n*(1 +
n)*Sqrt[Sec[e + f*x]])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$
\int \left(\sec^{n}\left(fx+e\right)\right)\sqrt{a-a\sec\left(fx+e\right)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(-a*(sec(e + f*x) - 1))*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n, x)
```

3.327 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}(d\sec(e+}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a^2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(1
+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/
f/(1+2*n)/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3899, 21, 3891, 69, 67}

$$\frac{2a^2(4n+1)\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x
]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]
*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a - a
*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
```

```
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(2a)\int \frac{(d\sec(e+fx))^n\left(-a\left(\frac{1}{2}+2n\right.}{\sqrt{a-a\sec}}}{1+2n}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a(1+4n))\int(d\sec(e+fx}{1+}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(a^3d(1+4n)\tan(e+fx))}{f(1+2n)\sqrt{a-a\,s}}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a^3(1+4n)(-\sec(e+fx)}{f(}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.39, size = 377, normalized size = 2.90

$$\frac{2^{-\frac{3}{2}+n}e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{-\frac{1}{2}+n}\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-e^{ifnx}(6+11n+6n^2+n^3)\,{}_2F_1\left(\frac{n}{2},\frac{3}{2}+n;\frac{2+n}{2};-e^{2i(e+fx)}\right)+3e^{i(e+f(1+n)x)}n(6+5n+n^2)\,{}_2F_1\left(\frac{1+n}{2},\frac{3}{2}+n;\frac{3+n}{2};-e^{2i(e+fx)}\right)+e^{2ie}n(1+n)\left(-3e^{if(2+n)x}(3+n)\,{}_2F_1\left(\frac{3}{2}+n,\frac{2+n}{2};\frac{4+n}{2};-e^{2i(e+fx)}\right)+e^{i(e+f(3+n)x)}(2+n)\,{}_2F_1\left(\frac{3}{2}+n,\frac{3+n}{2};\frac{5+n}{2};-e^{2i(e+fx)}\right)\right)\right)\sec^{-\frac{3}{2}-n}(e+fx)(d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}}{fn(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2^(-3/2 + n)*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^(-1/2 + n)*(1 + E^((2*I)*(e + f*x)))^(-1/2 + n)*Csc[(e + f*x)/
2]^3*(-(E^(I*f*n*x)*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1[n/2, 3/2 + n
, (2 + n)/2, -E^((2*I)*(e + f*x))]) + 3*E^(I*(e + f*(1 + n)*x))*n*(6 + 5*n
+ n^2)*Hypergeometric2F1[(1 + n)/2, 3/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x)
)] + E^((2*I)*e)*n*(1 + n)*(-3*E^(I*f*(2 + n)*x)*(3 + n)*Hypergeometric2F1[
3/2 + n, (2 + n)/2, (4 + n)/2, -E^((2*I)*(e + f*x))] + E^(I*(e + f*(3 + n)*
x))*(2 + n)*Hypergeometric2F1[3/2 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e +
 f*x))]))*Sec[e + f*x]^(-3/2 - n)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(
3/2))/(f*n*(1 + n)*(2 + n)*(3 + n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x +e \right)\right)^{n} \left(a -a \sec \left(f x +e \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec{\left(e + f x \right)}\right)^{n} \left(- a \left(\sec{\left(e + f x \right)} - 1\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
```

```
[Out] int((a - a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

3.328 $\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f/
((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3891, 69, 67}

$$\frac{2a\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*(d*Sec[e + f*x])^
n*Tan[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\, dx = -\frac{(a^2 d\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{a+ax}}\, dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(a^2(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\tan(e+fx))\,\mathrm{Subst}\Big(\int \frac{(\cdot}{\sqrt{}}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+}}$$

$$= \frac{2a\,{}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; 1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}(d\sec(e-}{f\sqrt{a-a\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.59, size = 236, normalized size = 3.42

$$\frac{2^{-\frac{1}{2}+n}e^{-\frac{1}{2}i(e+f(1+2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\left(e^{ifnx}(1+n)\,{}_2F_1\left(\frac{n}{2},\frac{1}{2}+n;\frac{2+n}{2};-e^{2i(e+fx)}\right)-e^{i(e+f(1+n)x)}n\,{}_2F_1\left(\frac{1}{2}+n,\frac{1+n}{2};\frac{3+n}{2};-e^{2i(e+fx)}\right)\right)\sec^{-\frac{1}{2}-n}(e+fx)(d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}}{fn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2^(-1/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 + E^
((2*I)*(e + f*x)))^(1/2 + n)*Csc[e/2 + (f*x)/2]*(E^(I*f*n*x)*(1 + n)*Hyperg
eometric2F1[n/2, 1/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))] - E^(I*(e + f*(1
 + n)*x))*n*Hypergeometric2F1[1/2 + n, (1 + n)/2, (3 + n)/2, -E^((2*I)*(e +
 f*x))])*Sec[e + f*x]^(-1/2 - n)*(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]
])/(E^((I/2)*(e + f*(1 + 2*n)*x))*f*n*(1 + n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^n \sqrt{a-a\sec(fx+e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec{(e+fx)})^n \sqrt{-a(\sec{(e+fx)}-1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a-\frac{a}{\cos(e+f\,x)}}\,\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

```
[Out] int((a - a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

3.329 $\int \sec^n(e+fx)(1+\sec(e+fx))^m \, dx$

Optimal. Leaf size=72

$$\frac{2^{\frac{1}{2}+m} F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f
*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3910, 138}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rubi steps

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2}; 1-n, \frac{1}{2}-m; \frac{3}{2}; 1-\sec(e+fx), \frac{1}{2}(1-\sec(e+fx))\right)\text{ta}}{f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2246 vs. $2(72) = 144$.
time = 14.41, size = 2246, normalized size = 31.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*Sec[e + f*x]^n*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*Appell
F1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-
1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n,
2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2
, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF
1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n
)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*Appel
lF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Ta
```

```
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 + m)*
AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(
Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[
(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
+ 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[
3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*(2
- n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1 + m
 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n, 2 -
 n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))
)/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m + n)
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n
)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos
[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m
 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appell
F1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Ta
n[(e + f*x)/2]^2)))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(1+\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\sec(e+fx)+1)^m \sec^n(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**m,x)
```

```
[Out] Integral((sec(e + f*x) + 1)**m*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)}^m\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^m*(1/cos(e + f*x))^n,x)
```

```
[Out] int((1/cos(e + f*x) + 1)^m*(1/cos(e + f*x))^n, x)
```

3.330 $\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(1-\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1-n,1/2,3/2+m,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(1-sec(f*x+e)
)^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3911, 138}

$$\frac{\sqrt{2}\tan(e+fx)(1-\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(1 - Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec[e
 + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[x^(m - 1/2)*(
(a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Rubi steps

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}x^{-\frac{1}{2}+m}}{\sqrt{2-x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m; 1-n, \frac{1}{2}; \frac{3}{2}+m; 1-\sec(e+fx), \frac{1}{2}(1-\sec(e+fx)\right.}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 255 vs. 2(89) = 178.
time = 2.53, size = 255, normalized size = 2.87

$$\frac{(3+2m)F_1\left(\frac{1}{2}+m; m+n, 1-n; \frac{3}{2}+m; \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1-\sec(e+fx))^m \sec^n(e+fx)\sin(e+fx)}{f(1+2m)\left((3+2m)F_1\left(\frac{1}{2}+m; m+n, 1-n; \frac{3}{2}+m; \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\left((-1+n)F_1\left(\frac{3}{2}+m; m+n, 2-n; \frac{5}{2}+m; \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m+n)F_1\left(\frac{3}{2}+m; 1+m+n, 1-n; \frac{5}{2}+m; \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

```
[Out] ((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*Sec[e + f*x]^n*Sin[e + f*x])/(f*(1 +
 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5/2 +
 m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 + m, 1
+ m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2))
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(1-\sec\left(fx+e\right)\right)^{m} \left(\sec^{n}\left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

```
[Out] int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="fricas")
[Out] integral((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - \sec(e + fx))^m \sec^n(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))**m*sec(f*x+e)**n,x)
[Out] Integral((1 - sec(e + f*x))**m*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="giac")
[Out] integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(1-\frac{1}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 1/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
[Out] int((1 - 1/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

3.331 $\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=88

$$\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))}{f}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(1+se
c(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3913, 3910, 138}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^mF_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f
*x])/f
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \sec^n(e+fx)(1+\mathrm{se}$$

$$= \frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\mathrm{Subst}\big(}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\big(}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2248 vs. $2(88) = 176$.
time = 6.27, size = 2248, normalized size = 25.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*Sec[e + f*x]^n*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(3*Ap
pellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1
[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(
e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*App
ellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
 + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*A
```

```
ppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 +
 m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*
Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3
*(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1
 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n,
 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/
5))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m
+ n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m
 + n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
 Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/
2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Ap
pellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
)*Tan[(e + f*x)/2]^2)))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^m \sec^n(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**m,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m \left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

3.332 $\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(a-a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1-n,1/2,3/2+m,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(a-a*sec(f*x+
e))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3913, 3911, 138}

$$\frac{\sqrt{2}\tan(e+fx)(a-a\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(a - a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec
[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[x^(m - 1/2)*(
(a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx = \left((1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\right)\int (1-\sec(e+fx))^m$$

$$= \frac{\left((1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}}{f\sqrt{1+\sec(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx)\right.}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

Mathematica [F]
time = 1.10, size = 0, normalized size = 0.00

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
[Out] Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m, x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\sec^n(fx+e)\right)(a-a\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-a(\sec{(e+fx)}-1))^m \sec^n{(e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**m,x)
[Out] Integral((-a*(sec(e + f*x) - 1))**m*sec(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

3.333 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1-n,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1+sec(f*x+e)
)^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3911, 138}

$$\frac{\sqrt{2}\tan(e+fx)(\sec(e+fx)+1)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(1 + Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[x^(m - 1/2)*(
(a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Rubi steps

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}x^{-\frac{1}{2}+m}}{\sqrt{2-x}}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1+\sec(e+fx),\frac{1}{2}(1+\sec(e+\right.}{f(1+2m)\sqrt{1-\sec(e+fx}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2248 vs. $2(85) = 170$.
time = 6.22, size = 2248, normalized size = 26.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(-Sec[e + f*x])^n*(Cos[(e + f*x)/
2]^2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*App
ellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*
((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(C
os[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[
3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*Appe
llF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
+ n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*Ap
pellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
```

```
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 +
m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*T
an[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/
2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*Appell
F1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*
(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1
+ m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n,
2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5
))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m +
 n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m
+ n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2
, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*App
ellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2)))
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(1+\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
[Out] int((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sec(e+fx))^n (\sec(e+fx)+1)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
[Out] Integral((-sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m \left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^m*(-1/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^m*(-1/cos(e + f*x))^n, x)
```

3.334 $\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx$

Optimal. Leaf size=70

$$\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f
*x+e)/f/(1-sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3910, 138}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rubi steps

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(1-x)^{-1+n}(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(70) = 140.
time = 0.54, size = 257, normalized size = 3.67

$$\frac{(3+2m)F_1\left(\frac{1}{2}+m;m+n,1-n;\frac{3}{2}+m;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1-\sec(e+fx))^m(-\sec(e+fx))^n\sin(e+fx)}{f(1+2m)\left((3+2m)F_1\left(\frac{1}{2}+m;m+n,1-n;\frac{3}{2}+m;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\left((-1+n)F_1\left(\frac{3}{2}+m;m+n,2-n;\frac{5}{2}+m;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m+n)F_1\left(\frac{3}{2}+m;1+m+n,1-n;\frac{5}{2}+m;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

```
[Out] ((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n*Sin[e + f*x])/(f*(
1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5/
2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 + m,
 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2))
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int (1-\sec(fx+e))^m(-\sec(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

```
[Out] int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sec(e+fx))^n (1-\sec(e+fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))**m*(-sec(f*x+e))**n,x)
[Out] Integral((-sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(1-\frac{1}{\cos(e+f\,x)}\right)^m \left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 1/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
[Out] int((1 - 1/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

3.335 $\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1-n,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+
e))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3913, 3911, 138}

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec
[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3911

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[x^(m - 1/2)*(
(a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (-\sec(e+fx))^{n}$$

$$= \frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Su}}{f\sqrt{1-\sec(e+fx}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1+\sec(e+fx),\frac{1}{2}(1+\sec(e+\right.}{f(1+2m)\sqrt{1-\sec(e+f}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2250 vs. $2(87) = 174$.
time = 6.27, size = 2250, normalized size = 25.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(-Sec[e + f*x])^n*(Cos[(e + f*x)/
2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(3
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^
n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2,
m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Co
s[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2,
 m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*
AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 +
```

```
n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n
)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(
1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m +
n)*Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*Ap
pellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*(
(-3*(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2
, 1 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m +
 n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
])/5))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*
(m + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1
+ m + n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]
) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n,
1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1
[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)
*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F]

time = 0.08, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(a+a\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sec{(e+fx)})^{n} (a(\sec{(e+fx)}+1))^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
[Out] Integral((-sec(e + f*x))**n*(a*(sec(e + f*x) + 1))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

3.336 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m \, dx$

Optimal. Leaf size=87

$$\frac{2^{\frac{1}{2}+m} F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))}{f}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1-se
c(f*x+e))^(-1/2-m)*(a-a*sec(f*x+e))^m*tan(f*x+e)/f
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3913, 3910, 138}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 - m)*(a - a*Sec[e + f*x])^m*Tan[e + f
*x])/f
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \left((1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\right)\int (1-\sec(e+fx)$$

$$= \frac{\left((1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))^m\tan(e+fx)\right)\mathrm{Su}}{f\sqrt{1+\sec(e+f}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx)\right.}{f}$$

Mathematica [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
[Out] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(fx+e\right)\right)^n\left(a-a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sec (e+fx))^n (-a(\sec (e+fx)-1))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
[Out] Integral((-sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos (e+f\,x)}\right)^m\,\left(-\frac{1}{\cos (e+f\,x)}\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

3.337 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

Optimal. Leaf size=79

$$-\frac{F_1\left(n;\frac{1}{2},\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] -AppellF1(n,1/2-m,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+
e)/f/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3912, 138}

$$-\frac{\tan(e+fx)(d\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx &= -\frac{(d\tan(e+fx))\text{Subst}\left(\int\frac{(dx)^{-1+n}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{F_1\left(n;\frac{1}{2},\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+f\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2248 vs. 2(79) = 158.

time = 6.21, size = 2248, normalized size = 28.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(d*Sec[e + f*x])^n*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*Ap
pellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1
[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(
e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*App
ellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
 + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*A
ppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 +
 m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*
Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3
```

```
*(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1
 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n,
 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/
5))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m
+ n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m
 + n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
 Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/
2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Ap
pellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
)*Tan[(e + f*x)/2]^2)))
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(1+\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
[Out] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n (\sec(e+fx)+1)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
[Out] Integral((d*sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m \left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^m*(d/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^m*(d/cos(e + f*x))^n, x)
```

3.338 $\int (1-\sec(e+fx))^m(d\sec(e+fx))^n\,dx$

Optimal. Leaf size=79

$$-\frac{F_1\left(n;\frac{1}{2}-m,\frac{1}{2};1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

```
[Out] -AppellF1(n,1/2,1/2-m,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+
e)/f/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3912, 138}

$$-\frac{\tan(e+fx)(d\sec(e+fx))^nF_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]
```

```
[Out] -((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (1-\sec(e+fx))^m(d\sec(e+fx))^n\,dx &= -\frac{(d\tan(e+fx))\text{Subst}\left(\int\frac{(1-x)^{-\frac{1}{2}+m}(dx)^{-1+n}}{\sqrt{1+x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{F_1\left(n;\frac{1}{2}-m,\frac{1}{2};1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+f}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(79) = 158.
time = 0.54, size = 257, normalized size = 3.25

$$\frac{(3+2m)F_1\left(\frac{1}{2}+m;m+n,1-n;\frac{3}{2}+m;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1-\sec(e+fx))^m(d\sec(e+fx))^n\sin(e+fx)}{f(1+2m)\left((3+2m)F_1\left(\frac{1}{2}+m;m+n,1-n;\frac{3}{2}+m;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\left((-1+n)F_1\left(\frac{3}{2}+m;m+n,2-n;\frac{5}{2}+m;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m+n)F_1\left(\frac{3}{2}+m;1+m+n,1-n;\frac{5}{2}+m;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]
```

```
[Out] ((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*
(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5
/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 + m
, 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (1-\sec(fx+e))^m (d\sec(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)
```

```
[Out] int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n (1-\sec(e+fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))**m*(d*sec(f*x+e))**n,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(1-\frac{1}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 1/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

```
[Out] int((1 - 1/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

3.339 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=95

$$-\frac{F_1\left(n;\frac{1}{2},\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+f$$
$$fn\sqrt{1-\sec(e+fx)}$$

```
[Out] -AppellF1(n,1/2-m,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*(1+sec(f
*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1-sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3913, 3912, 138}

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(d\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])
/(f*n*Sqrt[1 - Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (d\sec(e+fx))^n($$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)}{f\sqrt{1-\sec(e+f}}$$

$$= -\frac{F_1\left(n;\frac{1}{2},\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+}{fn\sqrt{1-s}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2250 vs. $2(95) = 190$.
time = 6.25, size = 2250, normalized size = 23.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(d*Sec[e + f*x])^n*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(
3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)
^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2,
 m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appe
llF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(C
os[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2
, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)
*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 +
 n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 -
```

```
n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n,
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^
(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m +
 n)*Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*A
ppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*
((-3*(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/
2, 1 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
 f*x)/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m
+ n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/
2])/5))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)
*(m + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1
 + m + n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2
]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF
1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n
)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F]

time = 0.08, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(fx+e\right)\right)^{n} \left(a+a \sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
[Out] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec{(e + fx)} + 1))^m (d\sec{(e + fx)})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*(d*sec(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos{(e + f\,x)}}\right)^m \left(\frac{d}{\cos{(e + f\,x)}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

3.340 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$

Optimal. Leaf size=96

$$-\frac{F_1\left(n;\frac{1}{2}-m,\frac{1}{2};1+n;\sec(e+fx),-\sec(e+fx)\right)(1-\sec(e+fx))^{-\frac{1}{2}-m}(d\sec(e+fx))^n(a-a\sec(e+f$$
$$\overline{fn\sqrt{1+\sec(e+fx)}}$$

```
[Out] -AppellF1(n,1/2,1/2-m,1+n,-sec(f*x+e),sec(f*x+e))*(1-sec(f*x+e))^(-1/2-m)*(
d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3913, 3912, 138}

$$-\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m(d\sec(e+fx))^nF_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(1 - Sec[e
 + f*x])^(-1/2 - m)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m*Tan[e + f*x])
/(f*n*Sqrt[1 + Sec[e + f*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \left((1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\right)\int (1-\sec(e+fx)$$

$$= -\frac{\left(d(1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))^m\tan(e+fx)\right)}{f\sqrt{1+\sec(e+f}}$$

$$= -\frac{F_1\left(n;\frac{1}{2}-m,\frac{1}{2};1+n;\sec(e+fx),-\sec(e+fx)\right)(1-\sec(e}{fn\sqrt{1+s}}$$

Mathematica [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
[Out] Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a-a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
[Out] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n\,(-a(\sec(e+fx)-1))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
[Out] Integral((d*sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

3.341 $\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=211

$$\frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)}+\frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+m)}+\frac{2^{\frac{1}{2}+m}m(5+3m}{}$$

```
[Out] (4+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(m^3+6*m^2+11*m+6)+sec(f*x+e)^2*(a+a*
sec(f*x+e))^m*tan(f*x+e)/f/(3+m)+2^(1/2+m)*m*(m^2+3*m+5)*hypergeom([1/2, 1/
2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*t
an(f*x+e)/f/(m^3+6*m^2+11*m+6)+m*(a+a*sec(f*x+e))^(1+m)*tan(f*x+e)/a/f/(m^2
+5*m+6)
```

Rubi [A]
time = 0.25, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3909, 4095, 4086, 3913, 3912, 71}

$$\frac{2^{m+\frac{1}{2}}m(m^2+3m+5)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)(m+2)(m+3)}+\frac{m\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af\,(m^2+5m+6)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}+\frac{(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((4 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)*(2 + m)*(3 + m)) +
 (Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + m)) + (2^(1/2
 + m)*m*(5 + 3*m + m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f
*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/
(f*(1 + m)*(2 + m)*(3 + m)) + (m*(a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])
/(a*f*(6 + 5*m + m^2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 3909

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(m + n - 1))), x] + Dist[d^2/(b*(m + n - 1)), Int
[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
 2] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+m)} + \frac{\int \sec^2(e+fx)(a \ldots}{\ldots}$$

$$= \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+m)} + \frac{m(a+a\sec(e+ \ldots}{af(6+ \ldots}$$

$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec \ldots}{f(3 \ldots}$$

$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec \ldots}{f(3 \ldots}$$

$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec \ldots}{f(3 \ldots}$$

$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec \ldots}{f(3 \ldots}$$

Mathematica [A]
time = 1.39, size = 154, normalized size = 0.73

$$\frac{(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\left(2^{\frac{3}{2}+m}m(5+3m+m^2)\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)+(1+\sec(e+fx))^{\frac{1}{2}+m}\left(4+m+m^2+m(1+2m)\sec(e+fx)+(2+5m+2m^2)\sec^2(e+fx)\right)\right)\tan(e+fx)}{f(2+m)(3+m)(1+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*(5 +
 3*m + m^2)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (
1 + Sec[e + f*x])^(1/2 + m)*(4 + m + m^2 + m*(1 + 2*m)*Sec[e + f*x] + (2 +
5*m + 2*m^2)*Sec[e + f*x]^2))*Tan[e + f*x])/(f*(2 + m)*(3 + m)*(1 + 2*m))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^4(fx+e)\right)(a+a\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec{(e+fx)}+1))^m \sec^4{(e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/cos(e + f*x)^4,x)
```

```
[Out] int((a + a/cos(e + f*x))^m/cos(e + f*x)^4, x)
```

3.342 $\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=155

$$-\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(2+3m+m^2\right)}+\frac{2^{\frac{1}{2}+m}\left(1+m+m^2\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+f}{f(1+m)(2+m)}$$

```
[Out] -(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(m^2+3*m+2)+2^(1/2+m)*(m^2+m+1)*hypergeom(
[1/2, 1/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x
+e))^m*tan(f*x+e)/f/(m^2+3*m+2)+(a+a*sec(f*x+e))^(1+m)*tan(f*x+e)/a/f/(2+m)
```

Rubi [A]
time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3885, 4086, 3913, 3912, 71}

$$\frac{2^{m+\frac{1}{2}}\left(m^2+m+1\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)(m+2)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f\left(m^2+3m+2\right)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(2 + 3*m + m^2))) + (2^(1/2 + m)
*(1 + m + m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m
)*(2 + m)) + ((a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(a*f*(2 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
```

```
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af(2+m)} + \frac{\int \sec(e+fx)(a(1+m) -}{}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(2+3m+m^2)} + \frac{(a+a\sec(e+fx))^{1+m}\tan}{af(2+m)}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(2+3m+m^2)} + \frac{(a+a\sec(e+fx))^{1+m}\tan}{af(2+m)}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(2+3m+m^2)} + \frac{(a+a\sec(e+fx))^{1+m}\tan}{af(2+m)}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(2+3m+m^2)} + \frac{2^{\frac{1}{2}+m}(1+m+m^2)\ {}_2F_1\left(\frac{1}{2},\right.}{}$$

Mathematica [A]
time = 0.67, size = 123, normalized size = 0.79

$$\frac{(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\left(2^{\frac{3}{2}+m}(1+m+m^2)\ {}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)+(1+\sec(e+fx))^{\frac{1}{2}+m}(-1+m+(1+2m)\sec(e+fx))\right)\tan(e+fx)}{f(2+m)(1+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*(1 + m
 + m^2)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (1 +
Sec[e + f*x])^(1/2 + m)*(-1 + m + (1 + 2*m)*Sec[e + f*x]))*Tan[e + f*x])/(f
*(2 + m)*(1 + 2*m))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(fx+e\right)\right) \left(a+a\sec\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m} \sec^{3}\left(e+fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/cos(e + f*x)^3,x)
[Out] int((a + a/cos(e + f*x))^m/cos(e + f*x)^3, x)
```

3.343 $\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=107

$$\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)}+\frac{2^{\frac{1}{2}+m}m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+}{f(1+m)}$$

```
[Out] (a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+m)+2^(1/2+m)*m*hypergeom([1/2, 1/2-m],[3
/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+
e)/f/(1+m)
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3883, 3913, 3912, 71}

$$\frac{2^{m+\frac{1}{2}}m\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)) + (2^(1/2 + m)*m*Hypergeo
metric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2
 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx &= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)}+\frac{m\int \sec(e+fx)(a+a\sec(e\\ &= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)}+\frac{(m(1+\sec(e+fx))^{-m}(a+\\ &= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)}-\frac{\left(m(1+\sec(e+fx))^{-\frac{1}{2}-m}(a\\ &= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)}+\frac{2^{\frac{1}{2}+m}m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1\end{aligned}$$

Mathematica [A]
time = 0.22, size = 95, normalized size = 0.89

$$\frac{(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\left(2^{\frac{3}{2}+m}m\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)+(1+\sec(e+fx))^{\frac{1}{2}+m}\right)\tan(e+fx)}{f+2fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*Hype
rgeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (1 + Sec[e + f*x]
)^(1/2 + m))*Tan[e + f*x])/(f + 2*f*m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\sec^2(fx+e)\right)(a+a\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^m \sec^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/cos(e + f*x)^2,x)
[Out] int((a + a/cos(e + f*x))^m/cos(e + f*x)^2, x)
```

3.344 $\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=73

$$\frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)}{f}$$

```
[Out] 2^(1/2+m)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(
-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3913, 3912, 71}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
+ Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/f
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \sec(e+fx)(1+\sec$$

$$= -\frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subs}}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}}{f}$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 1.00

$$\frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
+ Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*Tan[e + f*x])/f
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \sec(fx+e)(a+a\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e + fx) + 1))^m \sec(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e + f\,x)}\right)^m}{\cos(e + f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/cos(e + f*x),x)
[Out] int((a + a/cos(e + f*x))^m/cos(e + f*x), x)
```

3.345 $\int (a + a\sec(e + fx))^m \, dx$

Optimal. Leaf size=83

$$\frac{\sqrt{2}\, F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e)
)^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3864, 3863, 141}

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec[e
 + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e +
 f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (1+\sec(e+fx))^m\,dx$$

$$= -\frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+}}{\sqrt{1-x}}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(a+a\sec(e+}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 711 vs. 2(83) = 166.
time = 6.79, size = 711, normalized size = 8.57

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (30*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(
e + f*x)/2]^2*Cos[e + f*x]*(a*(1 + Sec[e + f*x]))^m*Sin[e + f*x]*(3*AppellF
1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/
2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1
+ m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
/(f*(45*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]^2
*Cos[(e + f*x)/2]^2*(1 + 2*m - 2*m*Cos[e + f*x] + Cos[2*(e + f*x)]) + 6*App
ellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x
)/2]^2*(-5*AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*(1 + 2*m - 2*(2 + m)*Cos[e + f*x] + Cos[2*(e + f*x)]) + 5*m*AppellF1[3/2,
 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + 2*m - 2*(2 +
m)*Cos[e + f*x] + Cos[2*(e + f*x)]) - 48*(2*AppellF1[5/2, m, 3, 7/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*m*AppellF1[5/2, 1 + m, 2, 7/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*(1 + m)*AppellF1[5/2, 2 + m, 1, 7/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cot[e + f*x]*Csc[e + f*x]*Sin[(
e + f*x)/2]^4) + 40*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])^2*Cos[e + f*x]*Sin[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int (a+a\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m,x)
```

```
[Out] int((a+a*sec(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec (e+fx)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*sec(e + f*x) + a)**m, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m,x)
[Out] int((a + a/cos(e + f*x))^m, x)
```

3.346 $\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

```
[Out] -AppellF1(1/2+m,2,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e
))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3913, 3912, 141}

$$-\frac{\sqrt{2}\,\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \cos(e+fx)(1+\sec($$

$$= -\frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3781 vs. $2(84) = 168$.
time = 17.03, size = 3781, normalized size = 45.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1 + m)*Cos[(e + f*x)/2]^3*Cos[e + f*x]*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^m*(a*(1 + Sec[e + f*x]))^m*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1
/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2,
m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m
, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (
2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(Appel
lF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*Appel
lF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3
/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/
2]^2)/3)))/(f*(2^m*Cos[(e + f*x)/2]^4*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*(
(-3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2) + (2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
```

```
 + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) - 3*2^m*Cos[(e + f*x)/2]^2*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^m*Sin[(e + f*x)/2]^2*((-3*AppellF1[1/2, m, 1, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1/
2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m
, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m,
 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (2
*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(Appell
F1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*Appell
F1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/
2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2)/3)) + 2^(1 + m)*Cos[(e + f*x)/2]^3*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m
*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3*AppellF1[1/2, m, 1, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sec[(e +
f*x)/2]^2*(-1/3*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (m*AppellF1[3/2, 1 + m, 1, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3))/(3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2) + (2*((-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[
3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/
2]^2*Tan[(e + f*x)/2])/3))/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3) + (3*AppellF1[1/2, m, 1, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(-2*(AppellF1[3/2,
 m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 +
m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2] + 3*(-1/3*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (m*AppellF1[3/2, 1 +
 m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/3) - 2*Tan[(e + f*x)/2]^2*((-6*AppellF1[5/2, m, 3, 7/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5
 + (3*m*AppellF1[5/2, 1 + m, 2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 - m*((-3*AppellF1[5/2, 1 + m, 2,
7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2])/5 + (3*(1 + m)*AppellF1[5/2, 2 + m, 1, 7/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/
2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m
, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m,
```

```
 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 -
(2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*((-2*A
ppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3/2,...
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \cos \left(f x +e \right) \left(a +a \sec \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec{\left(e + f x \right)} + 1\right)\right)^{m} \cos{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a/cos(e + f*x))^m,x)
[Out] int(cos(e + f*x)*(a + a/cos(e + f*x))^m, x)
```

3.347 $\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=98

$$-\frac{2F_1\left(\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;\frac{5}{2};\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^{3/2}(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx)}{3f\sqrt{1-\sec(e+fx)}}$$

```
[Out] -2/3*AppellF1(3/2,1/2-m,1/2,5/2,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^(3/2
)*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^(1
/2)
```

Rubi [A]
time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3912, 138}

$$-\frac{2\tan(e+fx)(d\sec(e+fx))^{3/2}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;\frac{5}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (-2*AppellF1[3/2, 1/2, 1/2 - m, 5/2, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^(3/2)*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e +
f*x])/(3*f*Sqrt[1 - Sec[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (d\sec(e+fx)$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx\right.}{f\sqrt{1-\sec(e+}}$$

$$= -\frac{2F_1\left(\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;\frac{5}{2};\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+}{3f\sqrt{1-}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2529 vs. $2(98) = 196$.
time = 15.58, size = 2529, normalized size = 25.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (-3*2^(1 + m)*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2)*(Cos[(e + f*x)/2]^2
*Sec[e + f*x])^m*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(-1 + Tan[(e
 + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^(1 + m)*AppellF1[
1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f
*x)/2]^2)/((-1 + Tan[(e + f*x)/2]^2)^2*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 +
 m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e
 + f*x])^m)/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 +
 m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
```

)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Sin[e + f*x]*Tan[(e + f*x)/2])/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*2^(1 + m)*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/2]*((AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((3/2 + m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*2^(1 + m)*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/2]*((AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*((AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((3/2 + m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*((-3*AppellF1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(3/2 + m)*AppellF1[5/2, 5/2 + m, 1/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3 + 2*m)*((3*AppellF1[5/2, 5/2 + m, 1/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(5/2 + m)*AppellF1[5/2, 7/2 + m, -1/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2) - (3*2^(1 + m)*m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))

Maple [F]

time = 0.10, size = 0, normalized size = 0.00

$$\int (d \sec (fx + e))^{\frac{3}{2}} (a + a \sec (fx + e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
[Out] int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m*d*sec(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**m,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(3/2),x)
[Out] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(3/2), x)
```

3.348 $\int \sqrt{d\sec(e+fx)}\,(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=96

$$-\frac{2F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\sec(e+fx),-\sec(e+fx)\right)\sqrt{d\sec(e+fx)}\,(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

```
[Out] -2*AppellF1(1/2,1/2-m,1/2,3/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-1/2-
m)*(a+a*sec(f*x+e))^m*(d*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(1-sec(f*x+e))^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3912, 138}

$$-\frac{2\tan(e+fx)\sqrt{d\sec(e+fx)}\,(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (-2*AppellF1[1/2, 1/2, 1/2 - m, 3/2, Sec[e + f*x], -Sec[e + f*x]]*Sqrt[d*Se
c[e + f*x]]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}\,(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \sqrt{d\sec(e+fx)}$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)}{f\sqrt{1-\sec(e+fx}}$$

$$= -\frac{2F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\sec(e+fx),-\sec(e+fx)\right)\sqrt{d\sec(e+}}{f\sqrt{1-\sec}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2225 vs. $2(96) = 192$.
time = 15.00, size = 2225, normalized size = 23.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
[Out] (2^(1 + m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sqrt[d*Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*
(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*Sqrt[Sec[(e + f*x)/2]^2]*(App
ellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((
AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)*((2^m*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[(e + f*x)/2]^2]*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(1/2 + m))/(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3) - (
2^m*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]^2)/(Sqrt[Se
c[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) + (2^(1 + m)*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]*(-1/6*(AppellF1[3/2
```

```
, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2]) + ((1/2 + m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)
)/(Sqrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) - (2^(1
+ m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]*(-1/6*(App
ellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((1/2 + m)*AppellF1[3/2, 3/2 + m, 1/2, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3 - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 - (Tan[(e +
f*x)/2]^2*((-9*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(1/2 + m)*Appell
F1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2])/5 - (1 + 2*m)*((-3*AppellF1[5/2, 3/2 + m, 3/2
, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/10 + (3*(3/2 + m)*AppellF1[5/2, 5/2 + m, 1/2, 7/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))/3))/(S
qrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)^2) + (2^(1 +
m)*(1/2 + m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 + m)*Tan[(e + f*x)/2]*(
-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[
e + f*x]*Tan[e + f*x]))/(Sqrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1
/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m
, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3
/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)/3))))
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \sqrt{d \sec \left(f x + e\right)}\, \left(a + a \sec \left(f x + e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^m \sqrt{d\sec(e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**m,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*sqrt(d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m \sqrt{\frac{d}{\cos(e+f\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(1/2), x)
```

3.349 $\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=96

$$\frac{2F_1\left(-\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] 2*AppellF1(-1/2,1/2-m,1/2,1/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-1/2-
m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3912, 138}

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*AppellF1[-1/2, 1/2, 1/2 - m, 1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec[
e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 - Sec[e
 + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \frac{(1+\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\Big(\int \frac{(1+x}{\sqrt{1-}}}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2F_1\left(-\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.37, size = 2424, normalized size = 25.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-3*2^(1 + m)*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 +
m)*(a*(1 + Sec[e + f*x]))^m*(Cos[2*(e + f*x)]*((1 + Sec[e + f*x])^m/(2*Sqrt
[Sec[e + f*x]]) - (I/2)*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^m*Sin[e + f*x
]) + ((1 + Sec[e + f*x])^m/2 + (I/2)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)])
/Sqrt[Sec[e + f*x]] + Sqrt[Sec[e + f*x]]*Sin[e + f*x]*((-1/2*I)*(1 + Sec[e
+ f*x])^m + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)])/2))*Tan[(e + f*x)/2])/(
f*(Sec[(e + f*x)/2]^2)^(3/2)*Sqrt[d*Sec[e + f*x]]*(1 + Sec[e + f*x])^m*(-3*
AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((-3*2^m*AppellF1[1/2, -1/2 + m, 3/2,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f
*x])^(-1/2 + m))/(Sqrt[Sec[(e + f*x)/2]^2]*(-3*AppellF1[1/2, -1/2 + m, 3/2,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2, -1/2 + m,
 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/
2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)) + (9*2^m*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 + m)*Tan[(e + f*
```

```
x)/2]^2)/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 3/2, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2, -1/2 + m, 5/2,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2
 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2)) - (3*2^(1 + m)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 + m)*Tan[(e + f*
x)/2]*(-1/2*(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + m)*AppellF1[3/
2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/3))/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2,
-1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[
3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*
m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2)) + (3*2^(1 + m)*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-
1/2 + m)*Tan[(e + f*x)/2]*((3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/
2] - 3*(-1/2*(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + m)*AppellF1[3
/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*
x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(3*((-3*AppellF1[5/2, -1/
2 + m, 7/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/2 + (3*(-1/2 + m)*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5
) + (1 - 2*m)*((-9*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(1/2 + m)*Ap
pellF1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*App
ellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2) - (3*2^(1 + m)*(-1/2 + m)*AppellF1[1/2
, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(-3/2 + m)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e
 + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/
((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F]

time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sec(fx + e))^m}{\sqrt{d\sec(fx + e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d*sec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(e+fx)+1))^m}{\sqrt{d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m/sqrt(d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(1/2), x)
```

3.350 $\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2F_1\left(-\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;-\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)}{3f\sqrt{1-\sec(e+fx)}\,(d\sec(e+fx))^{3/2}}$$

```
[Out] 2/3*AppellF1(-3/2,1/2-m,1/2,-1/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-1
/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(d*sec(f*x+e))^(3/2)/(1-sec(f*x+e))^(
1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3913, 3912, 138}

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;-\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}\,(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*AppellF1[-3/2, 1/2, 1/2 - m, -1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec
[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(3*f*Sqrt[1 - Se
c[e + f*x]]*(d*Sec[e + f*x])^(3/2))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \frac{(1+\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1+}{\sqrt{1-}}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2F_1\left(-\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;-\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a}{3f\sqrt{1-\sec(e+fx)}\,(d\sec(e+fx))^{3/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 19.87, size = 3349, normalized size = 34.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2^(1 + m)*Sec[e + f*x]^(3/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*(
a*(1 + Sec[e + f*x]))^m*((Cos[2*(e + f*x)]^3*Sqrt[Sec[e + f*x]]*(1 + Sec[e
+ f*x])^m)/4 + Cos[2*(e + f*x)]^2*Sqrt[Sec[e + f*x]]*((1 + Sec[e + f*x])^m/
2 + (I/4)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]) + Cos[2*(e + f*x)]*Sqrt[Se
c[e + f*x]]*((1 + Sec[e + f*x])^m/4 + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)
]^2)/4) + Sqrt[Sec[e + f*x]]*((-1/4*I)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)
] + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]^2)/2 + (I/4)*(1 + Sec[e + f*x])^
m*Sin[2*(e + f*x)]^3))*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2
)^(1/2 + m)*Tan[(e + f*x)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3
/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))/(3*f*(d*Sec[e + f*x])^(3/2)
*(1 + Sec[e + f*x])^m*((2^m*Sec[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2*Sec[e +
f*x])^(1/2 + m)*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1/2 + m)*Tan[(e + f*x
)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1
```

/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))/3 + (2^(1 + m)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1/2 + m)*Tan[(e + f*x)/2]) - (Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1/2 + m)*Tan[(e + f*x)/2]^2*((-3*AppellF1[5/2, -1/2 + m, 7/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (3*(-1/2 + m)*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) - (1/2 + m)*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1/2 + m)*Tan[(e + f*x)/2]^2*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x])/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (27*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*Tan[(e + f*x)/2])/(2*(Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (9*Cos[e + f*x]*((-5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((-1/2 + m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*((5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] - 3*((-5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((-1/2 + m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(5*((-21*AppellF1[5/2, -1/2 + m, 9/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(-1/2 + m)*AppellF1[5/2, 1/2 + m, 7/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (1 - 2*m)*((-3*AppellF1[5/2, 1/2 + m, 7/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (3*(1/2 + m)*AppellF1[5/2,

```
 3/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/
2]^2*Tan[(e + f*x)/2])/5))))/((Sec[(e + f*x)/2]...
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sec (fx + e))^{m}}{(d \sec (fx + e))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
[Out] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d^2*sec(f*x + e)^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec (e + fx) + 1))^{m}}{(d \sec (e + fx))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(3/2),x)
[Out] Integral((a*(sec(e + f*x) + 1))**m/(d*sec(e + f*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(3/2),x)
[Out] int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(3/2), x)
```

3.351 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{10a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a$$

[Out] 6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d

Rubi [A]

time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2715, 2719, 2720}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10a\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]

[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(21*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= \int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx \\
&= a\int \cos^{\frac{5}{2}}(c+dx)\,dx + a\int \cos^{\frac{7}{2}}(c+dx)\,dx \\
&= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(3a) \\
&= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+ \\
&= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10a\sqrt{\cos(c+dx)}\,\sin(}{21d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.10, size = 490, normalized size = 4.41

$$a\left(\sqrt{\cos(c+dx)}(1+\cos(c+dx))\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{3\cot(c)}{5d}+\frac{23\cos(dx)\sin(c)}{84d}+\frac{\cos(2dx)\sin(2c)}{10d}+\frac{\cos(3dx)\sin(3c)}{28d}+\frac{23\cos(c)\sin(dx)}{84d}+\frac{\cos(2c)\sin(2dx)}{10d}+\frac{\cos(3c)\sin(3dx)}{28d}\right)-\frac{5(1+\cos(c+dx))\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\mathrm{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\mathrm{ArcTan}(\cot(c)))}}{21d\sqrt{1+\cot^2(c)}}-\frac{3(1+\cos(c+dx))\csc(c)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{10d}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*Cot[c])/
(5*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[3
*d*x]*Sin[3*c])/(28*d) + (23*Cos[c]*Sin[d*x])/(84*d) + (Cos[2*c]*Sin[2*d*x]
)/(10*d) + (Cos[3*c]*Sin[3*d*x])/(28*d)) - (5*(1 + Cos[c + d*x])*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*(1 + Cos[c + d*x])*Csc[c]
*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
```

```
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [A]
time = 0.10, size = 270, normalized size = 2.43

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-528\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+25*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.04, size = 148, normalized size = 1.33

$$\frac{2\left(15\,a\cos\left(dx+c\right)^2+21\,a\cos\left(dx+c\right)+25\,a\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-25i\,\sqrt{2}\,a\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+25i\,\sqrt{2}\,a\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+63i\,\sqrt{2}\,a\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-63i\,\sqrt{2}\,a\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(15*a*cos(d*x + c)^2 + 21*a*cos(d*x + c) + 25*a)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Mupad [**B**]
time = 1.18, size = 87, normalized size = 0.78

$$-\frac{2\,a\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,a\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a*cos(c + d*x)^(9/2)*sin(c + d*x
)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

```
[Out] 6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2
/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2715, 2720, 2719}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx \\
&= a\int \cos^{\frac{3}{2}}(c+dx)\,dx + a\int \cos^{\frac{5}{2}}(c+dx)\,dx \\
&= \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \\
&= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.59, size = 232, normalized size = 2.67

$$\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{9(3\cos(c-dx-\text{ArcTan}(\tan(c)))+\cos(c+dx+\text{ArcTan}(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-20\cos(c+dx)\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec(dx-\text{ArcTan}(\cot(c)))\sin(c)+2\cos(c+dx)(-18\cot(c)+10\sin(c+dx)+3\sin(2(c+dx)))-18\cos(c)\csc(dx+\text{ArcTan}(\tan(c)))\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}\right)}{60d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((9*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Cos[
c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*
Sin[c] + 2*Cos[c + d*x]*(-18*Cot[c] + 10*Sin[c + d*x] + 3*Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.08, size = 219, normalized size = 2.52

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-28\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{}\right.}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 137, normalized size = 1.57

$$\frac{2\,(3\,a\cos(dx+c)+5\,a)\sqrt{\cos(dx+c)}\,\sin(dx+c)-5i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+9i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-9i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(3*a*cos(d*x + c) + 5*a)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqrt
(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(
2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2
)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 0.78, size = 80, normalized size = 0.92

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,a\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d
*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2719, 2715, 2720}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= \int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))\,dx\\ &= a\int \sqrt{\cos(c+dx)}\,dx + a\int \cos^{\frac{3}{2}}(c+dx)\,dx\\ &= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c\ldots}}\\ &= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c\ldots}{3d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.15, size = 222, normalized size = 3.64

$$\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(3\cos(c-dx-\mathrm{ArcTan}(\tan(c)))+\cos(c+dx+\mathrm{ArcTan}(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-4\cos(c+dx)\sqrt{\cos^2(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec(dx-\mathrm{ArcTan}(\cot(c)))\sin(c)-4\cos(c+dx)(3\cot(c)-\sin(c+dx))-6\cos(c)\csc(dx+\mathrm{ArcTan}(\tan(c)))\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\mathrm{ArcTan}(\tan(c)))}\right)}{12d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((3*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 4*Cos[c
 + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*S
in[c] - 4*Cos[c + d*x]*(3*Cot[c] - Sin[c + d*x]) - 6*Cos[c]*Csc[d*x + ArcTa
n[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[c + d
*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(107) = 214$.
time = 0.08, size = 225, normalized size = 3.69

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\right.}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 125, normalized size = 2.05

$$\frac{2a\sqrt{\cos(dx+c)}\,\sin(dx+c)-i\sqrt{2}\,a\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,a\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,a\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,a\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*a*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 0.76, size = 53, normalized size = 0.87

$$\frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*
d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

3.354 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4310, 2827, 2720, 2719}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))\,dx &= \int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx \\
&= a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + a\int \sqrt{\cos(c+dx)}\,dx \\
&= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d}
\end{aligned}
$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.82, size = 155, normalized size = 4.43

$$
\frac{a\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-2\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\,\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec(dx-\text{ArcTan}(\cot(c)))\sin(c)+\tan(dx+\text{ArcTan}(\tan(c)))-\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\tan(dx+\text{ArcTan}(\tan(c)))}{\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}}\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-2*Sqrt[Cos[d*
x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Tan[d*x + A
rcTan[Tan[c]]] - (HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Tan[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(
2*d)
```

Maple [**A**]
time = 0.10, size = 150, normalized size = 4.29

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\left(\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}}$
risch	$-\frac{i\sqrt{2}\,a\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d}-\frac{i\left(\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\text{EllipticF}}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 107, normalized size = 3.06

$$\frac{-i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sq
rt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{\cos(c+dx)}\,\sec(c+dx)\,dx+\int\sqrt{\cos(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c + d*x
)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.20, size = 27, normalized size = 0.77

$$\frac{2\,a\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*(ellipticE(c/2 + (d*x)/2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d
```

3.355 $\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}} \, dx$

Optimal. Leaf size=57

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2716, 2719, 2720}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a\int \sqrt{\cos(c+dx)}\,dx \\
&= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.04, size = 209, normalized size = 3.67

$$\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(4\cos(dx)\csc(c)-\frac{(3\cos(c-dx-\text{ArcTan}(\tan(c)))+\cos(c+dx+\text{ArcTan}(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-4\cos(c+dx)\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec(dx-\text{ArcTan}(\cot(c)))\sin(c)+2\cos(c)\csc(dx+\text{ArcTan}(\tan(c)))\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}\right)}{4d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*Csc[c] - ((3*Cos[c - d
*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[S
ec[c]^2] - 4*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x -
 ArcTan[Cot[c]]]*Sin[c] + 2*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[S
in[d*x + ArcTan[Tan[c]]]^2]))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.08, size = 148, normalized size = 2.60

method	result

default	$\frac{2a\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\sqrt{2\left(\right.}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.97, size = 156, normalized size = 2.74

$$\frac{-i\sqrt{2}\,a\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,a\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-i\sqrt{2}\,a\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,a\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2a\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) - I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*a*cos(d*x
 + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))) + 2*a*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c + d
*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 1.03, size = 60, normalized size = 1.05

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*sin(c + d*x)*hypergeom([-1/4, 1/
2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.356 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*
a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4310, 2827, 2716, 2720, 2719}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - a\int \sqrt{\cos(c+d} \\
&= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.13, size = 444, normalized size = 5.35

$$
a\left(\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\csc(c)\sec(c)}{d}+\frac{\sec(c)\sec^2(c+dx)\sin(dx)}{3d}+\frac{\sec(c)\sec(c+dx)(\sin(c)+3\sin(dx))}{3d}\right)-\frac{(1+\cos(c+dx))\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\text{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\text{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\,\sin(c)\sin(dx-\text{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\text{ArcTan}(\cot(c)))}}{3d\sqrt{1+\cot^2(c)}}+\frac{(1+\cos(c+dx))\csc(c)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sin(dx+\text{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\text{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\text{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\frac{\sin(dx+\text{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{}}{\sqrt{\cos(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{2d}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((Csc[c]*Sec[
c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c + d*x]*(Sin[
c] + 3*Sin[d*x]))/(3*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(3*d*Sqrt[1 + Cot[c]^2]) + ((1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
```

```
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(2*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(127) = 254$.
time = 0.10, size = 368, normalized size = 4.43

method	result
default	$-\frac{2a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2
*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.18, size = 175, normalized size = 2.11

$$\frac{-i\sqrt{2}a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2(3a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\sin(dx+c)}{3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqr
t(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**(
-3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.17, size = 87, normalized size = 1.05

$$\frac{2\,a\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\!\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\!\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d
*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2]
, 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.357 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)+
2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2716, 2719, 2720}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}(3a)\int \frac{1}{\cos^{\frac{3}{2}}(c-$$

$$= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}$$

$$= -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.16, size = 477, normalized size = 4.30

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((3*Csc[c]*Se
c[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*x]
^2*(3*Sin[c] + 5*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 9*Sin
[d*x]))/(15*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqr
```

```
t[1 + Cot[c]^2]) + (3*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
10*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(147) = 294$.

time = 0.11, size = 384, normalized size = 3.46

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\right.}}{40\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/40*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(
1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 188, normalized size = 1.69

$$\frac{-5i\sqrt{2}\,a\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5i\sqrt{2}\,a\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-9i\sqrt{2}\,a\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+9i\sqrt{2}\,a\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(9\,a\cos\left(dx+c\right)^{2}+5\,a\cos\left(dx+c\right)+3\,a\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a*cos(d*x + c)^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*
I*sqrt(2)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*a*cos(d*x + c)^2 + 5*a*cos(d*x + c
) + 3*a)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.28, size = 87, normalized size = 0.78

$$\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c +
 d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-5/4, 1/
2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))
```

3.358 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a*sin(d*x+c)/d/cos(d*x+c)^(7/2
)+2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)+10/21*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)
+6/5*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2716, 2720, 2719}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*a*Sin[c + d*x])/(5
*d*Cos[c + d*x]^(5/2)) + (10*a*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (6
*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \int \frac{a+a\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)}\,dx\\ &= a\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}(3a)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{7}(5a)\int \frac{1}{\cos^{\frac{5}{2}}(c}\\ &= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{21}(\\ &= -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.69, size = 294, normalized size = 2.18

$$\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left((189\cos(c)+85\cos(dx)-85\cos(2c+dx)+231\cos(c+2dx)+21\cos(3c+2dx)+25\cos(2c+3dx)-25\cos(4c+3dx)+63\cos(3c+4dx))\csc(c)-200\cos^4(c+dx)\sqrt{\cos^2(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec(dx-\mathrm{ArcTan}(\cot(c)))\sin(c)-\frac{126\cos^3(c+dx)\sec(c)\left(-2\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sin(dx+\mathrm{ArcTan}(\tan(c)))+(3\cos(c-dx-\mathrm{ArcTan}(\tan(c)))+\cos(c+dx+\mathrm{ArcTan}(\tan(c))))\csc(c)\sqrt{\sin^2(dx+\mathrm{ArcTan}(\tan(c)))}\right)}{\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\mathrm{ArcTan}(\tan(c)))}}\right)}{840d\cos^{\frac{7}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((189*Cos[c] + 85*Cos[d*x] - 85*Co
s[2*c + d*x] + 231*Cos[c + 2*d*x] + 21*Cos[3*c + 2*d*x] + 25*Cos[2*c + 3*d*
x] - 25*Cos[4*c + 3*d*x] + 63*Cos[3*c + 4*d*x])*Csc[c] - 200*Cos[c + d*x]^4
*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] -
 (126*Cos[c + d*x]^3*Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTa
```

```
n[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Ta
n[c]]]^2]))/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))))/(840*d*Cos
[c + d*x]^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(167) = 334$.
time = 0.13, size = 437, normalized size = 3.24

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\right.}}{40\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/40*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(
1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+44/105*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/112*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/84*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.71, size = 199, normalized size = 1.47

$$\frac{-25i\sqrt{2}\,a\cos\left(dx+c\right)^4 \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+25i\sqrt{2}\,a\cos\left(dx+c\right)^4 \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-63i\sqrt{2}\,a\cos\left(dx+c\right)^4 \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+63i\sqrt{2}\,a\cos\left(dx+c\right)^4 \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(63\,a\cos\left(dx+c\right)^3+25\,a\cos\left(dx+c\right)^2+21\,a\cos\left(dx+c\right)+15\,a\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/105*(-25*I*sqrt(2)*a*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) + 25*I*sqrt(2)*a*cos(d*x + c)^4*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*a*cos(d*x + c)^4*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 63*I*sqrt(2)*a*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(63*a*cos(d*x + c)^3 + 25*a*cos(
d*x + c)^2 + 21*a*cos(d*x + c) + 15*a)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Mupad [**B**]
time = 1.38, size = 87, normalized size = 0.64

$$\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c
+ d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-7/4, 1
/2], -3/4, cos(c + d*x)^2))/(7*d*cos(c + d*x)^(7/2)*(sin(c + d*x)^2)^(1/2))
```

3.359 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=147

$$\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{20a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+d\ldots}{45d}$$

```
[Out] 32/15*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+20/21*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/45*a^2*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+4/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*s
in(d*x+c)/d+20/21*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3854, 3856, 2720, 4130, 2719}

$$\frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{32a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{20a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (32*a^2*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a^2*EllipticF[(c + d*x)/2,
2])/(21*d) + (20*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a^2*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}\,dx \\
&= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \Big(2a^2 \\
&= \frac{4a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{7}\Big(1 \\
&= \frac{20a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \\
&= \frac{20a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \\
&= \frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} + \frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d} + \frac{20a^2\sqrt{\cos(c+dx)}}{21d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.13, size = 548, normalized size = 3.73

$$\cos^{\frac{5}{2}}(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-\frac{8\cot(c)}{15d}+\frac{23\cos(dx)\sin(c)}{84d}+\frac{37\cos(2dx)\sin(2c)}{360d}+\frac{\cos(3dx)\sin(3c)}{28d}+\frac{\cos(4dx)\sin(4c)}{144d}+\frac{23\cos(c)\sin(dx)}{84d}+\frac{37\cos(2c)\sin(2dx)}{360d}+\frac{\cos(3c)\sin(3dx)}{28d}+\frac{\cos(4c)\sin(4dx)}{144d}\right)-\frac{5\cos^2(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sec(dx-\mathrm{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\mathrm{ArcTan}(\cot(c)))}}{21d\sqrt{1+\cot^2(c)}}-\frac{4\cos^2(c+dx)\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{15d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((-8*Cot[c])
/(15*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (37*Cos[2*d*x]*Sin[2*c])/(360*d) +
(Cos[3*d*x]*Sin[3*c])/(28*d) + (Cos[4*d*x]*Sin[4*c])/(144*d) + (23*Cos[c]*S
in[d*x])/(84*d) + (37*Cos[2*c]*Sin[2*d*x])/(360*d) + (Cos[3*c]*Sin[3*d*x])/
(28*d) + (Cos[4*c]*Sin[4*d*x])/(144*d)) - (5*Cos[c + d*x]^2*Csc[c]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2
]^4*(a + a*Sec[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (4*Cos[c +
 d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d)
```

Maple [A]
time = 0.10, size = 260, normalized size = 1.77

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-960\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+608\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(560*cos
(1/2*d*x+1/2*c)^11-960*cos(1/2*d*x+1/2*c)^9+608*cos(1/2*d*x+1/2*c)^7-96*cos
(1/2*d*x+1/2*c)^5-205*cos(1/2*d*x+1/2*c)^3+75*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+93*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 175, normalized size = 1.19

$$-\frac{2\left(75i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-75i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-168i\sqrt{2}\,a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+168i\sqrt{2}\,a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\left(35\,a^2\cos(dx+c)^3+90\,a^2\cos(dx+c)^2+112\,a^2\cos(dx+c)+150\,a^2\right)\sqrt{\cos(dx+c)}\,\sin(dx+c)\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/315*(75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) - 168*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*I*sqrt(2)*a^2*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*a^2*c
os(d*x + c)^3 + 90*a^2*cos(d*x + c)^2 + 112*a^2*cos(d*x + c) + 150*a^2)*sqr
t(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

Mupad [B]
time = 1.18, size = 136, normalized size = 0.93

$$-\frac{2\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{4\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,a^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] - (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
+ d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^2*cos(c + d*x)^(9/2)*sin(c +
 d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1
/2)) - (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4,
 cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

3.360 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=121

$$\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{8a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{7d}+\frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\cdots$$

```
[Out] 12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a^2*cos(d*x+c)^(3/2)*sin(d*x
+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+8/7*a^2*sin(d*x+c)*cos(d*x+c)^(
1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3854, 3856, 2719, 4130, 2720}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2]
)/(7*d) + (8*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (4*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx\\
&= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \Big(2a\\
&= \frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}\Big(\\
&= \frac{8a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{7d} + \frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \\
&= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{7d} + 4a^2\cos^{\frac{3}{2}}\\
&= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d} + \frac{8a^2\sqrt{\cos(c+dx)}}{7d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.11, size = 516, normalized size = 4.26

$$\cos^{\frac{5}{2}}(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-\frac{3\cot(c)}{5d}+\frac{17\cos(dx)\sin(c)}{56d}+\frac{\cos(2dx)\sin(2c)}{10d}+\frac{\cos(3dx)\sin(3c)}{56d}+\frac{17\cos(c)\sin(dx)}{56d}+\frac{\cos(2c)\sin(2dx)}{10d}+\frac{\cos(3c)\sin(3dx)}{56d}\right)-\frac{2\cos^2(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sec(dx-\mathrm{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\mathrm{ArcTan}(\cot(c)))}}{7d\sqrt{1+\cot^2(c)}}-\frac{3\cos^2(c+dx)\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((-3*Cot[c])
/(5*d) + (17*Cos[d*x]*Sin[c])/(56*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[
3*d*x]*Sin[3*c])/(56*d) + (17*Cos[c]*Sin[d*x])/(56*d) + (Cos[2*c]*Sin[2*d*x
])/(10*d) + (Cos[3*c]*Sin[3*d*x])/(56*d)) - (2*Cos[c + d*x]^2*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^4*(a + a*Sec[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (3*Cos[c
+ d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Hypergeometri
cPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[
1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (
2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]
^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

Maple [A]
time = 0.09, size = 272, normalized size = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-39*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.35, size = 162, normalized size = 1.34

$$-\frac{2\left(10i\sqrt{2}a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(5a^2\cos(dx+c)^2+14a^2\cos(dx+c)+20a^2)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/35*(10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) - 10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*a^2*cos(d
*x + c)^2 + 14*a^2*cos(d*x + c) + 20*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/
d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Mupad [B]
time = 1.04, size = 129, normalized size = 1.07

$$\frac{2\left(a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}-\frac{4\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,a^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x)))
/(3*d) - (4*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,
 cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^2*cos(c + d*x)^(9/2)*
sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x
)^2)^(1/2))
```

3.361 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=95

$$\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

```
[Out] 16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c)^(3/2)*sin(d*x
+c)/d+4/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3854, 3856, 2720, 4130, 2719}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \Big(2a^2 \\
&= \frac{4a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\Big(\\
&= \frac{4a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\Big(\\
&= \frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} + \frac{4a^2\sqrt{\cos(c+dx)}\,\mathrm{s}}{3d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.86, size = 235, normalized size = 2.47

$$\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{12(3\cos(c-dx-\text{ArcTan}(\tan(c)))+\cos(c+dx+\text{ArcTan}(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-20\cos(c+dx)\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\,\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec(dx-\text{ArcTan}(\cot(c)))\sin(c)+\cos(c+dx)(-48\cot(c)+20\sin(c+dx)+3\sin(2(c+dx)))-24\cos(c)\csc(dx+\text{ArcTan}(\tan(c)))\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sqrt{\sec^2(c)}\,\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}\right)}{60d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]

[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((12*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-48*Cot[c] + 20*Sin[c + d*x] + 3*Sin[2*(c + d*x)]) - 24*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])

Maple [A]
time = 0.11, size = 250, normalized size = 2.63

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-12\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{15\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-13*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 149, normalized size = 1.57

$$-\frac{2\left(5i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-12i\sqrt{2}\,a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+12i\sqrt{2}\,a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-(3\,a^2\cos(dx+c)+10\,a^2)\sqrt{\cos(dx+c)}\,\sin(dx+c)\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^2*cos(d*x
 + c) + 10*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Mupad [**B**]
time = 0.92, size = 104, normalized size = 1.09

$$\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (4*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d*x
)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin
(c + d*x)^2)^(1/2))
```

3.362 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=67

$$\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A]
time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4349, 3873, 3856, 2719, 4130, 2720}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(2a^2\right.$$
$$= \frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \left(2a^2\right)\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}\left(4\right.$$
$$= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\left(4a^2\right)\int$$
$$= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\,\sin}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.36, size = 224, normalized size = 3.34

$$\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(3\cos(c-dx-\text{ArcTan}(\tan(c)))+\cos(c+dx+\text{ArcTan}(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-8\cos(c+dx)\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\,\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec(dx-\text{ArcTan}(\cot(c)))\sin(c)+2\cos(c+dx)(-6\cot(c)+\sin(c+dx))-6\cos(c)\csc(dx+\text{ArcTan}(\tan(c)))\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sqrt{\sec^2(c)}\,\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}\right)}{12d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((3*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 8*C
os[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometric
```

```
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]]*Sin[c] + 2*Cos[c + d*x]*(-6*Cot[c] + Sin[c + d*x]) - 6*Cos[c]*Csc[d*x +
ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan
[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[
c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(113) = 226$.
time = 0.08, size = 228, normalized size = 3.40

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.00, size = 134, normalized size = 2.00

$$\frac{2\left(a^2\sqrt{\cos(dx+c)}\sin(dx+c) - 2i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 2i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) + 3i\sqrt{2}\,a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 3i\sqrt{2}\,a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*sqrt(cos(d*x + c))*sin(d*x + c) - 2*I*sqrt(2)*a^2*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a^2*weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*s
qrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 0.88, size = 59, normalized size = 0.88

$$\frac{4\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{8\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (8*a^2*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

3.363 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=44

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4349, 3873, 3856, 2720, 4128}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*a^2*EllipticF[(c + d*x)/2, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
```

```
FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \left(2a\right.$$
$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(2a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.17, size = 39, normalized size = 0.89

$$\frac{2a^2\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*(2*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]/Sqrt[Cos[c + d*x]]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(68) = 136.
time = 0.09, size = 185, normalized size = 4.20

method	result
default	$-\frac{4a^2\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*a^2*(-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.90, size = 97, normalized size = 2.20

$$-\frac{2\left(i\sqrt{2}\,a^2\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-i\sqrt{2}\,a^2\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-a^2\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) - I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - a^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)dx+\int\sqrt{\cos\left(c+dx\right)}\sec^2\left(c+dx\right)dx+\int\sqrt{\cos\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c
+ d*x))*sec(c + d*x)**2, x) + Integral(sqrt(cos(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 1.21, size = 82, normalized size = 1.86

$$\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.364 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=91

$$-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3
/2)+4*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3853, 3856, 2719, 4131, 2720}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a^2*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\,dx \\
&= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,\left(a^2+a^2\sec^2(c+dx)\right)\,dx + \\
&= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{1}{3}\left(4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \\
&= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{1}{3}\left(4a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx-\left(2a^2\right)\int \\
&= -\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+}{d\sqrt{\cos(c+}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.15, size = 470, normalized size = 5.16

$$\cos^{\frac{3}{2}}(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{\csc(c)\sec(c)}{d}+\frac{\sec(c)\sec^2(c+dx)\sin(dx)}{6d}+\frac{\sec(c)\sec(c+dx)(\sin(c)+6\sin(dx))}{6d}\right)-\frac{2\cos^2(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sec(dx-\mathrm{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\,\sin(c)\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\mathrm{ArcTan}(\cot(c)))}}{3d\sqrt{1+\cot^2(c)}}+\frac{\cos^2(c+dx)\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Csc[c]*Sec
[c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*(Sin
[c] + 6*Sin[d*x]))/(6*d)) - (2*Cos[c + d*x]^2*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec
[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (Cos[c + d*x]^2*Csc[c]*Se
c[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - (
(Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(135) = 270$.

time = 0.11, size = 371, normalized size = 4.08

method	result
default	$-\frac{4a^2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*
c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-7*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 187, normalized size = 2.05

$$-\frac{2\left(2i\sqrt{2}\,a^2\cos\left(dx+c\right)^2\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-2i\sqrt{2}\,a^2\cos\left(dx+c\right)^2\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+3i\sqrt{2}\,a^2\cos\left(dx+c\right)^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3i\sqrt{2}\,a^2\cos\left(dx+c\right)^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-(6\,a^2\cos\left(dx+c\right)+a^2)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)\right)}{3\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)^2*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 - 3*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)*sqrt(
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{2\sec\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx+\int\frac{\sec^2\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx+\int\frac{1}{\sqrt{\cos\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(2*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(sec(c + d*x
)**2/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 1.27, size = 109, normalized size = 1.20

$$\frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*a^2*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*co
s(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.365 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*sin(d*x+c)/d/cos(d*x+c)
^(5/2)+4/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+16/5*a^2*sin(d*x+c)/d/cos(d*x+
c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3853, 3856, 2720, 4131, 2719}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)) + (16*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x
]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx + ($$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(2a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{3}(2a^2)\int \frac{1}{\sqrt{\cos(c}}$$

$$= \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c}{3d\cos^{\frac{3}{2}}(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.19, size = 503, normalized size = 4.16

$$\cos^{\frac{5}{2}}(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{4\csc(c)\sec(c)}{5d}+\frac{\sec(c)\sec^3(c+dx)\sin(dx)}{10d}+\frac{\sec(c)\sec^2(c+dx)(3\sin(c)+10\sin(dx))}{30d}+\frac{\sec(c)\sec(c+dx)(5\sin(c)+12\sin(dx))}{15d}\right)-\frac{\cos^2(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sec(dx-\mathrm{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\mathrm{ArcTan}(\cot(c)))}}{3d\sqrt{1+\cot^2(c)}}+\frac{2\cos^2(c+dx)\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((4*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c]*Sec[c + d*
x]^2*(3*Sin[c] + 10*Sin[d*x]))/(30*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 12
*Sin[d*x]))/(15*d)) - (Cos[c + d*x]^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x
])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (2*Cos[c + d*x]^2*Csc[c]*Sec[c/2
+ (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(157) = 314$.
time = 0.13, size = 386, normalized size = 3.19

method	result
default	$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\right.}}{12\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/12*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
```

```
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-4/5*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)-2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 202, normalized size = 1.67

$$-\frac{2\left(5i\sqrt{2}\,a^2\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-5i\sqrt{2}\,a^2\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+12i\sqrt{2}\,a^2\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-12i\sqrt{2}\,a^2\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\left(24\,a^2\cos\left(dx+c\right)^2+10\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)\right)}{15\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*I*sqrt(2)*a^2*cos(d*x + c)^3*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) - 12*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (24*a^2*cos(d*x + c)^2 + 10*a
^2*cos(d*x + c) + 3*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{2\sec\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{\sec^{2}\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{1}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(2*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(sec(c + d*
x)**2/cos(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**(-3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.38, size = 114, normalized size = 0.94

$$\frac{6\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+20\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+30\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{15\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

```
[Out] (6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*a^2*c
os(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30*a
^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))
/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.366 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$-\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)}+\frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^2*sin(d*x+c)/d/cos(d*x+c)
^(7/2)+4/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+8/7*a^2*sin(d*x+c)/d/cos(d*x+c
)^(3/2)+12/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3853, 3856, 2719, 4131, 2720}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}-\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Cos[c + d*x]^(5/2)) + (8*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(3/2
)) + (12*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx + \left(2\right.$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\left(6a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \mathrm{s}$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)} + \frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{7}$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)} + \frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{7}$$

$$= -\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d} + \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+}{5d\cos^{\frac{5}{2}}(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.23, size = 531, normalized size = 3.61

$$\cos^{\frac{5}{2}}(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{3\csc(c)\sec(c)}{5d}+\frac{\sec(c)\sec^4(c+dx)\sin(dx)}{14d}+\frac{\sec(c)\sec^2(c+dx)(7\sin(c)+10\sin(dx))}{35d}+\frac{\sec(c)\sec^3(c+dx)(5\sin(c)+14\sin(dx))}{70d}+\frac{\sec(c)\sec(c+dx)(10\sin(c)+21\sin(dx))}{35d}\right)-\frac{2\cos^2(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sec(dx-\mathrm{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\mathrm{ArcTan}(\cot(c)))}}{7d\sqrt{1+\cot^2(c)}}+\frac{3\cos^2(c+dx)\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((3*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c]*Sec[c + d*
x]^2*(7*Sin[c] + 10*Sin[d*x]))/(35*d) + (Sec[c]*Sec[c + d*x]^3*(5*Sin[c] +
14*Sin[d*x]))/(70*d) + (Sec[c]*Sec[c + d*x]*(10*Sin[c] + 21*Sin[d*x]))/(35*
d)) - (2*Cos[c + d*x]^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(7*d*Sqrt[1 + Cot[c]^2]) + (3*Cos[c + d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(
a + a*Sec[c + d*x])^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(179) = 358$.
time = 0.13, size = 439, normalized size = 2.99

method	result
default	$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}{224\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/224*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^4-1/14*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+31/70*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(
1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.13, size = 215, normalized size = 1.46

$$-\frac{2 \left(10 i \sqrt{2}\, a^{2} \cos \left(dx+c\right)^{4} \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)+i \sin \left(dx+c\right)\right)-10 i \sqrt{2}\, a^{2} \cos \left(dx+c\right)^{4} \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)-i \sin \left(dx+c\right)\right)+21 i \sqrt{2}\, a^{2} \cos \left(dx+c\right)^{4} \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)+i \sin \left(dx+c\right)\right)\right)-21 i \sqrt{2}\, a^{2} \cos \left(dx+c\right)^{4} \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)-i \sin \left(dx+c\right)\right)\right)-\left(42 a^{2} \cos \left(dx+c\right)^{3}+20 a^{2} \cos \left(dx+c\right)^{2}+14 a^{2} \cos \left(dx+c\right)+5 a^{2}\right) \sqrt{\cos \left(dx+c\right)} \sin \left(dx+c\right)\right)}{35 d \cos \left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/35*(10*I*sqrt(2)*a^2*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^2*cos(d*x + c)^4*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2)*a^2*cos(d*x + c)^4*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 21*I*sqrt(2)*a^2*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (42*a^2*cos(d*x + c)^3 + 20
*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 5*a^2)*sqrt(cos(d*x + c))*sin(d
*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)

Mupad [B]
time = 1.46, size = 114, normalized size = 0.78

$$\frac{30\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)+84\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)+70\,a^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{105\,d\,{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{1-{\cos\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^(5/2),x)

[Out] (30*a^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 84*a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 70*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))

3.367 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=147

$$\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{44a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{68a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}$$

```
[Out] 68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+44/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+68/45*a^3*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+6/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^3*cos(d*x+c)^(7/2)*s
in(d*x+c)/d+44/21*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3876, 3854, 3856, 2719, 2720}

$$\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{68a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{44a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (68*a^3*EllipticE[(c + d*x)/2, 2])/(15*d) + (44*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (44*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (68*a^3*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (6*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^3*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{7}{2}}(c+dx}\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx+\left(a^3\sqrt{\cos}\right.$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+$$

$$= \frac{44a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{68a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}$$

$$= \frac{18a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{44a^3\sqrt{\cos(c+dx)}}{21d}$$

$$= \frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{44a^3\sqrt{\cos(c+dx}}{21d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.12, size = 548, normalized size = 3.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((-17*Cot[c]
)/(30*d) + (97*Cos[d*x]*Sin[c])/(336*d) + (73*Cos[2*d*x]*Sin[2*c])/(720*d)
+ (3*Cos[3*d*x]*Sin[3*c])/(112*d) + (Cos[4*d*x]*Sin[4*c])/(288*d) + (97*Cos
[c]*Sin[d*x])/(336*d) + (73*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*Cos[3*c]*Sin[
3*d*x])/(112*d) + (Cos[4*c]*Sin[4*d*x])/(288*d)) - (11*Cos[c + d*x]^3*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) -
(17*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(60*d)
```

Maple [A]
time = 0.10, size = 260, normalized size = 1.77

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-600\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+212\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos
(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos
(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 175, normalized size = 1.19

$$-\frac{2\left(165i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-165i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-357i\sqrt{2}\,a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+357i\sqrt{2}\,a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\left(35\,a^3\cos(dx+c)^3+135\,a^3\cos(dx+c)^2+238\,a^3\cos(dx+c)+330\,a^3\right)\sqrt{\cos(dx+c)}\,\sin(dx+c)\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*a^3
*cos(d*x + c)^3 + 135*a^3*cos(d*x + c)^2 + 238*a^3*cos(d*x + c) + 330*a^3)*
sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

Mupad [B]
time = 1.15, size = 206, normalized size = 1.40

$$\frac{2\left(a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}-\frac{2\left(\frac{33\,a^3\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)}{\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{5\,a^3\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)}{\sqrt{\sin\left(c+d\,x\right)^2}}\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{77\,d}-\frac{2\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{104\,a^3\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{19}{4};\ \cos\left(c+d\,x\right)^2\right)}{385\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))
/(3*d) - (2*((33*a^3*cos(c + d*x)^(7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2
) - (5*a^3*cos(c + d*x)^(11/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2))*hyperg
eom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(77*d) - (2*a^3*cos(c + d*x)^(9/2)*
sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x
)^2)^(1/2)) - (104*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/
4], 19/4, cos(c + d*x)^2))/(385*d*(sin(c + d*x)^2)^(1/2))
```

3.368 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=121

$$\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{52a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{5d}$$

```
[Out] 28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+6/5*a^3*cos(d*x+c)^(3/2)*sin(d
*x+c)/d+2/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+52/21*a^3*sin(d*x+c)*cos(d*x+
c)^(1/2)/d
```

Rubi [A]

time = 0.17, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3876, 3854, 3856, 2720, 2719}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2, 2
])/(21*d) + (52*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (6*a^3*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(a^3\sqrt{\cos(\right.$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a}{}$$

$$= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{52a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{6a^3\cos^{\frac{3}{2}}(}{}$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{52a^3\sqrt{\cos(c+dx)}}{21d}$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{52a^3\sqrt{\cos(c+dx}}{21d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.11, size = 516, normalized size = 4.26

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((-7*Cot[c])
/(10*d) + (107*Cos[d*x]*Sin[c])/(336*d) + (3*Cos[2*d*x]*Sin[2*c])/(40*d) +
(Cos[3*d*x]*Sin[3*c])/(112*d) + (107*Cos[c]*Sin[d*x])/(336*d) + (3*Cos[2*c]
*Sin[2*d*x])/(40*d) + (Cos[3*c]*Sin[3*d*x])/(112*d)) - (13*Cos[c + d*x]^3*C
sc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec
[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]
) - (7*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((
HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[
c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]
))/(20*d)
```

Maple [A]
time = 0.11, size = 272, normalized size = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)+65*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.95, size = 162, normalized size = 1.34

$$-\frac{2\left(65i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-147i\sqrt{2}\,a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+147i\sqrt{2}\,a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\left(15\,a^3\cos(dx+c)^2+63\,a^3\cos(dx+c)+130\,a^3\right)\sqrt{\cos(dx+c)}\,\sin(dx+c)\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/105*(65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) - 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*I*sqrt(2)*a^3*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*a^3*c
os(d*x + c)^2 + 63*a^3*cos(d*x + c) + 130*a^3)*sqrt(cos(d*x + c))*sin(d*x +
 c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Mupad [B]
time = 1.03, size = 143, normalized size = 1.18

$$\frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}-\frac{6\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,a^3\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3
*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (6*a^3*cos(c + d*x)^(7/2)*sin(c + d*
x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)
) - (2*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(
c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.369 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d}+\frac{2a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

```
[Out] 36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.15, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3876, 3854, 3856, 2719, 2720}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2]
)/d + (2*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx$$
$$= \left(a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\left(a^3\sqrt{\cos$$
$$= \frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d}+\frac{2a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+a$$
$$= \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sqrt{\cos(c+dx)}\,\mathrm{s}}{d}$$
$$= \frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sqrt{\cos(c+dx)}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.96, size = 233, normalized size = 2.56

$$\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{9(3\cos(c-dx-\mathrm{ArcTan}(\tan(c)))+\cos(c+dx+\mathrm{ArcTan}(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-20\cos(c+dx)\sqrt{\cos^2(dx-\mathrm{ArcTan}(\cot(c)))}\,\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec(dx-\mathrm{ArcTan}(\cot(c)))\sin(c)+\cos(c+dx)(-36\cot(c)+10\sin(c+dx)+\sin(2(c+dx)))-18\cos(c)\csc(dx+\mathrm{ArcTan}(\tan(c)))\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sqrt{\sec^2(c)}\,\sqrt{\sin^2(dx+\mathrm{ArcTan}(\tan(c)))}\right)}{40d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((9*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*
Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c
]]]*Sin[c] + Cos[c + d*x]*(-36*Cot[c] + 10*Sin[c + d*x] + Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(40*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.10, size = 250, normalized size = 2.75

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(-4\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\cdots\right.}{5\sqrt{-2\,(\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 148, normalized size = 1.63

$$-\frac{2\left(5i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-9i\sqrt{2}\,a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+9i\sqrt{2}\,a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(a^3\cos(dx+c)+5a^3)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c)
 + 5*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Mupad [**B**]
time = 0.97, size = 104, normalized size = 1.14

$$\frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{d}-\frac{2\,a^3\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^3*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2)*
sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x
)^2)^(1/2))
```

3.370 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2
)+2/3*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3876, 3854, 3856, 2720, 2719, 3853}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\
&= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+d}}\right.\\
&= \left(a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\left(a^3\sqrt{\cos}\right.\\
&= \frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\left(3a^3\right)\int \frac{}{\sqrt{c}}\\
&= \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\\
&= \frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 4.91, size = 240, normalized size = 2.64

$$\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-3\cos(dx)\csc(c)-9\cos(2c+dx)\csc(c)+9\cos(c-dx-\mathrm{ArcTan}(\tan(c)))\cot(c)\sqrt{\sec^2(c)}+3\cos(c+dx+\mathrm{ArcTan}(\tan(c)))\cot(c)\sqrt{\sec^2(c)}-20\cos(c+dx)\sqrt{\cos^2(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec(dx-\mathrm{ArcTan}(\cot(c)))\sin(c)+\sin(2(c+dx))-6\cos(c)\csc(dx+\mathrm{ArcTan}(\tan(c)))\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\mathrm{ArcTan}(\tan(c)))}\right)}{24d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-3*Cos[d*x]*Csc[c] - 9*Cos[2*
c + d*x]*Csc[c] + 9*Cos[c - d*x - ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] + 3
*Cos[c + d*x + ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] - 20*Cos[c + d*x]*Sqrt
[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Sin[
2*(c + d*x)] - 6*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Ar
cTan[Tan[c]]]^2]))/(24*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.10, size = 172, normalized size = 1.89

method	result
default	$-\frac{4a^3\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{Ellip}}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 180, normalized size = 1.98

$$-\frac{2\left(5i\sqrt{2}\,a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(a^3\cos(dx+c)+3a^3)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c) + 3*a^3)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.02, size = 104, normalized size = 1.14

$$\frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{20\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (20*a^3*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c + d*
x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(
c + d*x)^2)^(1/2))
```

3.371 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=91

$$-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^3*sin(d*x+c)/d/cos(d*x+c)^(
3/2)+6*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3876, 3856, 2719, 2720, 3853}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (6*a^3*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sqrt{\sec(c+dx)}} + 3a^3\sqrt{\text{se}\cdots}\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(a^3\cdots\right.$$

$$= \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + a^3\int \sqrt{\cos(c+dx)}\,dx + \cdots$$

$$= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \cdots$$

$$= -\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^3\sin(c+dx\cdots}{3d\cos^{\frac{3}{2}}(c+dx\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.19, size = 479, normalized size = 5.26

$$\cos^{\frac{7}{2}}(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\left(-\frac{(-5+\cos(2c))\csc(c)\sec(c)}{8d}+\frac{\sec(c)\sec^2(c+dx)\sin(dx)}{12d}+\frac{\sec(c)\sec(c+dx)(\sin(c)+9\sin(dx))}{12d}\right)-\frac{5\cos^3(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sec(dx-\text{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\text{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\text{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\text{ArcTan}(\cot(c)))}}{6d\sqrt{1+\cot^2(c)}}+\frac{\cos^3(c+dx)\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sin(dx+\text{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\text{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\text{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\sin(dx+\text{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{\sqrt{\cos(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(-1/8*((-5 +
 Cos[2*c])*Csc[c]*Sec[c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*d) + (Se
c[c]*Sec[c + d*x]*(Sin[c] + 9*Sin[d*x]))/(12*d)) - (5*Cos[c + d*x]^3*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (C
os[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(135) = 270$.

time = 0.12, size = 371, normalized size = 4.08

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(18\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-10\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2
*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-10*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 187, normalized size = 2.05

$$-\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}a^3\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}a^3\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(9a^3\cos(dx+c)+a^3)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^3*cos(d*x + c)^2*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 - 3*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (9*a^3*cos(d*x + c) + a^3)*sqrt(
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sqrt{\cos(c+dx)}\sec(c+dx)\,dx+\int 3\sqrt{\cos(c+dx)}\sec^2(c+dx)\,dx+\int\sqrt{\cos(c+dx)}\sec^3(c+dx)\,dx+\int\sqrt{\cos(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(3*sqrt(cos(
c + d*x))*sec(c + d*x)**2, x) + Integral(sqrt(cos(c + d*x))*sec(c + d*x)**3
, x) + Integral(sqrt(cos(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 1.64, size = 126, normalized size = 1.38

$$\frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{6\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^3*ellipticF(c/2 + (d*x)/2, 2)))/d
 + (6*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/
2))
```

3.372 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=117

$$-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(
5/2)+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)+36/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(
1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3876, 3856, 2720, 3853, 2719}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2
])/d + (2*a^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a^3*Sin[c + d*x])
/(d*Cos[c + d*x]^(3/2)) + (36*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(a^3\sqrt{\sec(c+dx)} + 3a^3\sec^{\frac{3}{2}}(c+dx) + 3a^3$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,dx + \left(a^3\sqrt{\cos(c+dx)}\,\sqrt{}\right.$$

$$= \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + a^3\int \frac{1}{\sqrt{\cos(c+dx)}}\,d$$

$$= \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+}{d\cos^{\frac{3}{2}}(c+}$$

$$= -\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+}{d\cos^{\frac{3}{2}}(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.21, size = 501, normalized size = 4.28

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((9*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(20*d) + (Sec[c]*Sec[c + d
*x]^2*(Sin[c] + 5*Sin[d*x]))/(20*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 18*S
in[d*x]))/(20*d)) - (Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])
^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (9*Cos[c + d*x]^3*Csc[c]*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^3*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(157) = 314$.

time = 0.12, size = 386, normalized size = 3.30

method	result
default	$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{10\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 200, normalized size = 1.71

$$-\frac{2 \left(5 i \sqrt{2}\, a^{3} \cos \left(d x+c\right)^{3} \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(d x+c\right)+i \sin \left(d x+c\right)\right)-5 i \sqrt{2}\, a^{3} \cos \left(d x+c\right)^{3} \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(d x+c\right)-i \sin \left(d x+c\right)\right)+9 i \sqrt{2}\, a^{3} \cos \left(d x+c\right)^{3} \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(d x+c\right)+i \sin \left(d x+c\right)\right)\right)-9 i \sqrt{2}\, a^{3} \cos \left(d x+c\right)^{3} \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(d x+c\right)-i \sin \left(d x+c\right)\right)\right)-\left(18 a^{3} \cos \left(d x+c\right)^{2}+5 a^{3} \cos \left(d x+c\right)+a^{3}\right) \sqrt{\cos \left(d x+c\right)} \sin \left(d x+c\right)\right)}{5 d \cos \left(d x+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(5*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a^3*cos(d*x + c)^3*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 - 9*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (18*a^3*cos(d*x + c)^2 + 5*a^3*c
os(d*x + c) + a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3} \left(\int \frac{3 \sec \left(c+d x\right)}{\sqrt{\cos \left(c+d x\right)}}\, dx+\int \frac{3 \sec ^{2}\left(c+d x\right)}{\sqrt{\cos \left(c+d x\right)}}\, dx+\int \frac{\sec ^{3}\left(c+d x\right)}{\sqrt{\cos \left(c+d x\right)}}\, dx+\int \frac{1}{\sqrt{\cos \left(c+d x\right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(3*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(3*sec(c + d
*x)**2/sqrt(cos(c + d*x)), x) + Integral(sec(c + d*x)**3/sqrt(cos(c + d*x))
, x) + Integral(1/sqrt(cos(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Mupad [B]

time = 1.58, size = 154, normalized size = 1.32

$$\frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^3*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([-5/
4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1
/2))
```

3.373 $\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{28a^3}{5d\sqrt{c}}$$

```
[Out] -28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^3*sin(d*x+c)/d/cos(d*x+
c)^(7/2)+6/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)+52/21*a^3*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+28/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3876, 3853, 3856, 2719, 2720}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (52*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^
(3/2)) + (28*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx \\
&= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \left(a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\right. \\
&= \left(a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx+\left(a^3\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{s}}\right. \\
&= \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{1}{7} \\
&= \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+ \\
&= -\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+}{5d\cos^{\frac{5}{2}}(c+} \\
&= -\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(}{5d\cos^{\frac{5}{2}}(}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.24, size = 531, normalized size = 3.61

$$\cos^{\frac{7}{2}}(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\left(\frac{7\csc(c)\sec(c)}{10d}+\frac{\sec(c)\sec^4(c+dx)\sin(dx)}{28d}+\frac{\sec(c)\sec^3(c+dx)(5\sin(c)+21\sin(dx))}{140d}+\frac{\sec(c)\sec^2(c+dx)(63\sin(c)+130\sin(dx))}{420d}+\frac{\sec(c)\sec(c+dx)(65\sin(c)+147\sin(dx))}{210d}\right)-\frac{13\cos^3(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sec(dx-\mathrm{ArcTan}(\cot(c)))\sqrt{1-\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\mathrm{ArcTan}(\cot(c)))}\sqrt{1+\sin(dx-\mathrm{ArcTan}(\cot(c)))}}{84d\sqrt{1+\cot^2(c)}}+\frac{7\cos^3(c+dx)\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{ArcTan}(\tan(c)))\right)\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\mathrm{ArcTan}(\tan(c)))}\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\frac{\frac{\sin(dx+\mathrm{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1+\tan^2(c)}}+\frac{2\cos^2(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}{\cos^2(c)+\sin^2(c)}}{\sqrt{\cos(c)\cos(dx+\mathrm{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((7*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Sec[c]*Sec[c + d
*x]^3*(5*Sin[c] + 21*Sin[d*x]))/(140*d) + (Sec[c]*Sec[c + d*x]^2*(63*Sin[c]
 + 130*Sin[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(65*Sin[c] + 147*Sin[d*x])
)/(210*d)) - (13*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) + (7*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*
x)/2]^6*(a + a*Sec[c + d*x])^3*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(179) = 358$.

time = 0.14, size = 439, normalized size = 2.99

method	result
default	$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\right.}}{448\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-1/448*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^4-13/168*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+53/105*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/160*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^3-7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-
2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-7/20*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(c
```

```
os(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 215, normalized size = 1.46

$$-\frac{2\left(65i\sqrt{2}\,a^3\cos\left(dx+c\right)^4\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-65i\sqrt{2}\,a^3\cos\left(dx+c\right)^4\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+147i\sqrt{2}\,a^3\cos\left(dx+c\right)^4\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-147i\sqrt{2}\,a^3\cos\left(dx+c\right)^4\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-\left(294\,a^3\cos\left(dx+c\right)^3+130\,a^3\cos\left(dx+c\right)^2+63\,a^3\cos\left(dx+c\right)+15\,a^3\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)\right)}{105\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/105*(65*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 65*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a^3*cos(d*x + c)^4*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 147*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (294*a^3*cos(d*x + c)^3
+ 130*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 15*a^3)*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int\frac{3\sec\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{3\sec^{2}\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{\sec^{3}\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{1}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] a**3*(Integral(3*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(3*sec(c +
d*x)**2/cos(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**3/cos(c + d*x)**(3
/2), x) + Integral(cos(c + d*x)**(-3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.64, size = 145, normalized size = 0.99

$$\frac{\frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)}{7}+\frac{6\,a^3\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)}{5}+2\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+2\,a^3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

```
[Out] ((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*a
^3*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/
5 + 2*a^3*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d
*x)^2) + 2*a^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(
c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.374 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=128

$$\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad}+\frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad}-\frac{\cos^{\frac{3}{2}}(c-}{d(a+}$$

```
[Out] 21/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+7/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d-
cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))-5/3*sin(d*x+c)*cos(d*x+c)^(1
/2)/a/d
```

Rubi [A]
time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3904, 3872, 3854, 3856, 2719, 2720}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (21*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + (7*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(5*a*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c +
d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \cdots$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+\cdots}{d(a+a\sec(c+d\cdots}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+\cdots}{d(a+a\sec(c+d\cdots}$$

$$= \frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.69, size = 314, normalized size = 2.45

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}\,e^{-i(c+dx)}\left(63\left(1+e^{2i(c+dx)}\right)+63\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+25e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}+\frac{-96\cot(c)-30\csc(c)-20\cos(dx)\sin(c)+6\cos(2dx)\sin(2c)-30\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)-20\cos(c)\sin(dx)+6\cos(2c)\sin(2dx)}{d\sqrt{\cos(c+dx)}}\right)}{15a(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
)*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c
 + d*x)))/E^(I*(c + d*x))]) + (-96*Cot[c] - 30*Csc[c] - 20*Cos[d*x]*Sin[c]
+ 6*Cos[2*d*x]*Sin[2*c] - 30*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 20*Co
s[c]*Sin[d*x] + 6*Cos[2*c]*Sin[2*d*x])/(d*Sqrt[Cos[c + d*x]])))/(15*a*(1 +
Sec[c + d*x]))
```

Maple [A]
time = 0.09, size = 229, normalized size = 1.79

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)^4
+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.00, size = 208, normalized size = 1.62

$$\frac{2\left(6\cos\left(dx+c\right)^{2}-4\cos\left(dx+c\right)-25\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-25\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-63\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-63\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(2*(6*cos(d*x + c)^2 - 4*cos(d*x + c) - 25)*sqrt(cos(d*x + c))*sin(d*x
 + c) - 25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) - 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*(-I*sqrt(2)*c
os(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{a+\frac{a}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x)), x)
```

3.375 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=100

$$-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad}-\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] -3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d-si
n(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))
```

Rubi [A]

time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3904, 3872, 3854, 3856, 2720, 2719}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d
) + (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) - (Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}}{a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}}{2a}$$

$$= \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int\sqrt{\cos(c+dx)}\,dx}{2a}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c}{d(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.12, size = 292, normalized size = 2.92

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}+\frac{12\cot(c)+6\csc(c)+4\cos(dx)\sin(c)+6\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+4\cos(c)\sin(dx)}{d\sqrt{\cos(c+dx)}}\right)}{3a(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*
Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c +
 d*x)))/E^(I*(c + d*x))]) + (12*Cot[c] + 6*Csc[c] + 4*Cos[d*x]*Sin[c] + 6*S
ec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 4*Cos[c]*Sin[d*x])/(d*Sqrt[Cos[c +
d*x]])))/(3*a*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.08, size = 215, normalized size = 2.15

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-
8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 198, normalized size = 1.98

$$\frac{2\left(2\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-5\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))-9\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-9\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(I*sqrt(2)*
cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 9*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c)
+ a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}}{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x)), x)
```

3.376 $\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}$$

```
[Out] 3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))/cos(d*x+c)^(1
/2)
```

Rubi [A]

time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3904, 3872, 3856, 2719, 2720}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) - Sin
[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx \\
&= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3a}{2}+\frac{1}{2}a\ldots}{\sqrt{\sec(\ldots}}}{a^2} \\
&= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(\ldots}}{2a} \\
&= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{3\int \sqrt{\cos(c+dx)}\ldots}{2a} \\
&= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.70, size = 270, normalized size = 3.75

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}\,e^{-i(c+dx)}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}} - \frac{2\left(2\cot(c)+\csc(c)+\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)\right)}{d\sqrt{\cos(c+dx)}}\right)}{a(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec
[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*
x)))/E^(I*(c + d*x))]) - (2*(2*Cot[c] + Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*
Sin[(d*x)/2]))/(d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.09, size = 199, normalized size = 2.76

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\mathrm{E}\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 186, normalized size = 2.58

$$\frac{\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))-3\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-3\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))-2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x + c
) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*s
qrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\cos (c+dx)}}{\sec (c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos \left(c+d\,x\right)}}{a+\frac{a}{\cos \left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x)), x)
```

3.377 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=70

$$-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))/a/d+sin(d*x+c)/d/(a+a*sec(d*x+c))/cos(d*x+c)^(1/
2)
```

Rubi [A]
time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3905, 3872, 3856, 2719, 2720}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a*d)) + EllipticF[(c + d*x)/2, 2]/(a*d) + Sin[
c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Dist[d*((n - 1)/(a*b)), Int[(d*Csc[e + f*x])^(
n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[
a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$$

$$= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{2a}$$

$$= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{2a}$$

$$= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} - \frac{\int \sqrt{}}{}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{se}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.06, size = 262, normalized size = 3.74

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\left(1+e^{2i(c+dx)}+\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}+\frac{2\left(\csc(c)+\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)\right)}{d\sqrt{\cos(c+dx)}}\right)}{a(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^
((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c +
d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/
E^(I*(c + d*x))]) + (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]))/(
d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.09, size = 198, normalized size = 2.83

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{a\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{si}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.30, size = 184, normalized size = 2.63

$$\frac{\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c)
- I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(
cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))), x)
```

3.378 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=70

$$\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2
)
```

Rubi [A]

time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.261$, Rules used = {4349, 3903, 3872, 3856, 2719, 2720}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a*d) + EllipticF[(c + d*x)/2, 2]/(a*d) - Sin[c +
 d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\ldots\right.}}{a^{\ldots}}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\ldots\right.}}{2\ldots}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{\int \ldots}{\ldots}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.06, size = 263, normalized size = 3.76

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}\,e^{-i(c+dx)}\left(1+e^{2i(c+dx)}+\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}-\frac{2\left(\csc(c)+\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)\right)}{d\sqrt{\cos(c+dx)}}\right)}{a(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(
(2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d
*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E
^(I*(c + d*x))]) - (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]))/(d
*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.09, size = 200, normalized size = 2.86

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 184, normalized size = 2.63

$$\frac{\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) +
 I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*sqrt(
cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sec(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(cos(c + d*x)**(3/2)*sec(c + d*x) + cos(c + d*x)**(3/2)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))), x)
```

3.379 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=96

$$-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

```
[Out] -3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x
+c))+3*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3903, 3872, 3856, 2720, 3853, 2719}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) + (3
*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(3/2
)*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.}{2a}$$

$$= \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c}}}{2}$$

$$= -\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{}{d\cos}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.73, size = 303, normalized size = 3.16

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\left(2\cos\left(\frac{1}{2}(c-dx)\right)+\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)}{2d\cos^{\frac{3}{2}}(c+dx)}-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right)}{a(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*Cos[(c - d*x)/2] + Cos[(3*c + d*x)/2] + 3*Cos[(c +
 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(2*d*Cos[c + d*x]^(3/2)) -
((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E^(I*(c
 + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]
)))/(a*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.10, size = 253, normalized size = 2.64

method	result
default	$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\text{Ellip}\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/cos
(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 236, normalized size = 2.46

$$\frac{2\,(3\,\cos(dx+c)+2)\sqrt{\cos(dx+c)}\,\sin(dx+c)+\left(i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(-i\sqrt{2}\cos(dx+c)^2-i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\left(i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\left(-i\sqrt{2}\cos(dx+c)^2-i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + (I*sqrt(2)*co
s(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c)
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*
cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c
)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c
))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{5/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))), x)
```

3.380 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=124

$$\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+$$

```
[Out] 3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-sin
(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))-3*sin(d*x+c)/a/d/cos(d*x+c)^(1/
2)
```

Rubi [A]
time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3903, 3872, 3853, 3856, 2719, 2720}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d)
 + (5*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (3*Sin[c + d*x])/(a*d*Sqrt
[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{2a}$$

$$= \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c}$$

$$= \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c}$$

$$= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\,\mathrm{si}}{ad\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.68, size = 338, normalized size = 2.73

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(10\cos(\frac{1}{2}(c-dx))+8\cos(\frac{1}{2}(3c+dx))+4\cos(\frac{1}{2}(c+3dx))+5\cos(\frac{1}{2}(5c+3dx))+9\cos(\frac{1}{2}(3c+5dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec(\frac{1}{2}(c+dx))}{4d\cos^{\frac{5}{2}}(c+dx)}+\frac{2i\sqrt{2}\,e^{-i(c+dx)}\left(9(1+e^{2i(c+dx)})+9(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)})-5e^{i(c+dx)}(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)})\right)\sec(c+dx)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right)}{3a(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(-1/4*((10*Cos[(c - d*x)/2] + 8*Cos[(3*c + d*x)/2] + 4*
Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Csc[c
/2]*Sec[c/2]*Sec[(c + d*x)/2])/(d*Cos[c + d*x]^(5/2)) + ((2*I)*Sqrt[2]*(9*(
1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*
x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4,
 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^
((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a*(1 + Sec
[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(166) = 332.
time = 0.13, size = 413, normalized size = 3.33

method	result
default	$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(10\cos\left(\frac{dx}{2}+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/sin(1/2*d*x
+1/2*c)^3/cos(1/2*d*x+1/2*c)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*cos(1/2*d*x+1/
2*c)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-18*cos(1/2*d*x+1/2*c)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*cos(1
/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+9*cos(1/2*d*x+1/2*c)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.03, size = 258, normalized size = 2.08

$$-\frac{2 \left(9 \cos \left(dx+c\right)^{2}+4 \cos \left(dx+c\right)-2\right) \sqrt{\cos \left(dx+c\right)}\, \sin \left(dx+c\right)+5 \left(i \sqrt{2}\, \cos \left(dx+c\right)^{3}+i \sqrt{2}\, \cos \left(dx+c\right)^{2}\right) \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)+i \sin \left(dx+c\right)\right)+5 \left(-i \sqrt{2}\, \cos \left(dx+c\right)^{3}-i \sqrt{2}\, \cos \left(dx+c\right)^{2}\right) \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)-i \sin \left(dx+c\right)\right)+9 \left(-i \sqrt{2}\, \cos \left(dx+c\right)^{3}-i \sqrt{2}\, \cos \left(dx+c\right)^{2}\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)+i \sin \left(dx+c\right)\right)\right)+9 \left(i \sqrt{2}\, \cos \left(dx+c\right)^{3}+i \sqrt{2}\, \cos \left(dx+c\right)^{2}\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)-i \sin \left(dx+c\right)\right)\right)}{6 \left(ad \cos \left(dx+c\right)^{3}+ad \cos \left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(9*cos(d*x + c)^2 + 4*cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
+ c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^
3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
))) + 9*(I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*
cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))), x)
```

3.381 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=160

$$\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d}+\frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d}-\frac{3\cos^{\frac{3}{2}}(c}{a^2d(1}$$

```
[Out] 56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+56/15*cos(d*x+c)^(3/2)*sin(d*x+c)/
a^2/d-3*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)^(3/
2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-5*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3902, 4105, 3872, 3854, 3856, 2719, 2720}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{56\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d}-\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (56*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(a
^2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d) + (56*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(15*a^2*d) - (3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a^2*d*(1
+ Sec[c + d*x])) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*
x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{11a}{2}+\frac{7}{2}a\sec(c}{\sec^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{se}}}{3a^2}$$

$$= -\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{s}}\right.}{}$$

$$= -\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(15\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{a^2d(1+\sec(c}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{a^2d(1+\sec(c}$$

$$= \frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d} + 56\,\mathrm{co}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.39, size = 366, normalized size = 2.29

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{4i\sqrt{2}e^{-i(c+dx)}\left(56(1+e^{2i(c+dx)})+56(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+25e^{i(c+dx)}(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^2(c+dx)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}+\frac{2\left(-216\cot(c)-120\csc(c)-40\cos(dx)\sin(c)+6\cos(2dx)\sin(2c)-120\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+5\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)-40\cos(c)\sin(dx)+6\cos(2c)\sin(2dx)+5\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)}{5a^2(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(56*(1 + E^((2*I)*(c + d*x))) + 56*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
)*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*
(c + d*x)))/E^(I*(c + d*x))]) + (2*(-216*Cot[c] - 120*Csc[c] - 40*Cos[d*x]*
Sin[c] + 6*Cos[2*d*x]*Sin[2*c] - 120*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]
 + 5*Sec[c/2]*Sec[(c + d*x)/2]^3*Sin[(d*x)/2] - 40*Cos[c]*Sin[d*x] + 6*Cos[
2*c]*Sin[2*d*x] + 5*Sec[(c + d*x)/2]^2*Tan[c/2]))/(3*d*Cos[c + d*x]^(3/2)))
)/(5*a^2*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 283, normalized size = 1.77

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(96\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-150\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*d
*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a^2/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 288, normalized size = 1.80

$$\frac{2\left(6\cos(dx+c)^3-8\cos(dx+c)^2-94\cos(dx+c)-75\right)\sqrt{\cos(dx+c)}\sin(dx+c)-75\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-75\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-168\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-168\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{30\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(2*(6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 94*cos(d*x + c) - 75)*sqrt(c
os(d*x + c))*sin(d*x + c) - 75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos
(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 75*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2)
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 168*(-I*sqrt(
2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 168*(I*s
qrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZe
```

```
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*
d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^2,x)
[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^2, x)
```

3.382 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{10\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d}-\frac{7\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sqrt{c\cdots}}{3\cdots}$$

```
[Out] -7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*sin(d*x+c)*cos(d*x+c)^(1/2)/
a^2/d-7/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*x+c)*c
os(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3902, 4105, 3872, 3854, 3856, 2720, 2719}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*
a^2*d) + (10*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) - (7*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{9a}{2}+\frac{5}{2}a\,\mathrm{s}}{\sec^{\frac{3}{2}}(c+dx)(a}}{3a^2}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(7\sqrt{\cos(c+d}\right.}{}$$

$$= \frac{10\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} - \frac{7\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+dx)}}{3d(a+a\sec}$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} - \frac{7\sqrt{\cos(c+dx)}\,\sin(c-}{3a^2d(1+\sec(c+dx}$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} - 7$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.67, size = 341, normalized size = 2.47

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4i\sqrt{2}\,e^{-i(c+dx)}\left(21\left(1+e^{2i(c+dx)}\right)+21\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+10e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^2(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}+\frac{48\cot(c)+36\csc(c)+8\cos(dx)\sin(c)+36\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)-2\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+8\cos(c)\sin(dx)-2\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)}{d\cos^{\frac{3}{2}}(c+dx)}\right)}{3a^2(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 10*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]) + (48*Cot[c] + 36*Csc[c] + 8*Cos[d*x]*Sin[c]
 + 36*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 2*Sec[c/2]*Sec[(c + d*x)/2]^
3*Sin[(d*x)/2] + 8*Cos[c]*Sin[d*x] - 2*Sec[(c + d*x)/2]^2*Tan[c/2])/(d*Cos[
c + d*x]^(3/2))))/(3*a^2*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 270, normalized size = 1.96

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(16\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-}\right.}{6a^2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 278, normalized size = 2.01

$$\frac{2\left(2\cos(dx+c)^2+13\cos(dx+c)+10\right)\sqrt{\cos(dx+c)}\sin(dx+c)-10\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(2*(2*cos(d*x + c)^2 + 13*cos(d*x + c) + 10)*sqrt(cos(d*x + c))*sin(d*x + c) - 10*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 10*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassP

```
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^2, x)
```

3.383 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}$$

```
[Out] 4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*sin(d*x+c)/a^2/d/(1+sec(d*x+c))
/cos(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3902, 4105, 3872, 3856, 2719, 2720}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a^
2*d) - (5*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - S
in[c + d*x]/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int -}{3a^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}$$

$$= \frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.18, size = 374, normalized size = 3.34

$$\frac{4i\sqrt{2}\,e^{-i(c+dx)}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\left(1+e^{2i(c+dx)}\right)+12\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^2(c+dx)}{3d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\,(a+a\sec(c+dx))^2} + \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{8\cot\left(\frac{c}{2}\right)}{d}-\frac{8\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}+\frac{2\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{3d}+\frac{2\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\tan\left(\frac{c}{2}\right)}{3d}\right)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (((4*I)/3)*Sqrt[2]*Cos[c/2 + (d*x)/2]^4*(12*(1 + E^((2*I)*(c + d*x))) + 12*
(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/
2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*
I)*(c + d*x)))/E^(I*(c + d*x))]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/
2]^4*((-8*Cot[c/2])/d - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d + (2
*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(3*d) + (2*Sec[c/2 + (d*x)/2]^
2*Tan[c/2])/(3*d)))/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 257, normalized size = 2.29

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{6a^2\sqrt{-2\left(\sin\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*x
+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d*x+1
/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.26, size = 268, normalized size = 2.39

$$-\frac{2\,(6\,\cos(dx+c)+5)\sqrt{\cos(dx+c)}\,\sin(dx+c)+5\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+12\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+12\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(6*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(-I*sqrt(2
)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*cos(d*x + c)^2 + 2*I
*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c
) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x
+ c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^
2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\cos(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^2, x)
```

3.384 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a
+a*sec(d*x+c))^2+sin(d*x+c)/a^2/d/(1+sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3902, 4104, 3872, 3856, 2719, 2720}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a^2*d)) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2
*d) + Sin[c + d*x]/(a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c +
d*x]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.}{}$$

$$= \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sin(c+}{a^2d\sqrt{\cos(c+dx)}\,(1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.18, size = 656, normalized size = 6.02

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((-1/2*I)*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*((2*E^((2*I
)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*
Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d
*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Sec[c
 + d*x])^2 - (4*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*((4*Csc
```

```
[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (2*Sec[c/2]*Sec[c
/2 + (d*x)/2]^3*Sin[(d*x)/2])/(3*d) - (2*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*
d)))/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 257, normalized size = 2.36

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{6a^{2}\sqrt{-2\left(\sin^{4}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x+1/2
*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 268, normalized size = 2.46

$$\frac{2\left(3\cos\left(dx+c\right)+2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-2\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*(I*sqrt(2)*
cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 2*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*
sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) -
```

```
 I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) +
 I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c
) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sec^2(c+dx)+2\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x)**2 + 2*sqrt(cos(c + d*x))*sec(c
 + d*x) + sqrt(cos(c + d*x))), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2), x)
```

3.385 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

```
[Out] 1/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/a^2/d+1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4349, 3900, 21, 3856, 2720}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] EllipticF[(c + d*x)/2, 2]/(3*a^2*d) + Sin[c + d*x]/(3*d*Cos[c + d*x]^(3/2)*
(a + a*Sec[c + d*x])^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3900

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
```

```
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\ldots\right.}{\ldots}$$

$$= \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\ldots\right.}{6\ldots}$$

$$= \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{6a^2}$$

$$= \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

Mathematica [**A**]
time = 0.23, size = 63, normalized size = 1.11

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(77) = 154$.
time = 0.10, size = 188, normalized size = 3.30

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticF}\left(\right.\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.15, size = 150, normalized size = 2.63

$$\frac{\left(-i\sqrt{2}\cos\left(dx+c\right)^2-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{6\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*((-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x
 + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{3}{2}}\left(c+dx\right)\sec^2\left(c+dx\right)+2\cos^{\frac{3}{2}}\left(c+dx\right)\sec\left(c+dx\right)+\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(3/2)*sec(c + d*x)**2 + 2*cos(c + d*x)**(3/2)*sec
(c + d*x) + cos(c + d*x)**(3/2)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2), x)
```

3.386 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+
a*sec(d*x+c))^2-sin(d*x+c)/a^2/d/(1+sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3901, 4104, 3872, 3856, 2719, 2720}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2*d)
 - Sin[c + d*x]/(a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x
]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= -\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= -\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^2d} - \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\mathrm{sec}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.98, size = 312, normalized size = 2.86

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\left(7\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{4i\sqrt{2}\,e^{-i(c+dx)}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-2e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^2(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right)}{3a^2(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(-1/2*((7*Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 3*C
os[(c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(d*Cos[c + d*x]^(3
/2)) + ((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*S
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))] - 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^2)
/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*
(c + d*x))])))/(3*a^2*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 257, normalized size = 2.36

method	result

default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 268, normalized size = 2.46

$$-\frac{2\,(3\cos(dx+c)+4)\sqrt{\cos(dx+c)}\sin(dx+c)+2\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+2\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*cos(d*x + c) + 4)*sqrt(cos(d*x + c))*sin(d*x + c) + 2*(I*sqrt(2)
*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I
*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c)
 - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x +
c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2), x)
```

3.387 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))}-\frac{}{3d\cos^{\frac{5}{2}}}$$

```
[Out] -4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(3
/2)/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2+4*s
in(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4349, 3901, 4104, 3872, 3856, 2720, 3853, 2719}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (4*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - (5*Sin[c + d*x])/(3*a
^2*d*Cos[c + d*x]^(3/2)*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x
]^(5/2)*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{}{3d\,\mathrm{co}}$$

$$= -\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1}$$

$$= -\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.25, size = 393, normalized size = 2.89

$$-\frac{4i\sqrt{2}e^{-i(c+dx)}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\left(1+e^{2i(c+dx)}\right)+12\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^2(c+dx)}{3d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\,(a+a\sec(c+dx))^2}+\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{8\cot\left(\frac{c}{2}\right)\sec(c)}{d}+\frac{8\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}+\frac{2\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{3d}+\frac{8\sec(c)\sec(c+dx)\sin(dx)}{d}+\frac{2\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\tan\left(\frac{c}{2}\right)}{3d}\right)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (((-4*I)/3)*Sqrt[2]*Cos[c/2 + (d*x)/2]^4*(12*(1 + E^((2*I)*(c + d*x))) + 12
*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1
/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2
*I)*(c + d*x)))/E^(I*(c + d*x))]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)
/2]^4*((8*Cot[c/2]*Sec[c])/d + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])
/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(3*d) + (8*Sec[c]*Sec[c
 + d*x]*Sin[d*x])/d + (2*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Cos[c + d*
x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(176) = 352$.
time = 0.12, size = 405, normalized size = 2.98

method	result
default	$-\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(12\,\mathrm{EllipticE}\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(12*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(12*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*cos
(1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin
(1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 318, normalized size = 2.34

$$\frac{2\left(12\cos\left(dx+c\right)^2+19\cos\left(dx+c\right)+6\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(-i\sqrt{2}\cos\left(dx+c\right)^3-2i\sqrt{2}\cos\left(dx+c\right)^2-i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-5\left(i\sqrt{2}\cos\left(dx+c\right)^3+2i\sqrt{2}\cos\left(dx+c\right)^2+i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))-12\left(i\sqrt{2}\cos\left(dx+c\right)^3+2i\sqrt{2}\cos\left(dx+c\right)^2+i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-12\left(-i\sqrt{2}\cos\left(dx+c\right)^3-2i\sqrt{2}\cos\left(dx+c\right)^2-i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))}{6\left(a^2d\cos\left(dx+c\right)^3+2\,a^2d\cos\left(dx+c\right)^2+a^2d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(12*cos(d*x + c)^2 + 19*cos(d*x + c) + 6)*sqrt(cos(d*x + c))*sin(d*x
 + c) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(
2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
- 5*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(
```

```
d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 12*(I
*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x +
c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 12*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I
*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)
^2 + a^2*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2), x)
```

3.388 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(}$$

```
[Out] 7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(
3/2)-7/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(5/2)/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/
cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2-7*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3901, 4104, 3872, 3853, 3856, 2719, 2720}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) - (7*Sin[c + d*x])/(
a^2*d*Sqrt[Cos[c + d*x]]) - (7*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(5/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x
])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(}$$

$$= \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+s}$$

$$= \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+s}$$

$$= \frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{}{a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.21, size = 372, normalized size = 2.30

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(82\cos(\frac{1}{2}(c-dx))+65\cos(\frac{1}{2}(3c+dx))+68\cos(\frac{1}{2}(c+3dx))+37\cos(\frac{1}{2}(5c+3dx))+53\cos(\frac{1}{2}(3c+5dx))+10\cos(\frac{1}{2}(7c+5dx))+21\cos(\frac{1}{2}(5c+7dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec^3(\frac{1}{2}(c+dx))}{8d\cos^{\frac{7}{2}}(c+dx)} + \frac{4i\sqrt{2}\,e^{-i(c+dx)}\left(21(1+e^{2i(c+dx)})+21(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)})-10e^{i(c+dx)}(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)})\right)\sec^2(c+dx)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right)}{3a^2(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(-1/8*((82*Cos[(c - d*x)/2] + 65*Cos[(3*c + d*x)/2] + 6
8*Cos[(c + 3*d*x)/2] + 37*Cos[(5*c + 3*d*x)/2] + 53*Cos[(3*c + 5*d*x)/2] +
10*Cos[(7*c + 5*d*x)/2] + 21*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c
 + d*x)/2]^3)/(d*Cos[c + d*x]^(7/2)) + ((4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c
+ d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 10*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^
((2*I)*(c + d*x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*S
qrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a^2*(1 + Sec[c + d*x])
^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(198) = 396$.
time = 0.16, size = 413, normalized size = 2.55

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}+14\sqrt{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(6*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)+14*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3-22/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2
)-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 338, normalized size = 2.09

$$-\frac{2\left(21\cos\left(dx+c\right)^{3}+32\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+10\left(i\sqrt{2}\cos\left(dx+c\right)^{4}+2i\sqrt{2}\cos\left(dx+c\right)^{3}+i\sqrt{2}\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+10\left(-i\sqrt{2}\cos\left(dx+c\right)^{4}-2i\sqrt{2}\cos\left(dx+c\right)^{3}-i\sqrt{2}\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+21\left(-i\sqrt{2}\cos\left(dx+c\right)^{4}-2i\sqrt{2}\cos\left(dx+c\right)^{3}-i\sqrt{2}\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+21\left(i\sqrt{2}\cos\left(dx+c\right)^{4}+2i\sqrt{2}\cos\left(dx+c\right)^{3}+i\sqrt{2}\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{4}+2a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(21*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 8*cos(d*x + c) - 2)*sqrt(c
os(d*x + c))*sin(d*x + c) + 10*(I*sqrt(2)*cos(d*x + c)^4 + 2*I*sqrt(2)*cos(
d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 10*(-I*sqrt(2)*cos(d*x + c)^4 - 2*I*sqrt(2)*cos(d*x
 + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 21*(-I*sqrt(2)*cos(d*x + c)^4 - 2*I*sqrt(2)*cos(d*x +
c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c)^4 + 2*
I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x
 + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{9/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2), x)
```

3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{21\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d}+\frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d}-\frac{\cos}{5a}$$

```
[Out] 231/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d-21/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+77/10*cos(d*x+c)^(3/2)*sin(d*
x+c)/a^3/d-1/5*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/5*cos(d*x
+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-63/10*cos(d*x+c)^(3/2)*sin(d*x+
c)/d/(a^3+a^3*sec(d*x+c))-21/2*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d
```

Rubi [A]

time = 0.30, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3902, 4105, 3872, 3854, 3856, 2719, 2720}

$$-\frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{77\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10a^3d}-\frac{21\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{63\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10d\left(a^3\sec(c+dx)+a^3\right)}-\frac{4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (231*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (21*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) - (21*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) + (77*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(10*a^3*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d
*(a + a*Sec[c + d*x])^3) - (4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a +
a*Sec[c + d*x])^2) - (63*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{15a}{2}+\frac{9}{2}a\sec}{\sec^{\frac{5}{2}}(c+dx)(a+a}}{5a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}}{}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{63\cos^{\frac{3}{2}}(c+dx)\sin}{10d\left(a^3+a^3\sec(c\right.}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{63\cos^{\frac{3}{2}}(c+dx)\sin}{10d\left(a^3+a^3\sec(c\right.}$$

$$= -\frac{21\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{3}{2}}(c+dx)}{5d(a+a\sec}$$

$$= -\frac{21\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{3}{2}}(c+dx)}{5d(a+a\sec}$$

$$= \frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{21\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d} +$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.75, size = 391, normalized size = 1.89

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{42i\sqrt{2}\,e^{-i(c+dx)}\left(11\left(1+e^{2i(c+dx)}\right)+11\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+5e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^3(c+dx)}{\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}+\frac{-264\cot(c)-198\csc(c)+\frac{1}{16}\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-1210\sin\left(\frac{dx}{2}\right)+770\sin\left(c+\frac{dx}{2}\right)-840\sin\left(c+\frac{3dx}{2}\right)+150\sin\left(2c+\frac{3dx}{2}\right)-238\sin\left(2c+\frac{5dx}{2}\right)-40\sin\left(3c+\frac{5dx}{2}\right)-5\sin\left(3c+\frac{7dx}{2}\right)-5\sin\left(4c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)+\sin\left(5c+\frac{9dx}{2}\right)\right)}{\cos^{\frac{5}{2}}(c+dx)}\right)}{5a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((42*I)*Sqrt[2]*(11*(1 + E^((2*I)*(c + d*x))) + 11*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)])*Sec[c + d*x]^3)/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*
(c + d*x)))/E^(I*(c + d*x))]) + (-264*Cot[c] - 198*Csc[c] + (Sec[c/2]*Sec[(
c + d*x)/2]^5*(-1210*Sin[(d*x)/2] + 770*Sin[c + (d*x)/2] - 840*Sin[c + (3*d
*x)/2] + 150*Sin[2*c + (3*d*x)/2] - 238*Sin[2*c + (5*d*x)/2] - 40*Sin[3*c +
```

(5*d*x)/2] - 5*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2] + Sin[5*c + (9*d*x)/2]))/16)/Cos[c + d*x]^(5/2)))/(5*a^3*d*(1 + Sec[c + d*x])^3)

Maple [A]
time = 0.12, size = 296, normalized size = 1.43

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(64\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-288\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-76\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-210\sqrt{\ }\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1/2*d
*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*cos(1/2
*d*x+1/2*c)^2-1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 364, normalized size = 1.76

$$\frac{2\left(4\cos(dx+c)^4-8\cos(dx+c)^3-147\cos(dx+c)^2-238\cos(dx+c)-105\right)\sqrt{\cos(dx+c)}\sin(dx+c)-105\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-105\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-231\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-231\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{20\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/20*(2*(4*cos(d*x + c)^4 - 8*cos(d*x + c)^3 - 147*cos(d*x + c)^2 - 238*cos
(d*x + c) - 105)*sqrt(cos(d*x + c))*sin(d*x + c) - 105*(-I*sqrt(2)*cos(d*x
+ c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))
```

```
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 105*(I*sqrt(2)
*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I
*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 231*(
-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*
x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - 231*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*co
s(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x
 + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^3, x)
```

3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=181

$$-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{11\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d}-\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{2\sqrt{}}{3}$$

```
[Out] -119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*sin(d*x+c)*cos(d*x+c)^(
1/2)/a^3/d-1/5*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-2/3*sin(d*x
+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-119/30*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3902, 4105, 3872, 3854, 3856, 2720, 2719}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{119\sin(c+dx)\sqrt{\cos(c+dx)}}{30d\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2]
)/(2*a^3*d) + (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) - (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (2*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) - (119*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(30*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{13a}{2}+\frac{7}{2}a\,\mathrm{se}}{\sec^{\frac{3}{2}}(c+dx)(a+a}}{5a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sqrt{\cos(c+dx}}{30d\,(a^3+a^3\,\mathrm{s}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sqrt{\cos(c+dx}}{30d\,(a^3+a^3\,\mathrm{s}}$$

$$= \frac{11\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\mathrm{s}}{3ad(a+a\sec(c}$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+}{5d(a+a\sec(c+dx}$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{11\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.18, size = 375, normalized size = 2.07

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4i\sqrt{2}\,e^{-i(c+dx)}\left(119\left(1+e^{2i(c+dx)}\right)+119\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+55e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^3(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}+\frac{720\cot(c)+708\csc(c)+\frac{1}{4}\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(1061\sin\left(\frac{dx}{2}\right)-709\sin\left(c+\frac{dx}{2}\right)+715\sin\left(c+\frac{3dx}{2}\right)-170\sin\left(2c+\frac{3dx}{2}\right)+202\sin\left(2c+\frac{5dx}{2}\right)+25\sin\left(3c+\frac{5dx}{2}\right)+5\sin\left(3c+\frac{7dx}{2}\right)+5\sin\left(4c+\frac{7dx}{2}\right)\right)}{3d\cos^{\frac{5}{2}}(c+dx)}\right)}{5a^3(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x))) + 119*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*
I)*(c + d*x)))/E^(I*(c + d*x))]) + (720*Cot[c] + 708*Csc[c] + (Sec[c/2]*Sec
[(c + d*x)/2]^5*(1061*Sin[(d*x)/2] - 709*Sin[c + (d*x)/2] + 715*Sin[c + (3*
d*x)/2] - 170*Sin[2*c + (3*d*x)/2] + 202*Sin[2*c + (5*d*x)/2] + 25*Sin[3*c
```

+ (5*d*x)/2] + 5*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (7*d*x)/2]))/4)/(3*d*Co
s[c + d*x]^(5/2))))/(5*a^3*(1 + Sec[c + d*x])^3)

Maple [A]
time = 0.11, size = 283, normalized size = 1.56

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(160\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2*
d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*cos(
1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 354, normalized size = 1.96

$$\frac{2\left(20\cos(dx+c)^{3}+237\cos(dx+c)^{2}+376\cos(dx+c)+165\right)\sqrt{\cos(dx+c)}\sin(dx+c)-165\left(i\sqrt{2}\cos(dx+c)^{3}+3i\sqrt{2}\cos(dx+c)^{2}+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-165\left(-i\sqrt{2}\cos(dx+c)^{3}-3i\sqrt{2}\cos(dx+c)^{2}-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-357\left(i\sqrt{2}\cos(dx+c)^{3}+3i\sqrt{2}\cos(dx+c)^{2}+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-357\left(-i\sqrt{2}\cos(dx+c)^{3}-3i\sqrt{2}\cos(dx+c)^{2}-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{60\left(a^{3}d\cos(dx+c)^{3}+3a^{3}d\cos(dx+c)^{2}+3a^{3}d\cos(dx+c)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/60*(2*(20*cos(d*x + c)^3 + 237*cos(d*x + c)^2 + 376*cos(d*x + c) + 165)*s
qrt(cos(d*x + c))*sin(d*x + c) - 165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2
)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 165*(-I*sqrt(2)*cos(d*x + c)^3 -

```
3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 357*(I*sqrt(2)*cos(d*x
+ c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) - 357*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I
*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*
cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^3, x)
```

3.391 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3}-\frac{8\sin(c+dx}{15ad\sqrt{\cos(c+dx)}\,(a+a}$$

```
[Out] 49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/(a+a*sec(d*x+
c))^3/cos(d*x+c)^(1/2)-8/15*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1
/2)-13/6*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3902, 4105, 3872, 3856, 2719, 2720}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3\sec(c+dx)+a^3)}-\frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])/
(6*a^3*d) - Sin[c + d*x]/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3) -
(8*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) - (13*S
in[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+d}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+d}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+d}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+d}$$

$$= \frac{49E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.82, size = 357, normalized size = 2.30

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\left(806\cos\left(\frac{1}{2}(c-dx)\right)+664\cos\left(\frac{1}{2}(3c+dx)\right)+470\cos\left(\frac{1}{2}(c+3dx)\right)+265\cos\left(\frac{1}{2}(5c+3dx)\right)+117\cos\left(\frac{1}{2}(3c+5dx)\right)+30\cos\left(\frac{1}{2}(7c+5dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{4i\sqrt{2}\,e^{-i(c+dx)}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+65e^{i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^3(c+dx)}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right)}{15a^3(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-1/8*((806*Cos[(c - d*x)/2] + 664*Cos[(3*c + d*x)/2] +
 470*Cos[(c + 3*d*x)/2] + 265*Cos[(5*c + 3*d*x)/2] + 117*Cos[(3*c + 5*d*x)/
2] + 30*Cos[(7*c + 5*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(d*Cos[
c + d*x]^(5/2)) + ((4*I)*Sqrt[2]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])
*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(
c + d*x)))/E^(I*(c + d*x))]))))/(15*a^3*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.11, size = 270, normalized size = 1.74

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(348\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{60a^3\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*d
*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos(1/2
*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 344, normalized size = 2.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(2*(87*cos(d*x + c)^2 + 146*cos(d*x + c) + 65)*sqrt(cos(d*x + c))*sin
(d*x + c) + 65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*
I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c
)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*c
```

```
os(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(I*sqrt(
2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) +
 I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*
d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\cos(c+dx)}}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^3, x)
```

3.392 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}+\frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}\,(a+a\sec(c$$

```
[Out] -9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(3/
2)/(a+a*sec(d*x+c))^3+2/5*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2
)+1/2*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3902, 4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a^3\sec(c+dx)+a^3)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}+\frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (-9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^
3*d) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) + (2*Si
n[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x
]/(2*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+d}{5ad\sqrt{\cos(c+dx)}\,(a+}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+d}{5ad\sqrt{\cos(c+dx)}\,(a+}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+d}{5ad\sqrt{\cos(c+dx)}\,(a+}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+d}{5ad\sqrt{\cos(c+dx)}\,(a+}$$

$$= -\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{\sin(c+dx}{5d\cos^{\frac{3}{2}}(c+dx)(a+a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.23, size = 721, normalized size = 4.65

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (((-9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^3*((2*E^((
2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[
c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin
[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]
)/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) -
(2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I
)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Se
c[c + d*x])^3 - (2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/
```

2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*((36*Csc[c])/(5*d) + (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (12*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(5*d) - (12*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) + (2*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3)

Maple [A]
time = 0.11, size = 270, normalized size = 1.74

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}\right.}{20a^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(1/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 344, normalized size = 2.22

$$\frac{2\left(9\cos(dx+c)^2+12\cos(dx+c)+5\right)\sqrt{\cos(dx+c)}\sin(dx+c)-5\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-9\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-9\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{20\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/20*(2*(9*cos(d*x + c)^2 + 12*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x
 + c) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 -
3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 9*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x +
c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(-I*sqrt(2)*cos(d*x
 + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{\cos (c+dx)}\sec^3(c+dx)+3\sqrt{\cos (c+dx)}\sec^2(c+dx)+3\sqrt{\cos (c+dx)}\sec (c+dx)+\sqrt{\cos (c+dx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x)**3 + 3*sqrt(cos(c + d*x))*sec(c
 + d*x)**2 + 3*sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x)/a*
*3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos \left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos \left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3), x)
```

3.393 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}-\frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(}$$

```
[Out] -1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*sin(d*x+c)/d/cos(d*x+c)^(3/
2)/(a+a*sec(d*x+c))^3-1/15*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/
2)+1/6*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3900, 4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3\sec(c+dx)+a^3)}+\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}-\frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] -1/10*EllipticE[(c + d*x)/2, 2]/(a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*
d) + Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - Sin[c +
 d*x]/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x]/(6*
d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3900

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.68, size = 342, normalized size = 2.21

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{(14\cos(\frac{1}{2}(c-dx))+16\cos(\frac{1}{2}(3c+dx))+20\cos(\frac{1}{2}(c+3dx))-5\cos(\frac{1}{2}(5c+3dx))+3\cos(\frac{1}{2}(3c+5dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec^5(\frac{1}{2}(c+dx))}{8d\cos^{\frac{5}{2}}(c+dx)} - \frac{4i\sqrt{2}e^{-i(c+dx)}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)})+5e^{i(c+dx)}(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)})\right)\sec^3(c+dx)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}}\right)}{15a^3(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((14*Cos[(c - d*x)/2] + 16*Cos[(3*c + d*x)/2] + 20*Cos
[(c + 3*d*x)/2] - 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)/2])*Csc[c/2]
*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*d*Cos[c + d*x]^(5/2)) - ((4*I)*Sqrt[2]*(3*
(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d
*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4
, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 +
 E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(15*a^3*(1
 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.11, size = 270, normalized size = 1.74

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}{60a^3\cos}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 344, normalized size = 2.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(2*(3*cos(d*x + c)^2 + 14*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x
 + c) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 -
3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x +
```

```
c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x
 + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3), x)
```

3.394 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}-\frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+$$

```
[Out] 1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(3/2
)/(a+a*sec(d*x+c))^3-4/15*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2
)+1/6*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3901, 4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3\sec(c+dx)+a^3)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}-\frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
- Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - (4*Sin[c +
 d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x]/(6
*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx}{15ad\sqrt{\cos(c+dx)}\,(a+a}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx}{15ad\sqrt{\cos(c+dx)}\,(a+a}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx}{15ad\sqrt{\cos(c+dx)}\,(a+a}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx}{15ad\sqrt{\cos(c+dx)}\,(a+a}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.58, size = 342, normalized size = 2.21

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(4\cos(\frac{1}{2}(c-dx))+26\cos(\frac{1}{2}(3c+dx))+10\cos(\frac{1}{2}(c+3dx))+5\cos(\frac{1}{2}(5c+3dx))+3\cos(\frac{1}{2}(3c+5dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec^5(\frac{1}{2}(c+dx))}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{4i\sqrt{2}\,e^{-i(c+dx)}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-5e^{i(c+dx)}(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)\sec^3(c+dx)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right)}{15a^3(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-1/8*((4*Cos[(c - d*x)/2] + 26*Cos[(3*c + d*x)/2] + 10
*Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)/2])*Csc[
c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(d*Cos[c + d*x]^(5/2)) + ((4*I)*Sqrt[2]*(
3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c +
 d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1
 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(15*a^3*
(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.12, size = 270, normalized size = 1.74

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}}{60a^3\cos\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.71, size = 344, normalized size = 2.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(2*(3*cos(d*x + c)^2 + 4*cos(d*x + c) - 5)*sqrt(cos(d*x + c))*sin(d*x
 + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 -
3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x +
```

```
c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x
 + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3), x)
```

3.395 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}-\frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+$$

```
[Out] 9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(5/2
)/(a+a*sec(d*x+c))^3-2/5*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2
-9/10*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3901, 4104, 3872, 3856, 2719, 2720}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{9\sin(c+dx)}{10d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^3
*d) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3) - (2*Sin
[c + d*x])/(5*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) - (9*Sin[c + d
*x])/(10*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= \frac{9E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{2a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{se}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.21, size = 721, normalized size = 4.65

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (((9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^3*((2*E^((2
*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Sec
[c + d*x])^3 - (2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2
```

```
}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^3*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*((-36*C
sc[c])/(5*d) - (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (8*Sec
[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x
)/2]^5*Sin[(d*x)/2])/(5*d) - (8*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) - (2*S
ec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*
x])^3)
```

Maple [A]
time = 0.11, size = 268, normalized size = 1.73

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{20a^3\cos\left(\frac{dx}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.01, size = 344, normalized size = 2.22

$$-\frac{2\left(9\cos\left(dx+c\right)^2+22\cos\left(dx+c\right)+15\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+9\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+9\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{20\left(a^3d\cos\left(dx+c\right)^3+3a^3d\cos\left(dx+c\right)^2+3a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/20*(2*(9*cos(d*x + c)^2 + 22*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(d
*x + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x
 + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(I*sqrt(2)*cos(d
*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(
2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*
x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3), x)
```

3.396 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=181

$$-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}-\frac{}{15a}$$

```
[Out] -49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(
7/2)/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c
))^2-13/6*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a^3+a^3*sec(d*x+c))+49/10*sin(d*x+
c)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3901, 4104, 3872, 3856, 2720, 3853, 2719}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{13\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}-\frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (-49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])
/(6*a^3*d) + (49*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]
/(5*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d
*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2) - (13*Sin[c + d*x])/(6*d*Cos[c
+ d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{}{15a}$$

$$= -\frac{13F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c-}{5d\cos^{\frac{7}{2}}(c+dx)(a}$$

$$= -\frac{49E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.04, size = 372, normalized size = 2.06

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{(1284\cos(\frac{1}{2}(c-dx))+921\cos(\frac{1}{2}(3c+dx))+1243\cos(\frac{1}{2}(c+3dx))+374\cos(\frac{1}{2}(5c+3dx))+670\cos(\frac{1}{2}(3c+5dx))+65\cos(\frac{1}{2}(7c+5dx))+147\cos(\frac{1}{2}(5c+7dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec^5(\frac{1}{2}(c+dx))}{16d\cos^{\frac{7}{2}}(c+dx)} - \frac{4i\sqrt{2}e^{-i(c+dx)}\left(147(1+e^{2i(c+dx)})+147(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)})-65e^{i(c+dx)}(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)})\right)\sec^3(c+dx)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}}\right)}{15a^3(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 124
3*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2]
+ 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec
[(c + d*x)/2]^5)/(16*d*Cos[c + d*x]^(7/2)) - ((4*I)*Sqrt[2]*(147*(1 + E^((2
*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hype
rgeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 65*E^(I*(c + d*x))*(-
1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2,
5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*
```

```
I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(15*a^3*(1 + Sec[
c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(213) = 426$.
time = 0.13, size = 555, normalized size = 3.07

method	result
default	$-\frac{-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(65\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 394, normalized size = 2.18

$$\frac{2\left(147\cos(dx+c)^3+376\cos(dx+c)^2+295\cos(dx+c)+60\right)\sqrt{\cos(dx+c)}\sin(dx+c)-65\left(-i\sqrt{2}\cos(dx+c)^4-3i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65\left(i\sqrt{2}\cos(dx+c)^4+3i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-147\left(i\sqrt{2}\cos(dx+c)^4+3i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-147\left(-i\sqrt{2}\cos(dx+c)^4-3i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{60\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/60*(2*(147*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 295*cos(d*x + c) + 60)*sqrt(cos(d*x + c))*sin(d*x + c) - 65*(-I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 65*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 147*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 147*(-I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{9/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3),x)

[Out] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3), x)

3.397 $\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{\sin(c+ }{5d\cos^{\frac{9}{2}}(c+dx)(a+ }$$

```
[Out] 119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*sin(d*x+c)/a^3/d/cos(d*x
+c)^(3/2)-1/5*sin(d*x+c)/d/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3-2/3*sin(d*x+
c)/a/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2-119/30*sin(d*x+c)/d/cos(d*x+c)^(
5/2)/(a^3+a^3*sec(d*x+c))-119/10*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3901, 4104, 3872, 3853, 3856, 2719, 2720}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{119\sin(c+dx)}{30d\cos^{\frac{5}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+dx)}{3ad\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Cos[c + d*x]^(3/2)) - (119*Sin[c +
d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(9/2)*
(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(7/2)*(a + a
*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Cos[c + d*x]^(5/2)*(a^3 + a^3*
Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a}$$

$$= \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+}{5d\cos^{\frac{9}{2}}(c+dx)(a-}$$

$$= \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+}{5d\cos^{\frac{9}{2}}(c+dx)(a-}$$

$$= \frac{119E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{2a^3d} + \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.58, size = 402, normalized size = 1.94

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(5134\cos(\frac{1}{2}(c-dx))+4148\cos(\frac{1}{2}(3c+dx))+4664\cos(\frac{1}{2}(c+3dx))+2476\cos(\frac{1}{2}(5c+3dx))+3340\cos(\frac{1}{2}(3c+5dx))+944\cos(\frac{1}{2}(7c+5dx))+1620\cos(\frac{1}{2}(5c+7dx))+165\cos(\frac{1}{2}(9c+7dx))+357\cos(\frac{1}{2}(7c+9dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec^5(\frac{1}{2}(c+dx))}{96d\cos^{\frac{9}{2}}(c+dx)} + \frac{4i\sqrt{2}e^{-i(c+dx)}\left(119(1+e^{2i(c+dx)})+119(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)})-55e^{i(c+dx)}(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)})\right)\sec^3(c+dx)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}}\right)}{5a^3(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-1/96*((5134*Cos[(c - d*x)/2] + 4148*Cos[(3*c + d*x)/2
] + 4664*Cos[(c + 3*d*x)/2] + 2476*Cos[(5*c + 3*d*x)/2] + 3340*Cos[(3*c + 5
*d*x)/2] + 944*Cos[(7*c + 5*d*x)/2] + 1620*Cos[(5*c + 7*d*x)/2] + 165*Cos[(
9*c + 7*d*x)/2] + 357*Cos[(7*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)
/2]^5)/(d*Cos[c + d*x]^(9/2)) + ((4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x)
)) + 119*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1
[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 55*E^(I*(c + d*x))*(-1 + E^((2*I)*
c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I
```

```
)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(
1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(5*a^3*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.16, size = 453, normalized size = 2.19

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{32\sqrt{-2}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(1/5*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+32/1
5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3
+118/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+48*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1
/2*c)^2)^(1/2)-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 414, normalized size = 2.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(2*(357*cos(d*x + c)^4 + 906*cos(d*x + c)^3 + 695*cos(d*x + c)^2 + 12
0*cos(d*x + c) - 20)*sqrt(cos(d*x + c))*sin(d*x + c) + 165*(I*sqrt(2)*cos(d
*x + c)^5 + 3*I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + I*sqr
t(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + 165*(-I*sqrt(2)*cos(d*x + c)^5 - 3*I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqr
t(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 357*(-I*sqrt(2)*cos(d*x + c)^5 - 3*I*sqrt(
2)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + 357*(I*sqrt(2)*cos(d*x + c)^5 + 3*I*sqrt(2)*cos(d*x + c)^4 + 3*I*s
qrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x +
c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)
^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(11/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{11/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(11/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(11/2)*(a + a/cos(c + d*x))^3), x)
```

3.398 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{16a\sqrt{\cos(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}+\frac{12a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}+\frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 12/35*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a*cos(d*x+
c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+32/35*a*sin(d*x+c)/d/cos(d*x+c
)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/35*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*s
ec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4349, 3890, 3889}

$$\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\cos(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (32*a*Sin[c + d*x])/(35*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (1
6*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (12*
a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C
os[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\left(6\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.$$
$$= \frac{12a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{35}$$
$$= \frac{16a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{12a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} +$$
$$= \frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{16a\sqrt{\cos(c+dx)}\,\sin(}{35d\sqrt{a+a\sec(c+}}$$

Mathematica [A]
time = 0.22, size = 80, normalized size = 0.52

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a(1+\sec(c+dx))}\,(140\sin(c+dx)+42\sin(2(c+dx))+12\sin(3(c+dx))+5\sin(4(c+dx)))}{140d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(140*Sin[c + d*x] + 42*Sin[2
*(c + d*x)] + 12*Sin[3*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(140*d*(1 + Cos[c
+ d*x]))
```

Maple [A]
time = 0.60, size = 80, normalized size = 0.52

method	result	size
default	$-\frac{2\left(5\left(\cos^4(dx+c)\right)+\cos^3(dx+c)+2\left(\cos^2(dx+c)\right)+8\cos(dx+c)-16\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\sqrt{\cos}\,(dx+c)\right)}{35d\sin(dx+c)}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/35/d*(5*cos(d*x+c)^4+cos(d*x+c)^3+2*cos(d*x+c)^2+8*cos(d*x+c)-16)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(129) = 258$.
time = 0.83, size = 293, normalized size = 1.92

$$\frac{\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\cos\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)-105\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-35\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-7\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+10\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sin\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+35\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+105\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\right)\sqrt{a}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2*c)
*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7/2*
d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 3.00, size = 79, normalized size = 0.52

$$\frac{2\left(5\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)+16\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{7/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2), x)
```

3.399 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{8a\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/15*a*sin(d*x+
c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+8/15*a*sin(d*x+c)*cos(d*x+c)^(
1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4349, 3890, 3889}

$$\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*a*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (8
*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{1}{5}\left(4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.$$

$$= \frac{8a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{1}{15}$$

$$= \frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{8a\sqrt{\cos(c+dx)}\,\sin(c}{15d\sqrt{a+a\sec(c+}$$

Mathematica [A]
time = 0.18, size = 61, normalized size = 0.53

$$\frac{\sqrt{\cos(c+dx)}\,(19+8\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 +
Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

Maple [A]
time = 0.12, size = 70, normalized size = 0.61

method	result	size
default	$-\frac{2\left(3\left(\cos^{3}(dx+c)\right)+\cos^{2}(dx+c)+4\cos(dx+c)-8\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\sqrt{\cos}\,(dx+c)\right)}{15d\sin(dx+c)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(3*cos(d*x+c)^3+cos(d*x+c)^2+4*cos(d*x+c)-8)*(a*(1+cos(d*x+c))/cos(
d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(97) = 194.
time = 0.55, size = 203, normalized size = 1.77

$$\frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-30\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-5\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+6\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+30\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)\sqrt{a}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x +
 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 2.41, size = 69, normalized size = 0.60

$$\frac{2\left(3\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)+8\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2), x)
```

3.400 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 4/3*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+c)
*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4349, 3890, 3889}

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{3}\Big(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}$$
$$= \frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c-}{3d\sqrt{a+a\sec(c+dx}}$$

Mathematica [**A**]
time = 0.12, size = 49, normalized size = 0.64

$$\frac{2\sqrt{\cos(c+dx)}\,(2+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c
+ d*x)/2])/(3*d)
```

Maple [**A**]
time = 0.12, size = 58, normalized size = 0.75

method	result	size
default	$-\frac{2\left(\cos^{2}(dx+c)+\cos(dx+c)-2\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\sqrt{\cos}\,(dx+c)\right)}{3d\sin(dx+c)}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)^2+cos(d*x+c)-2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(
d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [**A**]
time = 0.54, size = 113, normalized size = 1.47

$$\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\right)\sqrt{a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 3.63, size = 57, normalized size = 0.74

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(\cos(dx+c)+2\right)\sqrt{\cos(dx+c)}\sin(dx+c)}{3\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) + 2)*sqrt(cos(d*x
 + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3433 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2), x)
```

3.401 $\int \sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4349, 3889}

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 39, normalized size = 1.08

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]

[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d

Maple [A]
time = 0.12, size = 50, normalized size = 1.39

method	result	size
default	$-\frac{2\left(\sqrt{\cos}\left(dx+c\right)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))}{d\sin(dx+c)}$	50
risch	$-\frac{2i\sqrt{\frac{a\left(e^{i(dx+c)}+1\right)^{2}}{e^{2i(dx+c)}+1}}\,\left(\sqrt{\cos}\left(dx+c\right)\right)\left(e^{i(dx+c)}-1\right)}{\left(e^{i(dx+c)}+1\right)d}$	69

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))/s
in(d*x+c)

Maxima [A]
time = 0.52, size = 20, normalized size = 0.56

$$\frac{2\,\sqrt{2}\,\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d

Fricas [A]
time = 1.92, size = 49, normalized size = 1.36

$$\frac{2\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c) + d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sqrt{\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2), x)
```

3.402 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4349, 3886, 221}

$$\frac{2\sqrt{a}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+}{\sqrt{a+a\sec(}}\right.}{d}$$

$$= \frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.17, size = 74, normalized size = 1.30

$$-\frac{2\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(47) = 94.
time = 0.12, size = 142, normalized size = 2.49

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{\cos}(dx+c)\right)\left(\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+}}}{}\right.\right.}{2d\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(arctan(1/4*(-2/
(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-arctan(1/4*(-2/(1+
cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2)))*(-2/(1+cos(d*x+c)))^
(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(47) = 94$.
time = 0.55, size = 241, normalized size = 4.23

$$\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))/d
```

Fricas [A]
time = 2.55, size = 180, normalized size = 3.16

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^{3}-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-7a\cos\left(dx+c\right)^{2}+8a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)}{2d}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-2a}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*
x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
[Out] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

3.403 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}+\frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

```
[Out] arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3888, 3886, 221}

$$\frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
```

```
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{}$$

$$= \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\text{Subst}}{}$$

$$= \frac{\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d} + \frac{}{d\cos^{\frac{3}{2}}(c}$$

Mathematica [A]
time = 0.26, size = 90, normalized size = 0.98

$$\frac{2a\left(\frac{1}{2}\cos(c+dx)+\frac{\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)}{2\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\right)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(Cos[c + d*x]/2 + ArcSin[Sqrt[1 - Sec[c + d*x]]]/(2*Sqrt[1 - Sec[c + d
*x]]*Sec[c + d*x]^(3/2)))*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)*Sqrt[a*(1 + S
ec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(78) = 156.
time = 0.16, size = 178, normalized size = 1.93

method	result
default	$-\frac{(-1+\cos(dx+c))\left(\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos}}}{}\right.\right.}{2d\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)^2\sqrt{\cos}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+
cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)-cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*
x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)+2*sin(d*x+c)*(-2/(1
+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(-2/(1+cos(d*x+c))
)^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(78) = 156$.
time = 0.57, size = 662, normalized size = 7.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*
```

```
d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)
/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 3.00, size = 325, normalized size = 3.53

$$\left[\frac{\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7a\cos(dx+c)^{2}+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)},\frac{\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2a}\right)+2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*((cos(d*x + c)^2 + cos(
d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

3.404 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=136

$$\frac{3\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d}+\frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+$$

```
[Out] 3/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+1/2*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)
)^(1/2)+3/4*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3888, 3886, 221}

$$\frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (3*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*Sin[c + d*x])/(2*d*Cos[c + d*x]^
(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (3*a*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
```

```
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}\left(3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \mathrm{s}$$

$$= \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{3\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d} + \frac{}{2d\cos^{\frac{5}{2}}(}$$

Mathematica [A]
time = 0.51, size = 100, normalized size = 0.74

$$\frac{2a\left(\frac{1}{8}\cos(c+dx)(2+3\cos(c+dx)) + \frac{3\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)}{8\sqrt{1-\sec(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)}\right)\sin(c+dx)}{d\cos^{\frac{7}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*((Cos[c + d*x]*(2 + 3*Cos[c + d*x]))/8 + (3*ArcSin[Sqrt[1 - Sec[c + d*
x]]])/(8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)))*Sin[c + d*x])/(d*Cos[c
 + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.14, size = 213, normalized size = 1.57

method	result
default	$\frac{\left(3\left(\cos^2(dx+c)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-3\left(\cos^2(dx+c)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(d}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(3*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(
d*x+c)+sin(d*x+c))*2^(1/2))-3*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*
x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+6*cos(d*x+c)*sin(d*x+c)*(-2
/(1+cos(d*x+c)))^(1/2)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/sin(d*x+
c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. 2(112) = 224.
time = 0.58, size = 1264, normalized size = 9.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(
2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1
)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), co
```

```
s(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*
c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 4.26, size = 355, normalized size = 2.61

$$\left[\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(3\cos(dx+c)+2\right)\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}, \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(3\cos(dx+c)+2\right)\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{8\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sqrt(
cos(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(co
s(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2),
 1/8*(2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sqrt(c
os(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3
+ d*cos(d*x + c)^2)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2), x)
```

3.405 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=161

$$\frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{104a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{26a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}+$$

```
[Out] 26/35*a^2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a^2*cos(
d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)/d/c
os(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+104/105*a^2*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3898, 21, 3890, 3889}

$$\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{26a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{104a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (208*a^2*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
+ (104*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]
) + (26*a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]
) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\Big(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+d$$
$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\Big(13a\sqrt{\cos(c+dx)}\sqrt{\sec(c+$$
$$= \frac{26a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} +$$
$$= \frac{104a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{35d\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{104a^2\sqrt{\cos(c+dx)}}{105d\sqrt{a+a\sec(}}$$

Mathematica [A]
time = 0.30, size = 72, normalized size = 0.45

$$\frac{a\sqrt{\cos(c+dx)}\,(494+253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*Co
s[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d)
```

Maple [A]
time = 0.12, size = 83, normalized size = 0.52

method	result	size
default	$-\frac{2\left(15\left(\cos^{4}\left(dx+c\right)\right)+24\left(\cos^{3}\left(dx+c\right)\right)+13\left(\cos^{2}\left(dx+c\right)\right)+52\cos\left(dx+c\right)-104\right)\left(\sqrt{\cos}\left(dx+c\right)\right)\sqrt{\frac{a\left(1+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\,a}{105d\sin\left(dx+c\right)}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/d*(15*cos(d*x+c)^4+24*cos(d*x+c)^3+13*cos(d*x+c)^2+52*cos(d*x+c)-104
)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(137) = 274$.
time = 0.56, size = 303, normalized size = 1.88

$$\frac{\sqrt{2}\left(735\,a\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+175\,a\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+63\,a\cos\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)-735\,a\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-175\,a\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-63\,a\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\sin\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+30\,a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+63\,a\sin\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+175\,a\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+735\,a\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\right)\sqrt{a}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*
x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 1
75*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 3.90, size = 84, normalized size = 0.52

$$\frac{2\left(15\,a\cos\left(dx+c\right)^{3}+39\,a\cos\left(dx+c\right)^{2}+52\,a\cos\left(dx+c\right)+104\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 52*a*cos(d*x + c) + 104*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2), x)
```

3.406 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2\cos^{\frac{3}{2}}(c+dx)(a$$

```
[Out] 2/5*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+8/5*a^2*sin(d*x+c)
/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)*cos(d*x+c)^(1/2
)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3897, 3894, 3889}

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*Cos
[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 3897

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Dist[a*(m/(b*d*(m + 1))), Int[(a + b*Csc[e
+ f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
```

```
x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\Big(3\sqrt{\cos(c-}$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2\cos^{\frac{3}{2}}(c+}{}$$
$$= \frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+}}{}$$

Mathematica [A]
time = 0.22, size = 60, normalized size = 0.52

$$\frac{a\sqrt{\cos(c+dx)}\,(13+6\cos(c+dx)+\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 +
Sec[c + d*x])]*Tan[(c + d*x)/2])/(5*d)
```

Maple [A]
time = 0.12, size = 71, normalized size = 0.61

method	result	size
default	$-\frac{2\left(\cos^{3}(dx+c)+2\left(\cos^{2}(dx+c)\right)+3\cos(dx+c)-6\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a}{5d\sin(dx+c)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(cos(d*x+c)^3+2*cos(d*x+c)^2+3*cos(d*x+c)-6)*cos(d*x+c)^(1/2)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(98) = 196$.
time = 0.55, size = 210, normalized size = 1.81

$$\frac{\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-20\,a\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)-5\,a\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)+2\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)+20\,a\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\right)\sqrt{a}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/
2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin
(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)))*sqrt(a)/d
```

Fricas [A]
time = 3.05, size = 72, normalized size = 0.62

$$\frac{2\left(a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+6\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 6*a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2), x)
```

3.407 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=79

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

```
[Out] 8/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+
c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4349, 3894, 3889}

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\Big(4a\sqrt{\cos}$$

$$= \frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+}}{}$$

Mathematica [A]
time = 0.18, size = 50, normalized size = 0.63

$$\frac{2a\sqrt{\cos(c+dx)}\,(5+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(
c + d*x)/2])/(3*d)
```

Maple [A]
time = 0.12, size = 61, normalized size = 0.77

method	result	size
default	$-\frac{2\left(\cos^2(dx+c)+4\cos(dx+c)-5\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a}{3d\sin(dx+c)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)^2+4*cos(d*x+c)-5)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos
(d*x+c))^(1/2)/sin(d*x+c)*a
```

Maxima [A]
time = 0.55, size = 38, normalized size = 0.48

$$\frac{\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqr
t(a)/d
```

Fricas [A]
time = 3.50, size = 61, normalized size = 0.77

$$\frac{2\left(a\cos\left(dx+c\right)+5\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(d*x + c) + 5*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2), x)
```

3.408 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^
(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3898, 21, 3886, 221}

$$\frac{2a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \left(2a\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.$$

$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{sec}\right.$$

$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.}{}$$

$$= \frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}{d}$$

Mathematica [A]
time = 0.15, size = 81, normalized size = 0.84

$$\frac{2a^2\left(\sqrt{1-\sec(c+dx)}+\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{d\sqrt{-1+\cos(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

[Out] (2*a^2*(Sqrt[1 - Sec[c + d*x]] + ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])
])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(82) = 164$.
time = 0.11, size = 172, normalized size = 1.79

method	result
default	$-\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c}}\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*cos(d*x+c)-4)/sin(d*x+c)*a

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(82) = 164$.
time = 0.55, size = 274, normalized size = 2.85

$$\frac{\sqrt{2}\left(\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+8\,a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Fricas [A]
time = 2.53, size = 298, normalized size = 3.10

$$\left[\frac{4\,a\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7\,a\cos(dx+c)^2+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2\left(d\cos\left(dx+c\right)+d\right)}, \frac{2\,a\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(a\cos\left(dx+c\right)+a\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)}{d\cos\left(dx+c\right)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
)/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(-a)*arctan(2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3433 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2), x)
```

3.409 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+\frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 3*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1
/2)
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3899, 21, 3886, 221}

$$\frac{3a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{}}{}$$

$$= \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left(3a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(3a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\text{Sub}}{}$$

$$= \frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d} + \frac{}{d\cos^{\frac{3}{2}}(}$$

Mathematica [A]
time = 0.35, size = 92, normalized size = 0.97

$$-\frac{a^2\left(-\sqrt{1-\sec(c+dx)}+\frac{3\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}}\right)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((a^2*(-Sqrt[1 - Sec[c + d*x]] + (3*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec[c
 + d*x]])*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(81) = 162$.
time = 0.13, size = 182, normalized size = 1.92

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-3\cos(dx+c)\arctan\left(\frac{\sqrt{-}}{}\right.\right.}{4d\sqrt{\cos(dx+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c)*arctan(1/4*(-2/(1+c
os(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)-3*cos(d*x+c)*a
rctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1
/2)+2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c)^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1143 vs. $2(81) = 162$.
time = 0.57, size = 1143, normalized size = 12.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
```

```
*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x + 3/
2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 4
*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*
x + 2*c))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*d)
```

<u>Fricas</u> [A]
time = 4.42, size = 337, normalized size = 3.55

$$\left[\frac{4\,a\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-4\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,(\cos(dx+c)-2)\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-7\,a\cos(dx+c)^{2}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{4\,\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)},\ \frac{2\,a\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)}{2\,\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + 3*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^
3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sq
rt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*(2*a*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a*cos(d*
x + c)^2 + a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*co
s(d*x + c) - 2*a))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec (c+dx)+1))^{\frac{3}{2}}}{\sqrt{\cos (c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos (c+d\,x)}\right)}^{3/2}}{\sqrt{\cos (c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
[Out] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

3.410 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d}+\frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{}{4}$$

```
[Out] 7/4*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+1/2*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+
c))^(1/2)+7/4*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4349, 3899, 21, 3888, 3886, 221}

$$\frac{7a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*Sin[c + d*x])/(2*d*Cos[c + d*x
]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (7*a^2*Sin[c + d*x])/(4*d*Cos[c + d*x]^
(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}\left(7a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d} + \frac{}{2d\cos^{\frac{5}{2}}}$$

Mathematica [A]

time = 0.45, size = 99, normalized size = 0.71

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(7\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^2(c+dx)-3\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)\right)}{8d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(7*Sqrt[2]*ArcTanh[Sqrt[2]*S
in[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7*Sin[(3*(c + d*x))/
2]))/(8*d*Cos[c + d*x]^(3/2))
```

Maple [A]

time = 0.14, size = 212, normalized size = 1.51

method	result
default	$-\frac{(-1+\cos(dx+c))\left(7(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-7(\cos^2(dx+c))\sqrt{2}\,\arctan\right.}{8d\cos(d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(7*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-7*cos(d*x+c)^2*2^(1/2)*arctan(1
/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+14*cos(d*x+
c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)/(-2/(1+cos(d*x+c)
))^(1/2)/sin(d*x+c)^2*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. $2(116) = 232$.
time = 0.63, size = 2244, normalized size = 16.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*sqr
t(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin(3/
2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(7/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(5/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a*co
s(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a
)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(
```

```
8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*
log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)*a*
cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(si...
```

Fricas [A]

time = 3.71, size = 369, normalized size = 2.64

$$\left[\frac{4\,\left(7\,a\cos\left(dx+c\right)+2\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+7\,\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-4\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,(\cos(dx+c)-2)\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-7\,a\cos(dx+c)^{2}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{16\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)},\ \frac{2\,\left(7\,a\cos\left(dx+c\right)+2\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+7\,\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)}{8\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sq
rt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^
2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x
+ c)^2), 1/8*(2*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(d*x + c
)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d
*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

3.411 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d}+\frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\cdots$$

```
[Out] 11/8*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+1/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x
+c))^(1/2)+11/12*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+1
1/8*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = {4349, 3899, 21, 3888, 3886, 221}

$$\frac{11a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{11a^2\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (11*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*Sin[c + d*x])/(3*d*Cos[c + d*
x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Sin[c + d*x])/(12*d*Cos[c + d*
x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Sin[c + d*x])/(8*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}\left(11a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{8d} + \frac{}{3d\,\mathrm{co}}$$

Mathematica [A]
time = 0.55, size = 112, normalized size = 0.62

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(66\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)+54\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)\right)}{96d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(66*Sqrt[2]*ArcTanh[Sqrt[2]*
Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] + 11*(Sin[(3*(c + d*
x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.15, size = 244, normalized size = 1.36

method	result

default	$\frac{\left(33(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-33(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96/d*(33*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos
(d*x+c)+sin(d*x+c))*2^(1/2))-33*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(
d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+66*sin(d*x+c)*cos(d*x+c)^
2*(-2/(1+cos(d*x+c)))^(1/2)+44*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2)+16*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^
(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(5/2)/sin(d*x+c)^2*(cos(d*x+c)^2
-1)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2361 vs. $2(150) = 300$.
time = 0.67, size = 2361, normalized size = 13.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2
*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*
a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos
(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
```

```
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d
*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x
 + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c)
+ a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a
*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*c
os(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin
(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x +
2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*
c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) +
 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 +
 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 1
8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos
(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d
*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4
*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqr
t(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x +
4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*
cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) +
3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*si
n(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*
x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sq
rt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2
)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x
 + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sqrt(a)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
```

```
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*d)
```

Fricas [A]
time = 4.26, size = 391, normalized size = 2.17

$$\left[\frac{4\left(33\,a\cos\left(dx+c\right)^{2}+22\,a\cos\left(dx+c\right)+8\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+33\left(a\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{3}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7\,a\cos(dx+c)^{2}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{96\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)},\ \frac{2\left(33\,a\cos\left(dx+c\right)^{2}+22\,a\cos\left(dx+c\right)+8\,a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+33\left(a\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{3}\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)}{48\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
[Out] [1/96*(4*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 33*(a*cos(d*x + c)^
4 + a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos
(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(33*a*cos(d*x + c)^2 + 22*a*cos(d*
x + c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 33*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*
x + c)^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
[Out] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2), x)
```

3.412 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=201

$$\frac{1168a^3\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{584a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{146a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+$$

```
[Out] 146/105*a^3*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+38/63*a^3*
cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1168/315*a^3*sin(d*x+c
)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+584/315*a^3*sin(d*x+c)*cos(d*x+
c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a^2*cos(d*x+c)^(7/2)*sin(d*x+c)*(a+a*
sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.28, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3898, 4100, 3890, 3889}

$$\frac{38a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{146a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{584a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{1168a^3\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (1168*a^3*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
 + (584*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]
]) + (146*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*
x]]) + (38*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a^2*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*
d)
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx \\
&= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{9d}+\frac{1}{9}\Big(2a\sqrt{\cos(} \\
&= \frac{38a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c-}}{9d} \\
&= \frac{146a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{38a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{584a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{146a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{105d\sqrt{a+a\sec(c+dx)}} \\
&= \frac{1168a^3\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{584a^3\sqrt{\cos(c+dx)}}{315d\sqrt{a+a\sec(}}
\end{aligned}
$$

Mathematica [A]

time = 0.25, size = 90, normalized size = 0.45

$$\frac{2a^2\sqrt{\cos(c+dx)}\,(584+292\cos(c+dx)+219\cos^2(c+dx)+130\cos^3(c+dx)+35\cos^4(c+dx))\,\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{315d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(584 + 292*Cos[c + d*x] + 219*Cos[c + d*x]^2 + 13
0*Cos[c + d*x]^3 + 35*Cos[c + d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*
x])/(315*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.12, size = 95, normalized size = 0.47

method	result
default	$-\frac{2\left(35\left(\cos^{5}(dx+c)\right)+95\left(\cos^{4}(dx+c)\right)+89\left(\cos^{3}(dx+c)\right)+73\left(\cos^{2}(dx+c)\right)+292\cos(dx+c)-584\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c}{\cos(dx+c)}}}{315d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*(35*cos(d*x+c)^5+95*cos(d*x+c)^4+89*cos(d*x+c)^3+73*cos(d*x+c)^2+2
92*cos(d*x+c)-584)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin
(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(171) = 342$.
time = 0.57, size = 422, normalized size = 2.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/
2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a^
2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
+ 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)
*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos
(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos
(9/2*d*x + 9/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 3.43, size = 105, normalized size = 0.52

$$\frac{2\left(35\,a^{2}\cos\left(dx+c\right)^{4}+130\,a^{2}\cos\left(dx+c\right)^{3}+219\,a^{2}\cos\left(dx+c\right)^{2}+292\,a^{2}\cos\left(dx+c\right)+584\,a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x + c
)^2 + 292*a^2*cos(d*x + c) + 584*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(c+d\,x\right)}^{9/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.413 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=156

$$\frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d}+\frac{2a\cos^{\frac{3}{2}}(c+d}{}$$

```
[Out] 2/7*a*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*cos(d*x+c)^(
5/2)*(a+a*sec(d*x+c))^(5/2)*sin(d*x+c)/d+64/21*a^3*sin(d*x+c)/d/cos(d*x+c)^
(1/2)/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec
(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3897, 3894, 3889}

$$\frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d}+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sin[c + d*x])/(21*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) +
(2*a*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2
*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 3897

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(m + 1))), x] + Dist[a*(m/(b*d*(m + 1))), Int[(a + b*Csc[e
+ f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}\Big(5\sqrt{\cos(c-$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2\cos^{\frac{5}{2}}(c+c}{}$$

$$= \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c}{}$$

$$= \frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Mathematica [A]
time = 0.27, size = 74, normalized size = 0.47

$$\frac{a^2\sqrt{\cos(c+dx)}\,(208+101\cos(c+dx)+24\cos(2(c+dx))+3\cos(3(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(208 + 101*Cos[c + d*x] + 24*Cos[2*(c + d*x)] + 3*C
os[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(42*d)
```

Maple [A]
time = 0.12, size = 85, normalized size = 0.54

method	result	size

default	$-\frac{2\left(3\left(\cos^{4}(dx+c)\right)+9\left(\cos^{3}(dx+c)\right)+11\left(\cos^{2}(dx+c)\right)+23\cos(dx+c)-46\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a^{2}}{21d\sin(dx+c)}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(3*cos(d*x+c)^4+9*cos(d*x+c)^3+11*cos(d*x+c)^2+23*cos(d*x+c)-46)*co
s(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(132) = 264$.
time = 0.56, size = 323, normalized size = 2.07

$$\frac{\sqrt{2}\left(315\,a^{2}\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+77\,a^{2}\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\,a^{2}\cos\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-315\,a^{2}\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\sin\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)-77\,a^{2}\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\sin\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)-21\,a^{2}\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\sin\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+6\,a^{2}\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\,a^{2}\sin\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+77\,a^{2}\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+315\,a^{2}\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\right)\sqrt{a}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*cos
(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*s
in(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Fricas [A]
time = 2.14, size = 92, normalized size = 0.59

$$\frac{2\left(3\,a^{2}\cos\left(dx+c\right)^{3}+12\,a^{2}\cos\left(dx+c\right)^{2}+23\,a^{2}\cos\left(dx+c\right)+46\,a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) +
46*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.414 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=119

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d}+\frac{2a\cos^{\frac{3}{2}}(c+d$$

```
[Out] 2/5*a*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+64/15*a^3*sin(d*
x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/15*a^2*sin(d*x+c)*cos(d*x
+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4349, 3894, 3889}

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) +
(2*a*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\Big(8a\sqrt{\cos(}$$
$$= \frac{16a^2\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2a\cos^{\frac{3}{2}}(c}{}$$
$$= \frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\,\sqrt{}}{}$$

Mathematica [A]
time = 0.26, size = 64, normalized size = 0.54

$$\frac{a^2\sqrt{\cos(c+dx)}\,(89+28\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
[Out] (a^2*Sqrt[Cos[c + d*x]]*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*
(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

Maple [A]
time = 0.12, size = 75, normalized size = 0.63

method	result	size
default	$-\frac{2\left(3\left(\cos^3(dx+c)\right)+11\left(\cos^2(dx+c)\right)+29\cos(dx+c)-43\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a^2}{15d\sin(dx+c)}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/15/d*(3*cos(d*x+c)^3+11*cos(d*x+c)^2+29*cos(d*x+c)-43)*cos(d*x+c)^(1/2)*
(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a^2
```

Maxima [A]
time = 0.53, size = 60, normalized size = 0.50

$$\frac{\left(3\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}\,a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A]
time = 3.38, size = 79, normalized size = 0.66

$$\frac{2\left(3\,a^{2}\cos\left(dx+c\right)^{2}+14\,a^{2}\cos\left(dx+c\right)+43\,a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 43*a^2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.415 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=138

$$\frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}+\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+14/3*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c
))^(1/2)+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3898, 4100, 3886, 221}

$$\frac{2a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (14*a^3*Sin[c + d*x])/(3*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
```

```
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\Big(2a\sqrt{\cos(}$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+}}{}$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+}}{}$$

$$= \frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{d}$$

Mathematica [A]
time = 0.25, size = 93, normalized size = 0.67

$$\frac{2a^3\left((8+\cos(c+dx))\sqrt{1-\sec(c+dx)}+3\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{3d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*((8 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*ArcSin[Sqrt[1 - Sec[c
 + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.14, size = 185, normalized size = 1.34

method	result
default	$-\frac{\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*
x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*2^(1/2)*arctan(1/4*(-
2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2+28*cos(d*x+c)-32)*(a*(1+cos(d*x+c))/cos
(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(116) = 232$.
time = 0.57, size = 593, normalized size = 4.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*cos
```

```
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*x
 + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*sqrt(a)/d
```

Fricas [A]
time = 3.72, size = 339, normalized size = 2.46

$$\left[\frac{4\,{\left(a^{2} \cos\left(dx+c\right)+8\,a^{2}\right)}\sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,{\left(a^{2} \cos\left(dx+c\right)+a^{2}\right)}\sqrt{a}\,\log\left(\frac{a \cos\left(dx+c\right)^{3}-4\,\sqrt{a}\,\sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,{\left(\cos\left(dx+c\right)-2\right)}\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-7\,a \cos\left(dx+c\right)^{2}+8\,a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)}{6\,{\left(d \cos\left(dx+c\right)+d\right)}}, \frac{2\,{\left(a^{2} \cos\left(dx+c\right)+8\,a^{2}\right)}\sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,{\left(a^{2} \cos\left(dx+c\right)+a^{2}\right)}\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{a \cos\left(dx+c\right)^{2}-a \cos\left(dx+c\right)-2\,a}\right)}{3\,{\left(d \cos\left(dx+c\right)+d\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a
*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(co
s(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(a^2*cos(d*x
+ c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
+ c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^{3/2} \left(a+\frac{a}{\cos(c+dx)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.416 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=132

$$\frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}+\frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+$$

```
[Out] 5*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1
/2)+a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3899, 4100, 3886, 221}

$$\frac{5a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
```

```
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}}{d\sqrt{\cos(c+}}$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}}{d\sqrt{\cos(c+}}$$

$$= \frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}}{d}$$

Mathematica [A]
time = 0.32, size = 90, normalized size = 0.68

$$\frac{a^3\left(5\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}+\sqrt{1-\sec(c+dx)}\,(2+\sec(c+dx))\right)\sin(c+dx)}{d\sqrt{-1+\cos(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqrt[1 - Sec[c
+ d*x]]*(2 + Sec[c + d*x]))*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.13, size = 197, normalized size = 1.49

method	result
default	$-\frac{\left(5\cos(dx+c)\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-5\cos(dx+c)\sin}{4d\,\mathrm{si}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(5*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1
+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)-5*cos(d*x+c)
*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*
(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)+8*cos(d*x+c)^2-4*cos(d*x+c)-4)*(
a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 11494 vs. $2(114) = 228$.
time = 0.64, size = 11494, normalized size = 87.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^2*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c) + 16*a^2*cos(1/2*d*x
 + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^3 + 8*a^2*sin(1/2*d*x + 1/2*c)^5 + 5*(sqrt
(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^4 + 10*(sqrt(2)*a^2*
```

log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 5
*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)
*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*c)^4 + (8*a^2*sin
(1/2*d*x + 1/2*c)^3 + (5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) + 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a^2
*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 5*(sqrt(2)*a^2*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2 + (5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) + 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 8*a^2*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 5*(sqrt(2)
*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2

```
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*c)^2 + 2*(8*a^2*cos(1/2*d*
x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*
sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)...
```

Fricas [A]
time = 2.74, size = 373, normalized size = 2.83

$$\left[\frac{4\,{\left(2\,a^{2} \cos\left(d x + c\right) + a^{2}\right)} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right) + 5\,{\left(a^{2} \cos\left(d x + c\right)^{2} + a^{2} \cos\left(d x + c\right)\right)} \sqrt{a} \log\left(\frac{a \cos\left(d x + c\right)^{3} - 4\,\sqrt{a} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} {\left(\cos\left(d x + c\right) - 2\right)} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right) - 7\,a \cos\left(d x + c\right)^{2} + 8\,a}{\cos\left(d x + c\right)^{3} + \cos\left(d x + c\right)^{2}}\right)}{4\,{\left(d \cos\left(d x + c\right)^{2} + d \cos\left(d x + c\right)\right)}}, \frac{2\,{\left(2\,a^{2} \cos\left(d x + c\right) + a^{2}\right)} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right) + 5\,{\left(a^{2} \cos\left(d x + c\right)^{2} + a^{2} \cos\left(d x + c\right)\right)} \sqrt{-a} \arctan\left(\frac{2\,\sqrt{-a} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right)}{a \cos\left(d x + c\right)^{2} - a \cos\left(d x + c\right) - 2\,a}\right)}{2\,{\left(d \cos\left(d x + c\right)^{2} + d \cos\left(d x + c\right)\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))
*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d
*x + c)), 1/2*(2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a^2*cos(
d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.417 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d}+\frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\,\sqrt{a+a\sec(c+dx)}}+$$

```
[Out] 19/4*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+9/4*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x
+c))^(1/2)+1/2*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A]

time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3899, 4101, 3886, 221}

$$\frac{19a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (19*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (9*a^3*Sin[c + d*x])/(4*d*Cos[c +
d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
```

```
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} -$$

$$= \frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d} + \frac{}{4d\,\mathrm{co}}$$

Mathematica [A]

time = 0.71, size = 95, normalized size = 0.68

$$\frac{a^2\sqrt{a(1+\sec(c+dx))}\left(-\frac{19\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}}+\sqrt{1-\sec(c+dx)}\,(11+2\sec(c+dx))\right)\tan\left(\frac{1}{2}(c+dx)\right)}{4d\sqrt{-1+\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Sec[c + d*x])]*((-19*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec[
c + d*x]] + Sqrt[1 - Sec[c + d*x]]*(11 + 2*Sec[c + d*x]))*Tan[(c + d*x)/2])
/(4*d*Sqrt[-1 + Cos[c + d*x]])
```

Maple [A]
time = 0.14, size = 216, normalized size = 1.54

method	result
default	$\frac{\left(19\left(\cos^2(dx+c)\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-19\left(\cos^2(dx+c)\right)\sqrt{2}\,\arctan\left(\sqrt{-\frac{2}{1+\cos(\cdots}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(19*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos
(d*x+c)+sin(d*x+c))*2^(1/2))-19*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(
d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+22*cos(d*x+c)*sin(d*x+c)*
(-2/(1+cos(d*x+c)))^(1/2)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos
(d*x+c))/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/sin(d
*x+c)^2*(cos(d*x+c)^2-1)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2826 vs. 2(116) = 232.
time = 3.22, size = 2826, normalized size = 20.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a^
2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*
```

c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 - 7
6*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)
^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)
- 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
 - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c

```
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2
*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22
*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*
(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) +
 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2...
```

Fricas [A]
time = 2.87, size = 385, normalized size = 2.75

$$\left[\frac{4\,{\left(11\,a^{2} \cos\left(dx + c\right) + 2\,a^{2}\right)} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \sqrt{\cos\left(dx + c\right)} \sin\left(dx + c\right) + 19\,{\left(a^{2} \cos\left(dx + c\right)^{3} + a^{2} \cos\left(dx + c\right)^{2}\right)} \sqrt{a} \log\left(\frac{a \cos\left(dx + c\right)^{3} - 4\,\sqrt{a} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} {\left(\cos\left(dx + c\right) - 2\right)} \sqrt{\cos\left(dx + c\right)} \sin\left(dx + c\right) - 7\,a \cos\left(dx + c\right)^{2} + 8\,a}{\cos\left(dx + c\right)^{3} + \cos\left(dx + c\right)^{2}}\right)}{16\,{\left(d \cos\left(dx + c\right)^{3} + d \cos\left(dx + c\right)^{2}\right)}}, \frac{2\,{\left(11\,a^{2} \cos\left(dx + c\right) + 2\,a^{2}\right)} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \sqrt{\cos\left(dx + c\right)} \sin\left(dx + c\right) + 19\,{\left(a^{2} \cos\left(dx + c\right)^{3} + a^{2} \cos\left(dx + c\right)^{2}\right)} \sqrt{-a} \arctan\left(\frac{2\,\sqrt{-a} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \sqrt{\cos\left(dx + c\right)} \sin\left(dx + c\right)}{a \cos\left(dx + c\right)^{2} - a \cos\left(dx + c\right) - 2\,a}\right)}{8\,{\left(d \cos\left(dx + c\right)^{3} + d \cos\left(dx + c\right)^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x
+ c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos
(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 +
d*cos(d*x + c)^2), 1/8*(2*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d*x + c)
^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(
d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

3.418 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d}+\frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 25/8*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+13/12*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d
*x+c))^(1/2)+25/8*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+
1/3*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)
```

Rubi [A]
time = 0.25, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4349, 3899, 4101, 3888, 3886, 221}

$$\frac{25a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (25*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (13*a^3*Sin[c + d*x])/(12*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (25*a^3*Sin[c + d*x])/(8*d*Cos[c +
 d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(3*d*Cos[c + d*x]^(5/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
```

```
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \mathrm{se}$$

$$= \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{8d} + \frac{}{12d\,\mathrm{co}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 5.33, size = 180, normalized size = 1.00

$$\frac{a^2(1+\cos(c+dx))^2\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(-75ie^{\frac{1}{2}i(c+dx)}\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-e^{2i(c+dx)}\right)-25ie^{\frac{3}{2}i(c+dx)}\cos^3(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{2i(c+dx)}\right)+(8+34\cos(c+dx)+75\cos^2(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{96d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^5*Sqrt[a*(1 + Sec[c + d*x])]*((-
75*I)*E^((I/2)*(c + d*x))*Cos[c + d*x]^3*Hypergeometric2F1[1/4, 1, 5/4, -E^
((2*I)*(c + d*x))] - (25*I)*E^(((3*I)/2)*(c + d*x))*Cos[c + d*x]^3*Hypergeo
metric2F1[3/4, 1, 7/4, -E^((2*I)*(c + d*x))] + (8 + 34*Cos[c + d*x] + 75*Co
s[c + d*x]^2)*Sin[(c + d*x)/2]))/(96*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.15, size = 244, normalized size = 1.36

method	result

default	$-\frac{(-1+\cos(dx+c))\left(75(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-75(\cos^3(dx+c))\arctan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(75*cos(d*x+c)^3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+
c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-75*cos(d*x+c)^3*2^(1/2)*arcta
n(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+150*sin(
d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+68*cos(d*x+c)*sin(d*x+c)*(-2/
(1+cos(d*x+c)))^(1/2)+16*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(5/2)/(-2/(1+cos(d*x+c)))^(
1/2)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3469 vs. $2(150) = 300$.
time = 0.79, size = 3469, normalized size = 19.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)
*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*
a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6
*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12
*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) -
 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 4
56*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*
d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(1/3*
```

arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^
2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*
x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*
a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*
x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sq
rt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7
5*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^
2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2
+ 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*
a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) +
a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*
sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*l
og(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin

```
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(
6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*...
```

Fricas [A]
time = 2.85, size = 411, normalized size = 2.28

$$\left[\frac{4\,{\left(75\,a^{2} \cos\left(dx+c\right)^{2} + 34\,a^{2} \cos\left(dx+c\right) + 8\,a^{2}\right)} \sqrt{\frac{a \cos\left(dx+c\right) + a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right) + 75\,{\left(a^{2} \cos\left(dx+c\right)^{4} + a^{2} \cos\left(dx+c\right)^{3}\right)} \sqrt{a} \log\left(\frac{a \cos\left(dx+c\right)^{3} - 4\,\sqrt{a} \sqrt{\frac{a \cos\left(dx+c\right) + a}{\cos\left(dx+c\right)}} {\left(\cos\left(dx+c\right) - 2\right)} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right) - 7\,a \cos\left(dx+c\right)^{2} + 8\,a}{\cos\left(dx+c\right)^{3} + \cos\left(dx+c\right)^{2}}\right)}{96\,{\left(d \cos\left(dx+c\right)^{4} + d \cos\left(dx+c\right)^{3}\right)}}, \frac{2\,{\left(75\,a^{2} \cos\left(dx+c\right)^{2} + 34\,a^{2} \cos\left(dx+c\right) + 8\,a^{2}\right)} \sqrt{\frac{a \cos\left(dx+c\right) + a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right) + 75\,{\left(a^{2} \cos\left(dx+c\right)^{4} + a^{2} \cos\left(dx+c\right)^{3}\right)} \sqrt{-a} \arctan\left(\frac{2\,\sqrt{-a} \sqrt{\frac{a \cos\left(dx+c\right) + a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{a \cos\left(dx+c\right)^{2} - a \cos\left(dx+c\right) - 2\,a}\right)}{48\,{\left(d \cos\left(dx+c\right)^{4} + d \cos\left(dx+c\right)^{3}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos(d
*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(75*a^2*cos(d*x + c)^2 +
 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*s
qrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d
*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

3.419 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64d}+\frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

```
[Out] 163/64*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d+17/24*a^3*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec
(d*x+c))^(1/2)+163/96*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1
/2)+163/64*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/4*a^2
*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A]
time = 0.29, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4349, 3899, 4101, 3888, 3886, 221}

$$\frac{163a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{163a^3\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (163*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (17*a^3*Sin[c + d*x])/(24*d*Cos[
c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Sin[c + d*x])/(96*d*Cos
[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Sin[c + d*x])/(64*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{4}\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64d} + \frac{}{24d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 5.56, size = 190, normalized size = 0.86

$$\frac{a^2(1+\cos(c+dx))^2\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(-489ie^{\frac{1}{2}i(c+dx)}\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-e^{2i(c+dx)}\right)-163ie^{\frac{3}{2}i(c+dx)}\cos^4(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{2i(c+dx)}\right)+\left(48+184\cos(c+dx)+326\cos^2(c+dx)+489\cos^3(c+dx)\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{768d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^5*Sqrt[a*(1 + Sec[c + d*x])]*((-
489*I)*E^((I/2)*(c + d*x))*Cos[c + d*x]^4*Hypergeometric2F1[1/4, 1, 5/4, -E
^((2*I)*(c + d*x))] - (163*I)*E^(((3*I)/2)*(c + d*x))*Cos[c + d*x]^4*Hyperg
eometric2F1[3/4, 1, 7/4, -E^((2*I)*(c + d*x))] + (48 + 184*Cos[c + d*x] + 3
26*Cos[c + d*x]^2 + 489*Cos[c + d*x]^3)*Sin[(c + d*x)/2]))/(768*d*Cos[c + d
*x]^(7/2))
```

Maple [A]
time = 0.12, size = 276, normalized size = 1.25

method	result
default	$\frac{\left(-489(\cos^4(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)+489(\cos^4(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+c}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768/d*(-489*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+
cos(d*x+c)-sin(d*x+c))*2^(1/2))+489*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+
cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+978*sin(d*x+c)*cos(d*
x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)+652*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x
+c)))^(1/2)+368*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+96*sin(d*x+
c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2/(1+co
s(d*x+c)))^(1/2)/cos(d*x+c)^(7/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3860 vs. $2(184) = 368$.
time = 0.84, size = 3860, normalized size = 17.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) +
 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x
+ 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*si
n(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x +
 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2
*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*
sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x
```

+ 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(sqrt(
2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x
+ 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 4
8*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2
*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c
) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4
*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*
x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x +
4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos
(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a
^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^
2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x +
2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*
x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2
*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*
x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*
d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2
+ 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16
*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x
+ 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x
+ 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*
x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*si
n(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x +
6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*
cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2
+ a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4
*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^

```
2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*
d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(
4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*c
os(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2
*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin...
```

Fricas [A]
time = 2.84, size = 437, normalized size = 1.99

$$\left[\frac{4\left(489\,a^{2}\cos\left(dx+c\right)^{3}+326\,a^{2}\cos\left(dx+c\right)^{2}+184\,a^{2}\cos\left(dx+c\right)+48\,a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+489\left(a^{2}\cos\left(dx+c\right)^{5}+a^{2}\cos\left(dx+c\right)^{4}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7a\cos(dx+c)^{2}+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{768\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)},\ \frac{2\left(489\,a^{2}\cos\left(dx+c\right)^{3}+326\,a^{2}\cos\left(dx+c\right)^{2}+184\,a^{2}\cos\left(dx+c\right)+48\,a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+489\left(a^{2}\cos\left(dx+c\right)^{5}+a^{2}\cos\left(dx+c\right)^{4}\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2a}\right)}{384\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*
x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(a)*log((
a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d
*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(c
os(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/
384*(2*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x +
 c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*c
os(d*x + c)^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)
```

3.420 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2/5*cos(d*x+c)^(3
/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+26/15*sin(d*x+c)/d/cos(d*x+c)^(1/2)
/(a+a*sec(d*x+c))^(1/2)-2/15*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4349, 3908, 4107, 4098, 3893, 212}

$$\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
 (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a
+ b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-4a\,\mathrm{se}}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+}}}{5a}$$

$$= -\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos}{5d\sqrt{}}$$

$$= \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos}{5d\sqrt{}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} + \frac{}{1}$$

Mathematica [A]
time = 0.32, size = 136, normalized size = 0.72

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(15\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}\,(3-\sec(c+dx)+13\sec^2(c+dx))\right)\sin(c+dx)}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
- Sec[c + d*x]]]*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c +
 d*x] + 13*Sec[c + d*x]^2))*Sin[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt
[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.13, size = 120, normalized size = 0.63

method	result

default	$\frac{\left(15 \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\, \sin(dx+c)-6\left(\cos^3(dx+c)\right)+8\left(\cos^2(dx+c)\right)-28\cos(dx+c)+26\right)}{15d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/d*(15*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)-6*cos(d*x+c)^3+8*cos(d*x+c)^2-28*cos(d*x+c)+26)*(a*(1
+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(156) = 312$.
time = 0.55, size = 357, normalized size = 1.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

Fricas [A]
time = 3.18, size = 324, normalized size = 1.71

$$\left[\frac{4\left(3\cos(dx+c)^2-\cos(dx+c)+13\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\frac{15\sqrt{2}\left(a\cos(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{30\left(ad\cos(dx+c)+ad\right)}, \frac{15\sqrt{2}\left(a\cos(dx+c)+a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+2\left(3\cos(dx+c)^2-\cos(dx+c)+13\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{15\left(ad\cos(dx+c)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(4*(3*cos(d*x + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c) +
 a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqr
t(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*cos(d*x
 + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.421 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1
/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/3*sin(d*x+c)/d/c
os(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a
+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3908, 4098, 3893, 212}

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a
+ b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a}{\sqrt{\sec(c+dx}}}{3a}$$

$$= -\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \left(\sqrt{c}\right.$$

$$= -\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{\left(2\sqrt{}\right.}{}$$

$$= \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} - \frac{}{3d}$$

Mathematica [A]

time = 0.21, size = 116, normalized size = 0.77

$$\frac{\sqrt{\cos(c+dx)}\left(2(1-\sec(c+dx))^{3/2}-3\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{3}{2}}(c+dx)\right)\sin(c+dx)}{3d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(1 - Sec[c + d*x])^(3/2) - 3*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2))*Sin[c + d*
x])/(3*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.13, size = 110, normalized size = 0.73

method	result
default	$-\frac{\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)+2\left(\cos^2(dx+c)\right)-4\cos(dx+c)+2\right)\sqrt{\frac{a(1+\cos(dx+}{\cos(dx+c)}}}{3d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2-4*cos(d*x+c)+2)*(a*(1+cos(d*x+c))/cos(d
*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(124) = 248.
time = 0.55, size = 282, normalized size = 1.87

$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)-3\sqrt{2}\log\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^2+\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^2+2\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+1\right)+3\sqrt{2}\log\left(\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^2+\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)^2-2\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+1\right)-2\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sqrt{2}\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)}{6\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
```

```
cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

Fricas [A]
time = 3.77, size = 300, normalized size = 1.99

$$\left[\frac{4 \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \left(\cos\left(dx+c\right)-1\right) \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)+\frac{3 \sqrt{2} \left(a \cos(dx+c)+a\right) \log\left(-\frac{\cos(dx+c)^2-\frac{2 \sqrt{2} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin(dx+c)}{\sqrt{a}}-2 \cos(dx+c)-3}{\cos(dx+c)^2+2 \cos(dx+c)+1}\right)}{\sqrt{a}}}{6 \left(ad \cos\left(dx+c\right)+ad\right)}, -\frac{3 \sqrt{2} \left(a \cos\left(dx+c\right)+a\right) \sqrt{-\frac{1}{a}} \arctan\left(\frac{\sqrt{2} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{-\frac{1}{a}} \sqrt{\cos\left(dx+c\right)}}{\sin(dx+c)}\right)-2 \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \left(\cos\left(dx+c\right)-1\right) \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{3 \left(ad \cos\left(dx+c\right)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sqrt(cos
(d*x + c))*sin(d*x + c) + 3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)
^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*(a*cos(d*x + c) +
a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-
1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c
) + a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{a \left(\sec\left(c+dx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.422 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+}}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2*sin(d*x+c)/d/co
s(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3897, 3893, 212}

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
 (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3897

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(m + 1))), x] + Dist[a*(m/(b*d*(m + 1))), Int[(a + b*Csc[e
+ f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} - \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int -$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\text{Sul}}{}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} +$$

Mathematica [A]
time = 0.10, size = 100, normalized size = 0.88

$$\frac{\left(2\sqrt{1-\sec(c+dx)}+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{d\sqrt{-1+\cos(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c
+ d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.11, size = 98, normalized size = 0.87

method	result
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-2\cos(dx+c)+2\right)}{d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(arctan(1/2*sin(d*
x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*cos(
d*x+c)+2)/sin(d*x+c)/a
```

Maxima [A]
time = 0.54, size = 104, normalized size = 0.92

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-4\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/(sq
rt(a)*d)
```

Fricas [A]
time = 3.01, size = 281, normalized size = 2.49

$$\left[\frac{\frac{\sqrt{2}\,(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)},\ \frac{\sqrt{2}\,(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{ad\cos(dx+c)+ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) -
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*c
os(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(
```

```
d*x + c)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{a\,(\sec{(c+dx)}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cos(d*x + c))/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.423 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4349, 3893, 212}

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{}}{\sqrt{}}\right.}{d}$$
$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\text{se}}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.08, size = 95, normalized size = 1.70

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt
[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*S
qrt[a*(1 + Sec[c + d*x])]))
```

Maple [A]
time = 0.12, size = 91, normalized size = 1.62

method	result	size
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos^2(dx+c)-1)}{d\sin(dx+c)^2 a}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*arctan(1/2*sin(d*x
+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(
d*x+c)^2-1)/a
```

Maxima [A]
time = 0.53, size = 90, normalized size = 1.61

$$\frac{\sqrt{2}\,\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\,\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{2\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Fricas [A]
time = 3.15, size = 160, normalized size = 2.86

$$\left[\frac{\sqrt{2}\,\log\left(-\frac{\cos(dx+c)^2-\frac{2\,\sqrt{2}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{\sqrt{a}}-2\,\cos(dx+c)-3}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)}{2\,\sqrt{a}\,d},\ -\frac{\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{-\frac{1}{a}}\,\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x +
 c))/sin(d*x + c))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.424 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d/a^(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/
2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1
/2)
```

Rubi [A]
time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4349, 3906, 3886, 221, 3893, 212}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]
]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3906

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e
 + f*x]], x], x] - Dist[a*(d/b), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= -\left(\left(\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx\right.\right.$$

$$= \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(}}{\sqrt{a+}}\right.}{d}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.09, size = 109, normalized size = 0.81

$$\frac{\left(-2\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\sqrt{\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]

[Out] ((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]])*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

Maple [A]
time = 0.14, size = 172, normalized size = 1.27

method	result
default	$-\frac{\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c}{4}\right.\right.}{d\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c)
)*2^(1/2))-2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d
*x+c))*2^(1/2))-2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*(a*(1+c
os(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^(1/2)/(-2/(1+cos(d*
x+c)))^(1/2)/sin(d*x+c)^2/a

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(110) = 220$.
time = 0.56, size = 476, normalized size = 3.53

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si

```
n(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

Fricas [A]
time = 3.11, size = 342, normalized size = 2.53

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2ad}, \frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c
) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x + c)
^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*s
qrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) +
sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sec(c+dx)+1)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos \left(c+d\,x\right)^{3/2}\,\sqrt{a+\frac{a}{\cos \left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.425 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=168

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d/a^(1/2)+arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2
)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/
2)+sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4349, 3907, 4108, 3893, 212, 3886, 221}

$$\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*Sqr
t[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[d^2/(b*(2*n - 3)), Int[(d
*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.$$

$$= \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= -\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.24, size = 145, normalized size = 0.86

$$\frac{\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)+2\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sin(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*
ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c +
d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.14, size = 212, normalized size = 1.26

method	result

default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(-\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}+\cos(dx+\right.}{2d\sqrt{\cos\left(d\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-cos(d*x+c)*arc
tan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2
)+cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))
*2^(1/2))*2^(1/2)+2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+4*cos(d*x+c)*arcta
n(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))/cos(d*x+c)^(1/2)/sin(d*x+c)^2/
(-2/(1+cos(d*x+c)))^(1/2)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(139) = 278$.
time = 0.57, size = 876, normalized size = 5.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
```

```
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
+ 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)
```

Fricas [A]
time = 3.23, size = 520, normalized size = 3.10

$$\left[\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3+4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{2\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{4\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)},\ -\frac{2\sqrt{2}\left(a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 2*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*log(-(cos(d*x + c)
^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1))/sqrt(a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/2*(2*sqrt(2)*(a
*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + (cos
(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.426 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=211

$$\frac{7\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 7/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d/a^(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(
1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^
(1/2)+1/2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)-1/4*sin(d*x+
c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4349, 3907, 4106, 4108, 3893, 212, 3886, 221}

$$-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (7*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(2*d*Cos[c + d*x]^(5/2)*
Sqrt[a + a*Sec[c + d*x]]) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a
*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[d^2/(b*(2*n - 3)), Int[(d
*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{7\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4\sqrt{a}\,d}$$

Mathematica [A]
time = 0.43, size = 178, normalized size = 0.84

$$\frac{\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(-\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)-8\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)+4\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)-\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sin(c+dx)}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(-ArcSin[Sqrt[1 - Sec[c + d*x]]] - 8
*ArcSin[Sqrt[Sec[c + d*x]]] + 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])
/Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) - Sq
rt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.15, size = 247, normalized size = 1.17

method	result

default	$\frac{(-1+\cos(dx+c))\left(-7(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)+7(\cos^2(dx+c))\sqrt{2}\,\arctan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(-7*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+7*cos(d*x+c)^2*2^(1/2)*arctan(1
/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+2*cos(d*x+c
)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+16*cos(d*x+c)^2*arctan(1/2*sin(d*x+c
)*(-2/(1+cos(d*x+c)))^(1/2))-4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*
x+c)^(3/2)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1646 vs. $2(172) = 344$.
time = 0.59, size = 1646, normalized size = 7.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2
)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*co
```

```
s(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
 + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(
sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d
*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) +
 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) +
2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)
```

Fricas [A]
time = 3.65, size = 550, normalized size = 2.61

$$\left[-\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)-\frac{8\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{16\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}, \frac{8\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)+7\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{8\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(c
os(d*x + c))*sin(d*x + c) - 7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log
((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos
(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/
```

```
(cos(d*x + c)^3 + cos(d*x + c)^2)) - 8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*
x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*
x + c)^2), 1/8*(8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-1/a)*
arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(
d*x + c))/sin(d*x + c)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 7*(cos(d*x + c)^3 + cos(d*x +
 c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/
(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.427 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}-\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{}{10ad}$$

```
[Out] -1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-15/4*arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+9/10*cos(d*x+c)^(3/2)*sin(d*x+
c)/a/d/(a+a*sec(d*x+c))^(1/2)+49/10*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*se
c(d*x+c))^(1/2)-13/10*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2
)
```

Rubi [A]

time = 0.41, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4349, 3902, 4107, 4098, 3893, 212}

$$-\frac{15\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{9\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{13\sin(c+dx)\sqrt{\cos(c+dx)}}{10ad\sqrt{a\sec(c+dx)+a}}+\frac{49\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d
) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (4
9*Sin[c + d*x])/(10*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (13*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) + (9*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{9a}{2}\cdots}{\sec^{\frac{5}{2}}(c+dx)\sqrt{\cdots}}}{2a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{9\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\cdots\right.}{\cdots}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} + \frac{9\cos^{\frac{3}{2}}(c+dx)\cdots}{10ad\sqrt{a+a\,\mathrm{s}\cdots}}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{13\cdots}{1\cdots}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{13\cdots}{1\cdots}$$

$$= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d} - \cdots$$

Mathematica [A]
time = 0.97, size = 152, normalized size = 0.64

$$\frac{75\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+\sqrt{1-\sec(c+dx)}\,(4(9+\cos^2(c+dx))\sin(c+dx)-2\sin(2(c+dx))+49\tan(c+dx))}{10d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos
[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(4
*(9 + Cos[c + d*x]^2)*Sin[c + d*x] - 2*Sin[2*(c + d*x)] + 49*Tan[c + d*x]))
/(10*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.14, size = 193, normalized size = 0.81

method	result
default	$-\frac{\left(75\left(\cos^2(dx+c)\right)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}-8\left(\cos^5(dx+c)\right)+24\left(\cos^4(dx+c)\right)-75 a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20/d*(75*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)
))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)-8*cos(d*x+c)^5+24*cos(d*x+c)^4-75*arcta
n(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)-96*cos(d*x+c)^3+54*cos(d*x+c)^2+124*cos(d*x+c)-98)*(a*(1+cos(d*x+c))/
cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^3/a^2
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]

time = 4.09, size = 400, normalized size = 1.69

$$\left[\frac{75\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left(4\cos(dx+c)^3-4\cos(dx+c)^2+36\cos(dx+c)+49\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{40\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, \frac{75\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+2\left(4\cos(dx+c)^3-4\cos(dx+c)^2+36\cos(dx+c)+49\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{20\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x +
c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/20*(75*sqrt
(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))
) + 2*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x + c) + 49)*sqrt((a*
```

```
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.428 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{}{6ad\sqrt{c}}$$

```
[Out] -1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(3/2)+11/4*arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-19/6*sin(d*x+c)/a/d/cos(d*x+c)
^(1/2)/(a+a*sec(d*x+c))^(1/2)+7/6*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(
d*x+c))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4349, 3902, 4107, 4098, 3893, 212}

$$\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (11*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - (19
*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (7*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{7a}{2}\ldots}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\ldots}}}{2a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx}\ldots\right.}{\ldots}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{\ldots}}{6\ldots}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{\ldots}}{6\ldots}$$

$$= \frac{11\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d} - \frac{\sqrt{c\ldots}}{2d\ldots}$$

Mathematica [A]
time = 0.67, size = 133, normalized size = 0.68

$$\frac{\left((-12+4\cos(c+dx)-19\sec(c+dx))\sqrt{1-\sec(c+dx)} - 33\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\right)\sin(c+dx)}{6d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-12 + 4*Cos[c + d*x] - 19*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 33*Sqrt
[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*
x)/2]^2*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(6*d*Sqrt[-1 + Cos[c + d*x]]*(a*(
1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.13, size = 183, normalized size = 0.93

method	result

default	$\frac{\left(33(\cos^2(dx+c))\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}+8(\cos^4(dx+c))-33\arctan\left(\frac{\sin(dx+c)\sqrt{\ldots}}{\ldots}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/d*(33*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))
)^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)+8*cos(d*x+c)^4-33*arctan(1/2*sin(d*x+c)*
(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-40*cos(d*x+
c)^3+18*cos(d*x+c)^2+52*cos(d*x+c)-38)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*
cos(d*x+c)^(1/2)/sin(d*x+c)^3/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33960 vs. $2(162) = 324$.
time = 0.94, size = 33960, normalized size = 172.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3
/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d*x +
 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 +
 sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 4*
sin(3/2*d*x + 3/2*c)^5 + 4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3
*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*
c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(7/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(cos(3*d*x + 3
*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(c
os(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^4 + 4*(2*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x +
 3/2*c) + 2*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) +
2*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 8*(cos(3*d*x
 + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) -
9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 2*(sin(3/2*d*x + 3/2*c)^2 + 3)*sin(3*d*x + 3*c) + 20*(co
```

s(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 7*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 18*(cos(3*d*x +
3*c)^2*cos(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2 + cos
(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c))
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 32*(2*cos(3*d*x + 3*c)
^2*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/2*c)*sin(3
*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 2*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c
)*sin(3/2*d*x + 3/2*c) + 2*(sin(3/2*d*x + 3/2*c)^2 + 3)*sin(3*d*x + 3*c) +
20*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 7*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2
)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 18*(cos(3*
d*x + 3*c)^2*cos(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2
 + cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)*sin(3/2*d*x + 3
/2*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 4*(2*cos(3/2*d*x
 + 3/2*c)^2 + 7)*sin(3/2*d*x + 3/2*c)^3 + 4*((2*sin(3/2*d*x + 3/2*c)^2 + 1)
*cos(3*d*x + 3*c)^2 + (2*sin(3/2*d*x + 3/2*c)^2 + 1)*sin(3*d*x + 3*c)^2 + 2
*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 2*(sin(3/2*d*
x + 3/2*c)^2 + 3)*cos(3*d*x + 3*c) - 20*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3
*c)^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(co
s(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 8*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin
(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x +
3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 18*(cos(3*d*x + 3
*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + cos(
3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c))*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(7/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 32*((2*sin(3/2*d*x + 3/
2*c)^2 + 1)*cos(3*d*x + 3*c)^2 + (2*sin(3/2*d*x + 3/2*c)^2 + 1)*sin(3*d*x +
 3*c)^2 + 2*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 2*
(sin(3/2*d*x + 3/2*c)^2 + 3)*cos(3*d*x + 3*c) - 20*(cos(3*d*x + 3*c)^2 + si
n(3*d*x + 3*c)^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 7*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 18*(cos(3*d*x + 3*c)^2*sin(3/2*d*x +
3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin
(3*d*x + 3*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c))*sin(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^3 + 4*(4*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)^
2*sin(3/2*d*x + 3/2*c) + (sin(3/2*d*x + 3/2*c)^3 + (cos(3/2*d*x + 3/2*c)^2
+ 1)*sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c)^2 + 24*(cos(3*d*x + 3*c)^2*sin(
3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x +
 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2

```
*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + ...
```

Fricas [A]
time = 4.61, size = 380, normalized size = 1.93

$$\left[\frac{33\,\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+4\left(4\,\cos\left(dx+c\right)^{2}-12\,\cos\left(dx+c\right)-19\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{24\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)},\ -\frac{33\,\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)-2\left(4\,\cos\left(dx+c\right)^{2}-12\,\cos\left(dx+c\right)-19\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 - 12*cos(d*x + c) - 19)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d
*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*sqrt(2)*(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(4*cos(d*x +
 c)^2 - 12*cos(d*x + c) - 19)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.429 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+d\cdots}$$

```
[Out] -1/2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-7/4*arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+5/2*sin(d*x+c)/a/d/cos(d*x+c)^(
1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3902, 4098, 3893, 212}

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
 - Sin[c + d*x]/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (5*Si
n[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+}}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d} - \frac{}{2d}$$

Mathematica [A]

time = 0.94, size = 138, normalized size = 0.88

$$\frac{\sqrt{\sec(c+dx)}\left(7\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)+(5+4\cos(c+dx))\sqrt{(-1+\cos(c+dx))\sec^2(c+dx)}\right)\sin(c+dx)}{2d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 -
 Sec[c + d*x]]]*Cos[(c + d*x)/2]^2*Sec[c + d*x] + (5 + 4*Cos[c + d*x])*Sqrt
[(-1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(2*d*Sqrt[-1 + Cos[c +
d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.12, size = 173, normalized size = 1.10

method	result
default	$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(7\left(\cos^2(dx+c)\right)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\right.}{4d\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(7*sin(d*x+c)*c
os(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x
+c)))^(1/2)-7*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-8*cos(d*x+c)^3+6*cos(d*x+c)^2+12*cos(d*x+c)-10)/si
n(d*x+c)^3/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 7176 vs. 2(128) = 256.
time = 0.61, size = 7176, normalized size = 45.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)
^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
```

in(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin(1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*sin(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + s

```
in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c
))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (
7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x +
 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)
 - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + ...
```

Fricas [A]
time = 3.15, size = 360, normalized size = 2.29

$$\left[\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos\left(dx+c\right)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2a\cos\left(dx+c\right)-3a}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)+4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(4\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}, \frac{7\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin\left(dx+c\right)}\right)+2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(4\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x + c
) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sq
rt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*(4*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.430 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

```
[Out] -1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+3/4*arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4349, 3896, 3893, 212}

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3896

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(3\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}}{}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(3\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [A]
time = 0.47, size = 131, normalized size = 1.12

$$-\frac{\left(2\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}+3\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)(1+\sec(c+dx))\right)\sin(c+dx)}{4ad\sqrt{-1+\cos(c+dx)}\,(1+\cos(c+dx))\sqrt{\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -1/4*((2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 3*Sqrt[2]*ArcTan[(Sqrt
[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x]))*Sin[c +
 d*x])/(a*d*Sqrt[-1 + Cos[c + d*x]]*(1 + Cos[c + d*x])*Sqrt[Sec[c + d*x]]*S
qrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 0.14, size = 138, normalized size = 1.18

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{\cos}(dx+c)\right)\left(\cos(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}+3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)-\right.}{4d\sin(dx+c)^{3}a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)*(-2/
(1+cos(d*x+c)))^(1/2)+3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*si
n(d*x+c)-(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*
(cos(d*x+c)^2-1)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(94) = 188$.
time = 0.56, size = 1031, normalized size = 8.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*si
n(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
+ c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(
d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2*
sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
```

```
))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c)
*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) +
 4*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(d
*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*sin
(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2*sq
rt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a)*d)
```

Fricas [A]
time = 2.44, size = 340, normalized size = 2.91

$$\left[\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)-4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}, -\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -
1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*si
n(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.431 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}+\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+1/4*arctanh(1/2*si
n(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3895, 3893, 212}

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) +
Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
```

```
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.}{}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(117) = 234.
time = 0.92, size = 248, normalized size = 2.12

$$\frac{\sqrt{\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\left(-\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos(c+dx)+2\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\cos(c+dx))+2\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\cos(c+dx))+\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+\cos(2(c+dx))\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\right)\sin(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[
c + d*x]])/Sqrt[1 - Sec[c + d*x]]]) - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c + d*x
]]]*(1 + Cos[c + d*x]) + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c + d*x]) +
Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*Sqrt[1 - Sec[c
 + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(
1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.13, size = 136, normalized size = 1.16

method	result
default	$\frac{\left(\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)-\cos(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}+\sqrt{-\frac{2}{1+\cos(dx+c)}}\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{4d\sin(dx+c)^{3}a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-cos(d*x+
c)*(-2/(1+cos(d*x+c)))^(1/2)+(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/c
os(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(c
os(d*x+c)^2-1)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15721 vs. $2(94) = 188$.
time = 1.13, size = 15721, normalized size = 134.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d
*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(
d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) +
3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3
/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3
*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1
)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*
x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(cos(3/2*d*x
+ 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + cos(d*x
 + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(3*d*x +
 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 9
6*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2*c)^2*sin(3/2*d
*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3
/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*
d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) +
 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
```

cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3
/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*(2*(3*cos(d*x + c) + 1)*cos(2*
d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
 + 3*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 2*(3*sin(3/2*d*x + 3/2*c)*si
n(d*x + c) + cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) + (3*cos(d*x + c)^2 + 3
*sin(d*x + c)^2 + 2*cos(d*x + c))*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/
2*c)*sin(d*x + c))*cos(3*d*x + 3*c) - 4*(6*(sin(2*d*x + 2*c) + sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^3 + (2*(3*cos(2*d*x + 2*c) + 3*cos(
d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1
)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x
+ 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x
+ c) + 1)*sin(3*d*x + 3*c) + 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)
*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x +
 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin
(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 1
8*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*(2*(3*cos(2*
d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*
(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x +
c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3
*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x
 + c)^2 + 6*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*s
in(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*x + c)*sin(3/2*d*x + 3/2*c
) - 8*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*sin(2*d*x + 2*c)^2*sin(3
/2*d*x + 3/2*c) - 16*(3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 3*cos(3/2*d
*x + 3/2*c)*sin(d*x + c) - sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c) - 48*(cos
(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c
) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*
x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*
c)*sin(d*x + c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 16*(cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + 3*sin(2*d*x + 2*c)*sin(3/2
*d*x + 3/2*c) + 3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c))
*sin(3*d*x + 3*c) - 48*(3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x +
 3/2*c))*sin(2*d*x + 2*c) - 8*(9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 - 1)*sin
(3/2*d*x + 3/2*c) - 48*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 3*(2*(3*cos(2*d*
x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3
*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)
^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c
)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x +
 c)^2 + 6*cos(d*x + c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d

```
*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*sin
(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*...
```

Fricas [A]
time = 2.29, size = 338, normalized size = 2.89

$$\left[\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+4\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)},-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)-2\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/
4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*
x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.432 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{c}}{2\sqrt{2}\,a^{3/2}d}$$

```
[Out] -1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*arcsinh(a^(1/2)
*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/
2)/d-5/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d
*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4349, 3901, 4108, 3893, 212, 3886, 221}

$$-\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2
)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.}{}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.}{}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.}{}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{3/2}d} -$$

Mathematica [A]

time = 0.80, size = 248, normalized size = 1.43

$$-\frac{\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(-5\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-5\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos(c+dx)+2\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\cos(c+dx))+10\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\cos(c+dx))+\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+\cos(2(c+dx))\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\right)\sin(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -1/4*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-5*Sqrt[2]*ArcTan[(Sqrt[2]*Sqr
t[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] - 5*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[S
ec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[
c + d*x]]]*(1 + Cos[c + d*x]) + 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c +
d*x]) + Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*Sqrt[1
 - Sec[c + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]

time = 0.13, size = 228, normalized size = 1.31

method	result

default	$-\frac{\left(2\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sin(dx+c)-2\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(}{4}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*
x+c))*2^(1/2))*sin(d*x+c)-2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1
+cos(d*x+c)-sin(d*x+c))*2^(1/2))*sin(d*x+c)-5*arctan(1/2*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2))*sin(d*x+c)+cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-(-2/(1+
cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos
(d*x+c)^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. $2(141) = 282$.
time = 0.59, size = 2122, normalized size = 12.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*c
os(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2
```

(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)

Fricas [A]
time = 2.51, size = 550, normalized size = 3.16

$$\left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)-4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{8\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}, \frac{5\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+4\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)-2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*c
os(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x
+ c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*a
rctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))/(a*sin(d*x + c))) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)
*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
 + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.433 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}$$

```
[Out] -1/2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)-3*arcsinh(a^(1/2)
*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/
2)/d+9/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d
*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+3/2*sin(d
*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4349, 3901, 4106, 4108, 3893, 212, 3886, 221}

$$\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{3\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-3*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + (9*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(5/
2)*(a + a*Sec[c + d*x])^(3/2)) + (3*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(3/2)
*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.}{}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}}{a^{3/2}d}$$

Mathematica [A]
time = 0.99, size = 242, normalized size = 1.13

$$\frac{\sqrt{\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\left(-9\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-9\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos(c+dx)+6\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(1+\cos(c+dx))+18\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(1+\cos(c+dx))+4\sqrt{(-1+\cos(c+dx))\sec^2(c+dx)}+6\cos(c+dx)\sqrt{(-1+\cos(c+dx))\sec^2(c+dx)}\right)\sin(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec
[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c
+ d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d
*x]]]*(1 + Cos[c + d*x]) + 18*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c + d*x])
 + 4*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2] + 6*Cos[c + d*x]*Sqrt[(-1 + C
os[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*
(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 0.14, size = 271, normalized size = 1.27

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(3\cos(dx+c)\sin(dx+c)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(3*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-3*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))-9*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+3*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-2*(-2/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/a^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(175) = 350$.
time = 0.87, size = 4934, normalized size = 23.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] -1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2

c)))^2 + 2(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sq
rt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*s
in(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqrt(
2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(
2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqr
t(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*
x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt
(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*
c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(
2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
 + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c)

```
+ 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +...
```

Fricas [A]
time = 3.08, size = 614, normalized size = 2.87

$$\left[\frac{9\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(3\cos(dx+c)+2\right)\sqrt{\cos(dx+c)}\sin(dx+c)+6\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3+4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(\cos(dx+c)-2\right)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{8\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}, -\frac{9\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(3\cos(dx+c)+2\right)\sqrt{\cos(dx+c)}\sin(dx+c)+6\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{4\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(
3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(cos(d*x + c)^3 + 2
*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -
1/4*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*
arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*co
s(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(cos(d*x + c)^3 + 2*cos
(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d
*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.434 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{17\sqrt{c}}{16ad}$$

```
[Out] -1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(5/2)-17/16*sin(d*x+c)*
cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(3/2)+163/32*arctanh(1/2*sin(d*x+c)*a
^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)-299/48*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(
a+a*sec(d*x+c))^(1/2)+95/48*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+
c))^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4349, 3902, 4105, 4107, 4098, 3893, 212}

$$\frac{163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{95\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{17\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (
17*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - (
299*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(95*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{11a}{2}+3}{\sec^{\frac{3}{2}}(c+dx)(a}}{4a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+d}\right.}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{95\sqrt{\cos(c+}}{48a^2d\sqrt{a+}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{2}{48a^2d\sqrt{\cos(c}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{2}{48a^2d\sqrt{\cos(c}}$$

$$= \frac{163\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d} - \frac{\sqrt{}}{4}$$

Mathematica [A]
time = 1.18, size = 144, normalized size = 0.61

$$-\frac{\left(1956\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}\,(160-32\cos(c+dx)+503\sec(c+dx)+299\sec^2(c+dx))\right)\sin(c+dx)}{96d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/96*((1956*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d
*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(160
 - 32*Cos[c + d*x] + 503*Sec[c + d*x] + 299*Sec[c + d*x]^2))*Sin[c + d*x])/
(d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]
time = 0.14, size = 244, normalized size = 1.03

method	result

default	$-\frac{(-1+\cos(dx+c))^2\left(489(\cos^2(dx+c))\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}+978\sin(dx+c)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96/d*(-1+cos(d*x+c))^2*(489*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c
)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)+978*sin(d*x+c)*cos(d
*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^
(1/2)+64*cos(d*x+c)^4+489*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*
(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-384*cos(d*x+c)^3-686*cos(d*x+c)^2+408*
cos(d*x+c)+598)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*
x+c)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 148823 vs. $2(196) = 392$.
time = 3.54, size = 148823, normalized size = 627.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(
3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*sin(3/2
*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3
*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 288*sin(3/2*d*x + 3/2*c)^5 + 32*(
cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x +
3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(11/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2
*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + si
n(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 8192*
(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x +
```

3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 4*(16*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 16*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 80*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c))*cos(9/2*d*x + 9/2*c) + 192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 128*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - (48*sin(3/2*d*x + 3/2*c)^2 - 751)*sin(3*d*x + 3*c) + 33*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(10/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 409*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 519*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 88*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 240*(cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2 + 4*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*cos(9/2*d*x + 9/2*c) + 5*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) - 3*sin(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 864*(16*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 16*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 80*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c))*cos(9/2*d*x + 9/2*c) + 128*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - (48*sin(3/2*d*x + 3/2*c)^2 - 751)*sin(3*d*x + 3*c) + 519*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 88*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 240*(cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2 + 4*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*cos(9/2*d*x + 9/2*c) + 5*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) - 3*sin(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^3 + 256*(16*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 16*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + 80*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*c) + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c))*cos(9/2*d*x + 9/2

```
*c) - (48*sin(3/2*d*x + 3/2*c)^2 - 751)*sin(3*d*x + 3*c) + 519*(cos(3*d*x +
 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 88*(cos(3*d*x + 3*c)^2 + sin(...
```

Fricas [A]
time = 3.20, size = 448, normalized size = 1.89

$$\left[\frac{489 \sqrt{2} \left(\cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \sqrt{a} \log\left(-\frac{a \cos(dx+c)^{2}-2 \sqrt{2} \sqrt{a} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin(dx+c)-2 a \cos(dx+c)-3 a}{\cos(dx+c)^{2}+2 \cos(dx+c)+1}\right)+4 \left(32 \cos\left(dx+c\right)^{3}-160 \cos\left(dx+c\right)^{2}-503 \cos\left(dx+c\right)-299\right) \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{192 \left(a^{3} d \cos\left(dx+c\right)^{3}+3 a^{3} d \cos\left(dx+c\right)^{2}+3 a^{3} d \cos\left(dx+c\right)+a^{3} d\right)}, -\frac{489 \sqrt{2} \left(\cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \sqrt{-a} \arctan\left(\frac{\sqrt{2} \sqrt{-a} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)}}{a \sin(dx+c)}\right)-2 \left(32 \cos\left(dx+c\right)^{3}-160 \cos\left(dx+c\right)^{2}-503 \cos\left(dx+c\right)-299\right) \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{96 \left(a^{3} d \cos\left(dx+c\right)^{3}+3 a^{3} d \cos\left(dx+c\right)^{2}+3 a^{3} d \cos\left(dx+c\right)+a^{3} d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 - 160*cos(d*
x + c)^2 - 503*cos(d*x + c) - 299)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt(2)*(cos(d*x + c)^3 +
3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) -
 2*(32*cos(d*x + c)^3 - 160*cos(d*x + c)^2 - 503*cos(d*x + c) - 299)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.435 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+\ldots}$$

```
[Out] -1/4*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2)-13/16*sin(d*x+c)/
a/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-75/32*arctanh(1/2*sin(d*x+c)*a^
(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+49/16*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+
a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4349, 3902, 4105, 4098, 3893, 212}

$$-\frac{75\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - Sin[c + d*x]/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - (13
*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (49
*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}\,(a+a\sec(c}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}\,(a+a\sec(c}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}\,(a+a\sec(c}$$

$$= -\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d} -$$

Mathematica [A]
time = 0.96, size = 141, normalized size = 0.72

$$\frac{150\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+\sqrt{1-\sec(c+dx)}\,(32\sin(c+dx)+(85+49\sec(c+dx))\tan(c+dx))}{16d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (150*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(
32*Sin[c + d*x] + (85 + 49*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[-1 + Cos
[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]
time = 0.14, size = 234, normalized size = 1.19

method	result

default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(75\left(\cos^2(dx+c)\right)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(75*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2))*(-2/(1+cos(d*x+c)))^(1/2)+150*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x
+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)+75*arctan(1/2*sin(
d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-64*c
os(d*x+c)^3-106*cos(d*x+c)^2+72*cos(d*x+c)+98)/sin(d*x+c)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 258456 vs. $2(162) = 324$.
time = 2.98, size = 258456, normalized size = 1311.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/32*(576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x +
5/2*c)^6 + 14400*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*
d*x + 3/2*c)^6 + 187500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^6 + 576*(75*
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2
*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^6 + 5184
*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*si
n(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^6 +
 262500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c)^2 + 7
7700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(
```

1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^4 + 2700
*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*
d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^6 - 2304*sin(1/2*d*x + 1/2*c)^7 + 9
6*(86*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c
) + 10275*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8768*cos(1/2*d*x + 1/2*c
)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^5 + 88800*(75*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*cos(1/2*d*x + 1/2*c) - 64*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(
3/2*d*x + 3/2*c)^5 + 96*(62*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c
))*sin(3/2*d*x + 3/2*c) + 2625*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 224
0*sin(1/2*d*x + 1/2*c)^2 - 1996)*sin(5/2*d*x + 5/2*c)^5 + 864*(1275*(log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1))*sin(1/2*d*x + 1/2*c) - 1088*sin(1/2*d*x + 1/2*c)^2 - 920)*sin(3/
2*d*x + 3/2*c)^5 - 16*(4144*cos(1/2*d*x + 1/2*c)^2 + 675)*sin(1/2*d*x + 1/2
c)^5 + 16((75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x +
 5/2*c)^2 + 25*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*
x + 3/2*c)^2 + 7500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (75*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + 9*(75*log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 300*(log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 256*sin(1/2*d*x + 1/2*c)^3 + 10*((75*l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*

```
d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 150*(lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)...
```

Fricas [A]
time = 3.04, size = 428, normalized size = 2.17

$$\left[\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+4\left(32\cos\left(dx+c\right)^{2}+85\cos\left(dx+c\right)+49\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}, \frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+2\left(32\cos\left(dx+c\right)^{2}+85\cos\left(dx+c\right)+49\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^2 + 85*cos(d*x +
 c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*cos(d*x + c)^2 + 85*
cos(d*x + c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.436 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}$$

[Out] -1/4*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2)-9/16*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+19/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)

Rubi [A]
time = 0.21, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3902, 4097, 3893, 212}

$$\frac{19\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{9\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]

[Out] (19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - (9*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
```

```
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4097

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n},
 x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m
, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{9\sin(}{16ad\cos^{\frac{3}{2}}(c+dx)(}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{9\sin(}{16ad\cos^{\frac{3}{2}}(c+dx)(}$$

$$= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec}}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]

time = 1.29, size = 168, normalized size = 1.07

$$-\frac{\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(9\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+38\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+13\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sin(c+dx)}{16d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -1/16*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(9*Sqrt[1 - Sec[c + d*x]]*Sec[
c + d*x]^(3/2) + 38*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2 + 13*Sqrt[-((-1 + Sec[c + d*
x])*Sec[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c +
 d*x]))^(5/2))
```

Maple [A]
time = 0.15, size = 200, normalized size = 1.27

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{\cos}(dx+c)\right)(-1+\cos(dx+c))^2\left(19\cos(dx+c)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)+13(\cos\right.}{16d\sin(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))
^2*(19*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2
))+13*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+19*arctan(1/2*sin(d*x+c)*(-2/(
1+cos(d*x+c)))^(1/2))*sin(d*x+c)-4*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-9*(
-2/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(1/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3049 vs. 2(128) = 256.
time = 0.99, size = 3049, normalized size = 19.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*
```

c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3
*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 +
 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(
1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x +
 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d
*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
+ 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4
c) + 104(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7/
2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10*
sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*si
n(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*s

```
in(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(1
/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) +
 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
- 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*co
s(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1...
```

Fricas [A]
time = 4.61, size = 408, normalized size = 2.60

$$\left[\frac{19 \sqrt{2} \left(\cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \sqrt{a} \log\left(-\frac{a \cos\left(dx+c\right)^{2}-2 \sqrt{2} \sqrt{a} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)-2 a \cos\left(dx+c\right)-3 a}{\cos\left(dx+c\right)^{2}+2 \cos\left(dx+c\right)+1}\right)-4 \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \left(13 \cos\left(dx+c\right)+9\right) \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{64 \left(a^{3} d \cos\left(dx+c\right)^{3}+3 a^{3} d \cos\left(dx+c\right)^{2}+3 a^{3} d \cos\left(dx+c\right)+a^{3} d\right)}, -\frac{19 \sqrt{2} \left(\cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)^{2}+3 \cos\left(dx+c\right)+1\right) \sqrt{-a} \arctan\left(\frac{\sqrt{2} \sqrt{-a} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)}}{a \sin\left(dx+c\right)}\right)+2 \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \left(13 \cos\left(dx+c\right)+9\right) \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{32 \left(a^{3} d \cos\left(dx+c\right)^{3}+3 a^{3} d \cos\left(dx+c\right)^{2}+3 a^{3} d \cos\left(dx+c\right)+a^{3} d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(13*cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(1
9*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(13*
cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.437 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}-\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}$$

```
[Out] -1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+5/16*sin(d*x+c)/a
/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+5/32*arctanh(1/2*sin(d*x+c)*a^(1
/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4349, 3896, 3895, 3893, 212}

$$\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 - Sin[c + d*x]/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + (5*Si
n[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
```

```
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3896

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(5\sqrt{\cos(c+dx)}\,\sqrt{s\right.}}{}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{5\sin(c}{16ad\cos^{\frac{3}{2}}(c+dx)(a}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{5\sin(c}{16ad\cos^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]
time = 3.89, size = 224, normalized size = 1.43

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(-8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)-\frac{5(1+\sec(c+dx))\left(2\sqrt{1-\sec(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)-(1+\sec(c+dx))\left(2\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)+2\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+2\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)\right)}{\sqrt{1-\sec(c+dx)}}\right)}{32d(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]

[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-8*Sec[c + d*x]^(5/2)*Sin[c + d*x] - (5*(1 + Sec[c + d*x])*(2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - (1 + Sec[c + d*x])*(2*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x]))/Sqrt[1 - Sec[c + d*x]]))/(32*d*(a*(1 + Sec[c + d*x]))^(5/2))

Maple [A]
time = 0.13, size = 198, normalized size = 1.26

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(5\cos(dx+c)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)-5\left(\cos^2(dx+c)\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}+5\arctan\left(\right.\right.}{16d\sqrt{-\frac{}{1+\mathrm{co}}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/16/d*(-1+cos(d*x+c))^2*(5*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-5*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+5*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+4*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/a^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2875 vs. 2(128) = 256.
time = 0.86, size = 2875, normalized size = 18.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x +

3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*c
) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x +
3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*c
os(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*d
*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 1
2*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*
(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*

```
c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c) +
 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*c
os(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2...
```

Fricas [A]
time = 3.72, size = 408, normalized size = 2.60

$$\left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(5\cos\left(dx+c\right)+1\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)},-\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)-2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(5\cos\left(dx+c\right)+1\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*(5*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(5*s
qrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(
d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.438 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}$$

```
[Out] 1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+3/16*sin(d*x+c)/a/
d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(1/
2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3895, 3893, 212}

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 + Sin[c + d*x]/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + (3*Si
n[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
```

```
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{3\sin(c+}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{3\sin(c+}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c-}{16\sqrt{2}a^{5/2}d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(157) = 314.
time = 1.44, size = 341, normalized size = 2.17

$$\frac{-3\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sin(c+dx)+14\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+6\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sin(c+dx)-6\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)-3\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)+6\,\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(\sin(c+dx)+(2+\sec(c+dx))\tan(c+dx))+6\,\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(\sin(c+dx)+(2+\sec(c+dx))\tan(c+dx))}{32a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))^2\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sin
[c + d*x] + 14*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 6*S
qrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] - 6*Sqrt[2]*ArcTan[(S
```

qrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + (2 + Sec[c + d*x])*Tan[c + d*x]) + 6*ArcSin[Sqrt[Sec[c + d*x]]]*(Sin[c + d*x] + (2 + Sec[c + d*x])*Tan[c + d*x]))/(32*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])^2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])])

Maple [A]
time = 0.13, size = 200, normalized size = 1.27

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(3\cos(dx+c)\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)-3\left(\cos^2(dx+c)\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}+3\arctan\left(\right.\right.}{16d\sqrt{-\frac{}{1+c}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/16/d*(-1+cos(d*x+c))^2*(3*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-3*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-4*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+7*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/a^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. 2(128) = 256.
time = 22.66, size = 84332, normalized size = 537.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(5*d*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2

c)))^2 + 10240(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c)
 + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x
 + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) +
 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x +
5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(
5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) +
 cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x +
 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5
*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/
2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/
2*d*x + 5/2*c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))
)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + co
s(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*
c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*co
s(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c
) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d
*x + 5/2*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2
 - 512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*
d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) + (
2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)*
sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(5*d*x +
 5*c)^2 + 2560*cos(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 1024*(20*cos(2*d*x
 + 2*c) + 10*cos(d*x + c) + 1)*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 1024
0*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*sin(4*d*x + 4*c)^2*sin(5/2
*d*x + 5/2*c) + 10240*sin(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*(5*(2*
sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c
)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d
*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*s
in(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x +
 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(
8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*si
n(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*
sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x
 + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin
(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4
*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(6/
5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*si
n(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x +
 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5
/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c
)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2560*(5*(2*sin(2*d
*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5

```
*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/
2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*
d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*s
in(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(2/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 512*(5*cos(4*d*x + 4*
c)^2*sin(5/2*d*x + 5/2*c) + 4*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*co
s(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 20*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5
/2*c) + 5*sin(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 20*sin(3*d*x + 3*c)^2*s
in(5/2*d*x + 5/2*c) + 2*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2
*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c)
+ 2*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 2*(5*(2*sin(
2*d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2*...
```

Fricas [**A**]
time = 3.28, size = 408, normalized size = 2.60

$$\left[\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos\left(dx+c\right)^{2}-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2a\cos\left(dx+c\right)-3a}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)+4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(3\cos\left(dx+c\right)+7\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}, -\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin\left(dx+c\right)}\right)-2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(3\cos\left(dx+c\right)+7\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*(3*cos(d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(3*s
qrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(
d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.439 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d}-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{}}{16\sqrt{2}a^{5/2}d}$$

```
[Out] -1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)-11/16*sin(d*x+c)/
a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*arcsinh(a^(1/2)*tan(d*x+c)/(a
+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d-43/32*arc
tanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$,
Rules used = {4349, 3901, 4104, 4108, 3893, 212, 3886, 221}

$$-\frac{43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{11\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - (43*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(5
/2)*(a + a*Sec[c + d*x])^(5/2)) - (11*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3
/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.}{}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+}{16ad\cos^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{5/2}d} -$$

Mathematica [A]
time = 1.28, size = 328, normalized size = 1.53

$$\frac{43\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sin(c+dx)-30\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-22\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,\sin(c+dx)+86\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)+43\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)-22\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(\sin(c+dx)+(2+\sec(c+dx))\tan(c+dx))-86\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(\sin(c+dx)+(2+\sec(c+dx))\tan(c+dx))}{32d\cos^{\frac{9}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sin
[c + d*x] - 30*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 22*
Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + 86*Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 43*Sqrt
[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x
]*Tan[c + d*x] - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + (2 + Sec
[c + d*x])*Tan[c + d*x]) - 86*ArcSin[Sqrt[Sec[c + d*x]]]*(Sin[c + d*x] + (2
 + Sec[c + d*x])*Tan[c + d*x]))/(32*d*Cos[c + d*x]^(9/2)*Sqrt[1 - Sec[c + d
*x]]*Sec[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(175) = 350.
time = 0.14, size = 396, normalized size = 1.85

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(16\cos(dx+c)\sin(dx+c)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-16\cos(dx+c)\sin(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/16/d*(-1+cos(d*x+c))^2*(16*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-16*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))-43*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+16*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)-16*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*sin(d*x+c)+11*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-43*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+4*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-15*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/a^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4988 vs. $2(175) = 350$.
time $= 0.87$, size $= 4988$, normalized size $= 23.31$

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(19*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d

```
*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*s
qrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4
*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x +
4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 +
 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqr
t(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4
*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*s
in(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2
)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2
*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2)
*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqr
t(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16
*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6
*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
```

```
c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x
 + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + ...
```

Fricas [A]
time = 3.79, size = 638, normalized size = 2.98

$$\left[\frac{43\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos\left(dx+c\right)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2a\cos\left(dx+c\right)-3a}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)-4\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(11\cos\left(dx+c\right)+15\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+32\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^{3}-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-7a\cos\left(dx+c\right)^{2}+8a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}, \frac{43\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin\left(dx+c\right)}\right)-2\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(11\cos\left(dx+c\right)+15\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+32\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-2a}\right)}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(11*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(d*x + c) + 32*(cos(d*x
 + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c
)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*
sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(11*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(
d*x + c) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt
(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(
d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.440 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d}+\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right.}{16\sqrt{2}\,a^{5/2}}$$

```
[Out] -1/4*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2)-15/16*sin(d*x+c)/
a/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)-5*arcsinh(a^(1/2)*tan(d*x+c)/(a
+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d+115/32*ar
ctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+35/16*sin(d*x+c)/a^2
/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4349, 3901, 4104, 4106, 4108, 3893, 212, 3886, 221}

$$\frac{115\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{35\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{15\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-5*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + (115*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*S
qrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^
(7/2)*(a + a*Sec[c + d*x])^(5/2)) - (15*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^
(5/2)*(a + a*Sec[c + d*x])^(3/2)) + (35*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
```

```
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.}{}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c}{16ad\cos^{\frac{5}{2}}(c+dx)(a}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c}{16ad\cos^{\frac{5}{2}}(c+dx)(a}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c}{16ad\cos^{\frac{5}{2}}(c+dx)(a}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c}{16ad\cos^{\frac{5}{2}}(c+dx)(a}$$

$$= -\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{a^{5/2}d}$$

Mathematica [A]
time = 1.70, size = 348, normalized size = 1.37

$$\frac{\sqrt{\sec(c+dx)}\left(-115\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sin(c+dx)+110\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+32\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+70\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sin(c+dx)-230\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)-115\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)+70\,\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)(\sin(c+dx)+(2+\sec(c+dx))\tan(c+dx))+230\,\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)(\sin(c+dx)+(2+\sec(c+dx))\tan(c+dx))\right)}{32d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[
1 - Sec[c + d*x]]]*Sin[c + d*x] + 110*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(
3/2)*Sin[c + d*x] + 32*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*
x] + 70*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] - 230*Sqrt[2
]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x]
- 115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*S
ec[c + d*x]*Tan[c + d*x] + 70*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x]
+ (2 + Sec[c + d*x])*Tan[c + d*x]) + 230*ArcSin[Sqrt[Sec[c + d*x]]]*(Sin[c
+ d*x] + (2 + Sec[c + d*x])*Tan[c + d*x])))/(32*d*Sqrt[-1 + Cos[c + d*x]]*(
a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(209) = 418$.
time = 0.15, size = 444, normalized size = 1.75

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(40\left(\cos^2(dx+c)\right)\sin(dx+c)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{}}{4}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(40*sin(d*x+c
)*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c
))*2^(1/2))*2^(1/2)-40*sin(d*x+c)*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)-115*sin(d*x+c)*cos(d*x+
c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+40*cos(d*x+c)*sin(d*x
+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*
2^(1/2))-40*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1
/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+35*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^
(1/2)-115*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2))+20*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-39*cos(d*x+c)*(-2/(1+cos(d*
x+c)))^(1/2)-16*(-2/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/sin(d*x+c)^5/(-
2/(1+cos(d*x+c)))^(1/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 9048 vs. $2(209) = 418$.
time = 4.42, size = 9048, normalized size = 35.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] -1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*(24*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 96*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 300*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(

```
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt
(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqr
t(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*s
qrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(
sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x
 + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt(2)*cos
(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*
c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*
d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*...
```

Fricas [A]
time = 3.87, size = 702, normalized size = 2.76

$$\left[\frac{115 \sqrt{2} \left(\cos(dx+c)^4+3 \cos(dx+c)^3+3 \cos(dx+c)^2+\cos(dx+c)\right) \sqrt{a} \log\left(-\frac{a \cos(dx+c)^2-2 \sqrt{2} \sqrt{a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)-2 a \cos(dx+c)-3 a}{\cos(dx+c)^2+2 \cos(dx+c)+1}\right)+4 \left(35 \cos(dx+c)^2+55 \cos(dx+c)+16\right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)+80 \left(\cos(dx+c)^4+3 \cos(dx+c)^3+3 \cos(dx+c)^2+\cos(dx+c)\right) \sqrt{a} \log\left(\frac{a \cos(dx+c)^3+4 \sqrt{a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} (\cos(dx+c)-2) \sqrt{\cos(dx+c)} \sin(dx+c)-7 a \cos(dx+c)^2+8 a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{64 \left(a^3 d \cos(dx+c)^4+3 a^3 d \cos(dx+c)^3+3 a^3 d \cos(dx+c)^2+a^3 d \cos(dx+c)\right)}, -\frac{115 \sqrt{2} \left(\cos(dx+c)^4+3 \cos(dx+c)^3+3 \cos(dx+c)^2+\cos(dx+c)\right) \sqrt{-a} \arctan\left(\frac{\sqrt{2} \sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)}}{a \sin(dx+c)}\right)-2 \left(35 \cos(dx+c)^2+55 \cos(dx+c)+16\right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)+80 \left(\cos(dx+c)^4+3 \cos(dx+c)^3+3 \cos(dx+c)^2+\cos(dx+c)\right) \sqrt{-a} \arctan\left(\frac{2 \sqrt{-a} \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{a \cos(dx+c)^2-a \cos(dx+c)-2 a}\right)}{32 \left(a^3 d \cos(dx+c)^4+3 a^3 d \cos(dx+c)^3+3 a^3 d \cos(dx+c)^2+a^3 d \cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x
 + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(35*cos(d*x + c)^2
+ 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x +
c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x +
 c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^3*d*
cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*co
```

```
s(d*x + c)), -1/32*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(35*cos(d
*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*
cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{9/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.441 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$

Optimal. Leaf size=244

$$-\frac{a^3(7-4n)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a^3(1-4n)\cos(e+fx)(d\cos(e+fx)}{f(1-n)(1-}$$

```
[Out] -a^3*(7-4*n)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2
)*sin(f*x+e)/f/(2-n)/n/(sin(f*x+e)^2)^(1/2)-a^3*(1-4*n)*cos(f*x+e)*(d*cos(f
*x+e))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/
(-n^2+1)/(sin(f*x+e)^2)^(1/2)+a^3*(5-2*n)*(d*cos(f*x+e))^n*tan(f*x+e)/f/(n^
2-3*n+2)+(d*cos(f*x+e))^n*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f/(2-n)
```

Rubi [A]
time = 0.28, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3899, 4082, 3872, 3857, 2722}

$$-\frac{a^3(7-4n)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a^3(1-4n)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}+\frac{a^3(5-2n)\tan(e+fx)(d\cos(e+fx))^n}{f(1-n)(2-n)}+\frac{\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)(d\cos(e+fx))^n}{f(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*(7 - 4*n)*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 - n)*n*Sqrt[Sin[e + f*x]^2])) - (a^3*(1
 - 4*n)*Cos[e + f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (
3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x
]^2]) + (a^3*(5 - 2*n)*(d*Cos[e + f*x])^n*Tan[e + f*x])/(f*(1 - n)*(2 - n))
 + ((d*Cos[e + f*x])^n*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(f*(2 - n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx = ((d\cos(e+fx))^n(d\sec(e+fx))^n)\int (d\sec(e+fx))^{-n}(a+a\sec$$
$$= \frac{(d\cos(e+fx))^n\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{f(2-n)} + \frac{(a(d\cos(e$$
$$= \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\,(a^3$$
$$= \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\,(a^3$$
$$= \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\,(a^3$$
$$= -\frac{a^3(7-4n)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e}{f(2-n)n\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.65, size = 308, normalized size = 1.26

$$\frac{i2^{-3-n}a^3\left(e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^n\cos^{3-n}(e+fx)(d\cos(e+fx))^n\left(\frac{8e^{3i(e+fx)}\,{}_2F_1\left(1,\frac{1}{2}(-1+n);\frac{5-n}{2};-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2(-3+n)}+\frac{12e^{2i(e+fx)}\,{}_2F_1\left(1,\frac{n}{2};2-\frac{n}{2};-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)(-2+n)}+\frac{6e^{i(e+fx)}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3-n}{2};-e^{2i(e+fx)}\right)}{-1+n}+\frac{\left(1+e^{2i(e+fx)}\right)\,{}_2F_1\left(1,\frac{2+n}{2};1-\frac{n}{2};-e^{2i(e+fx)}\right)}{n}\right)\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (I*2^(-3 - n)*a^3*((1 + E^((2*I)*(e + f*x)))/E^(I*(e + f*x)))^n*Cos[e + f*x
]^(3 - n)*(d*Cos[e + f*x])^n*((8*E^((3*I)*(e + f*x))*Hypergeometric2F1[1, (
-1 + n)/2, (5 - n)/2, -E^((2*I)*(e + f*x))])/((1 + E^((2*I)*(e + f*x)))^2*(
-3 + n)) + (12*E^((2*I)*(e + f*x))*Hypergeometric2F1[1, n/2, 2 - n/2, -E^((
2*I)*(e + f*x))])/((1 + E^((2*I)*(e + f*x)))*(-2 + n)) + (6*E^(I*(e + f*x))
*Hypergeometric2F1[1, (1 + n)/2, (3 - n)/2, -E^((2*I)*(e + f*x))])/(-1 + n)
 + ((1 + E^((2*I)*(e + f*x)))*Hypergeometric2F1[1, (2 + n)/2, 1 - n/2, -E^(
(2*I)*(e + f*x))])/n)*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x])^3)/f
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (d\cos(fx+e))^n\,(a+a\sec(fx+e))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*(d*cos(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int (d\cos(e+fx))^n\,dx+\int 3(d\cos(e+fx))^n\sec(e+fx)\,dx+\int 3(d\cos(e+fx))^n\sec^2(e+fx)\,dx+\int (d\cos(e+fx))^n\sec^3(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((d*cos(e + f*x))**n, x) + Integral(3*(d*cos(e + f*x))**n*sec
(e + f*x), x) + Integral(3*(d*cos(e + f*x))**n*sec(e + f*x)**2, x) + Integr
al((d*cos(e + f*x))**n*sec(e + f*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (d\cos(e+fx))^n \left(a+\frac{a}{\cos(e+fx)}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^3,x)
[Out] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^3, x)
```

3.442 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$

Optimal. Leaf size=179

$$-\frac{2a^2(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^2(1-2n)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1}{f(1-n)(1+n)\sqrt{}}$$

```
[Out] -2*a^2*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(
f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)-a^2*(1-2*n)*cos(f*x+e)*(d*cos(f*x+e))^n*hyp
ergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(-n^2+1)/(si
n(f*x+e)^2)^(1/2)+a^2*(d*cos(f*x+e))^n*tan(f*x+e)/f/(1-n)
```

Rubi [A]

time = 0.16, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4349, 3873, 3857, 2722, 4131}

$$-\frac{2a^2\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^2(1-2n)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}+\frac{a^2\tan(e+fx)(d\cos(e+fx))^n}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a^2*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f
*x]^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) - (a^2*(1 - 2*n)*Cos[e + f*
x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e +
f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (a^2*(d*Co
s[e + f*x])^n*Tan[e + f*x])/(f*(1 - n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx = ((d\cos(e+fx))^n(d\sec(e+fx))^n)\int (d\sec(e+fx))^{-n}(a+a\operatorname{sec}$$

$$= ((d\cos(e+fx))^n(d\sec(e+fx))^n)\int (d\sec(e+fx))^{-n}\left(a^2+a^2\right.$$

$$= \frac{a^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)} + \frac{\left(2a^2\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\cos(e+\right.}{d}$$

$$= -\frac{2a^2(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} -$$

$$= -\frac{2a^2(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.38, size = 266, normalized size = 1.49

$$\frac{i2^{-2-n}a^2e^{-i(e+fx)}\left(e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^{-1+n}\cos^{-n}(e+fx)(d\cos(e+fx))^n(1+\cos(e+fx))^2\left(4e^{2i(e+fx)}(-1+n)n\,{}_2F_1\left(1,\frac{n}{2};2-\frac{n}{2};-e^{2i(e+fx)}\right)+\left(1+e^{2i(e+fx)}\right)(-2+n)\left(4e^{i(e+fx)}n\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3-n}{2};-e^{2i(e+fx)}\right)+\left(1+e^{2i(e+fx)}\right)(-1+n)\,{}_2F_1\left(1,\frac{2+n}{2};1-\frac{n}{2};-e^{2i(e+fx)}\right)\right)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)}{f(-2+n)(-1+n)n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (I*2^(-2 - n)*a^2*((1 + E^((2*I)*(e + f*x)))/E^(I*(e + f*x)))^(-1 + n)*(d*C
os[e + f*x])^n*(1 + Cos[e + f*x])^2*(4*E^((2*I)*(e + f*x))*(-1 + n)*n*Hyper
```

```
geometric2F1[1, n/2, 2 - n/2, -E^((2*I)*(e + f*x))] + (1 + E^((2*I)*(e + f*
x)))*(-2 + n)*(4*E^(I*(e + f*x))*n*Hypergeometric2F1[1, (1 + n)/2, (3 - n)/
2, -E^((2*I)*(e + f*x))] + (1 + E^((2*I)*(e + f*x)))*(-1 + n)*Hypergeometri
c2F1[1, (2 + n)/2, 1 - n/2, -E^((2*I)*(e + f*x))]))*Sec[(e + f*x)/2]^4)/(E^
(I*(e + f*x))*f*(-2 + n)*(-1 + n)*n*Cos[e + f*x]^n)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^{n} \left(a+a\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
[Out] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(d*cos(f*x + e))^n
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \left(d\cos\left(e+fx\right)\right)^{n}\,dx + \int 2\left(d\cos\left(e+fx\right)\right)^{n}\sec\left(e+fx\right)dx + \int \left(d\cos\left(e+fx\right)\right)^{n}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((d*cos(e + f*x))**n, x) + Integral(2*(d*cos(e + f*x))**n*sec
(e + f*x), x) + Integral((d*cos(e + f*x))**n*sec(e + f*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(e+f\,x))^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^2,x)
```

```
[Out] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^2, x)
```

3.443 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx$

Optimal. Leaf size=132

$$-\frac{a(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a(d\cos(e+fx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+\right.}{df(1+n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -a*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x+
e)/f/n/(sin(f*x+e)^2)^(1/2)-a*(d*cos(f*x+e))^(1+n)*hypergeom([1/2, 1/2+1/2*
n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4310, 16, 2827, 2722}

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] -((a*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])
/(d*f*(1 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx &= \int (d\cos(e+fx))^n(a+a\cos(e+fx))\sec(e+fx)\,dx\\ &= d\int (d\cos(e+fx))^{-1+n}(a+a\cos(e+fx))\,dx\\ &= a\int (d\cos(e+fx))^n\,dx+(ad)\int (d\cos(e+fx))^{-1+n}\,dx\\ &= -\frac{a(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a}{}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 105, normalized size = 0.80

$$-\frac{a(d\cos(e+fx))^n\left((1+n)\csc(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)+n\cot(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\right)\sqrt{\sin^2(e+fx)}}{fn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] -((a*(d*Cos[e + f*x])^n*((1 + n)*Csc[e + f*x]*Hypergeometric2F1[1/2, n/2, (
2 + n)/2, Cos[e + f*x]^2] + n*Cot[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (d\cos(fx+e))^n\,(a+a\sec(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (d\cos{(e+fx)})^n\, dx + \int (d\cos{(e+fx)})^n \sec{(e+fx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e)),x)
```

```
[Out] a*(Integral((d*cos(e + f*x))**n, x) + Integral((d*cos(e + f*x))**n*sec(e +
f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\cos{(e+f\,x)})^n\left(a+\frac{a}{\cos{(e+f\,x)}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x)),x)
```

```
[Out] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x)), x)
```

3.444 $\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=178

$$\frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))}-\frac{\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{af\sqrt{\sin^2(e+fx)}}+\frac{(1+$$

```
[Out] (d*cos(f*x+e))^n*sin(f*x+e)/f/(a+a*sec(f*x+e))-cos(f*x+e)*(d*cos(f*x+e))^n*
hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(sin(f*
x+e)^2)^(1/2)+(1+n)*cos(f*x+e)^2*(d*cos(f*x+e))^n*hypergeom([1/2, 1+1/2*n],
[2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(2+n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4349, 3905, 3872, 3857, 2722}

$$-\frac{\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{af\sqrt{\sin^2(e+fx)}}+\frac{(n+1)\sin(e+fx)\cos^2(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(e+fx)\right)}{af(n+2)\sqrt{\sin^2(e+fx)}}+\frac{\sin(e+fx)(d\cos(e+fx))^n}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] ((d*Cos[e + f*x])^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - (Cos[e + f*x]*
(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x
]^2]*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) + ((1 + n)*Cos[e + f*x]^2*(d*
Cos[e + f*x])^n*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[e + f*x]^2
]*Sin[e + f*x])/(a*f*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3905

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Dist[d*((n - 1)/(a*b)), Int[(d*Csc[e + f*x])^(
n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[
a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx &= ((d\cos(e+fx))^n(d\sec(e+fx))^n)\int \frac{(d\sec(e+fx))^{-n}}{a+a\sec(e+fx)}\,dx\\
&= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{(d(1+n)(d\cos(e+fx))^n(d\sec(e+fx))^n)\int(d\,\mathrm{s}}{a^2}\\
&= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{((1+n)(d\cos(e+fx))^n(d\sec(e+fx))^n)\int(d\,\mathrm{sec}}{a}\\
&= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1+n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\cos(e+fx))^n\right)\int\left(\frac{\cos(}{}\right.}{a}\\
&= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2\right.}{af\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
[Out] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]), x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(fx+e))^{n}}{a+a\sec(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
[Out] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(d\cos(e+fx))^{n}}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e)),x)
[Out] Integral((d*cos(e + f*x))**n/(sec(e + f*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\cos(e+fx))^n}{a+\frac{a}{\cos(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
```

```
[Out] int((d*cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

3.445 $\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=215

$$\frac{2(2+n)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(3+2n)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1}{3a^2f\sqrt{\sin^2(}}$$

```
[Out] 2/3*(2+n)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*s
in(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-1/3*(3+2*n)*cos(f*x+e)*(d*cos(f*x+e))^
n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a^2/f/(si
n(f*x+e)^2)^(1/2)-2/3*(2+n)*(d*cos(f*x+e))^n*tan(f*x+e)/a^2/f/(1+sec(f*x+e)
)-1/3*(d*cos(f*x+e))^n*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A]
time = 0.28, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3902, 4105, 3872, 3857, 2722}

$$\frac{2(n+2)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(2n+3)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{2(n+2)\tan(e+fx)(d\cos(e+fx))^n}{3a^2f(\sec(e+fx)+1)}-\frac{\tan(e+fx)(d\cos(e+fx))^n}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*(2 + n)*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e
+ f*x]^2]*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - ((3 + 2*n)*Cos[e +
 f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e
 + f*x]^2]*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (2*(2 + n)*(d*Cos
[e + f*x])^n*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - ((d*Cos[e + f*x])
^n*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = ((d\cos(e+fx))^n(d\sec(e+fx))^n)\int \frac{(d\sec(e+fx))^{-n}}{(a+a\sec(e+fx))^2}\,dx$$

$$= -\frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{((d\cos(e+fx))^n(d\sec(e+fx))^n)\int \frac{(d\sec(e+f}{}}{3a^2}$$

$$= -\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - ((d\,c$$

$$= -\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - (2n($$

$$= -\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \Big(2n$$

$$= \frac{2(2+n)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - (3+2n)\,c$$

Mathematica [F]
time = 7.81, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
[Out] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(fx+e))^n}{(a+a\sec(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
[Out] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(d\cos(e+fx))^n}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*cos(e + f*x))**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**
2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,\cos(e+f\,x))^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

```
[Out] int((d*cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

3.446 $\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)}{d}+\frac{3b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 3/8*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+1/
4*b*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 3852, 3853, 3855}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*b*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec^4(c+dx)\,dx + b\int \sec^5(c+dx)\,dx$$
$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3b)\int \sec^3(c+dx)\,dx - \frac{a\text{Subst}\big(\int (}{}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+}{4d}$$
$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx}{8d}$$

Mathematica [A]
time = 0.27, size = 76, normalized size = 0.89

$$\frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{3b\left(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)\right)}{8d} + \frac{a\left(\tan(c+dx) + \frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
[Out] (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*b*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.09, size = 73, normalized size = 0.86

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$-\frac{i\left(9b\,e^{7i(dx+c)}+33b\,e^{5i(dx+c)}-48a\,e^{4i(dx+c)}-33b\,e^{3i(dx+c)}-64a\,e^{2i(dx+c)}-9b\,e^{i(dx+c)}-16a\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}-\frac{3b\ln\left(e^{i(dx+c)}-i\right)}{8d}$
norman	$\frac{-\frac{(8a-5b)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(8a+5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{(40a-9b)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{(40a+9b)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b*(-(-1/4*sec(d*x+c)^3-3/8*sec(d
*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.30, size = 95, normalized size = 1.12

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a-3\,b\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*b*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 4.09, size = 99, normalized size = 1.16

$$\frac{9\,b\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-9\,b\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,a\cos\left(dx+c\right)^{3}+9\,b\cos\left(dx+c\right)^{2}+8\,a\cos\left(dx+c\right)+6\,b\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*b*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*b*cos(d*x + c)^2 + 8*a*cos(d*x
 + c) + 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\sec^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(77) = 154.
time = 0.48, size = 164, normalized size = 1.93

$$\frac{9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-40\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+40\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] 1/24*(9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7 - 40*a*tan(1/2*d*x + 1/2*c)^5 - 9*b*tan(1/2*d*x + 1/2*c)^5 + 40*a*tan(1/2*d*x + 1/2*c)^3 - 9*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c) - 15*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d

Mupad [B]
time = 3.57, size = 152, normalized size = 1.79

$$\frac{\left(\frac{5b}{4}-2a\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{10a}{3}+\frac{3b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{3b}{4}-\frac{10a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(2a+\frac{5b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)}+\frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))/cos(c + d*x)^4,x)

[Out] (tan(c/2 + (d*x)/2)*(2*a + (5*b)/4) - tan(c/2 + (d*x)/2)^7*(2*a - (5*b)/4) - tan(c/2 + (d*x)/2)^3*((10*a)/3 - (3*b)/4) + tan(c/2 + (d*x)/2)^5*((10*a)/3 + (3*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*b*atanh(tan(c/2 + (d*x)/2)))/(4*d)

3.447 $\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{b\tan^3(c+dx)}{3d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/
3*b*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 3853, 3855, 3852}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (b*Tan[c + d*x]^3)/(3*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec^3(c+dx)\,dx + b\int \sec^4(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{b\text{Subst}\left(\int (1+\right.}{}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.17, size = 60, normalized size = 0.95

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{b\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.06, size = 60, normalized size = 0.95

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
risch	$-\frac{i\left(3a\,e^{5i(dx+c)}-12b\,e^{2i(dx+c)}-3\,e^{i(dx+c)}a-4b\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{\frac{(a-2b)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b*(-2/3-1/
3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 70, normalized size = 1.11

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)b-3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b - 3*a*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 3.13, size = 88, normalized size = 1.40

$$\frac{3\,a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,b\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+2\,b\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*b*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*b)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(57) = 114.
time = 0.47, size = 122, normalized size = 1.94

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^5 + 4*
b*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c) - 6*b*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 2.81, size = 109, normalized size = 1.73

$$\frac{(a-2b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+(-a-2b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a - 2*b) - tan(c/2 + (d*x)/2)*(a + 2*b) + (4*b*tan(c
/2 + (d*x)/2)^3)/3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + t
an(c/2 + (d*x)/2)^6 - 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.448 $\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3872, 3852, 8, 3853, 3855}

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx &= a\int \sec^2(c+dx)\,dx + b\int \sec^3(c+dx)\,dx\\ &= \frac{b\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}b\int \sec(c+dx)\,dx - \frac{a\text{Subst}(\int 1\,dx, x\end{aligned}$$

$$= \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.00

$$\frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Maple [A]
time = 0.05, size = 47, normalized size = 1.00

method	result	size
derivativedivides	$\frac{a\tan(dx+c)+b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	47
default	$\frac{a\tan(dx+c)+b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	47
norman	$\frac{\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(2a-b)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	96
risch	$-\frac{i\left(b\,e^{3i(dx+c)}-2a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-2a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{2d}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*tan(d*x+c)+b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)
)))
```

Maxima [A]
time = 0.30, size = 58, normalized size = 1.23

$$-\frac{b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-4a\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

Fricas [A]
time = 3.58, size = 74, normalized size = 1.57

$$\frac{b\cos(dx+c)^2\log(\sin(dx+c)+1)-b\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2a\cos(dx+c)+b)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*a*cos(d*x + c) + b)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(43) = 86.
time = 0.44, size = 107, normalized size = 2.28

$$\frac{b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/
d
```

Mupad [B]
time = 1.48, size = 85, normalized size = 1.81

$$\frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(2\,a-b)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a+b)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x)^2,x)
```

```
[Out] (b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(2*a - b) - tan(c/2
 + (d*x)/2)*(2*a + b))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 +
1))
```

3.449 $\int \sec(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3872, 3855, 3852, 8}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec(c+dx)\,dx + b\int \sec^2(c+dx)\,dx$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\tan(c+dx)}{d}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Maple [A]
time = 0.04, size = 30, normalized size = 1.25

method	result	size
derivativedivides	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+b\tan(dx+c)}{d}$	30
default	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+b\tan(dx+c)}{d}$	30
risch	$\frac{2ib}{d(\mathrm{e}^{2i(dx+c)}+1)} + \frac{a\ln(\mathrm{e}^{i(dx+c)}+i)}{d} - \frac{a\ln(\mathrm{e}^{i(dx+c)}-i)}{d}$	59
norman	$-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d} - \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+b*tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 29, normalized size = 1.21

$$\frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+b\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + b*tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.
time = 2.77, size = 60, normalized size = 2.50

$$\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2b\sin(dx+c)}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*b*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [A]
time = 2.62, size = 37, normalized size = 1.54

$$\begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+b\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\sec(c))\sec(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*tan(c + d*x))/d, Ne(d, 0
)), (x*(a + b*sec(c))*sec(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(24) = 48$.
time = 0.46, size = 63, normalized size = 2.62

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 0.79, size = 47, normalized size = 1.96

$$\frac{2a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
(d*x)/2)^2 - 1))
```

3.450 $\int (a+b\sec(c+dx))\,dx$

Optimal. Leaf size=16

$$ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

[Out] a*x+b*arctanh(sin(d*x+c))/d

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {3855}

$$ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b*Sec[c + d*x],x]

[Out] a*x + (b*ArcTanh[Sin[c + d*x]])/d

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\,dx &= ax+b\int \sec(c+dx)\,dx\\ &= ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Sec[c + d*x],x]

[Out] a*x + (b*ArcTanh[Sin[c + d*x]])/d

Maple [A]
time = 0.03, size = 24, normalized size = 1.50

method	result	size
default	$ax+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
derivativedivides	$\frac{(dx+c)a+b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
norman	$ax+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	40
risch	$ax-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+b/d*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 23, normalized size = 1.44

$$ax+\frac{b\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x, algorithm="maxima")
[Out] a*x + b*log(sec(d*x + c) + tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.
time = 3.02, size = 36, normalized size = 2.25

$$\frac{2\,adx+b\log\left(\sin\left(dx+c\right)+1\right)-b\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x, algorithm="fricas")
[Out] 1/2*(2*a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1))/d
```

Sympy [A]
time = 1.02, size = 41, normalized size = 2.56

$$ax+b\left(\begin{cases}\frac{\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{for}\ d\neq 0\\ \frac{x\left(\tan\left(c\right)\sec\left(c\right)+\sec^{2}\left(c\right)\right)}{\tan\left(c\right)+\sec\left(c\right)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(c
)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(16) = 32$.
time = 0.43, size = 49, normalized size = 3.06

$$ax + \frac{b\left(\log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx+c\right) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx+c\right) - 2\right|\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/4*b*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*
x + c) + sin(d*x + c) - 2)))/d
```

Mupad [B]
time = 0.80, size = 57, normalized size = 3.56

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cos(c + d*x),x)
```

```
[Out] (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.451 $\int \cos(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=15

$$bx+\frac{a\sin(c+dx)}{d}$$

[Out] b*x+a*sin(d*x+c)/d

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3872, 2717, 8}

$$\frac{a\sin(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]

[Out] b*x + (a*Sin[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sec(c+dx))\,dx &= a\int \cos(c+dx)\,dx+b\int 1\,dx\\ &= bx+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.73

$$bx+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]
[Out] b*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

Maple [A]
time = 0.05, size = 21, normalized size = 1.40

method	result	size
risch	$bx+\frac{a\sin(dx+c)}{d}$	16
derivativedivides	$\frac{a\sin(dx+c)+b(dx+c)}{d}$	21
default	$\frac{a\sin(dx+c)+b(dx+c)}{d}$	21
norman	$\frac{bx+bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*sin(d*x+c)+b*(d*x+c))
```

Maxima [A]
time = 0.26, size = 20, normalized size = 1.33

$$\frac{(dx+c)b+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] ((d*x + c)*b + a*sin(d*x + c))/d
```

Fricas [A]
time = 2.07, size = 17, normalized size = 1.13

$$\frac{bdx+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] (b*d*x + a*sin(d*x + c))/d
```

Sympy [A]
time = 1.12, size = 17, normalized size = 1.13

$$a\left(\begin{cases} x\cos(c) & \text{for}\ d=0 \\ \frac{\sin(c+dx)}{d} & \text{otherwise} \end{cases}\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] a*Piecewise((x*cos(c), Eq(d, 0)), (sin(c + d*x)/d, True)) + b*x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.
time = 0.44, size = 39, normalized size = 2.60

$$\frac{(dx+c)b + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*b + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

Mupad [B]
time = 0.74, size = 17, normalized size = 1.13

$$\frac{a\,\sin\left(c+d\,x\right)+b\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) + b*d*x)/d
```

3.452 $\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{ax}{2}+\frac{b\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x+b*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.210$, Rules used = {3872, 2715, 8, 2717}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*x)/2 + (b*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx = a\int \cos^2(c+dx)\,dx + b\int \cos(c+dx)\,dx$$
$$= \frac{b\sin(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx$$
$$= \frac{ax}{2} + \frac{b\sin(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.08, size = 35, normalized size = 0.92

$$\frac{4b\sin(c+dx)+a(2(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (4*b*Sin[c + d*x] + a*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.07, size = 38, normalized size = 1.00

method	result	size
risch	$\frac{ax}{2}+\frac{b\sin(dx+c)}{d}+\frac{a\sin(2dx+2c)}{4d}$	32
derivativedivides	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\sin(dx+c)b}{d}$	38
default	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\sin(dx+c)b}{d}$	38
norman	$\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax}{2}+\frac{ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{(a-2b)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+sin(d*x+c)*b)
```

Maxima [A]
time = 0.26, size = 34, normalized size = 0.89

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a+4\,b\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*b*sin(d*x + c))/d
```

Fricas [A]
time = 3.78, size = 29, normalized size = 0.76

$$\frac{adx + (a\cos(dx + c) + 2b)\sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + (a*cos(d*x + c) + 2*b)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\cos^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(34) = 68.
time = 0.43, size = 82, normalized size = 2.16

$$\frac{(dx + c)a - \frac{2\left(a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 2b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)^3
 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 + 1)^2)/d
```

Mupad [B]
time = 0.81, size = 31, normalized size = 0.82

$$\frac{a\,x}{2} + \frac{a\,\sin(2\,c + 2\,d\,x)}{4\,d} + \frac{b\,\sin(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*x)/2 + (a*sin(2*c + 2*d*x))/(4*d) + (b*sin(c + d*x))/d
```

3.453 $\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=54

$$\frac{bx}{2}+\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] 1/2*b*x+a*sin(d*x+c)/d+1/2*b*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 2713, 2715, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*x)/2 + (a*Sin[c + d*x])/d + (b*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = a\int \cos^3(c+dx)\,dx + b\int \cos^2(c+dx)\,dx$$
$$= \frac{b\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}b\int 1\,dx - \frac{a\text{Subst}\left(\int (1-x^2)\ dx, x, -\right.}{d}$$
$$= \frac{bx}{2} + \frac{a\sin(c+dx)}{d} + \frac{b\cos(c+dx)\sin(c+dx)}{2d} - \frac{a\sin^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.08, size = 57, normalized size = 1.06

$$\frac{b(c+dx)}{2d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} + \frac{b\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*Si
n[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.08, size = 49, normalized size = 0.91

method	result
risch	$\frac{bx}{2}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(3dx+3c)}{12d}+\frac{b\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\frac{(2a-b)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{bx}{2}+\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{3bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{bx\left(\tan^6\left(\frac{dx}{2}\right.\right.}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)+b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A]
time = 0.26, size = 46, normalized size = 0.85

$$-\frac{4\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)a-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)b}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*b)/d
```

Fricas [A]
time = 4.43, size = 42, normalized size = 0.78

$$\frac{3\,bdx + \left(2\,a\cos\left(dx+c\right)^{2} + 3\,b\cos\left(dx+c\right) + 4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*b*d*x + (2*a*cos(d*x + c)^2 + 3*b*cos(d*x + c) + 4*a)*sin(d*x + c))/
d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec\left(c+dx\right)\right)\cos^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(48) = 96.
time = 0.44, size = 98, normalized size = 1.81

$$\frac{3\,(dx+c)b + \frac{2\left(6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} + 4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} + 6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*b + 2*(6*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*
c)^5 + 4*a*tan(1/2*d*x + 1/2*c)^3 + 6*a*tan(1/2*d*x + 1/2*c) + 3*b*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 0.83, size = 55, normalized size = 1.02

$$\frac{b\,x}{2} + \frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d} + \frac{b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d} + \frac{a\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

```
[Out] (b*x)/2 + (2*a*sin(c + d*x))/(3*d) + (b*cos(c + d*x)*sin(c + d*x))/(2*d) +
(a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.454 $\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=76

$$\frac{3ax}{8}+\frac{b\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{b\sin^3(c+dx)}{3d}$$

```
[Out] 3/8*a*x+b*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*sin
(d*x+c)/d-1/3*b*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2715, 8, 2713}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (b*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (b*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = a\int \cos^4(c+dx)\,dx + b\int \cos^3(c+dx)\,dx$$
$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{b\text{Subst}\left(\int (1$$
$$= \frac{b\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+d}{4d}$$
$$= \frac{3ax}{8} + \frac{b\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\text{si}}{4d}$$

Mathematica [A]
time = 0.14, size = 73, normalized size = 0.96

$$\frac{3a(c+dx)}{8d} + \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) + (a*
Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.09, size = 60, normalized size = 0.79

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{3ax}{8}+\frac{3b\sin(dx+c)}{4d}+\frac{a\sin(4dx+4c)}{32d}+\frac{b\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{9ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{3ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3ax\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{(5a-8b)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(5a+8b)}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.26, size = 57, normalized size = 0.75

$$\frac{3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a-32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a - 32*(sin
(d*x + c)^3 - 3*sin(d*x + c))*b)/d
```

Fricas [A]
time = 3.00, size = 53, normalized size = 0.70

$$\frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^{3}+8\,b\cos\left(dx+c\right)^{2}+9\,a\cos\left(dx+c\right)+16\,b\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 + 9*a*cos(d*x + c)
 + 16*b)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\cos^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(68) = 136.
time = 0.44, size = 140, normalized size = 1.84

$$\frac{9\left(dx+c\right)a-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2*d*x + 1
/2*c)^7 - 9*a*tan(1/2*d*x + 1/2*c)^5 - 40*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*ta
```

```
n(1/2*d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2
*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 0.83, size = 75, normalized size = 0.99

$$\frac{3\,a\,x}{8}+\frac{2\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{3\,a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d}+\frac{a\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x)),x)
[Out] (3*a*x)/8 + (2*b*sin(c + d*x))/(3*d) + (3*a*cos(c + d*x)*sin(c + d*x))/(8*d
) + (a*cos(c + d*x)^3*sin(c + d*x))/(4*d) + (b*cos(c + d*x)^2*sin(c + d*x))
/(3*d)
```

3.455 $\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=92

$$\frac{3bx}{8}+\frac{a\sin(c+dx)}{d}+\frac{3b\cos(c+dx)\sin(c+dx)}{8d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

```
[Out] 3/8*b*x+a*sin(d*x+c)/d+3/8*b*cos(d*x+c)*sin(d*x+c)/d+1/4*b*cos(d*x+c)^3*sin
(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 2713, 2715, 8}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*b*x)/8 + (a*Sin[c + d*x])/d + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c
 + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx = a\int \cos^5(c+dx)\,dx + b\int \cos^4(c+dx)\,dx$$
$$= \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3b)\int \cos^2(c+dx)\,dx - \frac{a\text{Subst}\left(\int (1\right.}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c+d}{4d}$$
$$= \frac{3bx}{8} + \frac{a\sin(c+dx)}{d} + \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin}{4d}$$

Mathematica [A]
time = 0.09, size = 92, normalized size = 1.00

$$\frac{3b(c+dx)}{8d} + \frac{5a\sin(c+dx)}{8d} + \frac{b\sin(2(c+dx))}{4d} + \frac{5a\sin(3(c+dx))}{48d} + \frac{b\sin(4(c+dx))}{32d} + \frac{a\sin(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) + (5*a*Sin[c + d*x])/(8*d) + (b*Sin[2*(c + d*x)])/(4*
d) + (5*a*Sin[3*(c + d*x)])/(48*d) + (b*Sin[4*(c + d*x)])/(32*d) + (a*Sin[5
*(c + d*x)])/(80*d)
```

Maple [A]
time = 0.10, size = 70, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3bx}{8}+\frac{5a\sin(dx+c)}{8d}+\frac{a\sin(5dx+5c)}{80d}+\frac{b\sin(4dx+4c)}{32d}+\frac{5a\sin(3dx+3c)}{48d}+\frac{b\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3bx}{8}+\frac{116a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{15bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{15bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{15bx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{15bx\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{3bx\left(\tan^1\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.25, size = 69, normalized size = 0.75

$$\frac{32\left(3\sin\left(dx+c\right)^{5}-10\sin\left(dx+c\right)^{3}+15\sin\left(dx+c\right)\right)a+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)b}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a + 15*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b)/d
```

Fricas [A]
time = 2.62, size = 64, normalized size = 0.70

$$\frac{45\,bdx+\left(24\,a\cos\left(dx+c\right)^{4}+30\,b\cos\left(dx+c\right)^{3}+32\,a\cos\left(dx+c\right)^{2}+45\,b\cos\left(dx+c\right)+64\,a\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(45*b*d*x + (24*a*cos(d*x + c)^4 + 30*b*cos(d*x + c)^3 + 32*a*cos(d*x
 + c)^2 + 45*b*cos(d*x + c) + 64*a)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\cos^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**5, x)
```

Giac [A]
time = 0.44, size = 154, normalized size = 1.67

$$\frac{45\left(dx+c\right)b+\frac{2\left(120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+160\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-30\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+464\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+160\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+30\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(45*(d*x + c)*b + 2*(120*a*tan(1/2*d*x + 1/2*c)^9 - 75*b*tan(1/2*d*x
+ 1/2*c)^9 + 160*a*tan(1/2*d*x + 1/2*c)^7 - 30*b*tan(1/2*d*x + 1/2*c)^7 + 4
```

```
64*a*tan(1/2*d*x + 1/2*c)^5 + 160*a*tan(1/2*d*x + 1/2*c)^3 + 30*b*tan(1/2*d
*x + 1/2*c)^3 + 120*a*tan(1/2*d*x + 1/2*c) + 75*b*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 4.75, size = 113, normalized size = 1.23

$$\frac{3\,b\,x}{8}+\frac{\left(2\,a-\frac{5\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,a}{3}-\frac{b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{116\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\left(\frac{8\,a}{3}+\frac{b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a+\frac{5\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] (3*b*x)/8 + (tan(c/2 + (d*x)/2)*(2*a + (5*b)/4) + tan(c/2 + (d*x)/2)^3*((8*
a)/3 + b/2) + tan(c/2 + (d*x)/2)^9*(2*a - (5*b)/4) + tan(c/2 + (d*x)/2)^7*(
(8*a)/3 - b/2) + (116*a*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2)^2
+ 1)^5)
```

3.456 $\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{(5a^2+4b^2)\tan(c+dx)}{5d}+\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{ab\sec^3(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 3/4*a*b*arctanh(sin(d*x+c))/d+1/5*(5*a^2+4*b^2)*tan(d*x+c)/d+3/4*a*b*sec(d*
x+c)*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3*tan(d*x+c)/d+1/5*b^2*sec(d*x+c)^4*ta
n(d*x+c)/d+1/15*(5*a^2+4*b^2)*tan(d*x+c)^3/d
```

Rubi [A]

time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3873, 3853, 3855, 4131, 3852}

$$\frac{(5a^2+4b^2)\tan^3(c+dx)}{15d}+\frac{(5a^2+4b^2)\tan(c+dx)}{5d}+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan(c+dx)\sec^3(c+dx)}{2d}+\frac{3ab\tan(c+dx)\sec(c+dx)}{4d}+\frac{b^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) + ((5*a^2 + 4*b^2)*Tan[c + d*x])/(5*d)
+ (3*a*b*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a*b*Sec[c + d*x]^3*Tan[c + d*x
])/(2*d) + (b^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((5*a^2 + 4*b^2)*Tan[c
 + d*x]^3)/(15*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^5(c+dx)\,dx + \int \sec^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{ab\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{b^2\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{2}(3ab)$$
$$= \frac{3ab\sec(c+dx)\tan(c+dx)}{4d} + \frac{ab\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{b^2\sec^4}{}$$
$$= \frac{3ab\tanh^{-1}(\sin(c+dx))}{4d} + \frac{(5a^2+4b^2)\tan(c+dx)}{5d} + \frac{3ab\sec(c+dx}{4}$$

Mathematica [A]
time = 0.62, size = 118, normalized size = 0.87

$$\frac{ab\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{3ab(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx))}{4d} + \frac{a^2\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d} + \frac{b^2\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) + (3*a*b*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x]))/(4*d) + (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/
d + (b^2*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

Maple [A]
time = 0.09, size = 110, normalized size = 0.81

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+2ba\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-b^2}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+2ba\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-b^2}{d}$

risch	$-\frac{i\left(45ab\,e^{9i(dx+c)}+210ba\,e^{7i(dx+c)}-120a^2e^{6i(dx+c)}-280a^2e^{4i(dx+c)}-320b^2e^{4i(dx+c)}-210ab\,e^{3i(dx+c)}-200a^2e^{2i(dx+}\right.}{30d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{4\left(25a^2+29b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{\left(4a^2-5ba+4b^2\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{\left(4a^2+5ba+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{\left(16a^2-3ba+8b^2\right)\left(\tan^7\left(\frac{dx}{2}\right.\right.}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*b*a*(-(-1/4*sec(d*x+c)^3-3/8
*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b^2*(-8/15-1/5*sec(d
*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 132, normalized size = 0.98

$$\frac{40\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^2+8\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)b^2-15\,ab\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*(40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 8*(3*tan(d*x + c)^5 + 10*
tan(d*x + c)^3 + 15*tan(d*x + c))*b^2 - 15*a*b*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 3.68, size = 136, normalized size = 1.01

$$\frac{45\,ab\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-45\,ab\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(45\,ab\cos\left(dx+c\right)^3+8\left(5\,a^2+4\,b^2\right)\cos\left(dx+c\right)^4+30\,ab\cos\left(dx+c\right)+4\left(5\,a^2+4\,b^2\right)\cos\left(dx+c\right)^2+12\,b^2\right)\sin\left(dx+c\right)}{120\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/120*(45*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*b*cos(d*x + c)^5*
log(-sin(d*x + c) + 1) + 2*(45*a*b*cos(d*x + c)^3 + 8*(5*a^2 + 4*b^2)*cos(d
*x + c)^4 + 30*a*b*cos(d*x + c) + 4*(5*a^2 + 4*b^2)*cos(d*x + c)^2 + 12*b^2
)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\sec^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(123) = 246$.
time = 0.50, size = 272, normalized size = 2.01

$$\frac{45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-160\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-80\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+200\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+232\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-160\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*d*x +
1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 - 160*a^2*tan(1/2*d*x + 1/2*c)^7 +
 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 80*b^2*tan(1/2*d*x + 1/2*c)^7 + 200*a^2*ta
n(1/2*d*x + 1/2*c)^5 + 232*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a^2*tan(1/2*d*x
 + 1/2*c)^3 - 30*a*b*tan(1/2*d*x + 1/2*c)^3 - 80*b^2*tan(1/2*d*x + 1/2*c)^3
 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^2*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 3.82, size = 221, normalized size = 1.64

$$\frac{3\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\left(2\,a^2-\frac{5\,a\,b}{2}+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{16\,a^2}{3}+a\,b-\frac{8\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{20\,a^2}{3}+\frac{116\,b^2}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{16\,a^2}{3}-a\,b-\frac{8\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2+\frac{5\,a\,b}{2}+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] (3*a*b*atanh(tan(c/2 + (d*x)/2)))/(2*d) - (tan(c/2 + (d*x)/2)^5*((20*a^2)/3
 + (116*b^2)/15) + tan(c/2 + (d*x)/2)^9*(2*a^2 - (5*a*b)/2 + 2*b^2) - tan(c
/2 + (d*x)/2)^3*(a*b + (16*a^2)/3 + (8*b^2)/3) - tan(c/2 + (d*x)/2)^7*((16*
a^2)/3 - a*b + (8*b^2)/3) + tan(c/2 + (d*x)/2)*((5*a*b)/2 + 2*a^2 + 2*b^2))
/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/
2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.457 $\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=110

$$\frac{(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2ab\tan(c+dx)}{d}+\frac{(4a^2+3b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 1/8*(4*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/8*(4*a^2+3*b^2
)*sec(d*x+c)*tan(d*x+c)/d+1/4*b^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*b*tan(d*x
+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3873, 3852, 4131, 3853, 3855}

$$\frac{(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2+3b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a*b*Tan[c + d*x])/d + ((
4*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (2*a*b*Tan[c + d*x]^3)/(3*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^4(c+dx)\,dx + \int \sec^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(4a^2+3b^2\right)\int \sec^3(c+dx)\,dx - \frac{(2a}{}$$

$$= \frac{2ab\tan(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b^2\sec^3(c+}{}$$

$$= \frac{\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2ab\tan(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\mathrm{se}}{}$$

Mathematica [A]
time = 0.30, size = 82, normalized size = 0.75

$$\frac{3\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3\left(4a^2+3b^2\right)\sec(c+dx)+6b^2\sec^3(c+dx)+16ab\left(3+\tan^2(c+dx)\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(4*a^2 + 3*b^2)*
Sec[c + d*x] + 6*b^2*Sec[c + d*x]^3 + 16*a*b*(3 + Tan[c + d*x]^2)))/(24*d)
```

Maple [A]
time = 0.10, size = 111, normalized size = 1.01

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2ba\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+b^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(d}{8}\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2ba\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+b^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(d}{8}\right.\right.}{d}$

norman	$\frac{\frac{\left(4a^2-16ba+5b^2\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{\left(4a^2+16ba+5b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{\left(12a^2-80ba-9b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{\left(12a^2+80ba-9b^2\right)\left(\tan\right.}{12d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$
risch	$-\frac{i\left(12a^2e^{7i(dx+c)}+9b^2e^{7i(dx+c)}+12a^2e^{5i(dx+c)}+33b^2e^{5i(dx+c)}-96ba\,e^{4i(dx+c)}-12a^2e^{3i(dx+c)}-33b^2e^{3i(dx+c)}-128ab\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-2*b*a*(-
2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*t
an(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.26, size = 144, normalized size = 1.31

$$\frac{32\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)ab-3b^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-12a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b - 3*b^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.93, size = 133, normalized size = 1.21

$$\frac{3\left(4a^2+3b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(4a^2+3b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(32ab\cos\left(dx+c\right)^3+16ab\cos\left(dx+c\right)+3\left(4a^2+3b^2\right)\cos\left(dx+c\right)^2+6b^2\right)\sin\left(dx+c\right)}{48d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*a^2 + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 + 3
*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a*b*cos(d*x + c)^3 + 16
*a*b*cos(d*x + c) + 3*(4*a^2 + 3*b^2)*cos(d*x + c)^2 + 6*b^2)*sin(d*x + c))
/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(102) = 204$.
time = 0.48, size = 258, normalized size = 2.35

$$\frac{3\,(4\,a^2+3\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,a^2+3\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+80\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*a^2 + 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 + 3*b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 -
 48*a*b*tan(1/2*d*x + 1/2*c)^7 + 15*b^2*tan(1/2*d*x + 1/2*c)^7 - 12*a^2*tan
(1/2*d*x + 1/2*c)^5 + 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1
/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 + 9
*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*tan(1/2*d*x + 1/2*c) + 48*a*b*tan(1/2*
d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)
/d
```

Mupad [B]
time = 3.66, size = 184, normalized size = 1.67

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2+\frac{3\,b^2}{4}\right)}{d}+\frac{\left(a^2-4\,a\,b+\frac{5\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(-a^2+\frac{20\,a\,b}{3}+\frac{3\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-a^2-\frac{20\,a\,b}{3}+\frac{3\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(a^2+4\,a\,b+\frac{5\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(a^2 + (3*b^2)/4))/d + (tan(c/2 + (d*x)/2)^5*((2
0*a*b)/3 - a^2 + (3*b^2)/4) + tan(c/2 + (d*x)/2)*(4*a*b + a^2 + (5*b^2)/4)
+ tan(c/2 + (d*x)/2)^7*(a^2 - 4*a*b + (5*b^2)/4) - tan(c/2 + (d*x)/2)^3*((2
0*a*b)/3 + a^2 - (3*b^2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)
/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.458 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=80

$$\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{(3a^2+2b^2)\tan(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}+\frac{b^2\sec^2(c+dx)\tan(c+dx)}{3d}$$

```
[Out] a*b*arctanh(sin(d*x+c))/d+1/3*(3*a^2+2*b^2)*tan(d*x+c)/d+a*b*sec(d*x+c)*tan
(d*x+c)/d+1/3*b^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3873, 3853, 3855, 4131, 3852, 8}

$$\frac{(3a^2+2b^2)\tan(c+dx)}{3d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + ((3*a^2 + 2*b^2)*Tan[c + d*x])/(3*d) + (a*b
*Sec[c + d*x]*Tan[c + d*x])/d + (b^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^3(c+dx)\,dx + \int \sec^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{b^2\sec^2(c+dx)\tan(c+dx)}{3d} + (ab)\int \mathrm{s}$$
$$= \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{b^2\sec^2(c+a}{}$$
$$= \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{(3a^2+2b^2)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\,\mathrm{t}}{d}$$

Mathematica [A]
time = 0.25, size = 71, normalized size = 0.89

$$\frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{b^2\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[c + d*x]*Ta
n[c + d*x])/d + (b^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.06, size = 74, normalized size = 0.92

method	result
derivativedivides	$\frac{a^2\tan(dx+c)+2ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$

default	$\frac{a^{2}\tan(dx+c)+2ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^{2}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
norman	$\frac{\frac{4\left(3a^{2}+b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2\left(a^{2}-ba+b^{2}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(a^{2}+ba+b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-ba\ln$
risch	$-\frac{2i\left(3ba\,\mathrm{e}^{5i(dx+c)}-3a^{2}\mathrm{e}^{4i(dx+c)}-6a^{2}\mathrm{e}^{2i(dx+c)}-6b^{2}\mathrm{e}^{2i(dx+c)}-3ba\,\mathrm{e}^{i(dx+c)}-3a^{2}-2b^{2}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}+\frac{ba\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-ba\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*tan(d*x+c)+2*b*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(
d*x+c)))-b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 84, normalized size = 1.05

$$\frac{2\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)b^{2}-3ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^{2}\tan\left(dx+c\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^2 - 3*a*b*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*tan
(d*x + c))/d
```

Fricas [A]
time = 2.36, size = 100, normalized size = 1.25

$$\frac{3ab\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3ab\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3ab\cos\left(dx+c\right)+\left(3a^{2}+2b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)\sin\left(dx+c\right)}{6d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*b*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(3*a*b*cos(d*x + c) + (3*a^2 + 2*b^2)*cos(d*x + c)^2
 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^{2}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(76) = 152.
time = 0.46, size = 178, normalized size = 2.22

$$\frac{3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2*c
)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^2*t
an(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/
2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 3.07, size = 141, normalized size = 1.76

$$\frac{2\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\left(2\,a^2-2\,a\,b+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-4\,a^2-\frac{4\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2+2\,a\,b+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] (2*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*a*b
+ 2*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^2 + (4*b^2)/3) + tan(c/2 + (d*x)/2)*(2
*a*b + 2*a^2 + 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4
+ tan(c/2 + (d*x)/2)^6 - 1))
```

3.459 $\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=59

$$\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*(2*a^2+b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/2*b^2*sec(d*x+c)
*tan(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3873, 3852, 8, 4131, 3855}

$$\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (b^2
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^2(c+dx)\,dx + \int \sec(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{b^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(2a^2+b^2\right)\int \sec(c+dx)\,dx - \frac{(2ab)\text{S}}{}$$
$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2ab\tan(c+dx)}{d} + \frac{b^2\sec(c+dx)\,\text{ta}}{2d}$$

Mathematica [A]
time = 0.11, size = 45, normalized size = 0.76

$$\frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))+b(4a+b\sec(c+dx))\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + b*(4*a + b*Sec[c + d*x])*Tan[c + d*x
])/(2*d)
```

Maple [A]
time = 0.06, size = 69, normalized size = 1.17

method	result
derivativedivides	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2ba\tan(dx+c)+b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+2ba\tan(dx+c)+b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
norman	$\frac{\frac{b(4a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{b(4a-b)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{(2a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{(2a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{ib\left(b\,e^{3i(dx+c)}-4a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-4a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)b^2}{2d}+\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}+\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*b*a*tan(d*x+c)+b^2*(1/2*sec(d*x+c)*tan
(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 80, normalized size = 1.36

$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)-8\,ab\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b*tan(d
*x + c))/d
```

Fricas [A]
time = 2.93, size = 93, normalized size = 1.58

$$\frac{\left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,ab\cos\left(dx+c\right)+b^2\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 + b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a*b*cos(d*x + c) + b^2)*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^2\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(55) = 110.
time = 0.43, size = 129, normalized size = 2.19

$$\frac{\left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 + b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(4*a*b*tan(1/2*d*x + 1/2*c)^3 - b^2*tan(1
/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 1.53, size = 99, normalized size = 1.68

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2+b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(4\,a\,b-b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b^2+4\,a\,b\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(2*a^2 + b^2))/d - (tan(c/2 + (d*x)/2)^3*(4*a*b
- b^2) - tan(c/2 + (d*x)/2)*(4*a*b + b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan
(c/2 + (d*x)/2)^2 + 1))
```

3.460 $\int (a + b\sec(c + dx))^2 \, dx$

Optimal. Leaf size=33

$$a^2x + \frac{2ab\tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2\tan(c + dx)}{d}$$

```
[Out] a^2*x+2*a*b*arctanh(sin(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3858, 3855, 3852, 8}

$$a^2x + \frac{2ab\tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2\tan(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\,dx = a^2x + (2ab)\int \sec(c+dx)\,dx + b^2\int \sec^2(c+dx)\,dx$$
$$= a^2x + \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{b^2\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}$$
$$= a^2x + \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d}$$

Mathematica [A]
time = 0.08, size = 32, normalized size = 0.97

$$\frac{a^2dx + 2ab\tanh^{-1}(\sin(c+dx)) + b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*d*x + 2*a*b*ArcTanh[Sin[c + d*x]] + b^2*Tan[c + d*x])/d
```

Maple [A]
time = 0.04, size = 43, normalized size = 1.30

method	result	size
derivativedivides	$\frac{a^2(dx+c)+2ba\ln(\sec(dx+c)+\tan(dx+c))+b^2\tan(dx+c)}{d}$	43
default	$\frac{a^2(dx+c)+2ba\ln(\sec(dx+c)+\tan(dx+c))+b^2\tan(dx+c)}{d}$	43
risch	$a^2x+\frac{2ib^2}{d(e^{2i(dx+c)}+1)}-\frac{2ba\ln(e^{i(dx+c)}-i)}{d}+\frac{2ba\ln(e^{i(dx+c)}+i)}{d}$	69
norman	$\frac{a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2x-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{2ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(d*x+c)+2*b*a*ln(sec(d*x+c)+tan(d*x+c))+b^2*tan(d*x+c))
```

Maxima [A]
time = 0.25, size = 40, normalized size = 1.21

$$a^2x + \frac{2\,ab\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{b^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a*b*log(sec(d*x + c) + tan(d*x + c))/d + b^2*tan(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.
time = 2.95, size = 74, normalized size = 2.24

$$\frac{a^2dx\cos(dx+c) + ab\cos(dx+c)\log(\sin(dx+c)+1) - ab\cos(dx+c)\log(-\sin(dx+c)+1) + b^2\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cos(d*x + c) + a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*
x + c)*log(-sin(d*x + c) + 1) + b^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c+dx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(33) = 66$.
time = 0.45, size = 77, normalized size = 2.33

$$\frac{(dx+c)a^2 + 2ab\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 2ab\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

Mupad [B]
time = 0.87, size = 181, normalized size = 5.48

$$\frac{2a^2\operatorname{atan}\left(\frac{64a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+256a^4b^2} + \frac{256a^4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+256a^4b^2}\right)}{d} - \frac{2b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)} + \frac{4ab\operatorname{atanh}\left(\frac{128a^5b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128a^5b+512a^3b^3} + \frac{512a^3b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128a^5b+512a^3b^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))^2,x)

[Out] (2*a^2*atan((64*a^6*tan(c/2 + (d*x)/2))/(64*a^6 + 256*a^4*b^2) + (256*a^4*b^2*tan(c/2 + (d*x)/2))/(64*a^6 + 256*a^4*b^2)))/d - (2*b^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1)) + (4*a*b*atanh((128*a^5*b*tan(c/2 + (d*x)/2))/(128*a^5*b + 512*a^3*b^3) + (512*a^3*b^3*tan(c/2 + (d*x)/2))/(128*a^5*b + 512*a^3*b^3)))/d

3.461 $\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=33

$$2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}$$

```
[Out] 2*a*b*x+b^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3873, 8, 4130, 3855}

$$\frac{a^2\sin(c+dx)}{d}+2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] 2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int 1\,dx + \int \cos(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= 2abx + \frac{a^2\sin(c+dx)}{d} + b^2\int \sec(c+dx)\,dx$$
$$= 2abx + \frac{b^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.39

$$2abx + \frac{b^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\cos(dx)\sin(c)}{d} + \frac{a^2\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] 2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*Co
s[c]*Sin[d*x])/d
```

Maple [A]
time = 0.07, size = 43, normalized size = 1.30

method	result
derivativedivides	$\frac{a^{2} \sin(dx+c)+2ba(dx+c)+b^{2} \ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^{2} \sin(dx+c)+2ba(dx+c)+b^{2} \ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$2abx-\frac{i a^{2} e^{i(dx+c)}}{2d}+\frac{i a^{2} e^{-i(dx+c)}}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right) b^{2}}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right) b^{2}}{d}$
norman	$\frac{-2abx-\frac{2a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^{2}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+2abx\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b^{2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b^{2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*sin(d*x+c)+2*b*a*(d*x+c)+b^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.26, size = 51, normalized size = 1.55

$$\frac{4\,(dx+c)ab + b^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,a^2\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*a*b + b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 2.29, size = 52, normalized size = 1.58

$$\frac{4\,abdx + b^2\log\left(\sin\left(dx + c\right) + 1\right) - b^2\log\left(-\sin\left(dx + c\right) + 1\right) + 2\,a^2\sin\left(dx + c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*d*x + b^2*log(sin(d*x + c) + 1) - b^2*log(-sin(d*x + c) + 1) + 2
*a^2*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a + b\sec\left(c + dx\right)\right)^2\cos\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(33) = 66$.
time = 0.46, size = 78, normalized size = 2.36

$$\frac{2\,(dx + c)ab + b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a*b + b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

Mupad [B]
time = 0.85, size = 73, normalized size = 2.21

$$\frac{a^2\,\sin\left(c + d\,x\right)}{d} + \frac{2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x))/d + (2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.462 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}\left(a^2+2b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*(a^2+2*b^2)*x+2*a*b*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3873, 2717, 4130, 8}

$$\frac{1}{2}x\left(a^2+2b^2\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*
x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \cos(c+dx)\,dx + \int \cos^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{2ab\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}\left(a^2+2b^2\right)\int 1\,dx$$
$$= \frac{1}{2}\left(a^2+2b^2\right)x + \frac{2ab\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.92

$$\frac{2\left(a^2+2b^2\right)(c+dx)+8ab\sin(c+dx)+a^2\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
[Out] (2*(a^2 + 2*b^2)*(c + d*x) + 8*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)])/(4*
d)
```

Maple [A]
time = 0.09, size = 51, normalized size = 1.02

method	result
risch	$\frac{a^2 x}{2}+x\,b^2+\frac{2ab\sin(dx+c)}{d}+\frac{a^2\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ba\sin(dx+c)+b^2(dx+c)}{d}$
default	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ba\sin(dx+c)+b^2(dx+c)}{d}$
norman	$\frac{\left(-\frac{a^2}{2}-b^2\right)x+\left(-\frac{a^2}{2}-b^2\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{2}+b^2\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{2}+b^2\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2a^2\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*b*a*sin(d*x+c)+b^2*(d*
x+c))
```

Maxima [A]
time = 0.26, size = 47, normalized size = 0.94

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2+4\,(dx+c)b^2+8\,ab\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*b^2 + 8*a*b*sin(d*x
 + c))/d
```

Fricas [A]
time = 3.81, size = 40, normalized size = 0.80

$$\frac{(a^2+2\,b^2)dx+(a^2\cos{(dx+c)}+4\,ab)\sin{(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 2*b^2)*d*x + (a^2*cos(d*x + c) + 4*a*b)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec{(c+dx)})^2\cos^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(46) = 92.
time = 0.44, size = 96, normalized size = 1.92

$$\frac{(a^2+2\,b^2)(dx+c)-\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*b^2)*(d*x + c) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/
2*d*x + 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))/(
tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 0.85, size = 42, normalized size = 0.84

$$\frac{a^2\,x}{2}+b^2\,x+\frac{a^2\,\sin{(2\,c+2\,d\,x)}}{4\,d}+\frac{2\,a\,b\,\sin{(c+d\,x)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*x)/2 + b^2*x + (a^2*sin(2*c + 2*d*x))/(4*d) + (2*a*b*sin(c + d*x))/d
```

3.463 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=58

$$abx+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{ab\cos(c+dx)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}$$

```
[Out] a*b*x+(a^2+b^2)*sin(d*x+c)/d+a*b*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)
^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3873, 2715, 8, 4129, 3092}

$$\frac{(a^2+b^2)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin(c+dx)\cos(c+dx)}{d}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a*b*x + ((a^2 + b^2)*Sin[c + d*x])/d + (a*b*Cos[c + d*x]*Sin[c + d*x])/d -
(a^2*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \cos^2(c+dx)\,dx+\int \cos^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx\\ &= \frac{ab\cos(c+dx)\sin(c+dx)}{d}+(ab)\int 1\,dx+\int \cos(c+dx)\left(b^2+a^2\,\mathrm{c}\right.\\ &= abx+\frac{ab\cos(c+dx)\sin(c+dx)}{d}-\frac{\mathrm{Subst}\left(\int \left(a^2+b^2-a^2x^2\right)\,dx,x,\right.}{d}\\ &= abx+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{ab\cos(c+dx)\sin(c+dx)}{d}-\frac{a^2\sin^3(c}{3c}\end{aligned}$$

Mathematica [A]

time = 0.16, size = 59, normalized size = 1.02

$$\frac{3(3a^2+4b^2)\sin(c+dx)+a(12b(c+dx)+6b\sin(2(c+dx))+a\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*(3*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*b*(c + d*x) + 6*b*Sin[2*(c + d*x)]
+ a*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A]

time = 0.10, size = 63, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2\sin(dx+c)}{d}$
default	$\frac{\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2\sin(dx+c)}{d}$
risch	$abx+\frac{3a^2\sin(dx+c)}{4d}+\frac{\sin(dx+c)b^2}{d}+\frac{a^2\sin(3dx+3c)}{12d}+\frac{ba\sin(2dx+2c)}{2d}$
norman	$\frac{abx\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-abx-\frac{2\left(a^2-3ba-3b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2\left(a^2-ba+b^2\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(a^2+ba+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(a^2}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*b*a*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+b^2*sin(d*x+c))
```

Maxima [A]
time = 0.26, size = 60, normalized size = 1.03

$$-\frac{2\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)ab-6\,b^{2}\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a*b - 6*b^2*sin(d*x + c))/d
```

Fricas [A]
time = 3.35, size = 52, normalized size = 0.90

$$\frac{3\,abdx+\left(a^{2}\cos\left(dx+c\right)^{2}+3\,ab\cos\left(dx+c\right)+2\,a^{2}+3\,b^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*d*x + (a^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) + 2*a^2 + 3*b^2)*
sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^{2}\cos^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(56) = 112.
time = 0.49, size = 153, normalized size = 2.64

$$\frac{3\left(dx+c\right)ab+\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+2\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)*a*b + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d

Mupad [B]
time = 0.83, size = 72, normalized size = 1.24

$$\frac{2\,a^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{b^2\,\sin\left(c+d\,x\right)}{d}+a\,b\,x+\frac{a^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^2,x)

[Out] (2*a^2*sin(c + d*x))/(3*d) + (b^2*sin(c + d*x))/d + a*b*x + (a^2*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (a*b*cos(c + d*x)*sin(c + d*x))/d

3.464 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{1}{8}\left(3a^2+4b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2ab\sin}{}$$

```
[Out] 1/8*(3*a^2+4*b^2)*x+2*a*b*sin(d*x+c)/d+1/8*(3*a^2+4*b^2)*cos(d*x+c)*sin(d*x
+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3873, 2713, 4130, 2715, 8}

$$\frac{(3a^2+4b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2+4b^2\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + 4*b^2)*x)/8 + (2*a*b*Sin[c + d*x])/d + ((3*a^2 + 4*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*b*
Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \cos^3(c+dx)\,dx + \int \cos^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\left(3a^2+4b^2\right)\int \cos^2(c+dx)\,dx - \frac{(2a}{}$$
$$= \frac{2ab\sin(c+dx)}{d} + \frac{(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+}{}$$
$$= \frac{1}{8}\left(3a^2+4b^2\right)x + \frac{2ab\sin(c+dx)}{d} + \frac{(3a^2+4b^2)\cos(c+dx)\sin(c+}{8d}$$

Mathematica [A]
time = 0.19, size = 86, normalized size = 0.85

$$\frac{36a^2c+48b^2c+36a^2dx+48b^2dx+192ab\sin(c+dx)-64ab\sin^3(c+dx)+24(a^2+b^2)\sin(2(c+dx))+3a^2\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (36*a^2*c + 48*b^2*c + 36*a^2*d*x + 48*b^2*d*x + 192*a*b*Sin[c + d*x] - 64*
a*b*Sin[c + d*x]^3 + 24*(a^2 + b^2)*Sin[2*(c + d*x)] + 3*a^2*Sin[4*(c + d*x
)])/(96*d)
```

Maple [A]
time = 0.12, size = 89, normalized size = 0.88

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ba\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ba\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$

risch	$\frac{3a^2x}{8}+\frac{x\,b^2}{2}+\frac{3ab\sin(dx+c)}{2d}+\frac{a^2\sin(4dx+4c)}{32d}+\frac{ba\sin(3dx+3c)}{6d}+\frac{a^2\sin(2dx+2c)}{4d}+\frac{\sin(2dx+2c)b^2}{4d}$
norman	$\frac{\left(-\frac{3a^2}{8}-\frac{b^2}{2}\right)x+\left(-\frac{9a^2}{8}-\frac{3b^2}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{3a^2}{4}-b^2\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3a^2}{4}+b^2\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3a^2}{8}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*b
*a*(2+cos(d*x+c)^2)*sin(d*x+c)+b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A]
time = 0.26, size = 82, normalized size = 0.81

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a^2-64\,\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)ab+24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))b^2}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 64*(s
in(d*x + c)^3 - 3*sin(d*x + c))*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*b
^2)/d
```

Fricas [A]
time = 2.13, size = 77, normalized size = 0.76

$$\frac{3\,(3\,a^2+4\,b^2)dx+\left(6\,a^2\cos\left(dx+c\right)^3+16\,ab\cos\left(dx+c\right)^2+32\,ab+3\,(3\,a^2+4\,b^2)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^2 + 4*b^2)*d*x + (6*a^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2
 + 32*a*b + 3*(3*a^2 + 4*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\cos^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(93) = 186.

time = 0.43, size = 224, normalized size = 2.22

$$\frac{3\,(3\,a^2+4\,b^2)(dx+c)-\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-80\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^2 + 4*b^2)*(d*x + c) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*a
*b*tan(1/2*d*x + 1/2*c)^7 + 12*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*a^2*tan(1/2*d
*x + 1/2*c)^5 - 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*b^2*tan(1/2*d*x + 1/2*c)
^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*b^2*
tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*d*x +
 1/2*c) - 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]

time = 0.87, size = 93, normalized size = 0.92

$$\frac{3\,a^2\,x}{8}+\frac{b^2\,x}{2}+\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b\,\sin\left(c+d\,x\right)}{2\,d}+\frac{a\,b\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/8 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) + (a^2*sin(4*c + 4*d
*x))/(32*d) + (b^2*sin(2*c + 2*d*x))/(4*d) + (3*a*b*sin(c + d*x))/(2*d) + (
a*b*sin(3*c + 3*d*x))/(6*d)
```

3.465 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=111

$$\frac{3abx}{4}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{3ab\cos(c+dx)\sin(c+dx)}{4d}+\frac{ab\cos^3(c+dx)\sin(c+dx)}{2d}-\frac{(2a^2+b^2)\sin^3(c+$$

```
[Out] 3/4*a*b*x+(a^2+b^2)*sin(d*x+c)/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/2*a*b*co
s(d*x+c)^3*sin(d*x+c)/d-1/3*(2*a^2+b^2)*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5
/d
```

Rubi [A]
time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3873, 2715, 8, 4129, 3092, 380}

$$-\frac{(2a^2+b^2)\sin^3(c+dx)}{3d}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{a^2\sin^5(c+dx)}{5d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\cos(c+dx)}{4d}+\frac{3abx}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/4 + ((a^2 + b^2)*Sin[c + d*x])/d + (3*a*b*Cos[c + d*x]*Sin[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - ((2*a^2 + b^2)*Sin[c
+ d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
```

```
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \cos^4(c+dx)\,dx + \int \cos^5(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx \\
&= \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}(3ab)\int \cos^2(c+dx)\,dx + \int \cos^3(c \\
&= \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{4}(3ab) \\
&= \frac{3abx}{4} + \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} - \\
&= \frac{3abx}{4} + \frac{(a^2+b^2)\sin(c+dx)}{d} + \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 97, normalized size = 0.87

$$\frac{30(5a^2+8b^2)\sin(c+dx)-80b^2\sin^3(c+dx)+a(120b\sin(2(c+dx))+25a\sin(3(c+dx))+3(60b(c+dx)+5b\sin(4(c+dx))+a\sin(5(c+dx))))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (30*(5*a^2 + 8*b^2)*Sin[c + d*x] - 80*b^2*Sin[c + d*x]^3 + a*(120*b*Sin[2*(
c + d*x)] + 25*a*Sin[3*(c + d*x)] + 3*(60*b*(c + d*x) + 5*b*Sin[4*(c + d*x)
] + a*Sin[5*(c + d*x)])))/(240*d)
```

Maple [A]
time = 0.12, size = 95, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+2ba\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2\left(2+\cos^2(dx+c)\right)}{3}}{d}$
default	$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+2ba\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2\left(2+\cos^2(dx+c)\right)}{3}}{d}$
risch	$\frac{3abx}{4}+\frac{5a^2\sin(dx+c)}{8d}+\frac{3\sin(dx+c)b^2}{4d}+\frac{a^2\sin(5dx+5c)}{80d}+\frac{ba\sin(4dx+4c)}{16d}+\frac{5a^2\sin(3dx+3c)}{48d}+\frac{\sin(3dx+3c}{12d}$
norman	$\frac{-\frac{3abx}{4}-\frac{\left(4a^2-9ba+20b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{\left(4a^2-5ba+4b^2\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{\left(4a^2+5ba+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{\left(4a^2+9ba+20b^2\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*b*a*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b^2*(2+cos(d*x+c)^2
)*sin(d*x+c))
```

Maxima [A]
time = 0.26, size = 94, normalized size = 0.85

$$\frac{16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2+15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)ab-80\left(\sin(dx+c)^3-3\sin(dx+c)\right)b^2}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b - 80*(sin(d*x
+ c)^3 - 3*sin(d*x + c))*b^2)/d
```

Fricas [A]
time = 3.93, size = 86, normalized size = 0.77

$$\frac{45\,abdx+\left(12\,a^2\cos(dx+c)^4+30\,ab\cos(dx+c)^3+45\,ab\cos(dx+c)+4\left(4\,a^2+5\,b^2\right)\cos(dx+c)^2+32\,a^2+40\,b^2\right)\sin(dx+c)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(45*a*b*d*x + (12*a^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 + 45*a*b*
cos(d*x + c) + 4*(4*a^2 + 5*b^2)*cos(d*x + c)^2 + 32*a^2 + 40*b^2)*sin(d*x
+ c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \cos^5(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(101) = 202.
time = 0.48, size = 247, normalized size = 2.23

$$\frac{45\,(dx+c)ab + \frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 160\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 232\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 200\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 160\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*(d*x + c)*a*b + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*
d*x + 1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 + 80*a^2*tan(1/2*d*x + 1/2*c
)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 160*b^2*tan(1/2*d*x + 1/2*c)^7 + 232*
a^2*tan(1/2*d*x + 1/2*c)^5 + 200*b^2*tan(1/2*d*x + 1/2*c)^5 + 80*a^2*tan(1/
2*d*x + 1/2*c)^3 + 30*a*b*tan(1/2*d*x + 1/2*c)^3 + 160*b^2*tan(1/2*d*x + 1/
2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 0.89, size = 117, normalized size = 1.05

$$\frac{5\,a^2\,\sin\left(c+d\,x\right)}{8\,d}+\frac{3\,b^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,a\,b\,x}{4}+\frac{5\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{a^2\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{b^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{a\,b\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (5*a^2*sin(c + d*x))/(8*d) + (3*b^2*sin(c + d*x))/(4*d) + (3*a*b*x)/4 + (5*
a^2*sin(3*c + 3*d*x))/(48*d) + (a^2*sin(5*c + 5*d*x))/(80*d) + (b^2*sin(3*c
 + 3*d*x))/(12*d) + (a*b*sin(2*c + 2*d*x))/(2*d) + (a*b*sin(4*c + 4*d*x))/(
16*d)
```

3.466 $\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=189

$$\frac{a(4a^2+9b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(3a^4-52a^2b^2-16b^4)\tan(c+dx)}{30bd}-\frac{a(6a^2-71b^2)\sec(c+dx)\tan(c+d}{120d}$$

```
[Out] 1/8*a*(4*a^2+9*b^2)*arctanh(sin(d*x+c))/d-1/30*(3*a^4-52*a^2*b^2-16*b^4)*ta
n(d*x+c)/b/d-1/120*a*(6*a^2-71*b^2)*sec(d*x+c)*tan(d*x+c)/d-1/60*(3*a^2-16*
b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d-1/20*a*(a+b*sec(d*x+c))^3*tan(d*x+c)
/b/d+1/5*(a+b*sec(d*x+c))^4*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.22, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3925, 4087, 4082, 3872, 3855, 3852, 8}

$$\frac{a(4a^2+9b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(3a^2-16b^2)\tan(c+dx)(a+b\sec(c+dx))^2}{60bd}-\frac{a(6a^2-71b^2)\tan(c+dx)\sec(c+dx)}{120d}-\frac{(3a^4-52a^2b^2-16b^4)\tan(c+dx)}{30bd}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}-\frac{a\tan(c+dx)(a+b\sec(c+dx))^3}{20bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - ((3*a^4 - 52*a^2*b^2 - 16
*b^4)*Tan[c + d*x])/(30*b*d) - (a*(6*a^2 - 71*b^2)*Sec[c + d*x]*Tan[c + d*x
])/(120*d) - ((3*a^2 - 16*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*b*d
) - (a*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b*d) + ((a + b*Sec[c + d*x]
)^4*Tan[c + d*x])/(5*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5bd} + \frac{\int \sec(c+dx)(4b-a\sec(c+dx}{5b}$$
$$= -\frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{20bd} + \frac{(a+b\sec(c+dx))^4\tan(c+}{5bd}$$
$$= -\frac{(3a^2-16b^2)(a+b\sec(c+dx))^2\tan(c+dx)}{60bd} - \frac{a(a+b\sec(c+a}{20}$$
$$= -\frac{a(6a^2-71b^2)\sec(c+dx)\tan(c+dx)}{120d} - \frac{(3a^2-16b^2)(a+b\sec(}{60b}$$
$$= -\frac{a(6a^2-71b^2)\sec(c+dx)\tan(c+dx)}{120d} - \frac{(3a^2-16b^2)(a+b\sec(}{60b}$$
$$= \frac{a(4a^2+9b^2)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a(6a^2-71b^2)\sec(c+dx)\tan(}{120d}$$
$$= \frac{a(4a^2+9b^2)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{(3a^4-52a^2b^2-16b^4)\tan(c+}{30bd}$$

Mathematica [A]
time = 0.91, size = 120, normalized size = 0.63

$$\frac{15a(4a^2+9b^2)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(15a(4a^2+9b^2)\sec(c+dx)+90ab^2\sec^3(c+dx)+8b(15(3a^2+b^2)+5(3a^2+2b^2)\tan^2(c+dx)+3b^2\tan^4(c+dx))\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (15*a*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*a*(4*a^2 + 9
*b^2)*Sec[c + d*x] + 90*a*b^2*Sec[c + d*x]^3 + 8*b*(15*(3*a^2 + b^2) + 5*(3
*a^2 + 2*b^2)*Tan[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(120*d)
```

Maple [A]
time = 0.12, size = 148, normalized size = 0.78

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3b a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3b^2a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\right.\right.}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3b a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3b^2a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\right.\right.}{d}$
norman	$\frac{\frac{\left(4a^3-24b a^2+15b^2a-8b^3\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{\left(4a^3+24b a^2+15b^2a+8b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{\left(12a^3-96b a^2+9b^2a-16b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{6d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}$
risch	$-\frac{i\left(60a^3e^{9i(dx+c)}+135a b^2e^{9i(dx+c)}+120a^3e^{7i(dx+c)}+630a b^2e^{7i(dx+c)}-720a^2b e^{6i(dx+c)}-1680b a^2e^{4i(dx+c)}-640b^3e\right.}{60d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-3*b*a^2*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*b^2*a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x
+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b^3*(-8/15-1/5*sec(d*x+c)^4-
4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 181, normalized size = 0.96

$$\frac{240\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)a^{2}b+16\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)b^{3}-45\,ab^{2}\left(\frac{2\left(3\sin(dx+c)^{3}-5\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\sin(dx+c)^{2}+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,a^{3}\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3 - 45*a*b^2*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.24, size = 170, normalized size = 0.90

$$\frac{15\left(4\,a^{3}+9\,ab^{2}\right)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(4\,a^{3}+9\,ab^{2}\right)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(15\,a^{2}b+4\,b^{3}\right)\cos\left(dx+c\right)^{4}+90\,ab^{2}\cos\left(dx+c\right)+15\left(4\,a^{3}+9\,ab^{2}\right)\cos\left(dx+c\right)^{3}+24\,b^{3}+8\left(15\,a^{2}b+4\,b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(4*a^3 + 9*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^
3 + 9*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(15*a^2*b + 4*b^
3)*cos(d*x + c)^4 + 90*a*b^2*cos(d*x + c) + 15*(4*a^3 + 9*a*b^2)*cos(d*x +
c)^3 + 24*b^3 + 8*(15*a^2*b + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^{3}\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(177) = 354.
time = 0.49, size = 367, normalized size = 1.94

$$\frac{15\,(4\,a^3+9\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(4\,a^3+9\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+225\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-120\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+960\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-90\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+160\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1200\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-464\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+960\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+90\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+160\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-360\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-225\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-120\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(4*a^3 + 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
+ 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1/2
*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 225*a*b^2*tan(1/2*d*x + 1/2*c)^9
 - 120*b^3*tan(1/2*d*x + 1/2*c)^9 - 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 960*a^
2*b*tan(1/2*d*x + 1/2*c)^7 - 90*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 160*b^3*tan(
1/2*d*x + 1/2*c)^7 - 1200*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 464*b^3*tan(1/2*d*
x + 1/2*c)^5 + 120*a^3*tan(1/2*d*x + 1/2*c)^3 + 960*a^2*b*tan(1/2*d*x + 1/2
*c)^3 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*b^3*tan(1/2*d*x + 1/2*c)^3 -
60*a^3*tan(1/2*d*x + 1/2*c) - 360*a^2*b*tan(1/2*d*x + 1/2*c) - 225*a*b^2*ta
n(1/2*d*x + 1/2*c) - 120*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

Mupad [B]
time = 4.77, size = 258, normalized size = 1.37

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^3+\frac{9\,a\,b^2}{4}\right)}{d}-\frac{\left(-a^3+6\,a^2\,b-\frac{15\,a\,b^2}{4}+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(2\,a^3-16\,a^2\,b+\frac{3\,a\,b^2}{2}-\frac{8\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(20\,a^2\,b+\frac{116\,b^3}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-2\,a^3-16\,a^2\,b-\frac{3\,a\,b^2}{2}-\frac{8\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(a^3+6\,a^2\,b+\frac{15\,a\,b^2}{4}+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((9*a*b^2)/4 + a^3))/d - (tan(c/2 + (d*x)/2)^7*(
(3*a*b^2)/2 - 16*a^2*b + 2*a^3 - (8*b^3)/3) - tan(c/2 + (d*x)/2)^3*((3*a*b^
2)/2 + 16*a^2*b + 2*a^3 + (8*b^3)/3) + tan(c/2 + (d*x)/2)*((15*a*b^2)/4 + 6
*a^2*b + a^3 + 2*b^3) + tan(c/2 + (d*x)/2)^5*(20*a^2*b + (116*b^3)/15) - ta
n(c/2 + (d*x)/2)^9*((15*a*b^2)/4 - 6*a^2*b + a^3 - 2*b^3))/(d*(5*tan(c/2 +
(d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2
+ (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.467 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=130

$$\frac{3b(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(a^2+4b^2)\tan(c+dx)}{2d}+\frac{b(2a^2+3b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a(a+b\,\mathrm{se}}{}$$

```
[Out] 3/8*b*(4*a^2+b^2)*arctanh(sin(d*x+c))/d+1/2*a*(a^2+4*b^2)*tan(d*x+c)/d+1/8*
b*(2*a^2+3*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/4*a*(a+b*sec(d*x+c))^2*tan(d*x+c)
/d+1/4*(a+b*sec(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A]

time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3920, 4087, 4082, 3872, 3855, 3852, 8}

$$\frac{a(a^2+4b^2)\tan(c+dx)}{2d}+\frac{3b(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(2a^2+3b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}+\frac{a\tan(c+dx)(a+b\sec(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(a^2 + 4*b^2)*Tan[c +
d*x])/(2*d) + (b*(2*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(a +
 b*Sec[c + d*x])^2*Tan[c + d*x])/(4*d) + ((a + b*Sec[c + d*x])^3*Tan[c + d*
x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3920

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[
e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
0]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{3}{4}\int \sec(c+dx)(b+a\sec(c+\\ &= \frac{a(a+b\sec(c+dx))^2\tan(c+dx)}{4d}+\frac{(a+b\sec(c+dx))^3\tan(c+\\ &= \frac{b(2a^2+3b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a(a+b\sec(c+dx))^2\tan(c\\ &= \frac{b(2a^2+3b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a(a+b\sec(c+dx))^2\tan(c\\ &= \frac{3b(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(2a^2+3b^2)\sec(c+dx)\tan(c\\ &= \frac{3b(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(a^2+4b^2)\tan(c+dx)}{2d}+\frac{b(2\end{aligned}$$

Mathematica [A]

time = 0.47, size = 90, normalized size = 0.69

$$\frac{3b(4a^2+b^2)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3b(4a^2+b^2)\sec(c+dx)+2b^3\sec^3(c+dx)+8a(a^2+3b^2+b^2\tan^2(c+dx))\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(4*a^2 + b^2)*
Sec[c + d*x] + 2*b^3*Sec[c + d*x]^3 + 8*a*(a^2 + 3*b^2 + b^2*Tan[c + d*x]^2
)))/(8*d)
```

Maple [A]
time = 0.09, size = 125, normalized size = 0.96

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+3b\,a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3b^2a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+b^3\left(-\left(-\frac{(s}{}\right.\right.}{d}$
default	$\frac{a^3\tan(dx+c)+3b\,a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3b^2a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+b^3\left(-\left(-\frac{(s}{}\right.\right.}{d}$
norman	$\frac{-\frac{\left(8a^3-12b\,a^2+24b^2a-5b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{\left(8a^3+12b\,a^2+24b^2a+5b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{\left(24a^3-12b\,a^2+40b^2a+3b^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$
risch	$-\frac{i\left(12b\,a^2e^{7i(dx+c)}+3b^3e^{7i(dx+c)}-8a^3e^{6i(dx+c)}+12a^2b\,e^{5i(dx+c)}+11b^3e^{5i(dx+c)}-24a^3e^{4i(dx+c)}-48a\,b^2e^{4i(dx+c)}-12b\right.}{4d\left(e^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*tan(d*x+c)+3*b*a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+ta
n(d*x+c)))-3*b^2*a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^3*(-(-1/4*sec(d*x+c
)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.26, size = 158, normalized size = 1.22

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab^2-b^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-12a^2b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+16a^3\tan(dx+c)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2 - b^3*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*b*(2*sin(d*x + c)/(sin(d*x +
```

```
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x
 + c))/d
```

Fricas [A]
time = 2.90, size = 140, normalized size = 1.08

$$\frac{3\left(4 a^{2} b+b^{3}\right) \cos\left(d x+c\right)^{4} \log\left(\sin\left(d x+c\right)+1\right)-3\left(4 a^{2} b+b^{3}\right) \cos\left(d x+c\right)^{4} \log\left(-\sin\left(d x+c\right)+1\right)+2\left(8 a b^{2} \cos\left(d x+c\right)+8\left(a^{3}+2 a b^{2}\right) \cos\left(d x+c\right)^{3}+2 b^{3}+3\left(4 a^{2} b+b^{3}\right) \cos\left(d x+c\right)^{2}\right) \sin\left(d x+c\right)}{16 d \cos\left(d x+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(4*a^2*b + b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2*b +
 b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a*b^2*cos(d*x + c) + 8*(
a^3 + 2*a*b^2)*cos(d*x + c)^3 + 2*b^3 + 3*(4*a^2*b + b^3)*cos(d*x + c)^2)*s
in(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b \sec\left(c+d x\right)\right)^{3} \sec^{2}\left(c+d x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(120) = 240.
time = 0.50, size = 330, normalized size = 2.54

$$\frac{3\left(4 a^{2} b+b^{3}\right) \log\left(\left|\tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)+1\right|\right)-3\left(4 a^{2} b+b^{3}\right) \log\left(\left|\tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)-1\right|\right)-\frac{2\left(8 a^{3} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{7}-12 a^{2} b \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{7}+24 a b^{2} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{7}-5 b^{3} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{7}-24 a^{3} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5}+12 a^{2} b \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5}-40 a b^{2} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5}-3 b^{3} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5}+24 a^{3} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}+12 a^{2} b \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}+40 a b^{2} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}-3 b^{3} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}-8 a^{3} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)-12 a^{2} b \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)-24 a b^{2} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)-5 b^{3} \tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)\right)}{\left(\tan\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2}-1\right)^{4}}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(4*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2*b + b^
3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a^3*tan(1/2*d*x + 1/2*c)^7 - 1
2*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 5*b^3*ta
n(1/2*d*x + 1/2*c)^7 - 24*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 - 40*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^
5 + 24*a^3*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 40*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*tan(1/2*d
*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c) - 24*a*b^2*tan(1/2*d*x + 1/2*c)
 - 5*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 4.80, size = 226, normalized size = 1.74

$$\frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,a^2+b^2\right)}{4\,d}-\frac{\left(2\,a^3-3\,a^2\,b+6\,a\,b^2-\frac{5\,b^3}{4}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\left(-6\,a^3+3\,a^2\,b-10\,a\,b^2-\frac{3\,b^3}{4}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(6\,a^3+3\,a^2\,b+10\,a\,b^2-\frac{3\,b^3}{4}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(-2\,a^3-3\,a^2\,b-6\,a\,b^2-\frac{5\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (3*b*atanh(tan(c/2 + (d*x)/2))*(4*a^2 + b^2))/(4*d) - (tan(c/2 + (d*x)/2)^7
*(6*a*b^2 - 3*a^2*b + 2*a^3 - (5*b^3)/4) + tan(c/2 + (d*x)/2)^3*(10*a*b^2 +
 3*a^2*b + 6*a^3 - (3*b^3)/4) - tan(c/2 + (d*x)/2)^5*(10*a*b^2 - 3*a^2*b +
6*a^3 + (3*b^3)/4) - tan(c/2 + (d*x)/2)*(6*a*b^2 + 3*a^2*b + 2*a^3 + (5*b^3
)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*
x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.468 $\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=99

$$\frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2b(4a^2+b^2)\tan(c+dx)}{3d}+\frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b(a+b\sec(c+$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2/3*b*(4*a^2+b^2)*tan(d*x+c)/d+5/
6*a*b^2*sec(d*x+c)*tan(d*x+c)/d+1/3*b*(a+b*sec(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3915, 4082, 3872, 3855, 3852, 8}

$$\frac{2b(4a^2+b^2)\tan(c+dx)}{3d}+\frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{6d}+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*b*(4*a^2 + b^2)*Tan[c
+ d*x])/(3*d) + (5*a*b^2*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b*(a + b*Sec[c
 + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3915

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^
2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx\\ &= \frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{6}\\ &= \frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\\ &= \frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b}{}\\ &= \frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2b(4a^2+b^2)\tan(c+dx)}{3d}+\frac{5ab^2}{}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 70, normalized size = 0.71

$$\frac{(6a^3+9ab^2)\tanh^{-1}(\sin(c+dx))+b\tan(c+dx)\left(18a^2+6b^2+9ab\sec(c+dx)+2b^2\tan^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
[Out] ((6*a^3 + 9*a*b^2)*ArcTanh[Sin[c + d*x]] + b*Tan[c + d*x]*(18*a^2 + 6*b^2 +
 9*a*b*Sec[c + d*x] + 2*b^2*Tan[c + d*x]^2))/(6*d)
```

Maple [A]
time = 0.07, size = 96, normalized size = 0.97

method	result
derivativedivides	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3b\,a^2\tan(dx+c)+3b^2a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^3\left(-\frac{2}{3}-\frac{(\sec^2(c}{3}\right.}{d}$
default	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3b\,a^2\tan(dx+c)+3b^2a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^3\left(-\frac{2}{3}-\frac{(\sec^2(c}{3}\right.}{d}$
norman	$\frac{\frac{4b(9a^2+b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{b(6a^2-3ba+2b^2)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b(6a^2+3ba+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a(2a^2+3b^2)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2d}$
risch	$-\frac{ib(9ba\,e^{5i(dx+c)}-18a^2e^{4i(dx+c)}-36a^2e^{2i(dx+c)}-12b^2e^{2i(dx+c)}-9ba\,e^{i(dx+c)}-18a^2-4b^2)}{3d(e^{2i(dx+c)}+1)^3}-\frac{a^3\ln(e^{i(dx+c)}-i)}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*b*a^2*tan(d*x+c)+3*b^2*a*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b^3*(-2/3-1/3*sec(d*x+c)^2)*ta
n(d*x+c))
```

Maxima [A]
time = 0.26, size = 106, normalized size = 1.07

$$\frac{4\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)b^3-9\,ab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+36\,a^2b\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^3 - 9*a*b^2*(2*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3
*log(sec(d*x + c) + tan(d*x + c)) + 36*a^2*b*tan(d*x + c))/d
```

Fricas [A]
time = 2.26, size = 126, normalized size = 1.27

$$\frac{3\left(2\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(9\,ab^2\cos\left(dx+c\right)+2\,b^3+2\left(9\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3 +
 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(9*a*b^2*cos(d*x + c) +
 2*b^3 + 2*(9*a^2*b + 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^3\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(91) = 182.
time = 0.48, size = 205, normalized size = 2.07

$$\frac{3\,(2\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(18\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 + 3*
a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b*tan(1/2*d*x + 1/2*c
)^5 - 9*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*b^3*tan(1/2*d*x + 1/2*c)^5 - 36*a^
2*b*tan(1/2*d*x + 1/2*c)^3 - 4*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*b*tan(1/
2*d*x + 1/2*c) + 9*a*b^2*tan(1/2*d*x + 1/2*c) + 6*b^3*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 3.15, size = 157, normalized size = 1.59

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^3+3\,a\,b^2\right)}{d}-\frac{\left(6\,a^2\,b-3\,a\,b^2+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-12\,a^2\,b-\frac{4\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(6\,a^2\,b+3\,a\,b^2+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 + 2*a^3))/d - (tan(c/2 + (d*x)/2)^5*(6*
a^2*b - 3*a*b^2 + 2*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b + (4*b^3)/3) + ta
n(c/2 + (d*x)/2)*(3*a*b^2 + 6*a^2*b + 2*b^3))/(d*(3*tan(c/2 + (d*x)/2)^2 -
3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.469 $\int (a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=73

$$a^3x+\frac{b(6a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d}$$

```
[Out] a^3*x+1/2*b*(6*a^2+b^2)*arctanh(sin(d*x+c))/d+5/2*a*b^2*tan(d*x+c)/d+1/2*b^
2*(a+b*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3867, 3855, 3852, 8}

$$a^3x+\frac{b(6a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (b*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a*b^2*Tan[c + d*
x])/(2*d) + (b^2*(a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\,dx = \frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d} + \frac{1}{2}\int \left(2a^3+b\left(6a^2+b^2\right)\sec(c+dx)+5ab^2\,\mathrm{se}\right.$$
$$= a^3x + \frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d} + \frac{1}{2}\left(5ab^2\right)\int \sec^2(c+dx)\,dx + \frac{1}{2}\left(b\left(6a^2\right.\right.$$
$$= a^3x + \frac{b(6a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d} - \frac{(5a}{}$$
$$= a^3x + \frac{b(6a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5ab^2\tan(c+dx)}{2d} + \frac{b^2(a+b\sec(c+dx)}{2d}$$

Mathematica [A]
time = 0.17, size = 55, normalized size = 0.75

$$\frac{2a^3dx+b(6a^2+b^2)\tanh^{-1}(\sin(c+dx))+b^2(6a+b\sec(c+dx))\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3,x]
[Out] (2*a^3*d*x + b*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + b^2*(6*a + b*Sec[c + d
*x])*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.06, size = 82, normalized size = 1.12

method	result
derivativedivides	$\frac{a^3(dx+c)+3b\,a^2\ln(\sec(dx+c)+\tan(dx+c))+3b^2a\tan(dx+c)+b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^3(dx+c)+3b\,a^2\ln(\sec(dx+c)+\tan(dx+c))+3b^2a\tan(dx+c)+b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$a^3x-\frac{ib^2\left(b\,\mathrm{e}^{3i(dx+c)}-6a\,\mathrm{e}^{2i(dx+c)}-b\,\mathrm{e}^{i(dx+c)}-6a\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{3b\ln\left(\mathrm{e}^{i(dx+c)}-i\right)a^2}{d}-\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}+\frac{3b\ln\left(\mathrm{e}^{i(dx+c)}+\right.}{d}$
norman	$\frac{a^3x+a^3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b^2(b+6a)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-2a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2(6a-b)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{b(6a^2+b^2)\ln\left(\tan\left(\right.\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^3*(d*x+c)+3*b*a^2*ln(sec(d*x+c)+tan(d*x+c))+3*b^2*a*tan(d*x+c)+b^3*(
1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 93, normalized size = 1.27

$$a^3x - \frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d} + \frac{3\,a^2b\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{3\,ab^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/4*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1))/d + 3*a^2*b*log(sec(d*x + c) + tan(d*x + c))/d +
 3*a*b^2*tan(d*x + c)/d
```

Fricas [A]
time = 2.75, size = 112, normalized size = 1.53

$$\frac{4\,a^3dx\cos\left(dx+c\right)^2 + \left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - \left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(6\,ab^2\cos\left(dx+c\right)+b^3\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + (6*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (6*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a*
b^2*cos(d*x + c) + b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec\left(c + dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(67) = 134.
time = 0.44, size = 145, normalized size = 1.99

$$\frac{2\left(dx+c\right)a^3 + \left(6\,a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(6\,a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 + (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a*b^2*tan(1/2*d*
x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c) -
b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 0.95, size = 136, normalized size = 1.86

$$\frac{2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^3\,\sin\left(c+d\,x\right)}{2\,d\cos\left(c+d\,x\right)^2}+\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2)
 + (6*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*sin(
c + d*x))/(d*cos(c + d*x))
```

3.470 $\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=67

$$3a^2bx+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a(a^2-b^2)\sin(c+dx)}{d}+\frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d}$$

```
[Out] 3*a^2*b*x+3*a*b^2*arctanh(sin(d*x+c))/d+a*(a^2-b^2)*sin(d*x+c)/d+b^2*(a+b*s
ec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3927, 4132, 8, 4130, 3855}

$$\frac{a(a^2-b^2)\sin(c+dx)}{d}+3a^2bx+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*ArcTanh[Sin[c + d*x]])/d + (a*(a^2 - b^2)*Sin[c + d*x]
)/d + (b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d} + \int \cos(c+dx)\left(a\left(a^2-b^2\right)+3a^2 \right.$$

$$= \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d} + \left(3a^2b\right)\int 1\,dx + \int \cos(c+dx)\,($$

$$= 3a^2bx + \frac{a(a^2-b^2)\sin(c+dx)}{d} + \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d} +$$

$$= 3a^2bx + \frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a(a^2-b^2)\sin(c+dx)}{d} + \frac{b^2(a+}{}$$

Mathematica [A]
time = 0.35, size = 88, normalized size = 1.31

$$\frac{3ab\left(ac+adx-b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+a^3\sin(c+dx)+b^3\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*b*(a*c + a*d*x - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[c + d*x] + b^3*Tan[c + d*x])/
d
```

Maple [A]
time = 0.08, size = 57, normalized size = 0.85

method	result
derivativedivides	$\frac{a^3\sin(dx+c)+3b\,a^2(dx+c)+3b^2a\ln(\sec(dx+c)+\tan(dx+c))+b^3\tan(dx+c)}{d}$
default	$\frac{a^3\sin(dx+c)+3b\,a^2(dx+c)+3b^2a\ln(\sec(dx+c)+\tan(dx+c))+b^3\tan(dx+c)}{d}$
risch	$3a^2bx-\frac{ia^3e^{i(dx+c)}}{2d}+\frac{ia^3e^{-i(dx+c)}}{2d}+\frac{2ib^3}{d(e^{2i(dx+c)}+1)}-\frac{3a\ln(e^{i(dx+c)}-i)b^2}{d}+\frac{3a\ln(e^{i(dx+c)}+i)b^2}{d}$

norman	$\frac{3a^2bx-\frac{4a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(a^3-b^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(a^3+b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-3a^2bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3a^2bx\left(\tan^4\left(\frac{dx}{2}\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*sin(d*x+c)+3*b*a^2*(d*x+c)+3*b^2*a*ln(sec(d*x+c)+tan(d*x+c))+b^3*t
an(d*x+c))
```

Maxima [A]
time = 0.28, size = 66, normalized size = 0.99

$$\frac{6\,(dx+c)a^2b+3\,ab^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,a^3\sin\left(dx+c\right)+2\,b^3\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a^2*b + 3*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 2*a^3*sin(d*x + c) + 2*b^3*tan(d*x + c))/d
```

Fricas [A]
time = 2.92, size = 94, normalized size = 1.40

$$\frac{6\,a^2bdx\cos\left(dx+c\right)+3\,ab^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(a^3\cos\left(dx+c\right)+b^3\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^2*b*d*x*cos(d*x + c) + 3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a*b^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + b^3)*
sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^3\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*cos(c + d*x), x)
```

Giac [A]
time = 0.50, size = 131, normalized size = 1.96

$$\frac{3\,(dx+c)a^2b+3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a^2*b + 3*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b^2*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - b^3*tan
(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B]
time = 0.92, size = 97, normalized size = 1.45

$$\frac{a^3\,\sin\left(c+d\,x\right)}{d}+\frac{b^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{6\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x))/d + (b^3*sin(c + d*x))/(d*cos(c + d*x)) + (6*a^2*b*atan(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a*b^2*atanh(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)))/d
```

3.471 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}a\left(a^2+6b^2\right)x+\frac{b^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*(a^2+6*b^2)*x+b^3*arctanh(sin(d*x+c))/d+5/2*a^2*b*sin(d*x+c)/d+1/2*a^
2*cos(d*x+c)*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3926, 4132, 8, 4130, 3855}

$$\frac{1}{2}ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}+\frac{b^3\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*x)/2 + (b^3*ArcTanh[Sin[c + d*x]])/d + (5*a^2*b*Sin[c + d*
x])/(2*d) + (a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{2d} + \frac{1}{2}\int \cos(c+dx)\,(5a$$
$$= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{2d} + \frac{1}{2}\int \cos(c+dx)\,(5a$$
$$= \frac{1}{2}a\left(a^2+6b^2\right)x + \frac{5a^2b\sin(c+dx)}{2d} + \frac{a^2\cos(c+dx)(a+b\sec(c+dx$$
$$= \frac{1}{2}a\left(a^2+6b^2\right)x + \frac{b^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^2b\sin(c+dx)}{2d} + \frac{a^2\cos$$

Mathematica [A]
time = 0.16, size = 105, normalized size = 1.33

$$\frac{2a(a^2+6b^2)(c+dx) - 4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + 12a^2b\sin(c+dx) + a^3\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*a*(a^2 + 6*b^2)*(c + d*x) - 4*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 4*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*a^2*b*Sin[c + d*x]
 + a^3*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.08, size = 73, normalized size = 0.92

method	result
derivativedivides	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3b\,a^2\sin(dx+c)+3b^2a(dx+c)+b^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3b\,a^2\sin(dx+c)+3b^2a(dx+c)+b^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$

risch	$\frac{a^3x}{2}+3a\,b^2x-\frac{3ib\,a^2\mathrm{e}^{i(dx+c)}}{2d}+\frac{3ib\,a^2\mathrm{e}^{-i(dx+c)}}{2d}+\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{a^3\sin(2dx+2c}{4d}$
norman	$\frac{\left(\frac{1}{2}a^3+3b^2a\right)x+\left(\frac{1}{2}a^3+3b^2a\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^3-6b^2a\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{a^2(a+6b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a^2(a-6b)\left(\tan^7\left(\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*b*a^2*sin(d*x+c)+3*b^2
*a*(d*x+c)+b^3*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.26, size = 76, normalized size = 0.96

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3+12\,(dx+c)ab^2+2\,b^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+12\,a^2b\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a*b^2 + 2*b^3*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^2*b*sin(d*x + c))/d
```

Fricas [A]
time = 2.90, size = 72, normalized size = 0.91

$$\frac{b^3\log(\sin(dx+c)+1)-b^3\log(-\sin(dx+c)+1)+(a^3+6\,ab^2)dx+(a^3\cos(dx+c)+6\,a^2b)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*log(sin(d*x + c) + 1) - b^3*log(-sin(d*x + c) + 1) + (a^3 + 6*a*b^
2)*d*x + (a^3*cos(d*x + c) + 6*a^2*b)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))^3\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**2, x)
```

Giac [A]
time = 0.48, size = 137, normalized size = 1.73

$$\frac{2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(a^3+6\,ab^2)(dx+c)-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b^3*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (a^3 + 6*a*b^2)*(d*x + c) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3 -
 6*a^2*b*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 1.03, size = 123, normalized size = 1.56

$$\frac{a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a^2\,b\,\sin\left(c+d\,x\right)}{d}+\frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^3*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*sin(2*c + 2*d*x))/(4*d) + (3*a^2*b
*sin(c + d*x))/d + (6*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.472 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=100

$$\frac{1}{2}b\left(3a^2+2b^2\right)x+\frac{a(2a^2+9b^2)\sin(c+dx)}{3d}+\frac{7a^2b\cos(c+dx)\sin(c+dx)}{6d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx)}{3d}$$

```
[Out] 1/2*b*(3*a^2+2*b^2)*x+1/3*a*(2*a^2+9*b^2)*sin(d*x+c)/d+7/6*a^2*b*cos(d*x+c)
*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^2*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]

time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3926, 4132, 2717, 4130, 8}

$$\frac{a(2a^2+9b^2)\sin(c+dx)}{3d}+\frac{1}{2}bx\left(3a^2+2b^2\right)+\frac{7a^2b\sin(c+dx)\cos(c+dx)}{6d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (b*(3*a^2 + 2*b^2)*x)/2 + (a*(2*a^2 + 9*b^2)*Sin[c + d*x])/(3*d) + (7*a^2*b
*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x]
)*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\int \cos^2(c+dx)\,($$
$$= \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\int \cos^2(c+dx)\,($$
$$= \frac{a(2a^2+9b^2)\sin(c+dx)}{3d} + \frac{7a^2b\cos(c+dx)\sin(c+dx)}{6d} + \frac{a^2\cos^2(c}{}$$
$$= \frac{1}{2}b\left(3a^2+2b^2\right)x + \frac{a(2a^2+9b^2)\sin(c+dx)}{3d} + \frac{7a^2b\cos(c+dx)\sin(c}{6d}$$

Mathematica [A]
time = 0.13, size = 80, normalized size = 0.80

$$\frac{18a^2bc+12b^3c+18a^2bdx+12b^3dx+9a(a^2+4b^2)\sin(c+dx)+9a^2b\sin(2(c+dx))+a^3\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
[Out] (18*a^2*b*c + 12*b^3*c + 18*a^2*b*d*x + 12*b^3*d*x + 9*a*(a^2 + 4*b^2)*Sin[
c + d*x] + 9*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3b\,a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3b^2a\sin(dx+c)+b^3(dx+c)}{d}$
default	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3b\,a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3b^2a\sin(dx+c)+b^3(dx+c)}{d}$

risch	$\frac{3a^2bx}{2}+b^3x+\frac{3a^3\sin(dx+c)}{4d}+\frac{3\sin(dx+c)b^2a}{d}+\frac{a^3\sin(3dx+3c)}{12d}+\frac{3b\,a^2\sin(2dx+2c)}{4d}$
norman	$\frac{(\frac{3}{2}b\,a^2+b^3)x+(\frac{3}{2}b\,a^2+b^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{2}b\,a^2+b^3)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{2}b\,a^2+b^3)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-3b\,a^2-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*b*a^2*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+3*b^2*a*sin(d*x+c)+b^3*(d*x+c))
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.73

$$-\frac{4\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2b-12\left(dx+c\right)b^3-36\,ab^2\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^2*b - 12*(d*x + c)*b^3 - 36*a*b^2*sin(d*x + c))/d
```

Fricas [A]
time = 2.87, size = 66, normalized size = 0.66

$$\frac{3\left(3\,a^2b+2\,b^3\right)dx+\left(2\,a^3\cos\left(dx+c\right)^2+9\,a^2b\cos\left(dx+c\right)+4\,a^3+18\,ab^2\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(3*a^2*b + 2*b^3)*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^2*b*cos(d*x + c)
 + 4*a^3 + 18*a*b^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^3\cos^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**3, x)
```

Giac [A]
time = 0.47, size = 170, normalized size = 1.70

$$\frac{3\,(3\,a^2b+2\,b^3)(dx+c)+\frac{2\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(3*a^2*b + 2*b^3)*(d*x + c) + 2*(6*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^
2*b*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*a^3*tan(1/
2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2*d*x + 1/
2*c) + 9*a^2*b*tan(1/2*d*x + 1/2*c) + 18*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 0.86, size = 77, normalized size = 0.77

$$b^3\,x+\frac{3\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{3\,a^2\,b\,x}{2}+\frac{3\,a^2\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^3,x)
```

```
[Out] b^3*x + (3*a^3*sin(c + d*x))/(4*d) + (a^3*sin(3*c + 3*d*x))/(12*d) + (3*a^2
*b*x)/2 + (3*a^2*b*sin(2*c + 2*d*x))/(4*d) + (3*a*b^2*sin(c + d*x))/d
```

3.473 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=123

$$\frac{3}{8}a\left(a^2+4b^2\right)x+\frac{b(11a^2+4b^2)\sin(c+dx)}{4d}+\frac{3a(a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)(a+b\sec$$

```
[Out] 3/8*a*(a^2+4*b^2)*x+1/4*b*(11*a^2+4*b^2)*sin(d*x+c)/d+3/8*a*(a^2+4*b^2)*cos
(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*(a+b*sec(d*x+c))*sin(d*x+c)/d-3/4
*a^2*b*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3926, 4132, 2715, 8, 4129, 3092}

$$\frac{b(11a^2+4b^2)\sin(c+dx)}{4d}+\frac{3a(a^2+4b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}ax\left(a^2+4b^2\right)-\frac{3a^2b\sin^3(c+dx)}{4d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 + 4*b^2)*x)/8 + (b*(11*a^2 + 4*b^2)*Sin[c + d*x])/(4*d) + (3*a*(a
^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*(a + b*S
ec[c + d*x])*Sin[c + d*x])/(4*d) - (3*a^2*b*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
```

```
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3(c+dx)\,(\\ &= \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3(c+dx)\,(\\ &= \frac{3a(a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)(a+b\sec(c+}{4d}\\ &= \frac{3}{8}a\left(a^2+4b^2\right)x+\frac{3a(a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+}{}\\ &= \frac{3}{8}a\left(a^2+4b^2\right)x+\frac{b(11a^2+4b^2)\sin(c+dx)}{4d}+\frac{3a(a^2+4b^2)\cos(c+}{8d}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 100, normalized size = 0.81

$$\frac{8b(9a^2+4b^2)\sin(c+dx)+a(12a^2c+48b^2c+12a^2dx+48b^2dx+8(a^2+3b^2)\sin(2(c+dx))+8ab\sin(3(c+dx))+a^2\sin(4(c+dx)))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (8*b*(9*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*a^2*c + 48*b^2*c + 12*a^2*d*x + 4
8*b^2*d*x + 8*(a^2 + 3*b^2)*Sin[2*(c + d*x)] + 8*a*b*Sin[3*(c + d*x)] + a^2
*Sin[4*(c + d*x)]))/(32*d)
```

Maple [A]
time = 0.12, size = 102, normalized size = 0.83

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\,a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3b^2a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$
default	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\,a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3b^2a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$
risch	$\frac{3a^3x}{8}+\frac{3a\,b^2x}{2}+\frac{9a^2b\sin(dx+c)}{4d}+\frac{\sin(dx+c)b^3}{d}+\frac{a^3\sin(4dx+4c)}{32d}+\frac{b\,a^2\sin(3dx+3c)}{4d}+\frac{a^3\sin(2dx+2c)}{4d}+$
norman	$\frac{(\frac{3}{8}a^3+\frac{3}{2}b^2a)x+(-\frac{3}{2}a^3-6b^2a)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{3}{8}a^3-\frac{3}{2}b^2a)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{3}{8}a^3-\frac{3}{2}b^2a)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+b*a^2
*(2+cos(d*x+c)^2)*sin(d*x+c)+3*b^2*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+b^3*sin(d*x+c))
```

Maxima [A]
time = 0.26, size = 95, normalized size = 0.77

$$\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3-32\,(\sin(dx+c)^3-3\,\sin(dx+c))a^2b+24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))ab^2+32\,b^3\sin(dx+c)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 32*(sin
(d*x + c)^3 - 3*sin(d*x + c))*a^2*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a
*b^2 + 32*b^3*sin(d*x + c))/d
```

Fricas [A]
time = 3.08, size = 84, normalized size = 0.68

$$\frac{3\,(a^3+4\,ab^2)dx+\left(2\,a^3\cos(dx+c)^3+8\,a^2b\cos(dx+c)^2+16\,a^2b+8\,b^3+3\,(a^3+4\,ab^2)\cos(dx+c)\right)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a^3 + 4*a*b^2)*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^2*b*cos(d*x + c)^2
 + 16*a^2*b + 8*b^3 + 3*(a^3 + 4*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^3 \cos^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(113) = 226.
time = 0.47, size = 297, normalized size = 2.41

$$\frac{3\left(a^{3}+4ab^{2}\right)(dx+c)-\frac{2\left(5a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-24a^{2}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+12ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-8b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-3a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-40a^{2}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+12ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-24b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+3a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-40a^{2}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-12ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-24b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-5a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-24a^{2}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-12ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{4}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^3 + 4*a*b^2)*(d*x + c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^2
*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*b^3*tan(1/2
*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^2*b*tan(1/2*d*x + 1/2
*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 24*b^3*tan(1/2*d*x + 1/2*c)^5 + 3
*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 24*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*tan(1/2*d*x +
1/2*c) - 24*a^2*b*tan(1/2*d*x + 1/2*c) - 12*a*b^2*tan(1/2*d*x + 1/2*c) - 8*
b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 3.83, size = 250, normalized size = 2.03

$$\frac{\left(-\frac{5a^{3}}{4}+6a^{2}b-3ab^{2}+2b^{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\left(\frac{3a^{3}}{4}+10a^{2}b-3ab^{2}+6b^{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+\left(-\frac{3a^{3}}{4}+10a^{2}b+3ab^{2}+6b^{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+\left(\frac{5a^{3}}{4}+6a^{2}b+3ab^{2}+2b^{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}+\frac{3a\,\mathrm{atan}\left(\frac{3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^{2}+4b^{2}\right)}{4\left(\frac{3a^{3}}{4}+3ab^{2}\right)}\right)\left(a^{2}+4b^{2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(3*a*b^2 + 10*a^2*b - (3*a^3)/4 + 6*b^3) - tan(c/2 +
(d*x)/2)^7*(3*a*b^2 - 6*a^2*b + (5*a^3)/4 - 2*b^3) + tan(c/2 + (d*x)/2)^5*(
10*a^2*b - 3*a*b^2 + (3*a^3)/4 + 6*b^3) + tan(c/2 + (d*x)/2)*(3*a*b^2 + 6*a
^2*b + (5*a^3)/4 + 2*b^3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2
)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atan((3*a*
tan(c/2 + (d*x)/2)*(a^2 + 4*b^2))/(4*(3*a*b^2 + (3*a^3)/4)))*(a^2 + 4*b^2))
/(4*d)
```

3.474 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=160

$$\frac{1}{8}b(9a^2+4b^2)\,x+\frac{a(4a^2+15b^2)\sin(c+dx)}{5d}+\frac{b(9a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{11a^2b\cos^3(c+dx)\sin(c}{20d}$$

```
[Out] 1/8*b*(9*a^2+4*b^2)*x+1/5*a*(4*a^2+15*b^2)*sin(d*x+c)/d+1/8*b*(9*a^2+4*b^2)
*cos(d*x+c)*sin(d*x+c)/d+11/20*a^2*b*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a^2*cos(
d*x+c)^4*(a+b*sec(d*x+c))*sin(d*x+c)/d-1/15*a*(4*a^2+15*b^2)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3926, 4132, 2713, 4130, 2715, 8}

$$-\frac{a(4a^2+15b^2)\sin^3(c+dx)}{15d}+\frac{a(4a^2+15b^2)\sin(c+dx)}{5d}+\frac{b(9a^2+4b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx(9a^2+4b^2)+\frac{11a^2b\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (b*(9*a^2 + 4*b^2)*x)/8 + (a*(4*a^2 + 15*b^2)*Sin[c + d*x])/(5*d) + (b*(9*a
^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (11*a^2*b*Cos[c + d*x]^3*Sin
[c + d*x])/(20*d) + (a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])*Sin[c + d*x])/
(5*d) - (a*(4*a^2 + 15*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
```

```
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4(c+dx)\,(\\ &= \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4(c+dx)\,(\\ &= \frac{11a^2b\cos^3(c+dx)\sin(c+dx)}{20d}+\frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))}{5d}\\ &= \frac{a(4a^2+15b^2)\sin(c+dx)}{5d}+\frac{b(9a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}+\\ &= \frac{1}{8}b\left(9a^2+4b^2\right)x+\frac{a(4a^2+15b^2)\sin(c+dx)}{5d}+\frac{b(9a^2+4b^2)\cos(c+}{8d}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 130, normalized size = 0.81

$$\frac{540a^2bc+240b^3c+540a^2bdx+240b^3dx+60a(5a^2+18b^2)\sin(c+dx)+120(3a^2b+b^3)\sin(2(c+dx))+50a^3\sin(3(c+dx))+120ab^2\sin(3(c+dx))+45a^2b\sin(4(c+dx))+6a^3\sin(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

[Out] (540*a^2*b*c + 240*b^3*c + 540*a^2*b*d*x + 240*b^3*d*x + 60*a*(5*a^2 + 18*b^2)*Sin[c + d*x] + 120*(3*a^2*b + b^3)*Sin[2*(c + d*x)] + 50*a^3*Sin[3*(c + d*x)] + 120*a*b^2*Sin[3*(c + d*x)] + 45*a^2*b*Sin[4*(c + d*x)] + 6*a^3*Sin[5*(c + d*x)])/(480*d)

Maple [A]
time = 0.14, size = 123, normalized size = 0.77

method	result
derivativedivides	$\frac{\frac{a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3b\,a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b^2a\left(2+\cos^2(dx\right.}{d}$
default	$\frac{\frac{a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3b\,a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b^2a\left(2+\cos^2(dx\right.}{d}$
risch	$\frac{9a^2bx}{8}+\frac{b^3x}{2}+\frac{5a^3\sin(dx+c)}{8d}+\frac{9\sin(dx+c)b^2a}{4d}+\frac{a^3\sin(5dx+5c)}{80d}+\frac{3b\,a^2\sin(4dx+4c)}{32d}+\frac{5a^3\sin(3dx+3c)}{48d}+$
norman	$\frac{(\frac{9}{8}b\,a^2+\frac{1}{2}b^3)x+(-\frac{45}{8}b\,a^2-\frac{5}{2}b^3)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{45}{8}b\,a^2-\frac{5}{2}b^3)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{9}{8}b\,a^2+\frac{1}{2}b^3)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*b*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+b^2*a*(2+cos(d*x+c)^2)*sin(d*x+c)+b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

Maxima [A]
time = 0.26, size = 119, normalized size = 0.74

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3+45\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)a^2b-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)ab^2+120\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)b^3}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*b^3)/d

Fricas [A]
time = 3.71, size = 110, normalized size = 0.69

$$\frac{15\left(9\,a^2b+4\,b^3\right)dx+\left(24\,a^3\cos(dx+c)^4+90\,a^2b\cos(dx+c)^3+64\,a^3+240\,ab^2+8\left(4\,a^3+15\,ab^2\right)\cos(dx+c)^2+15\left(9\,a^2b+4\,b^3\right)\cos(dx+c)\right)\sin(dx+c)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(9*a^2*b + 4*b^3)*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^2*b*cos(d*x
 + c)^3 + 64*a^3 + 240*a*b^2 + 8*(4*a^3 + 15*a*b^2)*cos(d*x + c)^2 + 15*(9*
a^2*b + 4*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(148) = 296.
time = 0.51, size = 332, normalized size = 2.08

$$\frac{15\,(9\,a^2b+4\,b^3)(dx+c)+\frac{2\left(120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-225\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+360\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+160\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-90\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+960\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-120\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+464\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1200\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+160\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+90\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+960\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+225\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+360\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+60\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(9*a^2*b + 4*b^3)*(d*x + c) + 2*(120*a^3*tan(1/2*d*x + 1/2*c)^9 -
 225*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*b
^3*tan(1/2*d*x + 1/2*c)^9 + 160*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*a^2*b*tan(1
/2*d*x + 1/2*c)^7 + 960*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*b^3*tan(1/2*d*x
+ 1/2*c)^7 + 464*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*a*b^2*tan(1/2*d*x + 1/2*
c)^5 + 160*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 9
60*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*a^3*
tan(1/2*d*x + 1/2*c) + 225*a^2*b*tan(1/2*d*x + 1/2*c) + 360*a*b^2*tan(1/2*d
*x + 1/2*c) + 60*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/
d
```

Mupad [B]
time = 3.93, size = 287, normalized size = 1.79

$$\frac{\left(2\,a^3-\frac{15\,a^2\,b}{4}+6\,a\,b^2-b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,a^3}{3}-\frac{3\,a^2\,b}{2}+16\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{116\,a^3}{15}+20\,a\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{8\,a^3}{3}+\frac{3\,a^2\,b}{2}+16\,a\,b^2+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^3+\frac{15\,a^2\,b}{4}+6\,a\,b^2+b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{b\,\mathrm{atan}\left(\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a^2+4\,b^2\right)}{4\,\left(\frac{9\,a^2\,b}{4}+b^3\right)}\right)\,\left(9\,a^2+4\,b^2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(16*a*b^2 + (3*a^2*b)/2 + (8*a^3)/3 + 2*b^3) + tan(c/
2 + (d*x)/2)^9*(6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - b^3) + tan(c/2 + (d*x)/2)^
7*(16*a*b^2 - (3*a^2*b)/2 + (8*a^3)/3 - 2*b^3) + tan(c/2 + (d*x)/2)*(6*a*b^
2 + (15*a^2*b)/4 + 2*a^3 + b^3) + tan(c/2 + (d*x)/2)^5*(20*a*b^2 + (116*a^3
)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
(d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (b*atan
((b*tan(c/2 + (d*x)/2)*(9*a^2 + 4*b^2))/(4*((9*a^2*b)/4 + b^3)))*(9*a^2 + 4
*b^2))/(4*d)
```

3.475 $\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=185

$$\frac{1}{16}a\left(5a^2+18b^2\right)x+\frac{b(17a^2+6b^2)\sin(c+dx)}{6d}+\frac{a(5a^2+18b^2)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a(5a^2+18b^2)\cos^3(c-}{24d}$$

```
[Out] 1/16*a*(5*a^2+18*b^2)*x+1/6*b*(17*a^2+6*b^2)*sin(d*x+c)/d+1/16*a*(5*a^2+18*
b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*a*(5*a^2+18*b^2)*cos(d*x+c)^3*sin(d*x+c)/
d+1/6*a^2*cos(d*x+c)^5*(a+b*sec(d*x+c))*sin(d*x+c)/d-1/3*b*(5*a^2+b^2)*sin(
d*x+c)^3/d+13/30*a^2*b*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.17, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3926, 4132, 2715, 8, 4129, 3092, 380}

$$-\frac{b(5a^2+b^2)\sin^3(c+dx)}{3d}+\frac{b(17a^2+6b^2)\sin(c+dx)}{6d}+\frac{a(5a^2+18b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a(5a^2+18b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}ax\left(5a^2+18b^2\right)+\frac{13a^2b\sin^5(c+dx)}{30d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(5*a^2 + 18*b^2)*x)/16 + (b*(17*a^2 + 6*b^2)*Sin[c + d*x])/(6*d) + (a*(5
*a^2 + 18*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(5*a^2 + 18*b^2)*Cos[
c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])*
Sin[c + d*x])/(6*d) - (b*(5*a^2 + b^2)*Sin[c + d*x]^3)/(3*d) + (13*a^2*b*Si
n[c + d*x]^5)/(30*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{6d} + \frac{1}{6}\int \cos^5(c+dx)$$

$$= \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{6d} + \frac{1}{6}\int \cos^5(c+dx)$$

$$= \frac{a(5a^2+18b^2)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a^2\cos^5(c+dx)(a+b\sec}{6d}$$

$$= \frac{a(5a^2+18b^2)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a(5a^2+18b^2)\cos^3(c+dx}{24d}$$

$$= \frac{1}{16}a\left(5a^2+18b^2\right)x + \frac{a(5a^2+18b^2)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a(5a}{}$$

$$= \frac{1}{16}a\left(5a^2+18b^2\right)x + \frac{b(17a^2+6b^2)\sin(c+dx)}{6d} + \frac{a(5a^2+18b^2)\mathrm{co}}{}$$

Mathematica [A]
time = 0.34, size = 159, normalized size = 0.86

$$\frac{300a^3c+1080ab^2c+300a^3dx+1080ab^2dx+360b(5a^2+2b^2)\sin(c+dx)+45(5a^3+16ab^2)\sin(2(c+dx))+300a^2b\sin(3(c+dx))+80b^3\sin(3(c+dx))+45a^3\sin(4(c+dx))+90ab^2\sin(4(c+dx))+36a^2b\sin(5(c+dx))+5a^3\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (300*a^3*c + 1080*a*b^2*c + 300*a^3*d*x + 1080*a*b^2*d*x + 360*b*(5*a^2 + 2
*b^2)*Sin[c + d*x] + 45*(5*a^3 + 16*a*b^2)*Sin[2*(c + d*x)] + 300*a^2*b*Sin
[3*(c + d*x)] + 80*b^3*Sin[3*(c + d*x)] + 45*a^3*Sin[4*(c + d*x)] + 90*a*b^
2*Sin[4*(c + d*x)] + 36*a^2*b*Sin[5*(c + d*x)] + 5*a^3*Sin[6*(c + d*x)])/(9
60*d)
```

Maple [A]
time = 0.14, size = 145, normalized size = 0.78

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3b\,a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3}{d}$
default	$\frac{a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3b\,a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3}{d}$
risch	$\frac{5a^3x}{16}+\frac{9a\,b^2x}{8}+\frac{15a^2b\sin(dx+c)}{8d}+\frac{3\sin(dx+c)b^3}{4d}+\frac{a^3\sin(6dx+6c)}{192d}+\frac{3b\,a^2\sin(5dx+5c)}{80d}+\frac{3a^3\sin(4dx+4c)}{64d}$
norman	$\frac{(\frac{5}{16}a^3+\frac{9}{8}b^2a)x+(-\frac{25}{8}a^3-\frac{45}{4}b^2a)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{5}{4}a^3-\frac{9}{2}b^2a)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{5}{4}a^3-\frac{9}{2}b^2a)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*b*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*b
^2*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b^3*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.26, size = 145, normalized size = 0.78

$$-\frac{5\,(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c))a^3-192\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))a^2b-90\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))ab^2+320\,(\sin(dx+c)^3-3\,\sin(dx+c))b^3}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*a^3 - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d
*x + c))*a^2*b - 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))
*a*b^2 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*b^3)/d
```

Fricas [A]
time = 2.64, size = 132, normalized size = 0.71

$$\frac{15\,(5\,a^3+18\,ab^2)dx+\left(40\,a^3\cos\left(dx+c\right)^5+144\,a^2b\cos\left(dx+c\right)^4+10\,(5\,a^3+18\,ab^2)\cos\left(dx+c\right)^3+384\,a^2b+160\,b^3+16\,(12\,a^2b+5\,b^3)\cos\left(dx+c\right)^2+15\,(5\,a^3+18\,ab^2)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(5*a^3 + 18*a*b^2)*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^2*b*cos(d
*x + c)^4 + 10*(5*a^3 + 18*a*b^2)*cos(d*x + c)^3 + 384*a^2*b + 160*b^3 + 16
*(12*a^2*b + 5*b^3)*cos(d*x + c)^2 + 15*(5*a^3 + 18*a*b^2)*cos(d*x + c))*si
n(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(171) = 342.
time = 0.46, size = 431, normalized size = 2.33

$$\frac{15\,(5\,a^3+18\,ab^2)(dx+c)-\frac{2\left(165\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-720\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+450\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-240\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1680\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+630\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-880\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+450\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3744\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+180\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1440\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-450\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3744\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-180\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1440\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-1680\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-630\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-880\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-165\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-720\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-450\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-240\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^3 + 18*a*b^2)*(d*x + c) - 2*(165*a^3*tan(1/2*d*x + 1/2*c)^11
 - 720*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 450*a*b^2*tan(1/2*d*x + 1/2*c)^11 -
240*b^3*tan(1/2*d*x + 1/2*c)^11 - 25*a^3*tan(1/2*d*x + 1/2*c)^9 - 1680*a^2*
b*tan(1/2*d*x + 1/2*c)^9 + 630*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 880*b^3*tan(1
/2*d*x + 1/2*c)^9 + 450*a^3*tan(1/2*d*x + 1/2*c)^7 - 3744*a^2*b*tan(1/2*d*x
 + 1/2*c)^7 + 180*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 1440*b^3*tan(1/2*d*x + 1/2
*c)^7 - 450*a^3*tan(1/2*d*x + 1/2*c)^5 - 3744*a^2*b*tan(1/2*d*x + 1/2*c)^5
- 180*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 1440*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*a
```

```
^3*tan(1/2*d*x + 1/2*c)^3 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 630*a*b^2*t
an(1/2*d*x + 1/2*c)^3 - 880*b^3*tan(1/2*d*x + 1/2*c)^3 - 165*a^3*tan(1/2*d*
x + 1/2*c) - 720*a^2*b*tan(1/2*d*x + 1/2*c) - 450*a*b^2*tan(1/2*d*x + 1/2*c
) - 240*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 3.30, size = 350, normalized size = 1.89

$$\frac{\left(-\frac{11\,a^3}{8}+6\,a^2\,b-\frac{15\,a\,b^2}{4}+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{5\,a^3}{24}+14\,a^2\,b-\frac{21\,a\,b^2}{4}+\frac{22\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{15\,a^3}{4}+\frac{156\,a^2\,b}{5}-\frac{3\,a\,b^2}{2}+12\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{15\,a^3}{4}+\frac{156\,a^2\,b}{5}+\frac{3\,a\,b^2}{2}+12\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{5\,a^3}{24}+14\,a^2\,b+\frac{21\,a\,b^2}{4}+\frac{22\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{11\,a^3}{8}+6\,a^2\,b+\frac{15\,a\,b^2}{4}+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\,\mathrm{atan}\left(\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,a^2+18\,b^2\right)}{8\,\left(\frac{5\,a^3}{8}+\frac{9\,a\,b^2}{4}\right)}\right)\,\left(5\,a^2+18\,b^2\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((21*a*b^2)/4 + 14*a^2*b - (5*a^3)/24 + (22*b^3)/3) -
 tan(c/2 + (d*x)/2)^11*((15*a*b^2)/4 - 6*a^2*b + (11*a^3)/8 - 2*b^3) + tan(
c/2 + (d*x)/2)^9*(14*a^2*b - (21*a*b^2)/4 + (5*a^3)/24 + (22*b^3)/3) + tan(
c/2 + (d*x)/2)^5*((3*a*b^2)/2 + (156*a^2*b)/5 + (15*a^3)/4 + 12*b^3) - tan(
c/2 + (d*x)/2)^7*((3*a*b^2)/2 - (156*a^2*b)/5 + (15*a^3)/4 - 12*b^3) + tan(
c/2 + (d*x)/2)*((15*a*b^2)/4 + 6*a^2*b + (11*a^3)/8 + 2*b^3))/(d*(6*tan(c/2
 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(
c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) +
(a*atan((a*tan(c/2 + (d*x)/2)*(5*a^2 + 18*b^2))/(8*((9*a*b^2)/4 + (5*a^3)/8
)))*(5*a^2 + 18*b^2))/(8*d)
```

3.476 $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=244

$$\frac{(8a^4+36a^2b^2+5b^4)\tanh^{-1}(\sin(c+dx))}{16d}-\frac{a(4a^4-121a^2b^2-128b^4)\tan(c+dx)}{60bd}-\frac{(8a^4-178a^2b^2-75b^4)}{24}$$

```
[Out] 1/16*(8*a^4+36*a^2*b^2+5*b^4)*arctanh(sin(d*x+c))/d-1/60*a*(4*a^4-121*a^2*b
^2-128*b^4)*tan(d*x+c)/b/d-1/240*(8*a^4-178*a^2*b^2-75*b^4)*sec(d*x+c)*tan(
d*x+c)/d-1/120*a*(4*a^2-53*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d-1/120*(4*
a^2-25*b^2)*(a+b*sec(d*x+c))^3*tan(d*x+c)/b/d-1/30*a*(a+b*sec(d*x+c))^4*tan
(d*x+c)/b/d+1/6*(a+b*sec(d*x+c))^5*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.32, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3925, 4087, 4082, 3872, 3855, 3852, 8}

$$-\frac{(4a^2-25b^2)\tan(c+dx)(a+b\sec(c+dx))^3}{120bd}-\frac{a(4a^2-53b^2)\tan(c+dx)(a+b\sec(c+dx))^2}{120bd}-\frac{a(4a^4-121a^2b^2-128b^4)\tan(c+dx)}{60bd}+\frac{(8a^4+36a^2b^2+5b^4)\tanh^{-1}(\sin(c+dx))}{16d}-\frac{(8a^4-178a^2b^2-75b^4)\tan(c+dx)\sec(c+dx)}{240d}+\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}-\frac{a\tan(c+dx)(a+b\sec(c+dx))^4}{30bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]])/(16*d) - (a*(4*a^4 - 1
21*a^2*b^2 - 128*b^4)*Tan[c + d*x])/(60*b*d) - ((8*a^4 - 178*a^2*b^2 - 75*b
^4)*Sec[c + d*x]*Tan[c + d*x])/(240*d) - (a*(4*a^2 - 53*b^2)*(a + b*Sec[c +
 d*x])^2*Tan[c + d*x])/(120*b*d) - ((4*a^2 - 25*b^2)*(a + b*Sec[c + d*x])^3
*Tan[c + d*x])/(120*b*d) - (a*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(30*b*d)
 + ((a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{(a+b\sec(c+dx))^5\tan(c+dx)}{6bd} + \frac{\int \sec(c+dx)(5b-a\sec(c+dx}{6b}$$

$$= -\frac{a(a+b\sec(c+dx))^4\tan(c+dx)}{30bd} + \frac{(a+b\sec(c+dx))^5\tan(c+}{6bd}$$

$$= -\frac{(4a^2-25b^2)(a+b\sec(c+dx))^3\tan(c+dx)}{120bd} - \frac{a(a+b\sec(c+}{30}$$

$$= -\frac{a(4a^2-53b^2)(a+b\sec(c+dx))^2\tan(c+dx)}{120bd} - \frac{(4a^2-25b^2)(a}{}$$

$$= -\frac{(8a^4-178a^2b^2-75b^4)\sec(c+dx)\tan(c+dx)}{240d} - \frac{a(4a^2-53b^2)}{}$$

$$= -\frac{(8a^4-178a^2b^2-75b^4)\sec(c+dx)\tan(c+dx)}{240d} - \frac{a(4a^2-53b^2)}{}$$

$$= \frac{(8a^4+36a^2b^2+5b^4)\tanh^{-1}(\sin(c+dx))}{16d} - \frac{(8a^4-178a^2b^2-75b}{2}$$

$$= \frac{(8a^4+36a^2b^2+5b^4)\tanh^{-1}(\sin(c+dx))}{16d} - \frac{a(4a^4-121a^2b^2-12}{60bd}$$

Mathematica [A]
time = 1.01, size = 154, normalized size = 0.63

$$\frac{15(8a^4+36a^2b^2+5b^4)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(15(8a^4+36a^2b^2+5b^4)\sec(c+dx)+10b^2(36a^2+5b^2)\sec^3(c+dx)+40b^4\sec^5(c+dx)+64ab(15(a^2+b^2)+5(a^2+2b^2)\tan^2(c+dx)+3b^2\tan^4(c+dx))\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(
8*a^4 + 36*a^2*b^2 + 5*b^4)*Sec[c + d*x] + 10*b^2*(36*a^2 + 5*b^2)*Sec[c +
d*x]^3 + 40*b^4*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2) + 5*(a^2 + 2*b^2)*T
an[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(240*d)
```

Maple [A]
time = 0.13, size = 209, normalized size = 0.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-4*b*a^3*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*b^2*a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d
*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*b^3*a*(-8/15-1/5*sec(d*x
+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+b^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c
)^3-5/16*sec(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 275, normalized size = 1.13

$$\frac{640\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)a^{3}b+128\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)ab^{3}-5\,b^{4}\left(\frac{2\left(15\sin(dx+c)^{5}-40\sin(dx+c)^{3}+33\sin(dx+c)\right)}{\sin(dx+c)^{6}-3\sin(dx+c)^{4}+3\sin(dx+c)^{2}-1}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)-180\,a^{2}b^{2}\left(\frac{2\left(3\sin(dx+c)^{3}-5\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\sin(dx+c)^{2}+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-120\,a^{4}\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3*b + 128*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3 - 5*b^4*(2*(15*sin(d*x + c)^5
- 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 +
 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 180*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2
*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) -
 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.32, size = 217, normalized size = 0.89

$$\frac{15\left(8\,a^{4}+36\,a^{2}b^{2}+5\,b^{4}\right)\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\,a^{4}+36\,a^{2}b^{2}+5\,b^{4}\right)\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(128\left(5\,a^{3}b+4\,ab^{3}\right)\cos\left(dx+c\right)^{5}+192\,ab^{3}\cos\left(dx+c\right)+15\left(8\,a^{4}+36\,a^{2}b^{2}+5\,b^{4}\right)\cos\left(dx+c\right)^{4}+40\,b^{4}+64\left(5\,a^{3}b+4\,ab^{3}\right)\cos\left(dx+c\right)^{3}+10\left(36\,a^{2}b^{2}+5\,b^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{480\,d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1)
 - 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) +
2*(128*(5*a^3*b + 4*a*b^3)*cos(d*x + c)^5 + 192*a*b^3*cos(d*x + c) + 15*(8*
a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 + 40*b^4 + 64*(5*a^3*b + 4*a*b^3)*
cos(d*x + c)^3 + 10*(36*a^2*b^2 + 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*c
os(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^{4}\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(230) = 460.
time = 0.53, size = 592, normalized size = 2.43

$$15\left(8\,a^{4}+36\,a^{2}b^{2}+5\,b^{4}\right)\log\left(\left|\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)+1\right|\right)-15\left(8\,a^{4}+36\,a^{2}b^{2}+5\,b^{4}\right)\log\left(\left|\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(120\,a^{4}\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)^{11}-960\,a^{3}b\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)^{11}+900\,a^{2}b^{2}\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)^{11}-960\,ab^{3}\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)^{11}+165\,b^{4}\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)^{11}-360\,a^{4}\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)^{9}+3520\,a^{3}b\ldots\right.}{\ldots}$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12
0*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2
*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 165*b^4*
tan(1/2*d*x + 1/2*c)^11 - 360*a^4*tan(1/2*d*x + 1/2*c)^9 + 3520*a^3*b*tan(1
/2*d*x + 1/2*c)^9 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 2240*a*b^3*tan(1/
2*d*x + 1/2*c)^9 + 25*b^4*tan(1/2*d*x + 1/2*c)^9 + 240*a^4*tan(1/2*d*x + 1/
2*c)^7 - 5760*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*
c)^7 - 4992*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^4*tan(1/2*d*x + 1/2*c)^7 +
 240*a^4*tan(1/2*d*x + 1/2*c)^5 + 5760*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*a
^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 4992*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 450*b^4
*tan(1/2*d*x + 1/2*c)^5 - 360*a^4*tan(1/2*d*x + 1/2*c)^3 - 3520*a^3*b*tan(1
/2*d*x + 1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2240*a*b^3*tan(1/
2*d*x + 1/2*c)^3 + 25*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*a^4*tan(1/2*d*x + 1/
2*c) + 960*a^3*b*tan(1/2*d*x + 1/2*c) + 900*a^2*b^2*tan(1/2*d*x + 1/2*c) +
960*a*b^3*tan(1/2*d*x + 1/2*c) + 165*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 4.88, size = 370, normalized size = 1.52

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^4+\frac{9\,a^2\,b^2}{2}+\frac{5\,b^4}{8}\right)}{d}+\frac{\left(a^4-8\,a^3\,b+\frac{15\,a^2\,b^2}{2}-8\,a\,b^3+\frac{11\,b^4}{8}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+\left(-3\,a^4+\frac{88\,a^3\,b}{3}-\frac{21\,a^2\,b^2}{2}+\frac{56\,a\,b^3}{3}+\frac{5\,b^4}{24}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+\left(2\,a^4-48\,a^3\,b+3\,a^2\,b^2-\frac{208\,a\,b^3}{5}+\frac{15\,b^4}{4}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\left(2\,a^4+48\,a^3\,b+3\,a^2\,b^2+\frac{208\,a\,b^3}{5}+\frac{15\,b^4}{4}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-3\,a^4-\frac{88\,a^3\,b}{3}-\frac{21\,a^2\,b^2}{2}-\frac{56\,a\,b^3}{3}+\frac{5\,b^4}{24}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(a^4+8\,a^3\,b+\frac{15\,a^2\,b^2}{2}+8\,a\,b^3+\frac{11\,b^4}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/cos(c + d*x)^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(a^4 + (5*b^4)/8 + (9*a^2*b^2)/2))/d + (tan(c/2
+ (d*x)/2)^9*((56*a*b^3)/3 + (88*a^3*b)/3 - 3*a^4 + (5*b^4)/24 - (21*a^2*b^
2)/2) - tan(c/2 + (d*x)/2)^3*((56*a*b^3)/3 + (88*a^3*b)/3 + 3*a^4 - (5*b^4)
/24 + (21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((208*a*b^3)/5 + 48*a^3*b + 2*
a^4 + (15*b^4)/4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(2*a^4 - 48*a^3*b - (2
08*a*b^3)/5 + (15*b^4)/4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3
*b + a^4 + (11*b^4)/8 + (15*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^11*(a^4 - 8*a^
3*b - 8*a*b^3 + (11*b^4)/8 + (15*a^2*b^2)/2))/(d*(15*tan(c/2 + (d*x)/2)^4 -
 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8
 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.477 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=179

$$\frac{ab(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2(3a^4+28a^2b^2+4b^4)\tan(c+dx)}{15d}+\frac{ab(6a^2+29b^2)\sec(c+dx)\tan(c+d}{30d}$$

```
[Out] 1/2*a*b*(4*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2/15*(3*a^4+28*a^2*b^2+4*b^4)*t
an(d*x+c)/d+1/30*a*b*(6*a^2+29*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*(3*a^2+4*b
^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/5*a*(a+b*sec(d*x+c))^3*tan(d*x+c)/d+1
/5*(a+b*sec(d*x+c))^4*tan(d*x+c)/d
```

Rubi [A]
time = 0.21, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3920, 4087, 4082, 3872, 3855, 3852, 8}

$$\frac{ab(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3a^2+4b^2)\tan(c+dx)(a+b\sec(c+dx))^2}{15d}+\frac{ab(6a^2+29b^2)\tan(c+dx)\sec(c+dx)}{30d}+\frac{2(3a^4+28a^2b^2+4b^4)\tan(c+dx)}{15d}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}+\frac{a\tan(c+dx)(a+b\sec(c+dx))^3}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a^4 + 28*a^2*b^2
+ 4*b^4)*Tan[c + d*x])/(15*d) + (a*b*(6*a^2 + 29*b^2)*Sec[c + d*x]*Tan[c +
d*x])/(30*d) + ((3*a^2 + 4*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(15*d)
 + (a*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(5*d) + ((a + b*Sec[c + d*x])^4*
Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3920

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[
e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
0]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5d} + \frac{4}{5}\int \sec(c+dx)(b+a\sec(c+dx$$
$$= \frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{5d} + \frac{(a+b\sec(c+dx))^4\tan(c+dx}{5d}$$
$$= \frac{(3a^2+4b^2)(a+b\sec(c+dx))^2\tan(c+dx)}{15d} + \frac{a(a+b\sec(c+dx))^3}{5d}$$
$$= \frac{ab(6a^2+29b^2)\sec(c+dx)\tan(c+dx)}{30d} + \frac{(3a^2+4b^2)(a+b\sec(c+}{15d}$$
$$= \frac{ab(6a^2+29b^2)\sec(c+dx)\tan(c+dx)}{30d} + \frac{(3a^2+4b^2)(a+b\sec(c+}{15d}$$
$$= \frac{ab(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{ab(6a^2+29b^2)\sec(c+dx)\tan}{30d}$$
$$= \frac{ab(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2(3a^4+28a^2b^2+4b^4)\tan(c+}{15d}$$

Mathematica [A]
time = 0.80, size = 125, normalized size = 0.70

$$\frac{15ab(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(30(a^4+6a^2b^2+b^4)+15ab(4a^2+3b^2)\sec(c+dx)+30ab^3\sec^3(c+dx)+20b^2(3a^2+b^2)\tan^2(c+dx)+6b^4\tan^4(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (15*a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(30*(a^4 + 6*a
^2*b^2 + b^4) + 15*a*b*(4*a^2 + 3*b^2)*Sec[c + d*x] + 30*a*b^3*Sec[c + d*x]
^3 + 20*b^2*(3*a^2 + b^2)*Tan[c + d*x]^2 + 6*b^4*Tan[c + d*x]^4))/(30*d)
```

Maple [A]
time = 0.11, size = 162, normalized size = 0.91

method	result
derivativedivides	$\frac{a^4\tan(dx+c)+4b\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-6b^2a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4b^3a\left(-\left(\right.\right.}{}$
default	$\frac{a^4\tan(dx+c)+4b\,a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-6b^2a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4b^3a\left(-\left(\right.\right.}{}$
norman	$\frac{-\frac{4\left(45a^4+150b^2a^2+29b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{\left(2a^4-4b\,a^3+12b^2a^2-5b^3a+2b^4\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(2a^4+4b\,a^3+12b^2a^2+5b^3a+2b^4\right)\mathrm{ta}}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$
risch	$\frac{i\left(-60b\,a^3e^{9i(dx+c)}-45a\,b^3e^{9i(dx+c)}+30a^4e^{8i(dx+c)}-120a^3b\,e^{7i(dx+c)}-210a\,b^3e^{7i(dx+c)}+120a^4e^{6i(dx+c)}+360a^2b^2e^{6i(d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^4*tan(d*x+c)+4*b*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-6*b^2*a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*b^3*a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))

Maxima [A]
time = 0.26, size = 195, normalized size = 1.09

$$\frac{120\,\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{2}b^{2}+4\,\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)b^{4}-15\,ab^{3}\left(\frac{2\,\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,a^{3}b\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+60\,a^{4}\tan\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")

[Out] 1/60*(120*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2 + 4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^4 - 15*a*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c))/d

Fricas [A]
time = 2.38, size = 182, normalized size = 1.02

$$\frac{15\,\left(4\,a^{3}b+3\,ab^{3}\right)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\,\left(4\,a^{3}b+3\,ab^{3}\right)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(30\,ab^{3}\cos\left(dx+c\right)+2\,\left(15\,a^{4}+60\,a^{2}b^{2}+8\,b^{4}\right)\cos\left(dx+c\right)^{4}+6\,b^{4}+15\,\left(4\,a^{3}b+3\,ab^{3}\right)\cos\left(dx+c\right)^{3}+4\,\left(15\,a^{2}b^{2}+2\,b^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{60\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/60*(15*(4*a^3*b + 3*a*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^3*b + 3*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*a*b^3*cos(d*x + c) + 2*(15*a^4 + 60*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 6*b^4 + 15*(4*a^3*b + 3*a*b^3)*cos(d*x + c)^3 + 4*(15*a^2*b^2 + 2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^{4}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(167) = 334.
time = 0.51, size = 461, normalized size = 2.58

$$\frac{15\,(4\,a^3b+3\,ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(4\,a^3b+3\,ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+30\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-480\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+30\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-40\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+180\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+600\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+116\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-480\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-30\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+60\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+75\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+30\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(15*(4*a^3*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*a^4*tan(1/2*d*x +
1/2*c)^9 - 60*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*
c)^9 - 75*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 - 12
0*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*a^2*b
^2*tan(1/2*d*x + 1/2*c)^7 + 30*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 40*b^4*tan(1/
2*d*x + 1/2*c)^7 + 180*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 116*b^4*tan(1/2*d*x + 1/2*c)^5 - 120*a^4*tan(1/2*d*x + 1/2*c)
^3 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3
- 30*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*
tan(1/2*d*x + 1/2*c) + 60*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 75*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 4.97, size = 304, normalized size = 1.70

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,a^3\,b+3\,a\,b^3\right)}{d}-\frac{\left(2\,a^4-4\,a^3\,b+12\,a^2\,b^2-5\,a\,b^3+2\,b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-8\,a^4+8\,a^3\,b-32\,a^2\,b^2+2\,a\,b^3-\frac{8\,b^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(12\,a^4+40\,a^2\,b^2+\frac{116\,b^4}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-8\,a^4-8\,a^3\,b-32\,a^2\,b^2-2\,a\,b^3-\frac{8\,b^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^4+4\,a^3\,b+12\,a^2\,b^2+5\,a\,b^3+2\,b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/cos(c + d*x)^2,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(3*a*b^3 + 4*a^3*b))/d - (tan(c/2 + (d*x)/2)^5*(
12*a^4 + (116*b^4)/15 + 40*a^2*b^2) + tan(c/2 + (d*x)/2)^9*(2*a^4 - 4*a^3*b
 - 5*a*b^3 + 2*b^4 + 12*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(2*a*b^3 + 8*a^3*b
+ 8*a^4 + (8*b^4)/3 + 32*a^2*b^2) - tan(c/2 + (d*x)/2)^7*(8*a^4 - 8*a^3*b -
 2*a*b^3 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)*(5*a*b^3 + 4*a^3*b
+ 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d
*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*
x)/2)^10 - 1))
```

3.478 $\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=146

$$\frac{(8a^4+24a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab(19a^2+16b^2)\tan(c+dx)}{6d}+\frac{b^2(26a^2+9b^2)\sec(c+dx)\tan(c+}{24d}$$

```
[Out] 1/8*(8*a^4+24*a^2*b^2+3*b^4)*arctanh(sin(d*x+c))/d+1/6*a*b*(19*a^2+16*b^2)*
tan(d*x+c)/d+1/24*b^2*(26*a^2+9*b^2)*sec(d*x+c)*tan(d*x+c)/d+7/12*a*b*(a+b*
sec(d*x+c))^2*tan(d*x+c)/d+1/4*b*(a+b*sec(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A]

time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3915, 4087, 4082, 3872, 3855, 3852, 8}

$$\frac{ab(19a^2+16b^2)\tan(c+dx)}{6d}+\frac{b^2(26a^2+9b^2)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(8a^4+24a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}+\frac{7ab\tan(c+dx)(a+b\sec(c+dx))^2}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*b*(19*a^2 +
 16*b^2)*Tan[c + d*x])/(6*d) + (b^2*(26*a^2 + 9*b^2)*Sec[c + d*x]*Tan[c + d
*x])/(24*d) + (7*a*b*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (b*(a +
b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3915

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^
2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{b(a+b\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{4}\int \sec(c+dx)(a+b\sec(c+dx$$

$$= \frac{7ab(a+b\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{b(a+b\sec(c+dx))^3\tan(c+$$

$$= \frac{b^2(26a^2+9b^2)\sec(c+dx)\tan(c+dx)}{24d} + \frac{7ab(a+b\sec(c+dx))^2\tan}{12d}$$

$$= \frac{b^2(26a^2+9b^2)\sec(c+dx)\tan(c+dx)}{24d} + \frac{7ab(a+b\sec(c+dx))^2\tan}{12d}$$

$$= \frac{(8a^4+24a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b^2(26a^2+9b^2)\sec(c+d}{24d}$$

$$= \frac{(8a^4+24a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{ab(19a^2+16b^2)\tan(c+}{6d}$$

Mathematica [A]

time = 0.55, size = 101, normalized size = 0.69

$$\frac{3(8a^4+24a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))+b\tan(c+dx)\left(9b(8a^2+b^2)\sec(c+dx)+6b^3\sec^3(c+dx)+32a(3(a^2+b^2)+b^2\tan^2(c+dx))\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]] + b*Tan[c + d*x]*(9*b
*(8*a^2 + b^2)*Sec[c + d*x] + 6*b^3*Sec[c + d*x]^3 + 32*a*(3*(a^2 + b^2) +
b^2*Tan[c + d*x]^2)))/(24*d)
```

Maple [A]
time = 0.10, size = 147, normalized size = 1.01

method	result
derivativedivides	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4b\,a^3\tan(dx+c)+6b^2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4b^3a\left(-\frac{2}{3}-\frac{(\mathrm{se}\cdots}{}\right.}{d}$
default	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4b\,a^3\tan(dx+c)+6b^2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4b^3a\left(-\frac{2}{3}-\frac{(\mathrm{se}\cdots}{}\right.}{d}$
norman	$\frac{-\frac{b(32a^3-24b\,a^2+32b^2a-5b^3)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{b(32a^3+24b\,a^2+32b^2a+5b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{b(288a^3-72b\,a^2+160b^2a+9b^3)(\tan^5\cdots}{12d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$
risch	$-\frac{ib(72b\,a^2\mathrm{e}^{7i(dx+c)}+9b^3\mathrm{e}^{7i(dx+c)}-96a^3\mathrm{e}^{6i(dx+c)}+72a^2b\,\mathrm{e}^{5i(dx+c)}+33b^3\mathrm{e}^{5i(dx+c)}-288a^3\mathrm{e}^{4i(dx+c)}-192a\,b^2\mathrm{e}^{4i(dx+c)}\cdots}{12d(\mathrm{e}^{2i(dx\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*ln(sec(d*x+c)+tan(d*x+c))+4*b*a^3*tan(d*x+c)+6*b^2*a^2*(1/2*sec(d*
x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-4*b^3*a*(-2/3-1/3*sec(d*x+c)
^2)*tan(d*x+c)+b^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(s
ec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.26, size = 180, normalized size = 1.23

$$\frac{64\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab^3-3b^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-72a^2b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+48a^4\log(\sec(dx+c)+\tan(dx+c))+192a^3b\tan(dx+c)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^3 - 3*b^4*(2*(3*sin(d*x + c)
^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^2*b^2*(2*sin(d*x + c)/(sin(d*
```

```
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4*log
(sec(d*x + c) + tan(d*x + c)) + 192*a^3*b*tan(d*x + c))/d
```

Fricas [A]
time = 2.59, size = 163, normalized size = 1.12

$$\frac{3\,(8\,a^4+24\,a^2b^2+3\,b^4)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(8\,a^4+24\,a^2b^2+3\,b^4)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(32\,ab^3\cos\left(dx+c\right)+6\,b^4+32\,(3\,a^3b+2\,ab^3)\cos\left(dx+c\right)^3+9\,(8\,a^2b^2+b^4)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(
32*a*b^3*cos(d*x + c) + 6*b^4 + 32*(3*a^3*b + 2*a*b^3)*cos(d*x + c)^3 + 9*(
8*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^4\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(136) = 272.
time = 0.46, size = 360, normalized size = 2.47

$$\frac{3\,(8\,a^4+24\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(8\,a^4+24\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(96\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-288\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-160\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+288\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+160\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-96\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(96*a^
3*b*tan(1/2*d*x + 1/2*c)^7 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 96*a*b^3*t
an(1/2*d*x + 1/2*c)^7 - 15*b^4*tan(1/2*d*x + 1/2*c)^7 - 288*a^3*b*tan(1/2*d
*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a*b^3*tan(1/2*d*x +
 1/2*c)^5 - 9*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*b*tan(1/2*d*x + 1/2*c)^3
 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 9
*b^4*tan(1/2*d*x + 1/2*c)^3 - 96*a^3*b*tan(1/2*d*x + 1/2*c) - 72*a^2*b^2*ta
n(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c) - 15*b^4*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 4.92, size = 245, normalized size = 1.68

$$\frac{\left(-8\,a^3\,b+6\,a^2\,b^2-8\,a\,b^3+\frac{5\,b^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(24\,a^3\,b-6\,a^2\,b^2+\frac{40\,a\,b^3}{3}+\frac{3\,b^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-24\,a^3\,b-6\,a^2\,b^2-\frac{40\,a\,b^3}{3}+\frac{3\,b^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(8\,a^3\,b+6\,a^2\,b^2+8\,a\,b^3+\frac{5\,b^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^4+6\,a^2\,b^2+\frac{3\,b^4}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/cos(c + d*x),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3*b + (5*b^4)/4 + 6*a^2*b^2) - tan(c/2 +
 (d*x)/2)^7*(8*a*b^3 + 8*a^3*b - (5*b^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)/2
)^3*((40*a*b^3)/3 + 24*a^3*b - (3*b^4)/4 + 6*a^2*b^2) + tan(c/2 + (d*x)/2)^
5*((40*a*b^3)/3 + 24*a^3*b + (3*b^4)/4 - 6*a^2*b^2))/(d*(6*tan(c/2 + (d*x)/
2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)
^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(2*a^4 + (3*b^4)/4 + 6*a^2*b^2))/d
```

3.479 $\int (a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=107

$$a^4x+\frac{2ab(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2(17a^2+2b^2)\tan(c+dx)}{3d}+\frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d}+\frac{b^2(a+b$$

```
[Out] a^4*x+2*a*b*(2*a^2+b^2)*arctanh(sin(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*tan(d*
x+c)/d+4/3*a*b^3*sec(d*x+c)*tan(d*x+c)/d+1/3*b^2*(a+b*sec(d*x+c))^2*tan(d*x
+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3867, 4133, 3855, 3852, 8}

$$a^4x+\frac{b^2(17a^2+2b^2)\tan(c+dx)}{3d}+\frac{2ab(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\tan(c+dx)\sec(c+dx)}{3d}+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4,x]
```

```
[Out] a^4*x + (2*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Tan[c + d*x])/(3*d) + (4*a*b^3*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (b^2*(
a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^4\,dx = \frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{3}\int (a+b\sec(c+dx))\left(3a^3+b\left(9a^2+2b^2\right.\right.$$

$$= \frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d} + \frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{6}\int \left(6a^4\right.$$

$$= a^4x + \frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d} + \frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d} + \left(2ab\right.$$

$$= a^4x + \frac{2ab(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{d} + \frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d} + \frac{b^2(a}{}$$

$$= a^4x + \frac{2ab(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{d} + \frac{b^2(17a^2+2b^2)\tan(c+dx)}{3d} + \frac{4ab^3 s}{}$$

Mathematica [A]
time = 0.32, size = 77, normalized size = 0.72

$$\frac{3a^4dx + 6ab(2a^2+b^2)\tanh^{-1}(\sin(c+dx)) + 3b^2(6a^2+b^2+2ab\sec(c+dx))\tan(c+dx) + b^4\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4,x]
[Out] (3*a^4*d*x + 6*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + 3*b^2*(6*a^2 + b^2
 + 2*a*b*Sec[c + d*x])*Tan[c + d*x] + b^4*Tan[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.08, size = 109, normalized size = 1.02

method	result
derivativedivides	$\frac{a^4(dx+c)+4b\,a^3\ln(\sec(dx+c)+\tan(dx+c))+6b^2a^2\tan(dx+c)+4b^3a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b}{d}$

default	$\frac{a^4(dx+c)+4b\,a^3\ln(\sec(dx+c)+\tan(dx+c))+6b^2a^2\tan(dx+c)+4b^3a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^4\left(\right.}{d}$
risch	$a^4x-\frac{4ib^2\left(3ba\,e^{5i(dx+c)}-9a^2e^{4i(dx+c)}-18a^2e^{2i(dx+c)}-3b^2e^{2i(dx+c)}-3ba\,e^{i(dx+c)}-9a^2-b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{4b\,a^3\ln\left(e^{i(dx+c)}-i\right)}{d}$
norman	$\frac{a^4x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^4x+3a^4x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3a^4x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4b^2\left(18a^2+b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2b^2\left(6a^2-2ba+b^2\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(d*x+c)+4*b*a^3*ln(sec(d*x+c)+tan(d*x+c))+6*b^2*a^2*tan(d*x+c)+4*b
^3*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b^4*(-2/3-1/
3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 121, normalized size = 1.13

$$a^4x+\frac{(\tan(dx+c)^3+3\tan(dx+c))b^4}{3d}-\frac{ab^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{d}+\frac{4a^3b\log(\sec(dx+c)+\tan(dx+c))}{d}+\frac{6a^2b^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^4/d - a*b^3*(2*sin(d*x + c)
/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d +
4*a^3*b*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^2*b^2*tan(d*x + c)/d
```

Fricas [A]
time = 2.47, size = 138, normalized size = 1.29

$$\frac{3a^4dx\cos(dx+c)^3+3(2a^3b+ab^3)\cos(dx+c)^3\log(\sin(dx+c)+1)-3(2a^3b+ab^3)\cos(dx+c)^3\log(-\sin(dx+c)+1)+(6ab^3\cos(dx+c)+b^4+2(9a^2b^2+b^4)\cos(dx+c)^2)\sin(dx+c)}{3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cos(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(sin(
d*x + c) + 1) - 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 (6*a*b^3*cos(d*x + c) + b^4 + 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x
+ c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(101) = 202.
time = 0.46, size = 221, normalized size = 2.07

$$\frac{3\,(dx+c)a^4+6\,(2\,a^3b+ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,(2\,a^3b+ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-36\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b^2
*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*
x + 1/2*c)^5 - 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^4*tan(1/2*d*x + 1/2*
c)^3 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6*a*b^3*tan(1/2*d*x + 1/2*c) + 3*b
^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 1.03, size = 185, normalized size = 1.73

$$\frac{2\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{b^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}+\frac{4\,a\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a\,b^3\,\sin\left(c+d\,x\right)}{d\,{\cos\left(c+d\,x\right)}^2}+\frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4,x)
```

```
[Out] (2*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^4*sin(c + d*x)
)/(3*d*cos(c + d*x)) + (b^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (4*a*b^3*a
tanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a^3*b*atanh(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a*b^3*sin(c + d*x))/(d*cos(c + d*x)^2)
+ (6*a^2*b^2*sin(c + d*x))/(d*cos(c + d*x))
```

3.480 $\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=104

$$4a^3bx+\frac{b^2(12a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(2a^2-b^2)\sin(c+dx)}{2d}+\frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{3ab}{}$$

```
[Out] 4*a^3*b*x+1/2*b^2*(12*a^2+b^2)*arctanh(sin(d*x+c))/d+1/2*a^2*(2*a^2-b^2)*si
n(d*x+c)/d+1/2*b^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+3*a*b^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3927, 4161, 4132, 8, 4130, 3855}

$$4a^3bx+\frac{a^2(2a^2-b^2)\sin(c+dx)}{2d}+\frac{b^2(12a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab^3\tan(c+dx)}{d}+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] 4*a^3*b*x + (b^2*(12*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*a^2
- b^2)*Sin[c + d*x])/(2*d) + (b^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d
) + (3*a*b^3*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{1}{2}\int \cos(c+dx)(a+b\sec(c+\\ &= \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{3ab^3\tan(c+dx)}{d}+\frac{1}{2}\int \cos(c\\ &= \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{3ab^3\tan(c+dx)}{d}+\frac{1}{2}\int \cos(c\\ &= 4a^3bx+\frac{a^2(2a^2-b^2)\sin(c+dx)}{2d}+\frac{b^2(a+b\sec(c+dx))^2\sin(c+d\\ &= 4a^3bx+\frac{b^2(12a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(2a^2-b^2)\sin(c+d\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(104) = 208.
time = 0.55, size = 280, normalized size = 2.69

$$\frac{\sec^2(c+dx)\left(8a^3bc+8a^3bdx-12a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\cos(2(c+dx))\left(8a^3(c+dx)-b(12a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b(12a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a^4+2b^4)\sin(c+dx)+8ab^3\sin(2(c+dx))+a^4\sin(3(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^2*(8*a^3*b*c + 8*a^3*b*d*x - 12*a^2*b^2*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] - b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^2
*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^4*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + b*Cos[2*(c + d*x)]*(8*a^3*(c + d*x) - b*(12*a^2 + b^2)*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*(12*a^2 + b^2)*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + (a^4 + 2*b^4)*Sin[c + d*x] + 8*a*b^3*Sin[2*(c
+ d*x)] + a^4*Sin[3*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.11, size = 96, normalized size = 0.92

method	result
derivativedivides	$\frac{a^4 \sin(dx+c)+4b\,a^3(dx+c)+6b^2a^2\ln(\sec(dx+c)+\tan(dx+c))+4b^3a\tan(dx+c)+b^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan}{2}\right.}{d}$
default	$\frac{a^4 \sin(dx+c)+4b\,a^3(dx+c)+6b^2a^2\ln(\sec(dx+c)+\tan(dx+c))+4b^3a\tan(dx+c)+b^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan}{2}\right.}{d}$
risch	$4a^3bx-\frac{ia^4e^{i(dx+c)}}{2d}+\frac{ia^4e^{-i(dx+c)}}{2d}-\frac{ib^3\left(b\,e^{3i(dx+c)}-8a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-8a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{6\ln\left(e^{i(dx+c)}+i\right)b^2a^2}{d}+$
norman	$\frac{\frac{\left(2a^4-8b^3a+b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(6a^4+8b^3a-b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-4a^3bx-\frac{\left(2a^4+8b^3a+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{\left(6a^4-8b^3a-b^4\right)\left(\tan\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*sin(d*x+c)+4*b*a^3*(d*x+c)+6*b^2*a^2*ln(sec(d*x+c)+tan(d*x+c))+4*b
^3*a*tan(d*x+c)+b^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)
)))
```

Maxima [A]
time = 0.26, size = 115, normalized size = 1.11

$$\frac{16\,(dx+c)a^3b-b^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^2b^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+4\,a^4\sin\left(dx+c\right)+16\,ab^3\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a^3*b - b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*b^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a*b^3*tan(d*x + c))/d
```

Fricas [A]
time = 3.20, size = 130, normalized size = 1.25

$$\frac{16\,a^3bdx\cos\left(dx+c\right)^2+\left(12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,a^4\cos\left(dx+c\right)^2+8\,ab^3\cos\left(dx+c\right)+b^4\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a^3*b*d*x*cos(d*x + c)^2 + (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) - (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
+ 2*(2*a^4*cos(d*x + c)^2 + 8*a*b^3*cos(d*x + c) + b^4)*sin(d*x + c))/(d*co
s(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^4 \cos(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*cos(c + d*x), x)
```

Giac [A]
time = 0.45, size = 179, normalized size = 1.72

$$\frac{8(dx + c)a^3b + \frac{4a^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1} + (12a^2b^2 + b^4)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (12a^2b^2 + b^4)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(8ab^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - b^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 8ab^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a^3*b + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2
 + 1) + (12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (12*a^2*b^2
 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a*b^3*tan(1/2*d*x + 1/2*c
)^3 - b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a*b^3*tan(1/2*d*x + 1/2*c) - b^4*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 1.04, size = 152, normalized size = 1.46

$$\frac{a^4\sin(c + dx)}{d} + \frac{b^4\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{b^4\sin(c + dx)}{2d\cos(c + dx)^2} + \frac{12a^2b^2\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{8a^3b\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{4ab^3\sin(c + dx)}{d\cos(c + dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (a^4*sin(c + d*x))/d + (b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
 + (b^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (12*a^2*b^2*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d + (4*a*b^3*sin(c + d*x))/(d*cos(c + d*x))
```

3.481 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=108

$$\frac{1}{2}a^2\left(a^2+12b^2\right)x+\frac{4ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3b\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$

```
[Out] 1/2*a^2*(a^2+12*b^2)*x+4*a*b^3*arctanh(sin(d*x+c))/d+3*a^3*b*sin(d*x+c)/d+1
/2*a^2*cos(d*x+c)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/2*b^2*(a^2-2*b^2)*tan(d
*x+c)/d
```

Rubi [A]

time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3926, 4161, 4132, 8, 4130, 3855}

$$\frac{3a^3b\sin(c+dx)}{d}-\frac{b^2(a^2-2b^2)\tan(c+dx)}{2d}+\frac{1}{2}a^2x\left(a^2+12b^2\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{4ab^3\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a^2*(a^2 + 12*b^2)*x)/2 + (4*a*b^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*b*Sin
[c + d*x])/d + (a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d)
 - (b^2*(a^2 - 2*b^2)*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{1}{2}\int \cos(c+dx)(\ldots$$

$$= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{b^2(a^2-2b^2)\tan(\ldots}{2d}$$

$$= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{b^2(a^2-2b^2)\tan(\ldots}{2d}$$

$$= \frac{1}{2}a^2\left(a^2+12b^2\right)x + \frac{3a^3b\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)(a+b\sec(c\ldots}{2d}$$

$$= \frac{1}{2}a^2\left(a^2+12b^2\right)x + \frac{4ab^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^3b\sin(c+dx)}{d} + \ldots$$

Mathematica [A]
time = 0.71, size = 119, normalized size = 1.10

$$\frac{2a\left(a\left(a^2+12b^2\right)(c+dx)-8b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+16a^3b\sin(c+dx)+a^4\sin(2(c+dx))+4b^4\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

[Out] (2*a*(a*(a^2 + 12*b^2)*(c + d*x) - 8*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 8*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*a^3*b*Sin[c + d*x] + a^4*Sin[2*(c + d*x)] + 4*b^4*Tan[c + d*x])/(4*d)

Maple [A]
time = 0.10, size = 87, normalized size = 0.81

method	result
derivativedivides	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4b\,a^3\sin(dx+c)+6b^2a^2(dx+c)+4b^3a\ln(\sec(dx+c)+\tan(dx+c))+b^4\tan(dx+c)}{d}$
default	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4b\,a^3\sin(dx+c)+6b^2a^2(dx+c)+4b^3a\ln(\sec(dx+c)+\tan(dx+c))+b^4\tan(dx+c)}{d}$
risch	$\frac{a^4x}{2}+6a^2b^2x-\frac{ia^4e^{2i(dx+c)}}{8d}-\frac{2ib\,a^3e^{i(dx+c)}}{d}+\frac{2ib\,a^3e^{-i(dx+c)}}{d}+\frac{ia^4e^{-2i(dx+c)}}{8d}+\frac{2ib^4}{d\left(e^{2i(dx+c)}+1\right)}+\frac{4b^3c}{}$
norman	$\frac{(-\frac{1}{2}a^4-6b^2a^2)x+(-\frac{1}{2}a^4-6b^2a^2)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{1}{2}a^4+6b^2a^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{1}{2}a^4+6b^2a^2)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*b*a^3*sin(d*x+c)+6*b^2*a^2*(d*x+c)+4*b^3*a*ln(sec(d*x+c)+tan(d*x+c))+b^4*tan(d*x+c))

Maxima [A]
time = 0.26, size = 90, normalized size = 0.83

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+24\,(dx+c)a^2b^2+8\,ab^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+16\,a^3b\sin(dx+c)+4\,b^4\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^2*b^2 + 8*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^3*b*sin(d*x + c) + 4*b^4*tan(d*x + c))/d

Fricas [A]
time = 3.49, size = 116, normalized size = 1.07

$$\frac{4\,ab^3\cos(dx+c)\log(\sin(dx+c)+1)-4\,ab^3\cos(dx+c)\log(-\sin(dx+c)+1)+(a^4+12\,a^2b^2)dx\cos(dx+c)+(a^4\cos(dx+c)^2+8\,a^3b\cos(dx+c)+2\,b^4)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")

[Out] 1/2*(4*a*b^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a*b^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4 + 12*a^2*b^2)*d*x*cos(d*x + c) + (a^4*cos(d*x + c)^2 + 8*a^3*b*cos(d*x + c) + 2*b^4)*sin(d*x + c))/(d*cos(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec(c + dx))^4 \cos^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*cos(c + d*x)**2, x)
```

Giac [A]
time = 0.47, size = 170, normalized size = 1.57

$$\frac{8\,ab^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 8\,ab^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{4\,b^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1} + \left(a^4 + 12\,a^2 b^2\right)(dx + c) - \frac{2\left(a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 8\,a^3 b \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 8\,a^3 b \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*a*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a*b^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 4*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
 + (a^4 + 12*a^2*b^2)*(d*x + c) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*b*t
an(1/2*d*x + 1/2*c)^3 - a^4*tan(1/2*d*x + 1/2*c) - 8*a^3*b*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 1.03, size = 150, normalized size = 1.39

$$\frac{a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{b^4 \sin(c + d\,x)}{d \cos(c + d\,x)} + \frac{12\,a^2\,b^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{a^4 \cos(c + d\,x) \sin(c + d\,x)}{2\,d} + \frac{4\,a^3\,b \sin(c + d\,x)}{d} + \frac{8\,a\,b^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^4*sin(c + d*x))/(d
*cos(c + d*x)) + (12*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
 + (a^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (4*a^3*b*sin(c + d*x))/d + (8*a*
b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.482 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=115

$$2ab\left(a^2+2b^2\right)x+\frac{b^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\left(2a^2+17b^2\right)\sin(c+dx)}{3d}+\frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}+\frac{a^2\cos^2(}{}$$

```
[Out] 2*a*b*(a^2+2*b^2)*x+b^4*arctanh(sin(d*x+c))/d+1/3*a^2*(2*a^2+17*b^2)*sin(d*
x+c)/d+4/3*a^3*b*cos(d*x+c)*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^2*(a+b*sec(d*x+
c))^2*sin(d*x+c)/d
```

Rubi [A]

time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3926, 4159, 4132, 8, 4130, 3855}

$$\frac{4a^3b\sin(c+dx)\cos(c+dx)}{3d}+\frac{a^2\left(2a^2+17b^2\right)\sin(c+dx)}{3d}+2abx\left(a^2+2b^2\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}+\frac{b^4\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] 2*a*b*(a^2 + 2*b^2)*x + (b^4*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^
2)*Sin[c + d*x])/(3*d) + (4*a^3*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (a^2*C
os[c + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int \cos^2(c+dx\\ &= \frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}\\ &= \frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}\\ &= 2ab\left(a^2+2b^2\right)x+\frac{a^2(2a^2+17b^2)\sin(c+dx)}{3d}+\frac{4a^3b\cos(c+dx)\,\mathrm{si}}{3d}\\ &= 2ab\left(a^2+2b^2\right)x+\frac{b^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(2a^2+17b^2)\sin(c+}{3d}\end{aligned}$$

Mathematica [A]

time = 0.17, size = 128, normalized size = 1.11

$$\frac{24ab(a^2+2b^2)(c+dx)-12b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+9a^2(a^2+8b^2)\sin(c+dx)+12a^3b\sin(2(c+dx))+a^4\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (24*a*b*(a^2 + 2*b^2)*(c + d*x) - 12*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*b^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*a^2*(a^2 + 8*b
^2)*Sin[c + d*x] + 12*a^3*b*Sin[2*(c + d*x)] + a^4*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]
time = 0.11, size = 98, normalized size = 0.85

method	result
derivativedivides	$\frac{\frac{a^4(2+\cos^2(dx+c))\sin(dx+c)}{3}+4b\,a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6b^2a^2\sin(dx+c)+4b^3a(dx+c)+b^4\ln(\sec(dx+c)+\tan(d}{d}$
default	$\frac{\frac{a^4(2+\cos^2(dx+c))\sin(dx+c)}{3}+4b\,a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6b^2a^2\sin(dx+c)+4b^3a(dx+c)+b^4\ln(\sec(dx+c)+\tan(d}{d}$
risch	$2a^3bx+4a\,b^3x-\frac{3ia^4e^{i(dx+c)}}{8d}-\frac{3ie^{i(dx+c)}b^2a^2}{d}+\frac{3ia^4e^{-i(dx+c)}}{8d}+\frac{3ie^{-i(dx+c)}b^2a^2}{d}+\frac{\ln(e^{i(dx+c)}+i)b^4}{d}-$
norman	$\frac{(-2b\,a^3-4b^3a)x+(-6b\,a^3-12b^3a)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2b\,a^3+4b^3a)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6b\,a^3+12b^3a)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4*b*a^3*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+6*b^2*a^2*sin(d*x+c)+4*b^3*a*(d*x+c)+b^4*ln(sec(d*x+c)+tan(
d*x+c)))
```

Maxima [A]
time = 0.26, size = 102, normalized size = 0.89

$$-\frac{2\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)a^4-6\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^3b-24\left(dx+c\right)ab^3-3\,b^4\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-36\,a^2b^2\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^3*b - 24*(d*x + c)*a*b^3 - 3*b^4*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1)) - 36*a^2*b^2*sin(d*x + c))/d
```

Fricas [A]
time = 3.43, size = 98, normalized size = 0.85

$$\frac{3\,b^4\log\left(\sin\left(dx+c\right)+1\right)-3\,b^4\log\left(-\sin\left(dx+c\right)+1\right)+12\left(a^3b+2\,ab^3\right)dx+2\left(a^4\cos\left(dx+c\right)^2+6\,a^3b\cos\left(dx+c\right)+2\,a^4+18\,a^2b^2\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*log(sin(d*x + c) + 1) - 3*b^4*log(-sin(d*x + c) + 1) + 12*(a^3*b
 + 2*a*b^3)*d*x + 2*(a^4*cos(d*x + c)^2 + 6*a^3*b*cos(d*x + c) + 2*a^4 + 18
*a^2*b^2)*sin(d*x + c))/d
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec{(c+dx)})^4\cos^3{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*cos(c + d*x)**3, x)
```

Giac [A]

time = 0.47, size = 212, normalized size = 1.84

$$\frac{3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+6\,(a^3b+2\,ab^3)(dx+c)+\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b^4*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 6*(a^3*b + 2*a*b^3)*(d*x + c) + 2*(3*a^4*tan(1/2*d*x + 1/2*
c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 +
 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^4*t
an(1/2*d*x + 1/2*c) + 6*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]

time = 1.11, size = 158, normalized size = 1.37

$$\frac{3\,a^4\,\sin\left(c+d\,x\right)}{4\,d}+\frac{2\,b^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a^3\,b\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d}+\frac{8\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (3*a^4*sin(c + d*x))/(4*d) + (2*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/d + (a^4*sin(3*c + 3*d*x))/(12*d) + (a^3*b*sin(2*c + 2*d*x))/d + (6*
a^2*b^2*sin(c + d*x))/d + (8*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (4*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.483 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=145

$$\frac{1}{8}\left(3a^4+24a^2b^2+8b^4\right)x+\frac{4ab(2a^2+3b^2)\sin(c+dx)}{3d}+\frac{a^2(3a^2+22b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{5a^3b\cos^2(c+$$

```
[Out] 1/8*(3*a^4+24*a^2*b^2+8*b^4)*x+4/3*a*b*(2*a^2+3*b^2)*sin(d*x+c)/d+1/8*a^2*(
3*a^2+22*b^2)*cos(d*x+c)*sin(d*x+c)/d+5/6*a^3*b*cos(d*x+c)^2*sin(d*x+c)/d+1
/4*a^2*cos(d*x+c)^3*(a+b*sec(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3926, 4159, 4132, 2717, 4130, 8}

$$\frac{5a^3b\sin(c+dx)\cos^2(c+dx)}{6d}+\frac{4ab(2a^2+3b^2)\sin(c+dx)}{3d}+\frac{a^2(3a^2+22b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}+\frac{1}{8}x\left(3a^4+24a^2b^2+8b^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((3*a^4 + 24*a^2*b^2 + 8*b^4)*x)/8 + (4*a*b*(2*a^2 + 3*b^2)*Sin[c + d*x])/(
3*d) + (a^2*(3*a^2 + 22*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a^3*b*Co
s[c + d*x]^2*Sin[c + d*x])/(6*d) + (a^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3(c+dx\\ &= \frac{5a^3b\cos^2(c+dx)\sin(c+dx)}{6d}+\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\\ &= \frac{5a^3b\cos^2(c+dx)\sin(c+dx)}{6d}+\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\\ &= \frac{4ab(2a^2+3b^2)\sin(c+dx)}{3d}+\frac{a^2(3a^2+22b^2)\cos(c+dx)\sin(c+dx\\ &= \frac{1}{8}\left(3a^4+24a^2b^2+8b^4\right)x+\frac{4ab(2a^2+3b^2)\sin(c+dx)}{3d}+\frac{a^2(3a^2+\end{aligned}$$

Mathematica [A]
time = 0.24, size = 104, normalized size = 0.72

$$\frac{12(3a^4+24a^2b^2+8b^4)(c+dx)+96ab(3a^2+4b^2)\sin(c+dx)+24a^2(a^2+6b^2)\sin(2(c+dx))+32a^3b\sin(3(c+dx))+3a^4\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

[Out] (12*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(c + d*x) + 96*a*b*(3*a^2 + 4*b^2)*Sin[c + d*x] + 24*a^2*(a^2 + 6*b^2)*Sin[2*(c + d*x)] + 32*a^3*b*Sin[3*(c + d*x)] + 3*a^4*Sin[4*(c + d*x)])/(96*d)

Maple [A]
time = 0.12, size = 116, normalized size = 0.80

method	result
derivativedivides	$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4b\,a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6b^2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4b\,a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6b^2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{3a^4x}{8}+3a^2b^2x+x\,b^4+\frac{3a^3b\sin(dx+c)}{d}+\frac{4\sin(dx+c)b^3a}{d}+\frac{a^4\sin(4dx+4c)}{32d}+\frac{b\,a^3\sin(3dx+3c)}{3d}+\frac{a^4\sin(2c}{4c}$
norman	$\frac{(-\frac{3}{8}a^4-3b^2a^2-b^4)x+(-\frac{9}{8}a^4-9b^2a^2-3b^4)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{9}{8}a^4-9b^2a^2-3b^4)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{3}{8}a^4-3b^2a^2-}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*b*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+6*b^2*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*b^3*a*sin(d*x+c)+b^4*(d*x+c))

Maxima [A]
time = 0.27, size = 109, normalized size = 0.75

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4-128\,(\sin(dx+c)^3-3\,\sin(dx+c))a^3b+144\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2b^2+96\,(dx+c)b^4+384\,ab^3\sin(dx+c)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="maxima")

[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 128*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3*b + 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b^2 + 96*(d*x + c)*b^4 + 384*a*b^3*sin(d*x + c))/d

Fricas [A]
time = 3.29, size = 96, normalized size = 0.66

$$\frac{3\,(3\,a^4+24\,a^2b^2+8\,b^4)dx+\left(6\,a^4\cos(dx+c)^3+32\,a^3b\cos(dx+c)^2+64\,a^3b+96\,ab^3+9\,(a^4+8\,a^2b^2)\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="fricas")

```
[Out] 1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^3*b
*cos(d*x + c)^2 + 64*a^3*b + 96*a*b^3 + 9*(a^4 + 8*a^2*b^2)*cos(d*x + c))*s
in(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(135) = 270.
time = 0.47, size = 318, normalized size = 2.19

$$\frac{3\,(3\,a^4+24\,a^2b^2+8\,b^4)(dx+c)-\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-96\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-96\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-160\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-288\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-160\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-288\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(15*a^4*tan(1/2*d*x + 1/
2*c)^7 - 96*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^
7 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^4*tan(1/2*d*x + 1/2*c)^5 - 160*a^
3*b*tan(1/2*d*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*a*b^3*
tan(1/2*d*x + 1/2*c)^5 + 9*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*a^3*b*tan(1/2*d
*x + 1/2*c)^3 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*a*b^3*tan(1/2*d*x +
 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c) - 96*a^3*b*tan(1/2*d*x + 1/2*c) - 7
2*a^2*b^2*tan(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^4)/d
```

Mupad [**B**]
time = 0.88, size = 123, normalized size = 0.85

$$\frac{3\,a^4\,x}{8}+b^4\,x+3\,a^2\,b^2\,x+\frac{a^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^4\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{3\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{4\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,a^3\,b\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (3*a^4*x)/8 + b^4*x + 3*a^2*b^2*x + (a^4*sin(2*c + 2*d*x))/(4*d) + (a^4*sin
(4*c + 4*d*x))/(32*d) + (a^3*b*sin(3*c + 3*d*x))/(3*d) + (3*a^2*b^2*sin(2*c
 + 2*d*x))/(2*d) + (4*a*b^3*sin(c + d*x))/d + (3*a^3*b*sin(c + d*x))/d
```

3.484 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=173

$$\frac{1}{2}ab\left(3a^2+4b^2\right)x+\frac{\left(4a^4+29a^2b^2+5b^4\right)\sin(c+dx)}{5d}+\frac{ab\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a^3b\cos^3(c+dx)}{5d}$$

```
[Out] 1/2*a*b*(3*a^2+4*b^2)*x+1/5*(4*a^4+29*a^2*b^2+5*b^4)*sin(d*x+c)/d+1/2*a*b*(
3*a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/d+3/5*a^3*b*cos(d*x+c)^3*sin(d*x+c)/d+1/
5*a^2*cos(d*x+c)^4*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/15*a^2*(4*a^2+27*b^2)*
sin(d*x+c)^3/d
```

Rubi [A]
time = 0.25, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3926, 4159, 4132, 2715, 8, 4129, 3092}

$$\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{5d}-\frac{a^2\left(4a^2+27b^2\right)\sin^3(c+dx)}{15d}+\frac{ab\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}abx\left(3a^2+4b^2\right)+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}+\frac{\left(4a^4+29a^2b^2+5b^4\right)\sin(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*b*(3*a^2 + 4*b^2)*x)/2 + ((4*a^4 + 29*a^2*b^2 + 5*b^4)*Sin[c + d*x])/(5*
d) + (a*b*(3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^3*b*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d) + (a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*
Sin[c + d*x])/(5*d) - (a^2*(4*a^2 + 27*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
```

```
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4(c+dx\\
&= \frac{3a^3b\cos^3(c+dx)\sin(c+dx)}{5d}+\frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\\
&= \frac{3a^3b\cos^3(c+dx)\sin(c+dx)}{5d}+\frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\\
&= \frac{ab(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a^3b\cos^3(c+dx)\sin(c+\\
&= \frac{1}{2}ab(3a^2+4b^2)\,x+\frac{ab(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a^3b\,c\\
&= \frac{1}{2}ab(3a^2+4b^2)\,x+\frac{(4a^4+29a^2b^2+5b^4)\sin(c+dx)}{5d}+\frac{ab(3a^2+4
\end{aligned}
$$

Mathematica [A]

time = 0.52, size = 133, normalized size = 0.77

$$\frac{30(5a^4+36a^2b^2+8b^4)\sin(c+dx)+a(360a^2bc+480b^3c+360a^2bdx+480b^3dx+240b(a^2+b^2)\sin(2(c+dx))+5(5a^3+24ab^2)\sin(3(c+dx))+30a^2b\sin(4(c+dx))+3a^3\sin(5(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (30*(5*a^4 + 36*a^2*b^2 + 8*b^4)*Sin[c + d*x] + a*(360*a^2*b*c + 480*b^3*c
+ 360*a^2*b*d*x + 480*b^3*d*x + 240*b*(a^2 + b^2)*Sin[2*(c + d*x)] + 5*(5*a
^3 + 24*a*b^2)*Sin[3*(c + d*x)] + 30*a^2*b*Sin[4*(c + d*x)] + 3*a^3*Sin[5*(
c + d*x)]))/(240*d)
```

Maple [A]

time = 0.14, size = 138, normalized size = 0.80

method	result
derivativedivides	$\frac{\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4ba^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2b^2a^2\left(2+\cos^2(dx\right.}{d}$
default	$\frac{\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4ba^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2b^2a^2\left(2+\cos^2(dx\right.}{d}$
risch	$\frac{3a^3bx}{2}+2a\,b^3x+\frac{5a^4\sin(dx+c)}{8d}+\frac{9\sin(dx+c)b^2a^2}{2d}+\frac{\sin(dx+c)b^4}{d}+\frac{a^4\sin(5dx+5c)}{80d}+\frac{b\,a^3\sin(4dx+4c)}{8d}+$
norman	$\frac{\left(-\frac{3}{2}ba^3-2b^3a\right)x+\left(\frac{3}{2}ba^3+2b^3a\right)x\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-9ba^3-12b^3a\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-3ba^3-4b^3a\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*b*a^3*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*b^2*a^2*(2+cos(d*x+
c)^2)*sin(d*x+c)+4*b^3*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b^4*sin(
d*x+c))
```

Maxima [A]

time = 0.26, size = 133, normalized size = 0.77

$$\frac{8\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^4+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)a^3b-240\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2b^2+120\left(2dx+2c+\sin(2dx+2c)\right)ab^3+120b^4\sin(dx+c)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 + 15*
(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3*b - 240*(sin(d*
x + c)^3 - 3*sin(d*x + c))*a^2*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a
*b^3 + 120*b^4*sin(d*x + c))/d
```

Fricas [A]
time = 2.81, size = 121, normalized size = 0.70

$$\frac{15\,(3\,a^3b+4\,ab^3)dx+\left(6\,a^4\cos\left(dx+c\right)^4+30\,a^3b\cos\left(dx+c\right)^3+16\,a^4+120\,a^2b^2+30\,b^4+4\,(2\,a^4+15\,a^2b^2)\cos\left(dx+c\right)^2+15\,(3\,a^3b+4\,ab^3)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(3*a^3*b + 4*a*b^3)*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^3*b*cos(d*x
 + c)^3 + 16*a^4 + 120*a^2*b^2 + 30*b^4 + 4*(2*a^4 + 15*a^2*b^2)*cos(d*x +
c)^2 + 15*(3*a^3*b + 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(161) = 322.
time = 0.46, size = 425, normalized size = 2.46

$$\frac{15\,(3\,a^3b+4\,ab^3)(dx+c)+\frac{2\left(30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+30\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+40\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+480\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-120\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+120\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+116\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+600\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+180\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+480\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+75\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+60\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+30\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(15*(3*a^3*b + 4*a*b^3)*(d*x + c) + 2*(30*a^4*tan(1/2*d*x + 1/2*c)^9 -
 75*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*
a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 + 40*a^4*tan(1
/2*d*x + 1/2*c)^7 - 30*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 480*a^2*b^2*tan(1/2*d
*x + 1/2*c)^7 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 120*b^4*tan(1/2*d*x + 1/
2*c)^7 + 116*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x + 1/2*c)^
5 + 180*b^4*tan(1/2*d*x + 1/2*c)^5 + 40*a^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^3
*b*tan(1/2*d*x + 1/2*c)^3 + 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^3*
tan(1/2*d*x + 1/2*c)^3 + 120*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*tan(1/2*d*
x + 1/2*c) + 75*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*d*x + 1/2*
```

```
c) + 60*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 3.82, size = 330, normalized size = 1.91

$$\frac{(2a^4-5a^3b+12a^2b^2-4ab^3+2b^4)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{8a^4}{3}-2a^3b+32a^2b^2-8ab^3+8b^4\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{116a^4}{15}+40a^2b^2+12b^4\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{8a^4}{3}+2a^3b+32a^2b^2+8ab^3+8b^4\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2a^4+5a^3b+12a^2b^2+4ab^3+2b^4)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{a\,b\,\mathrm{atan}\left(\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3a^2+4b^2)}{3a^3b+4ab^3}\right)(3a^2+4b^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*((116*a^4)/15 + 12*b^4 + 40*a^2*b^2) + tan(c/2 + (d*x
)/2)^9*(2*a^4 - 5*a^3*b - 4*a*b^3 + 2*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2
)^3*(8*a*b^3 + 2*a^3*b + (8*a^4)/3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (d*x)/
2)^7*((8*a^4)/3 - 2*a^3*b - 8*a*b^3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (d*x)
/2)*(4*a*b^3 + 5*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)
/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (a*b*atan((a*b*tan(c/2 + (d*x)/2)*(
3*a^2 + 4*b^2))/(4*a*b^3 + 3*a^3*b))*(3*a^2 + 4*b^2))/d
```

3.485 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=213

$$\frac{1}{16}\left(5a^4+36a^2b^2+8b^4\right)x+\frac{4ab(4a^2+5b^2)\sin(c+dx)}{5d}+\frac{(5a^4+36a^2b^2+8b^4)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^2(5a$$

```
[Out] 1/16*(5*a^4+36*a^2*b^2+8*b^4)*x+4/5*a*b*(4*a^2+5*b^2)*sin(d*x+c)/d+1/16*(5*
a^4+36*a^2*b^2+8*b^4)*cos(d*x+c)*sin(d*x+c)/d+1/24*a^2*(5*a^2+32*b^2)*cos(d
*x+c)^3*sin(d*x+c)/d+7/15*a^3*b*cos(d*x+c)^4*sin(d*x+c)/d+1/6*a^2*cos(d*x+c
)^5*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-4/15*a*b*(4*a^2+5*b^2)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.27, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3926, 4159, 4132, 2713, 4130, 2715, 8}

$$\frac{7a^3b\sin(c+dx)\cos^4(c+dx)}{15d}-\frac{4ab(4a^2+5b^2)\sin^3(c+dx)}{15d}+\frac{4ab(4a^2+5b^2)\sin(c+dx)}{5d}+\frac{a^2(5a^2+32b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}+\frac{(5a^4+36a^2b^2+8b^4)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(5a^4+36a^2b^2+8b^4)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((5*a^4 + 36*a^2*b^2 + 8*b^4)*x)/16 + (4*a*b*(4*a^2 + 5*b^2)*Sin[c + d*x])/
(5*d) + ((5*a^4 + 36*a^2*b^2 + 8*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (
a^2*(5*a^2 + 32*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (7*a^3*b*Cos[c +
 d*x]^4*Sin[c + d*x])/(15*d) + (a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*S
in[c + d*x])/(6*d) - (4*a*b*(4*a^2 + 5*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{6d} + \frac{1}{6}\int \cos^5(c+dx$$
$$= \frac{7a^3b\cos^4(c+dx)\sin(c+dx)}{15d} + \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$
$$= \frac{7a^3b\cos^4(c+dx)\sin(c+dx)}{15d} + \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$
$$= \frac{a^2(5a^2+32b^2)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{7a^3b\cos^4(c+dx)\sin(c-}{15d}$$
$$= \frac{4ab(4a^2+5b^2)\sin(c+dx)}{5d} + \frac{(5a^4+36a^2b^2+8b^4)\cos(c+dx)\sin(}{16d}$$
$$= \frac{1}{16}\left(5a^4+36a^2b^2+8b^4\right)x + \frac{4ab(4a^2+5b^2)\sin(c+dx)}{5d} + \frac{(5a^4+3}{}$$

Mathematica [A]

time = 0.46, size = 156, normalized size = 0.73

$$\frac{60(5a^4+36a^2b^2+8b^4)(c+dx)+480ab(5a^2+6b^2)\sin(c+dx)+15(15a^4+96a^2b^2+16b^4)\sin(2(c+dx))+80ab(5a^2+4b^2)\sin(3(c+dx))+45a^2(a^2+4b^2)\sin(4(c+dx))+48a^3b\sin(5(c+dx))+5a^4\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (60*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(c + d*x) + 480*a*b*(5*a^2 + 6*b^2)*Sin[c
+ d*x] + 15*(15*a^4 + 96*a^2*b^2 + 16*b^4)*Sin[2*(c + d*x)] + 80*a*b*(5*a^2
 + 4*b^2)*Sin[3*(c + d*x)] + 45*a^2*(a^2 + 4*b^2)*Sin[4*(c + d*x)] + 48*a^3
*b*Sin[5*(c + d*x)] + 5*a^4*Sin[6*(c + d*x)])/(960*d)
```

Maple [A]

time = 0.10, size = 174, normalized size = 0.82

method	result
derivativedivides	$\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4b\,a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{}$
default	$\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4b\,a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{}$
risch	$\frac{5a^4x}{16}+\frac{9a^2b^2x}{4}+\frac{x\,b^4}{2}+\frac{5a^3b\sin(dx+c)}{2d}+\frac{3\sin(dx+c)b^3a}{d}+\frac{a^4\sin(6dx+6c)}{192d}+\frac{b\,a^3\sin(5dx+5c)}{20d}+\frac{3a^4\sin(}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*b*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*b
^2*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*b^3
*a*(2+cos(d*x+c)^2)*sin(d*x+c)+b^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A]
time = 0.26, size = 170, normalized size = 0.80

$$-\frac{5\,(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c))a^4-256\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))a^3b-180\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^2b^2+1280\,(\sin(dx+c)^3-3\,\sin(dx+c))ab^3-240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))b^4}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d
*x + c))*a^3*b - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*a^2*b^2 + 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^3 - 240*(2*d*x + 2*c
 + sin(2*d*x + 2*c))*b^4)/d
```

Fricas [A]
time = 2.57, size = 150, normalized size = 0.70

$$\frac{15\,(5\,a^4+36\,a^2b^2+8\,b^4)dx+\left(40\,a^4\cos(dx+c)^5+192\,a^3b\cos(dx+c)^4+512\,a^3b+640\,ab^3+10\,(5\,a^4+36\,a^2b^2)\cos(dx+c)^3+64\,(4\,a^3b+5\,ab^3)\cos(dx+c)^2+15\,(5\,a^4+36\,a^2b^2+8\,b^4)\cos(dx+c)\right)\sin(dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*d*x + (40*a^4*cos(d*x + c)^5 + 192*a
^3*b*cos(d*x + c)^4 + 512*a^3*b + 640*a*b^3 + 10*(5*a^4 + 36*a^2*b^2)*cos(d
*x + c)^3 + 64*(4*a^3*b + 5*a*b^3)*cos(d*x + c)^2 + 15*(5*a^4 + 36*a^2*b^2
+ 8*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(199) = 398.
time = 0.50, size = 550, normalized size = 2.58

$$\frac{15\,(5\,a^4+36\,a^2b^2+8\,b^4)(dx+c)-\frac{2\left(165\,a^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}-960\,a^3b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}+900\,a^2b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}-960\,ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}+120\,b^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}-25\,a^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9-2240\,a^3b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9+1260\,a^2b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9-3520\,ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9+360\,b^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9+450\,a^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-4992\,a^3b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7+360\,a^2b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-5760\,ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7+240\,b^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-450\,a^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-4992\,a^3b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-360\,a^2b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-5760\,ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-240\,b^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+25\,a^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-2240\,a^3b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-1260\,a^2b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-3520\,ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-360\,b^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-165\,a^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-960\,a^3b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-900\,a^2b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-960\,ab^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-120\,b^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)\right)}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(165*a^4*tan(1/2*d*x +
 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2*b^2*tan(1/2*d*x +
1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*b^4*tan(1/2*d*x + 1/2*c
)^11 - 25*a^4*tan(1/2*d*x + 1/2*c)^9 - 2240*a^3*b*tan(1/2*d*x + 1/2*c)^9 +
1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 3520*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 3
60*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*a^3*b
*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*a*b^3*t
an(1/2*d*x + 1/2*c)^7 + 240*b^4*tan(1/2*d*x + 1/2*c)^7 - 450*a^4*tan(1/2*d*
x + 1/2*c)^5 - 4992*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 360*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 - 5760*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 240*b^4*tan(1/2*d*x + 1/2*
c)^5 + 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*a^3*b*tan(1/2*d*x + 1/2*c)^3 -
1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 3
60*b^4*tan(1/2*d*x + 1/2*c)^3 - 165*a^4*tan(1/2*d*x + 1/2*c) - 960*a^3*b*ta
n(1/2*d*x + 1/2*c) - 900*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*a*b^3*tan(1/2*d
*x + 1/2*c) - 120*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)
/d
```

Mupad [B]
time = 1.09, size = 214, normalized size = 1.00

$$\frac{5\,a^4\,x}{16}+\frac{b^4\,x}{2}+\frac{9\,a^2\,b^2\,x}{4}+\frac{15\,a^4\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{3\,a^4\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}+\frac{a^4\,\sin\left(6\,c+6\,d\,x\right)}{192\,d}+\frac{b^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{5\,a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a^3\,b\,\sin\left(5\,c+5\,d\,x\right)}{20\,d}+\frac{3\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{3\,a^2\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}+\frac{3\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{5\,a^3\,b\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (5*a^4*x)/16 + (b^4*x)/2 + (9*a^2*b^2*x)/4 + (15*a^4*sin(2*c + 2*d*x))/(64*
d) + (3*a^4*sin(4*c + 4*d*x))/(64*d) + (a^4*sin(6*c + 6*d*x))/(192*d) + (b^
4*sin(2*c + 2*d*x))/(4*d) + (a*b^3*sin(3*c + 3*d*x))/(3*d) + (5*a^3*b*sin(3
*c + 3*d*x))/(12*d) + (a^3*b*sin(5*c + 5*d*x))/(20*d) + (3*a^2*b^2*sin(2*c
+ 2*d*x))/(2*d) + (3*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (3*a*b^3*sin(c + d*
x))/d + (5*a^3*b*sin(c + d*x))/(2*d)
```

3.486 $\int (a + b\sec(c + dx))^5\, dx$

Optimal. Leaf size=158

$$a^5x+\frac{b(40a^4+40a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab^2(53a^2+20b^2)\tan(c+dx)}{6d}+\frac{b^3(58a^2+9b^2)\sec(c+dx)}{24d}$$

```
[Out] a^5*x+1/8*b*(40*a^4+40*a^2*b^2+3*b^4)*arctanh(sin(d*x+c))/d+1/6*a*b^2*(53*a
^2+20*b^2)*tan(d*x+c)/d+1/24*b^3*(58*a^2+9*b^2)*sec(d*x+c)*tan(d*x+c)/d+11/
12*a*b^2*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/4*b^2*(a+b*sec(d*x+c))^3*tan(d*x
+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3867, 4141, 4133, 3855, 3852, 8}

$$a^5x+\frac{ab^2(53a^2+20b^2)\tan(c+dx)}{6d}+\frac{b^3(58a^2+9b^2)\tan(c+dx)\sec(c+dx)}{24d}+\frac{b(40a^4+40a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{11ab^2\tan(c+dx)(a+b\sec(c+dx))^2}{12d}+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^5,x]
```

```
[Out] a^5*x + (b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*
b^2*(53*a^2 + 20*b^2)*Tan[c + d*x])/(6*d) + (b^3*(58*a^2 + 9*b^2)*Sec[c + d
*x]*Tan[c + d*x])/(24*d) + (11*a*b^2*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(
12*d) + (b^2*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
```

```
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^5\,dx &= \frac{b^2(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int (a+b\sec(c+dx))^2\left(4a^3+3b(4a^2+b\right.\\ &= \frac{11ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{b^2(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\\ &= \frac{b^3(58a^2+9b^2)\sec(c+dx)\tan(c+dx)}{24d}+\frac{11ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{12d}\\ &= a^5x+\frac{b^3(58a^2+9b^2)\sec(c+dx)\tan(c+dx)}{24d}+\frac{11ab^2(a+b\sec(c+dx))^2\tan(c}{12d}\\ &= a^5x+\frac{b(40a^4+40a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^3(58a^2+9b^2)\sec(c+dx)}{24d}\\ &= a^5x+\frac{b(40a^4+40a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab^2(53a^2+20b^2)\tan(c+}{6d}\end{aligned}$$

Mathematica [A]
time = 0.62, size = 114, normalized size = 0.72

$$\frac{24a^5dx+3b(40a^4+40a^2b^2+3b^4)\tanh^{-1}(\sin(c+dx))+3b^2(40a(2a^2+b^2)+b(40a^2+3b^2)\sec(c+dx)+2b^3\sec^3(c+dx))\tan(c+dx)+40ab^4\tan^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^5,x]
```

```
[Out] (24*a^5*d*x + 3*b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]] + 3*b
^2*(40*a*(2*a^2 + b^2) + b*(40*a^2 + 3*b^2)*Sec[c + d*x] + 2*b^3*Sec[c + d*
x]^3)*Tan[c + d*x] + 40*a*b^4*Tan[c + d*x]^3)/(24*d)
```

Maple [A]
time = 0.10, size = 160, normalized size = 1.01

method	result
derivativedivides	$\frac{a^5(dx+c)+5b\,a^4\ln(\sec(dx+c)+\tan(dx+c))+10b^2a^3\tan(dx+c)+10b^3a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-}{d}$
default	$\frac{a^5(dx+c)+5b\,a^4\ln(\sec(dx+c)+\tan(dx+c))+10b^2a^3\tan(dx+c)+10b^3a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-}{d}$
norman	$a^5x+a^5x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^5x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a^5x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^5x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{5b^2\left(16a^3-8b\,a^2+8b^2a-b\right.}{4d}$
risch	$a^5x-\frac{ib^2\left(120b\,a^2e^{7i(dx+c)}+9b^3e^{7i(dx+c)}-240a^3e^{6i(dx+c)}+120a^2b\,e^{5i(dx+c)}+33b^3e^{5i(dx+c)}-720a^3e^{4i(dx+c)}-240a\,b^2\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5*(d*x+c)+5*b*a^4*ln(sec(d*x+c)+tan(d*x+c))+10*b^2*a^3*tan(d*x+c)+10
*b^3*a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-5*b^4*a*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^5*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))
*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 198, normalized size = 1.25

$$a^5x+\frac{5\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)ab^4}{3\,d}-\frac{b^5\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{16\,d}-\frac{5\,a^2b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{2\,d}+\frac{5\,a^4b\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{10\,a^3b^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] a^5*x + 5/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^4/d - 1/16*b^5*(2*(3*sin(
d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*lo
g(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))/d - 5/2*a^2*b^3*(2*sin(d*x +
 c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d
 + 5*a^4*b*log(sec(d*x + c) + tan(d*x + c))/d + 10*a^3*b^2*tan(d*x + c)/d
```

Fricas [A]
time = 2.18, size = 183, normalized size = 1.16

$$\frac{48\,a^5dx\cos\left(dx+c\right)^4+3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(40\,ab^4\cos\left(dx+c\right)+6\,b^5+80\left(3\,a^3b^2+ab^4\right)\cos\left(dx+c\right)^3+3\left(40\,a^2b^3+3\,b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a^5*d*x*cos(d*x + c)^4 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*x
 + c)^4*log(sin(d*x + c) + 1) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(40*a*b^4*cos(d*x + c) + 6*b^5 + 80*(3*a^3
*b^2 + a*b^4)*cos(d*x + c)^3 + 3*(40*a^2*b^3 + 3*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^5\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(148) = 296.
time = 0.47, size = 380, normalized size = 2.41

$$\frac{24\,(dx+c)a^5 + 3\,(40\,a^4b + 40\,a^2b^3 + 3\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(40\,a^4b + 40\,a^2b^3 + 3\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(240\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 120\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 120\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 720\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 120\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 200\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 9\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 720\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 120\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 200\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 9\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 240\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 120\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 120\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 15\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a^5 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(240*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*a^2*b^3*tan(1/
2*d*x + 1/2*c)^7 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^7 - 15*b^5*tan(1/2*d*x +
1/2*c)^7 - 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 120*a^2*b^3*tan(1/2*d*x + 1
/2*c)^5 - 200*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 9*b^5*tan(1/2*d*x + 1/2*c)^5 +
 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 +
200*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*b^5*tan(1/2*d*x + 1/2*c)^3 - 240*a^3*b
^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a*b^4*tan(
1/2*d*x + 1/2*c) - 15*b^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

Mupad [B]
time = 1.36, size = 274, normalized size = 1.73

$$\frac{2\,a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{3\,b^5\,\sin\left(c+d\,x\right)}{8\,d\cos\left(c+d\,x\right)^2} + \frac{b^5\,\sin\left(c+d\,x\right)}{4\,d\cos\left(c+d\,x\right)^4} + \frac{10\,a\,b^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)} + \frac{5\,a\,b^4\,\sin\left(c+d\,x\right)}{3\,d\cos\left(c+d\,x\right)^3} + \frac{10\,a^3\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)} + \frac{5\,a^2\,b^3\,\sin\left(c+d\,x\right)}{d\cos\left(c+d\,x\right)^2} - \frac{b^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}}{4\,d} - \frac{a^2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,10{}\mathrm{i}}{d} - \frac{a^4\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,10{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^5,x)
```

```
[Out] (2*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b^5*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/(4*d) + (3*b^5*sin(c + d*x))/(8*d*co
s(c + d*x)^2) + (b^5*sin(c + d*x))/(4*d*cos(c + d*x)^4) - (a^2*b^3*atan((si
n(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*10i)/d - (a^4*b*atan((sin(c/2 + (d
*x)/2)*1i)/cos(c/2 + (d*x)/2))*10i)/d + (10*a*b^4*sin(c + d*x))/(3*d*cos(c
+ d*x)) + (5*a*b^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (10*a^3*b^2*sin(c +
 d*x))/(d*cos(c + d*x)) + (5*a^2*b^3*sin(c + d*x))/(d*cos(c + d*x)^2)
```

3.487 $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=157

$$-\frac{a(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d}+\frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d}-\frac{a\sec(c+}{}$$

```
[Out] -1/2*a*(2*a^2+b^2)*arctanh(sin(d*x+c))/b^4/d+2*a^4*arctanh((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4/d/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(3*a^2+2*b^2)
*tan(d*x+c)/b^3/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2/d+1/3*sec(d*x+c)^2*tan(d*
x+c)/b/d
```

Rubi [A]

time = 0.33, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3936, 4177, 4167, 4083, 3855, 3916, 2738, 214}

$$\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\,\sqrt{a+b}}-\frac{a(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d}-\frac{a\tan(c+dx)\sec(c+dx)}{2b^2d}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/2*(a*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^4*d) + (2*a^4*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + (
(3*a^2 + 2*b^2)*Tan[c + d*x])/(3*b^3*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*
b^2*d) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec^2(c+dx)(2a+2b\sec(c+dx)-3a\sec^2(c+dx))}{a+b\sec(c+dx)}\,dx}{3b}$$

$$= -\frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec(c+dx)(-3a^2+ab\sec(c+}{a+b\sec}}{}$$

$$= \frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd}$$

$$= \frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd}$$

$$= -\frac{a(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan}{2b^2d}$$

$$= -\frac{a(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan}{2b^2d}$$

$$= -\frac{a(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d} + \frac{(3a^2+}{}$$

Mathematica [A]
time = 2.79, size = 258, normalized size = 1.64

$$\frac{-\frac{24a^4\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{1}{2}\sec^3(c+dx)\left(9a(2a^2+b^2)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+3a(2a^2+b^2)\cos(3(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+4b(3a^2+4b^2-3ab\cos(c+dx)+(3a^2+2b^2)\cos(2(c+dx)))\sin(c+dx)\right)}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-24*a^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (Sec[c + d*x]^3*(9*a*(2*a^2 + b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*a*(2*
a^2 + b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*b*(3*a^2 + 4*b^2 - 3*a*b*Cos[c +
 d*x] + (3*a^2 + 2*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/2)/(12*b^4*d)
```

Maple [A]
time = 0.26, size = 252, normalized size = 1.61

method	result

derivativedivides	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-a-b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2a^{2}+ba+2b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\left(2a^{2}+b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^{4}}+\frac{2a^{4}\operatorname{arctanh}\left(\frac{(a-b)}{\sqrt{(a+}}\right.}{b^{4}\sqrt{(a+b)}}}{d}$
default	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-a-b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2a^{2}+ba+2b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\left(2a^{2}+b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^{4}}+\frac{2a^{4}\operatorname{arctanh}\left(\frac{(a-b)}{\sqrt{(a+}}\right.}{b^{4}\sqrt{(a+b)}}}{d}$
risch	$\frac{i\left(3ba\,e^{5i(dx+c)}+6a^{2}e^{4i(dx+c)}+12a^{2}e^{2i(dx+c)}+12b^{2}e^{2i(dx+c)}-3ba\,e^{i(dx+c)}+6a^{2}+4b^{2}\right)}{3b^{3}d\left(e^{2i(dx+c)}+1\right)^{3}}-\frac{a^{3}\ln\left(e^{i(dx+c)}+i\right)}{b^{4}d}-\frac{a\ln(e^{i(}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a-b)/b^2/(tan(1/2*d*x+1/2*c)+1)^
2-1/2*(2*a^2+a*b+2*b^2)/b^3/(tan(1/2*d*x+1/2*c)+1)-1/2*a*(2*a^2+b^2)/b^4*ln
(tan(1/2*d*x+1/2*c)+1)+2*a^4/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/3/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/b^
2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*a^2+a*b+2*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)
+1/2*a*(2*a^2+b^2)/b^4*ln(tan(1/2*d*x+1/2*c)-1))
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

<u>Fricas</u> [A]
time = 3.41, size = 557, normalized size = 3.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(a^2 - b^2)*a^4*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
```

```
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(2*a
^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^5 - a^3
*b^2 - a*b^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*a^2*b^3 - 2*b^5
+ 2*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*
x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/12*(12*sqrt(-a^
2 + b^2)*a^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c)))*cos(d*x + c)^3 - 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log
(sin(d*x + c) + 1) + 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(-sin(d*
x + c) + 1) + 2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(d*x
+ c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d
*cos(d*x + c)^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(140) = 280.
time = 0.53, size = 286, normalized size = 1.82

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^4}{\sqrt{-a^2+b^2}\,b^4}-\frac{3(2a^3+ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}+\frac{3(2a^3+ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}-\frac{2\left(6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3b^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^4/(sqrt(-a^
2 + b^2)*b^4) - 3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 +
3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(6*a^2*tan(1/2
*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)
^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*t
an(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

Mupad [**B**]
time = 2.45, size = 1021, normalized size = 6.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))),x)

[Out] -((9*a^4*cos(c + d*x)*atanh((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*s
in(c/2 + (d*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 +
(d*x)/2) - 2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)
/2)*(a^2 - b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(
c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))/(b*c
os(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2) +
2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2) - 2
*a^3*b*(a^2 - b^2)))))/2 - (3*b^3*sin(c + d*x)*(a^2 - b^2)^(1/2))/2 - (b^3*
sin(3*c + 3*d*x)*(a^2 - b^2)^(1/2))/2 + (3*a^4*cos(3*c + 3*d*x)*atanh((8*a^
6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d*x)/2) + b^6*sin(c/2 +
(d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) - 2*a*b^5*sin(c/2 + (d*x
)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 5*a^2*b^4*sin(c
/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^
4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/
2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2) + 2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a
^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2) - 2*a^3*b*(a^2 - b^2)))))/2 + (3*a^3
*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*(a^2 - b^2)^
(1/2))/2 - (3*a^2*b*sin(c + d*x)*(a^2 - b^2)^(1/2))/4 + (9*a^3*cos(c + d*x)
*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 - b^2)^(1/2))/2 + (3*a*b
^2*sin(2*c + 2*d*x)*(a^2 - b^2)^(1/2))/4 - (3*a^2*b*sin(3*c + 3*d*x)*(a^2 -
b^2)^(1/2))/4 + (9*a*b^2*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*(a^2 - b^2)^(1/2))/4 + (3*a*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2))*cos(3*c + 3*d*x)*(a^2 - b^2)^(1/2))/4)/(3*b^4*d*((3*cos(c + d*x)
)/4 + cos(3*c + 3*d*x)/4)*(a^2 - b^2)^(1/2))

3.488 $\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec(c+dx)\tan(c+dx)}{2bd}$$

```
[Out] 1/2*(2*a^2+b^2)*arctanh(sin(d*x+c))/b^3/d-2*a^3*arctanh((a-b)^(1/2)*tan(1/2
*d*x+1/2*c)/(a+b)^(1/2))/b^3/d/(a-b)^(1/2)/(a+b)^(1/2)-a*tan(d*x+c)/b^2/d+1
/2*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3936, 4167, 4083, 3855, 3916, 2738, 214}

$$-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*a^3*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*Tan
[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)(a+b\sec(c+dx)-2a\sec^2(c+dx))}{a+b\sec(c+dx)}\,dx}{2b}$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)(ab+(2a^2+b^2)\sec(c+dx))}{a+b\sec(c+dx)}\,dx}{2b^2}$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{a^3\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b^3} + \frac{(2a^2+b^2)\int \mathrm{s}}{2}$$

$$= \frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{a^3}{}$$

$$= \frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{(2a}{}$$

$$= \frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d} - \frac{a\tan(c+}{b^2d}$$

Mathematica [A]
time = 1.19, size = 238, normalized size = 2.00

$$\frac{\frac{8a^3\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{b^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - 4ab\tan(c+dx)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((8*a^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^
2] - 4*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*b^2*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] + 4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ 2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2/(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^2 - b^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a*b*T
an[c + d*x])/(4*b^3*d)
```

Maple [A]
time = 0.21, size = 192, normalized size = 1.61

method	result
derivativedivides	$\frac{-\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{-2a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{\left(2a^2+b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^3} - \frac{2a^3\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}} + \frac{}{2b}}{d}$

default	$\frac{-\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-2a-b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(2a^{2}+b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^{3}}-\frac{2a^{3}\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{3}\sqrt{(a+b)(a-b)}}+\frac{}{2b(\mathrm{t}}}{d}$
risch	$-\frac{i\left(b\,\mathrm{e}^{3i(dx+c)}+2a\,\mathrm{e}^{2i(dx+c)}-b\,\mathrm{e}^{i(dx+c)}+2a\right)}{d\,b^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)a^{2}}{b^{3}d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2bd}+\frac{a^{3}\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^{2}+ib^{2}+b}{a\sqrt{a^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/b/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)+1
)+1/2*(2*a^2+b^2)/b^3*ln(tan(1/2*d*x+1/2*c)+1)-2*a^3/b^3/((a+b)*(a-b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+1/2/b/(tan(1/2*d*x+
1/2*c)-1)^2-1/2*(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)-1)+1/2/b^3*(-2*a^2-b^2)*ln
(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(106) = 212$.
time = 3.13, size = 485, normalized size = 4.08

$$\left[\frac{2\sqrt{a^{2}-b^{2}}a^{3}\cos(dx+c)^{2}\log\left(\frac{2ab\cos(dx+c)-(a^{2}-2b^{2})\cos(dx+c)^{2}-2\sqrt{a^{2}-b^{2}}(b\cos(dx+c)+a)\sin(dx+c)+2a^{2}-b^{2}}{a^{2}\cos(dx+c)^{2}+2ab\cos(dx+c)+b^{2}}\right)+(2a^{4}-a^{2}b^{2}-b^{4})\cos(dx+c)^{2}\log(\sin(dx+c)+1)-(2a^{4}-a^{2}b^{2}-b^{4})\cos(dx+c)^{2}\log(-\sin(dx+c)+1)+2(a^{2}b^{2}-b^{4}-2(a^{3}b-ab^{3})\cos(dx+c))\sin(dx+c)}{4(a^{2}b^{3}-b^{5})d\cos(dx+c)^{2}},-\frac{4\sqrt{-a^{2}+b^{2}}a^{3}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}(b\cos(dx+c)+a)}{(a^{2}-b^{2})\sin(dx+c)}\right)\cos(dx+c)^{2}-(2a^{4}-a^{2}b^{2}-b^{4})\cos(dx+c)^{2}\log(\sin(dx+c)+1)+(2a^{4}-a^{2}b^{2}-b^{4})\cos(dx+c)^{2}\log(-\sin(dx+c)+1)-2(a^{2}b^{2}-b^{4}-2(a^{3}b-ab^{3})\cos(dx+c))\sin(dx+c)}{4(a^{2}b^{3}-b^{5})d\cos(dx+c)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a^2 - b^2)*a^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) - (a^2 -
 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c
) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*a^4
- a^2*b^2 - b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^4 - a^2*b^2 -
b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^2*b^2 - b^4 - 2*(a^3*b -
a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2), -1/4
```

```
*(4*sqrt(-a^2 + b^2)*a^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (2*a^4 - a^2*b^2 - b^4)*cos(d*x +
c)^2*log(sin(d*x + c) + 1) + (2*a^4 - a^2*b^2 - b^4)*cos(d*x + c)^2*log(-si
n(d*x + c) + 1) - 2*(a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*
x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.53, size = 211, normalized size = 1.77

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3}{\sqrt{-a^2+b^2}\,b^3}-\frac{(2a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3}+\frac{(2a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^3}-\frac{2\left(2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3/(sqrt(-a^
2 + b^2)*b^3) - (2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + (2*a
^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a*tan(1/2*d*x + 1/2
*c)^3 + b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

Mupad [B]
time = 1.65, size = 1002, normalized size = 8.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

```
[Out] sin(c + d*x)/(2*b*d*(cos(2*c + 2*d*x)/2 + 1/2)) + atanh(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2))/(2*b*d*(cos(2*c + 2*d*x)/2 + 1/2)) + (a^2*atanh(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d*(cos(2*c + 2*d*x)/2 + 1/2)) + (atan
h(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(2*b*d*(cos(2*c
+ 2*d*x)/2 + 1/2)) - (a*sin(2*c + 2*d*x))/(2*b^2*d*(cos(2*c + 2*d*x)/2 + 1/
```

2)) - (a^3*atan(((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) - 2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))*1i)/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2) + 2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2) - 2*a^3*b*(a^2 - b^2))))*1i)/(b^3*d*(a^2 - b^2)^(1/2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(b^3*d*(cos(2*c + 2*d*x)/2 + 1/2)) - (a^3*cos(2*c + 2*d*x)*atan(((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) - 2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))*1i)/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2) + 2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2) - 2*a^3*b*(a^2 - b^2))))*1i)/(b^3*d*(a^2 - b^2)^(1/2)*(cos(2*c + 2*d*x)/2 + 1/2))

3.489 $\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d}+\frac{\tan(c+dx)}{bd}$$

```
[Out] -a*arctanh(sin(d*x+c))/b^2/d+2*a^2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)+tan(d*x+c)/b/d
```

Rubi [A]
time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3875, 3874, 3855, 3916, 2738, 214}

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) + (2*a^2*ArcTanh[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + Tan[c + d*x]/(b*d
)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\tan(c+dx)}{bd}-\frac{a\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx}{b}\\
&= \frac{\tan(c+dx)}{bd}-\frac{a\int \sec(c+dx)\,dx}{b^2}+\frac{a^2\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b^2}\\
&= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\tan(c+dx)}{bd}+\frac{a^2\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^3}\\
&= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\tan(c+dx)}{bd}+\frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(\right.\right.}{b^3d}\\
&= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d}+\frac{\tan(c+dx)}{bd}
\end{aligned}$$

Mathematica [A]
time = 0.41, size = 115, normalized size = 1.35

$$\frac{-\frac{2a^2\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-2*a^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b
^2] + a*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + b*Tan[c + d*x])/(b^2*d)
```

Maple [A]
time = 0.14, size = 123, normalized size = 1.45

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{b^2}}{d}$
default	$\frac{-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{b^2}}{d}$
risch	$\frac{2i}{db\left(e^{2i(dx+c)}+1\right)}+\frac{a^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,b^2}-\frac{a^2\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,b^2}+\frac{a\,l}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b/(tan(1/2*d*x+1/2*c)+1)-a/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2*a^2/b^2/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/
b/(tan(1/2*d*x+1/2*c)-1)+a/b^2*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(76) = 152$.
time = 3.77, size = 392, normalized size = 4.61

$$\left[\frac{\sqrt{a^2-b^2}\,a^2\cos\left(dx+c\right)\log\left(\frac{2\,ab\cos\left(dx+c\right)-\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2+2\,\sqrt{a^2-b^2}\left(b\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)+2\,a^2-b^2}{a^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+b^2}\right)-\left(a^3-ab^2\right)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^3-ab^2\right)\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a^2b-b^3\right)\sin\left(dx+c\right)}{2\left(a^2b^2-b^4\right)d\cos\left(dx+c\right)},\ \frac{2\,\sqrt{-a^2+b^2}\,a^2\arctan\left(-\frac{\sqrt{-a^2+b^2}\left(b\cos\left(dx+c\right)+a\right)}{\left(a^2-b^2\right)\sin\left(dx+c\right)}\right)\cos\left(dx+c\right)-\left(a^3-ab^2\right)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^3-ab^2\right)\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a^2b-b^3\right)\sin\left(dx+c\right)}{2\left(a^2b^2-b^4\right)d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*a^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b
^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^3 - a*b^
2)*cos(d*x + c)*log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(-sin
(d*x + c) + 1) + 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x +
 c)), 1/2*(2*sqrt(-a^2 + b^2)*a^2*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (a^3 - a*b^2)*cos(d*x + c)*
log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(-sin(d*x + c) + 1) +
 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 152, normalized size = 1.79

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^2}{\sqrt{-a^2+b^2}\,b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2}+\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^2/(sqrt(-a^2 + b
^2)*b^2) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 + a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*
b))/d
```

Mupad [B]
time = 1.21, size = 119, normalized size = 1.40

$$\frac{\tan(c+d\,x)}{b\,d}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d}-\frac{a^2\,\mathrm{atan}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}\right)\,2\mathrm{i}}{b^2\,d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))),x)
```

```
[Out] tan(c + d*x)/(b*d) - (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^
2*d) - (a^2*atan((a*sin(c/2 + (d*x)/2)*1i - b*sin(c/2 + (d*x)/2)*1i)/(cos(c
/2 + (d*x)/2)*(a^2 - b^2)^(1/2)))*2i)/(b^2*d*(a^2 - b^2)^(1/2))
```

3.490 $\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}$$

```
[Out] arctanh(sin(d*x+c))/b/d-2*a*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1
/2))/b/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3874, 3855, 3916, 2738, 214}

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b*d) - (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
```

```
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\int \sec(c+dx)\,dx}{b} - \frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{a\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^2}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{(2a)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 102, normalized size = 1.50

$$\frac{\frac{2a\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
 - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]])/(b*d)
```

Maple [A]
time = 0.13, size = 83, normalized size = 1.22

method	result

derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
risch	$\frac{a\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,db}-\frac{a\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,db}+\frac{\ln(e^{i(dx+c)}+i)}{bd}-\frac{\ln(e^{i(dx}}{bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b*ln(tan(1/2*d*x+1/2*c)+1)-2*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/b*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 3.16, size = 290, normalized size = 4.26

$$\left[\frac{\sqrt{a^2-b^2}\,a\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+(a^2-b^2)\log(\sin(dx+c)+1)-(a^2-b^2)\log(-\sin(dx+c)+1)}{2\,(a^2b-b^3)d},\ -\frac{2\,\sqrt{-a^2+b^2}\,a\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^2-b^2)\log(\sin(dx+c)+1)+(a^2-b^2)\log(-\sin(dx+c)+1)}{2\,(a^2b-b^3)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*a*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^2 - b^2)*log(sin(d*x +
c) + 1) - (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d), -1/2*(2*sq
rt(-a^2 + b^2)*a*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)
```

```
*sin(d*x + c))) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log(-sin(
d*x + c) + 1))/((a^2*b - b^3)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(59) = 118.
time = 0.48, size = 120, normalized size = 1.76

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{\sqrt{-a^2+b^2}\,b}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/(sqrt(-a^2 + b^
2)*b) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b)/d
```

Mupad [B]
time = 1.06, size = 186, normalized size = 2.74

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{2\,a\,\mathrm{atanh}\left(\frac{2\,a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)-2\,a^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)+2\,a^3\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,\left(a\,b-b^2\right)}\right)}{b\,d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))),x)
```

```
[Out] (2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*a*atanh((2*a^2*
sin(c/2 + (d*x)/2)*(a^2 - b^2) - 2*a^4*sin(c/2 + (d*x)/2) + b^2*sin(c/2 + (
d*x)/2)*(a^2 - b^2) + 2*a^3*b*sin(c/2 + (d*x)/2) - 2*a*b*sin(c/2 + (d*x)/2)
*(a^2 - b^2))/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(a*b - b^2))))/(b*d*(
a^2 - b^2)^(1/2))
```

3.491 $\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,d}$$

```
[Out] 2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3916, 2738, 214}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
 + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
[Out] (-2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*
d)
```

Maple [A]
time = 0.07, size = 44, normalized size = 0.90

method	result	size
derivativedivides	$\frac{2\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
default	$\frac{2\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.81, size = 185, normalized size = 3.78

$$\left[\frac{\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,\sqrt{a^2-b^2}\,d}, \frac{\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)}{(a^2-b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 -
b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + b^2))/(sqrt(a^2 - b^2)*d), sqrt(-a^2 + b^2)*arctan(-s
qrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))/((a^2 - b^
2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 77, normalized size = 1.57

$$-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*d)
```

Mupad [B]
time = 0.89, size = 40, normalized size = 0.82

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))),x)
```

```
[Out] (2*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d*(a + b)^(1/2
)*(a - b)^(1/2))
```

3.492 $\int \frac{1}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d}$$

```
[Out] x/a-2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/d/(a-b)^(1/2)
/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3868, 2738, 214}

$$\frac{x}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-1),x]
```

```
[Out] x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a -
 b]*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \frac{x}{a} - \frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a}$$

$$= \frac{x}{a} - \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

$$= \frac{x}{a} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 1.02

$$\frac{\frac{c}{d}+x+\frac{2b\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-1),x]
```

```
[Out] (c/d + x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt
[a^2 - b^2]*d))/a
```

Maple [A]
time = 0.09, size = 65, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	65
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	65
risch	$\frac{x}{a}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,da}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,da}$	152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(2/a*arctan(tan(1/2*d*x+1/2*c))-2*b/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.71, size = 230, normalized size = 3.90

$$\left[\frac{2\,(a^2-b^2)dx+\sqrt{a^2-b^2}\,b\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,(a^3-ab^2)d},\ \frac{(a^2-b^2)dx-\sqrt{-a^2+b^2}\,b\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)}{(a^3-ab^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/((a^3 -
 a*b^2)*d), ((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*
(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/((a^3 - a*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sec{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(50) = 100.
time = 0.48, size = 218, normalized size = 3.69

$$\frac{\frac{\left(\sqrt{-a^2+b^2}\,(a-2\,b)|-a+b|-\sqrt{-a^2+b^2}\,|a||-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b+\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)}{(a^2-2\,ab+b^2)a^2+(a^2b-2\,ab^2+b^3)|a|}+\frac{\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b-\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)(a-2\,b+|a|)}{a^2-b|a|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((sqrt(-a^2 + b^2)*(a - 2*b)*abs(-a + b) - sqrt(-a^2 + b^2)*abs(a)*abs(-a +
 b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-
(b + sqrt((a + b)*(a - b) + b^2))/(a - b))))/((a^2 - 2*a*b + b^2)*a^2 + (a^
2*b - 2*a*b^2 + b^3)*abs(a)) + (pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(t
an(1/2*d*x + 1/2*c)/sqrt(-(b - sqrt((a + b)*(a - b) + b^2))/(a - b))))*(a -
 2*b + abs(a))/(a^2 - b*abs(a)))/d
```

Mupad [B]
time = 1.09, size = 186, normalized size = 3.15

$$\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)+2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)-2\,a\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,\left(a\,b-a^2\right)}\right)}{a\,d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x)),x)
```

```
[Out] (2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*b*atanh((2*b^4*s
in(c/2 + (d*x)/2) + a^2*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 2*b^2*sin(c/2 + (d
*x)/2)*(a^2 - b^2) - 2*a*b^3*sin(c/2 + (d*x)/2) - 2*a*b*sin(c/2 + (d*x)/2)*
(a^2 - b^2))/(a*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(a*b - a^2))))/(a*d*(a
^2 - b^2)^(1/2))
```

3.493 $\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{bx}{a^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}\,d}+\frac{\sin(c+dx)}{ad}$$

```
[Out] -b*x/a^2+sin(d*x+c)/a/d+2*b^2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^
(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3938, 12, 3868, 2738, 214}

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*x)/a^2) + (2*b^2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + Sin[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
```

```
] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\sin(c+dx)}{ad}-\frac{\int \frac{b}{a+b\sec(c+dx)}\,dx}{a}\\ &= \frac{\sin(c+dx)}{ad}-\frac{b\int \frac{1}{a+b\sec(c+dx)}\,dx}{a}\\ &= -\frac{bx}{a^2}+\frac{\sin(c+dx)}{ad}+\frac{b\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2}\\ &= -\frac{bx}{a^2}+\frac{\sin(c+dx)}{ad}+\frac{(2b)\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}\\ &= -\frac{bx}{a^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}\,d}+\frac{\sin(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 72, normalized size = 0.95

$$\frac{-b(c+dx)-\frac{2b^2\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a\sin(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
[Out] (-(b*(c + d*x)) - (2*b^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + a*Sin[c + d*x])/(a^2*d)
```

Maple [A]
time = 0.11, size = 97, normalized size = 1.28

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^2}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^2}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}}}{d}$
risch	$-\frac{bx}{a^2}-\frac{i e^{i(dx+c)}}{2ad}+\frac{i e^{-i(dx+c)}}{2ad}+\frac{b^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^2}-\frac{b^2\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-}}{a\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^2*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+b*arctan(tan(1/
2*d*x+1/2*c)))+2*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.07, size = 277, normalized size = 3.64

$$\left[\frac{\sqrt{a^2-b^2}\,b^2\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-2\,(a^2b-b^3)dx+2\,(a^3-ab^2)\sin(dx+c)}{2\,(a^4-a^2b^2)d},\ \frac{\sqrt{-a^2+b^2}\,b^2\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^2b-b^3)dx+(a^3-ab^2)\sin(dx+c)}{(a^4-a^2b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*b^2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
 c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
```

```
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^2*b - b^3)*d*x + 2*
(a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d), (sqrt(-a^2 + b^2)*b^2*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^
2*b - b^3)*d*x + (a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(c + dx \right)}}{a + b \sec{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x)
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A]

time = 0.48, size = 126, normalized size = 1.66

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{\sqrt{-a^2+b^2}\,a^2}-\frac{(dx+c)b}{a^2}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^2/(sqrt(-a^2 + b
^2)*a^2) - (d*x + c)*b/a^2 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^
2 + 1)*a))/d
```

Mupad [B]

time = 1.31, size = 395, normalized size = 5.20

$$\frac{a^3\sin(c+dx)}{d\,(a^4-a^2b^2)}+\frac{2b^3\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d\,(a^4-a^2b^2)}-\frac{ab^2\sin(c+dx)}{d\,(a^4-a^2b^2)}-\frac{2a^2b\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d\,(a^4-a^2b^2)}+\frac{b^2\operatorname{atan}\left(\frac{-a^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,1i+b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-b^2)^{3/2}\,2i+b^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,2i-a^2b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,3i+a^3b^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,1i+a^4b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,1i}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^6-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^2+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^4}\right)\sqrt{a^2-b^2}\,2i}{d\,(a^4-a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x)),x)
[Out] (a^3*sin(c + d*x))/(d*(a^4 - a^2*b^2)) + (2*b^3*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) - (a*b^2*sin(c + d*x))/(d*(a^4 - a^2*
b^2)) + (b^2*atan((b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin(c/
2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2
)*2i - a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(c/2 +
(d*x)/2)*(a^2 - b^2)^(1/2)*1i + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*
1i)/(a^6*cos(c/2 + (d*x)/2) + a^2*b^4*cos(c/2 + (d*x)/2) - 2*a^4*b^2*cos(c/
2 + (d*x)/2)))*(a^2 - b^2)^(1/2)*2i)/(d*(a^4 - a^2*b^2)) - (2*a^2*b*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2))
```

3.494 $\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{(a^2+2b^2)\,x}{2a^3}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}\,d}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

```
[Out] 1/2*(a^2+2*b^2)*x/a^3-b*sin(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)/a/d-2*b^
3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(a+
b)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3938, 4189, 4004, 3916, 2738, 214}

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\sin(c+dx)}{a^2d}+\frac{x(a^2+2b^2)}{2a^3}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - (b*Sin[c + d*x])/(a^2*d) + (
Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\cos(c+dx)\sin(c+dx)}{2ad}+\frac{\int \frac{\cos(c+dx)\left(-2b+a\sec(c+dx)+b\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2a}\\
&= -\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\int \frac{-a^2-2b^2-ab\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{2a^2}\\
&= \frac{(a^2+2b^2)\,x}{2a^3}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{b^3\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^3}\\
&= \frac{(a^2+2b^2)\,x}{2a^3}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{b^2\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^3}\\
&= \frac{(a^2+2b^2)\,x}{2a^3}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{(2b^2)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+(1-\cdots}\right.}{\cdots}\\
&= \frac{(a^2+2b^2)\,x}{2a^3}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}\,d}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\,\mathrm{s}\cdots}{2a\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 97, normalized size = 0.88

$$\frac{2(a^2+2b^2)(c+dx)+\frac{8b^3\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-4ab\sin(c+dx)+a^2\sin(2(c+dx))}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 + 2*b^2)*(c + d*x) + (8*b^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] - 4*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)])
/(4*a^3*d)
```

Maple [A]
time = 0.13, size = 138, normalized size = 1.25

method	result
derivativedivides	$\frac{\frac{2\left(\frac{\left(-\frac{1}{2}a^2-ba\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2-ba\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2+2b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3}-\frac{2b^3\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b}}\right)}{a^3\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{\frac{2\left(\frac{\left(-\frac{1}{2}a^2-ba\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2-ba\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2+2b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3}-\frac{2b^3\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b}}\right)}{a^3\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{x}{2a}+\frac{x\,b^2}{a^3}+\frac{ib\,\mathrm{e}^{i(dx+c)}}{2a^2d}-\frac{ib\,\mathrm{e}^{-i(dx+c)}}{2a^2d}+\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^3}-\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2}{a\sqrt{}}\right)}{\sqrt{a^2-b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^3*(((-1/2*a^2-b*a)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2-b*a)*tan(1/2*d*x+
1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+2*b^2)*arctan(tan(1/2*d*x+1/2*c
)))-2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.88, size = 334, normalized size = 3.04

$$\left[\frac{\sqrt{a^2-b^2}\,b^3\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+(a^4+a^2b^2-2\,b^4)dx-(2\,a^3b-2\,ab^3-(a^4-a^2b^2)\cos(dx+c))\sin(dx+c)}{2\,(a^5-a^3b^2)d},-\frac{2\,\sqrt{-a^2+b^2}\,b^3\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^4+a^2b^2-2\,b^4)dx+(2\,a^3b-2\,ab^3-(a^4-a^2b^2)\cos(dx+c))\sin(dx+c)}{2\,(a^5-a^3b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
 c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^4 + a^2*b^2 - 2*b^4)*
d*x - (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^
5 - a^3*b^2)*d), -1/2*(2*sqrt(-a^2 + b^2)*b^3*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4 + a^2*b^2 - 2*b^4)*d*x
+ (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 -
a^3*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 178, normalized size = 1.62

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^3}{\sqrt{-a^2+b^2}\,a^3}-\frac{(a^2+2\,b^2)(dx+c)}{a^3}+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^3/(sqrt(-a^
```

```
2 + b^2)*a^3) - (a^2 + 2*b^2)*(d*x + c)/a^3 + 2*(a*tan(1/2*d*x + 1/2*c)^3 +
 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

Mupad [B]
time = 1.81, size = 592, normalized size = 5.38

$$\frac{a\left(\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)+\frac{\sin(2c+2dx)}{4}\right)}{d\,(a^2-b^2)}-\frac{b\,\sin(c+dx)}{d\,(a^2-b^2)}+\frac{b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-\frac{b^2\,\sin(2c+2dx)}{4}}{a\,d\,(a^2-b^2)}+\frac{b^3\,\sin(c+dx)}{a^2\,d\,(a^2-b^2)}-\frac{2\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^3\,d\,(a^2-b^2)}-\frac{b^3\,\mathrm{atan}\left(\frac{\left(8\,b^7\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-b^2)^{3/2}-a^9\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}+8\,b^9\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}-8\,a^2\,b^7\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}-3\,a^4\,b^5\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}+3\,a^5\,b^4\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}+2\,a^6\,b^3\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}-2\,a^7\,b^2\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}+a^8\,b\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(a\,b^2-a^3)\,(4\,b^5\,(a^2-b^2)+2\,a\,b^6-a^7+4\,b^7-2\,a^2\,b^5+a^3\,b^4-2\,a^4\,b^3-2\,a^5\,b^2+2\,a^2\,b^3\,(a^2-b^2)+2\,a\,b^4\,(a^2-b^2))}\right)2\mathrm{i}}{a^3\,d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (a*(atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + sin(2*c + 2*d*x)/4))/(d*(
a^2 - b^2)) - (b*sin(c + d*x))/(d*(a^2 - b^2)) + (b^2*atan(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)) - (b^2*sin(2*c + 2*d*x))/4)/(a*d*(a^2 - b^2)) - (b^3
*atan(((8*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - a^9*sin(c/2 + (d*x)/2)
*(a^2 - b^2)^(1/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 8*a^2*b^7
*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 -
 b^2)^(1/2) + 3*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2*a^6*b^3*si
n(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 2*a^7*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^
2)^(1/2) + a^8*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))*1i)/(cos(c/2 + (d*x)
/2)*(a*b^2 - a^3)*(4*b^5*(a^2 - b^2) + 2*a*b^6 - a^7 + 4*b^7 - 2*a^2*b^5 +
a^3*b^4 - 2*a^4*b^3 - 2*a^5*b^2 + 2*a^2*b^3*(a^2 - b^2) + 2*a*b^4*(a^2 - b^
2))))*2i)/(a^3*d*(a^2 - b^2)^(1/2)) + (b^3*sin(c + d*x))/(a^2*d*(a^2 - b^2)
) - (2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d*(a^2 - b^2))
```

3.495 $\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{b(a^2+2b^2)\,x}{2a^4}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}\,d}+\frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d}-\frac{b\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{\text{co}}{}$$

```
[Out] -1/2*b*(a^2+2*b^2)*x/a^4+1/3*(2*a^2+3*b^2)*sin(d*x+c)/a^3/d-1/2*b*cos(d*x+c
)*sin(d*x+c)/a^2/d+1/3*cos(d*x+c)^2*sin(d*x+c)/a/d+2*b^4*arctanh((a-b)^(1/2
)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3938, 4189, 4004, 3916, 2738, 214}

$$\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{bx(a^2+2b^2)}{2a^4}+\frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d}+\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/2*(b*(a^2 + 2*b^2)*x)/a^4 + (2*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + ((2*a^2 + 3*b^2)*Sin[c +
d*x])/(3*a^3*d) - (b*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (Cos[c + d*x]^2
*Sin[c + d*x])/(3*a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\cos^2(c+dx)\sin(c+dx)}{3ad} + \frac{\int \frac{\cos^2(c+dx)(-3b+2a\sec(c+dx)+2b\sec^2(c+dx))}{a+b\sec(c+dx)}\,dx}{3a}$$

$$= -\frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{\int \frac{\cos(c+dx)(-2(2a^2+3b^2)-ab}{a+b\sec}}{6}$$

$$= \frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3ad} +$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+}{}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+}{}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+}{}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d} -$$

Mathematica [A]

time = 0.35, size = 122, normalized size = 0.82

$$\frac{-6b(a^2+2b^2)(c+dx) - \frac{24b^4\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 3a(3a^2+4b^2)\sin(c+dx) - 3a^2b\sin(2(c+dx)) + a^3\sin(3(c+dx))}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
[Out] (-6*b*(a^2 + 2*b^2)*(c + d*x) - (24*b^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sin[c + d*x] - 3*a
^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*a^4*d)
```

Maple [A]

time = 0.16, size = 179, normalized size = 1.21

method	result
derivativedivides	$-\frac{\frac{2\left(\frac{\left(-a^3-\frac{1}{2}b\,a^2-b^2a\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-2b^2a\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^3-b^2a+\frac{1}{2}b\,a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{b\left(a^2+2b^2\right)\arctan(\text{t}}{2}\right.}{a^4}}{d}$

default	$-\frac{\frac{2\left(\frac{\left(-a^{3}-\frac{1}{2} b\, a^{2}-b^{2} a\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3} a^{3}-2b^{2} a\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^{3}-b^{2} a+\frac{1}{2} b\, a^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{b\left(a^{2}+2b^{2}\right)\arctan\left(\tan\right.}{2}\right.}{a^{4}}}{d}$
risch	$-\frac{bx}{2a^{2}}-\frac{b^{3}x}{a^{4}}-\frac{3i\mathrm{e}^{i(dx+c)}}{8ad}-\frac{i\mathrm{e}^{i(dx+c)}b^{2}}{2a^{3}d}+\frac{3i\mathrm{e}^{-i(dx+c)}}{8ad}+\frac{i\mathrm{e}^{-i(dx+c)}b^{2}}{2a^{3}d}+\frac{b^{4}\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^{2}-ib^{2}+b\sqrt{a^{2}-\right.}{a\sqrt{a^{2}-b^{2}}}}{\sqrt{a^{2}-b^{2}}\, d\, a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^4*(((-a^3-1/2*b*a^2-b^2*a)*tan(1/2*d*x+1/2*c)^5+(-2/3*a^3-2*b^2*a
)*tan(1/2*d*x+1/2*c)^3+(-a^3-b^2*a+1/2*b*a^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/
2*d*x+1/2*c)^2)^3+1/2*b*(a^2+2*b^2)*arctan(tan(1/2*d*x+1/2*c)))+2*b^4/a^4/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.98, size = 401, normalized size = 2.71

$$\left[\frac{3\sqrt{a^{2}-b^{2}}\, b^{4}\log\left(\frac{2\, ab\cos(dx+c)-(a^{2}-2b^{2})\cos(dx+c)^{2}+2\sqrt{a^{2}-b^{2}}(b\cos(dx+c)+a)\sin(dx+c)+2a^{2}-b^{2}}{a^{2}\cos(dx+c)^{2}+2ab\cos(dx+c)+b^{2}}\right)-3(a^{4}b+a^{2}b^{3}-2b^{5})dx+\left(4a^{5}+2a^{3}b^{2}-6ab^{4}+2(a^{5}-a^{3}b^{2})\cos(dx+c)^{2}-3(a^{4}b-a^{2}b^{3})\cos(dx+c)\right)\sin(dx+c)}{6(a^{6}-a^{4}b^{2})d}, \frac{6\sqrt{-a^{2}+b^{2}}\, b^{4}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}(b\cos(dx+c)+a)}{(a^{2}-b^{2})\sin(dx+c)}\right)-3(a^{4}b+a^{2}b^{3}-2b^{5})dx+\left(4a^{5}+2a^{3}b^{2}-6ab^{4}+2(a^{5}-a^{3}b^{2})\cos(dx+c)^{2}-3(a^{4}b-a^{2}b^{3})\cos(dx+c)\right)\sin(dx+c)}{6(a^{6}-a^{4}b^{2})d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 - b^2)*b^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(a^4*b + a^2*b^3 - 2
*b^5)*d*x + (4*a^5 + 2*a^3*b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x + c)^2
 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d), 1/6
*(6*sqrt(-a^2 + b^2)*b^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c))) - 3*(a^4*b + a^2*b^3 - 2*b^5)*d*x + (4*a^5 + 2*a^3*
```

```
b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b^3)*cos(
d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 249, normalized size = 1.68

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^4}{\sqrt{-a^2+b^2}\,a^4}-\frac{3\left(a^2b+2b^3\right)(dx+c)}{a^4}+\frac{2\left(6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+4a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^4/(sqrt(-a^
2 + b^2)*a^4) - 3*(a^2*b + 2*b^3)*(d*x + c)/a^4 + 2*(6*a^2*tan(1/2*d*x + 1/
2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*a^
2*tan(1/2*d*x + 1/2*c)^3 + 12*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*
x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

Mupad [B]
time = 2.06, size = 654, normalized size = 4.42

$$\frac{\frac{b^2\sin(c+dx)}{4}-\frac{b^2\sin(3c+3dx)}{12}}{a\,d\,(a^2-b^2)}-\frac{b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)+\frac{b\sin(2c+2dx)}{4}}{d\,(a^2-b^2)}+\frac{a\left(\frac{3\sin(c+dx)}{4}+\frac{\sin(3c+3dx)}{12}\right)}{d\,(a^2-b^2)}-\frac{b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-\frac{b^3\sin(2c+2dx)}{4}}{a^2\,d\,(a^2-b^2)}-\frac{b^4\sin(c+dx)}{a^3\,d\,(a^2-b^2)}+\frac{2b^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^4\,d\,(a^2-b^2)}+\frac{b^4\,\mathrm{atan}\left(\frac{\left(8b^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-b^2)^{3/2}-a^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}+8b^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}-8a^2b^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}-3a^4b^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}+3a^5b^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}+2a^6b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}-2a^7b^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}+a^8b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\right)1i}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(ab^2-a^3)\left(4b^5(a^2-b^2)+2ab^6-a^7+4b^7-2a^2b^5+a^3b^4-2a^4b^3-2a^5b^2+2a^2b^3(a^2-b^2)+2ab^4(a^2-b^2)\right)}\right)2i}{a^4\,d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] ((b^2*sin(c + d*x))/4 - (b^2*sin(3*c + 3*d*x))/12)/(a*d*(a^2 - b^2)) - (b*a
tan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (b*sin(2*c + 2*d*x))/4)/(d*(a^
2 - b^2)) + (a*((3*sin(c + d*x))/4 + sin(3*c + 3*d*x)/12))/(d*(a^2 - b^2))
- (b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (b^3*sin(2*c + 2*d*x))
/4)/(a^2*d*(a^2 - b^2)) + (b^4*atan(((8*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^
(3/2) - a^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*b^9*sin(c/2 + (d*x)/2)
*(a^2 - b^2)^(1/2) - 8*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^4
*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 3*a^5*b^4*sin(c/2 + (d*x)/2)*(a
```

```
^2 - b^2)^(1/2) + 2*a^6*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 2*a^7*b^
2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^8*b*sin(c/2 + (d*x)/2)*(a^2 - b^
2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(4*b^5*(a^2 - b^2) + 2*a*b^
6 - a^7 + 4*b^7 - 2*a^2*b^5 + a^3*b^4 - 2*a^4*b^3 - 2*a^5*b^2 + 2*a^2*b^3*(
a^2 - b^2) + 2*a*b^4*(a^2 - b^2))))*2i)/(a^4*d*(a^2 - b^2)^(1/2)) - (b^4*si
n(c + d*x))/(a^3*d*(a^2 - b^2)) + (2*b^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/(a^4*d*(a^2 - b^2))
```

3.496 $\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=193

$$\frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\,\sqrt{a+b}\,d}-\frac{b(2a^2+3b^2)\sin(c+dx)}{3a^4d}+\frac{(3a^2+4b^2)\cos(c+d\ldots}{8a^3d}$$

```
[Out] 1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5-1/3*b*(2*a^2+3*b^2)*sin(d*x+c)/a^4/d+1/8*
(3*a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/d-1/3*b*cos(d*x+c)^2*sin(d*x+c)/a^2
/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a/d-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/a^5/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3938, 4189, 4004, 3916, 2738, 214}

$$-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\sin(c+dx)\cos^2(c+dx)}{3a^2d}-\frac{b(2a^2+3b^2)\sin(c+dx)}{3a^4d}+\frac{(3a^2+4b^2)\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{x(3a^4+4a^2b^2+8b^4)}{8a^5}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) - (b*(2*a^2 + 3*
b^2)*Sin[c + d*x])/(3*a^4*d) + ((3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/
(8*a^3*d) - (b*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (Cos[c + d*x]^3*Sin
[c + d*x])/(4*a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int \frac{\cos^3(c+dx)(-4b+3a\sec(c+dx)+3b\sec^2(c+dx))}{a+b\sec(c+dx)}\,dx}{4a}$$

$$= -\frac{b\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \frac{\cos^2(c+dx)(-3(3a^2+4b^2)-}{a+b\,\text{se}}}{1}$$

$$= \frac{(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{\cos^3(c+dx)\,\text{s}}{4ad}$$

$$= -\frac{b(2a^2+3b^2)\sin(c+dx)}{3a^4d} + \frac{(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b\cos^2(c+dx)}{3a^2}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{b(2a^2+3b^2)\sin(c+dx)}{3a^4d} + \frac{(3a^2+4b^2)\cos(c+dx)\sin(c}{8a^3d}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{b(2a^2+3b^2)\sin(c+dx)}{3a^4d} + \frac{(3a^2+4b^2)\cos(c+dx)\sin(c}{8a^3d}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{b(2a^2+3b^2)\sin(c+dx)}{3a^4d} + \frac{(3a^2+4b^2)\cos(c+dx)\sin(c}{8a^3d}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\,\sqrt{a+b}\,d} - \frac{b(2a^2+3b^2)\sin(c+}{3a^4d}$$

Mathematica [A]
time = 0.61, size = 153, normalized size = 0.79

$$\frac{12(3a^4+4a^2b^2+8b^4)(c+dx) + \frac{192b^5\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 24ab(3a^2+4b^2)\sin(c+dx) + 24a^2(a^2+b^2)\sin(2(c+dx)) - 8a^3b\sin(3(c+dx)) + 3a^4\sin(4(c+dx))}{96a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(c + d*x) + (192*b^5*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*Si
n[c + d*x] + 24*a^2*(a^2 + b^2)*Sin[2*(c + d*x)] - 8*a^3*b*Sin[3*(c + d*x)]
 + 3*a^4*Sin[4*(c + d*x)])/(96*a^5*d)
```

Maple [A]
time = 0.16, size = 256, normalized size = 1.33

method	result

derivativedivides	$\frac{\frac{2\left(\left(-\frac{5}{8}a^4-b\,a^3-\frac{1}{2}b^2a^2-b^3a\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3}{8}a^4-\frac{5}{3}b\,a^3-3b^3a-\frac{1}{2}b^2a^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{3}{8}a^4+\frac{1}{2}b^2a^2-\frac{5}{3}b\,a^3-3b^3a\right)\left(\tan^3 \right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{a^5}$
default	$\frac{\frac{2\left(\left(-\frac{5}{8}a^4-b\,a^3-\frac{1}{2}b^2a^2-b^3a\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3}{8}a^4-\frac{5}{3}b\,a^3-3b^3a-\frac{1}{2}b^2a^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{3}{8}a^4+\frac{1}{2}b^2a^2-\frac{5}{3}b\,a^3-3b^3a\right)\left(\tan^3 \right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{a^5}$
risch	$\frac{3x}{8a}+\frac{x\,b^2}{2a^3}+\frac{x\,b^4}{a^5}+\frac{3ib\,\mathrm{e}^{i(dx+c)}}{8a^2d}+\frac{ib^3\mathrm{e}^{i(dx+c)}}{2a^4d}-\frac{3ib\,\mathrm{e}^{-i(dx+c)}}{8a^2d}-\frac{ib^3\mathrm{e}^{-i(dx+c)}}{2a^4d}+\frac{b^5\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{\ }}{a\sqrt{a^2}}\right.}{\sqrt{a^2-b^2}\,d\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^5*(((-5/8*a^4-b*a^3-1/2*b^2*a^2-b^3*a)*tan(1/2*d*x+1/2*c)^7+(3/8*a
^4-5/3*b*a^3-3*b^3*a-1/2*b^2*a^2)*tan(1/2*d*x+1/2*c)^5+(-3/8*a^4+1/2*b^2*a^
2-5/3*b*a^3-3*b^3*a)*tan(1/2*d*x+1/2*c)^3+(5/8*a^4+1/2*b^2*a^2-b*a^3-b^3*a)
*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+1/8*(3*a^4+4*a^2*b^2+8*b^4)
*arctan(tan(1/2*d*x+1/2*c)))-2*b^5/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.58, size = 482, normalized size = 2.50

$$\left[\frac{12\sqrt{a^2-b^2}\,b^5\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+3\,(3\,a^6+a^4b^2+4\,a^2b^4-8\,b^6)dx-\left(16\,a^5b+8\,a^3b^3-24\,ab^5-6\,(a^6-a^4b^2)\cos(dx+c)^3+8\,(a^5b-a^3b^3)\cos(dx+c)^2-3\,(3\,a^6+a^4b^2-4\,a^2b^4)\cos(dx+c)\right)\sin(dx+c)}{24\,(a^7-a^5b^2)d},\ -\frac{24\sqrt{-a^2+b^2}\,b^5\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-3\,(3\,a^6+a^4b^2+4\,a^2b^4-8\,b^6)dx+\left(16\,a^5b+8\,a^3b^3-24\,ab^5-6\,(a^6-a^4b^2)\cos(dx+c)^3+8\,(a^5b-a^3b^3)\cos(dx+c)^2-3\,(3\,a^6+a^4b^2-4\,a^2b^4)\cos(dx+c)\right)\sin(dx+c)}{24\,(a^7-a^5b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(12*sqrt(a^2 - b^2)*b^5*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d
*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b
```

```
^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*(3*a^6 + a^4*b^2 +
 4*a^2*b^4 - 8*b^6)*d*x - (16*a^5*b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a^4*b
^2)*cos(d*x + c)^3 + 8*(a^5*b - a^3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a^4*b^
2 - 4*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d), -1/24*(24*s
qrt(-a^2 + b^2)*b^5*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) - 3*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*d*x + (16*a^5*
b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a^4*b^2)*cos(d*x + c)^3 + 8*(a^5*b - a^
3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a^4*b^2 - 4*a^2*b^4)*cos(d*x + c))*sin(d
*x + c))/((a^7 - a^5*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(174) = 348.
time = 0.49, size = 393, normalized size = 2.04

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^5}{\sqrt{-a^2+b^2}\,a^5}-\frac{3\left(3a^4+4a^2b^2+8b^4\right)(dx+c)}{a^5}+\frac{2\left(15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-9a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+40a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+72b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+9a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+40a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+72b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4a^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^5/(sqrt(-
a^2 + b^2)*a^5) - 3*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 + 2*(15*a^3*t
an(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*
d*x + 1/2*c)^7 + 24*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^3*tan(1/2*d*x + 1/2*c)
^5 + 40*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72
*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b*tan(1
/2*d*x + 1/2*c)^3 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*b^3*tan(1/2*d*x +
1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) - 12
*a*b^2*tan(1/2*d*x + 1/2*c) + 24*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
1/2*c)^2 + 1)^4*a^4))/d
```

Mupad [B]
time = 3.42, size = 2678, normalized size = 13.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(a + b/cos(c + d*x)),x)

[Out] (b^5*atan(((b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) + (b^5*((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 - (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2) + (b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) - (b^5*((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 + (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2))/((96*a*b^13 - 64*b^14 - 96*a^2*b^12 + 104*a^3*b^11 - 104*a^4*b^10 + 88*a^5*b^9 - 48*a^6*b^8 + 33*a^7*b^7 - 18*a^8*b^6 + 9*a^9*b^5)/a^12 - (b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) + (b^5*((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 - (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2))/(a^7 - a^5*b^2)))/(a^7 - a^5*b^2) + (b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) - (b^5*((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 + (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2))/(a^7 - a^5*b^2)))/(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2)*2i)/(d*(a^7 - a^5*b^2)) - (atan((((((((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 - (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*8i + a^2*b^2*4i)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(16*a^13))*(a^4*3i + b^4*8i + a^2*b^2*4i))/(8*a^5) + (tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8))*(a^4*3i + b^4*8i + a^2*b^2*4i)*1i)/(8*a^5) - (((((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*8i + a^2*b^2*4i)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(16*a^13))*(a^4*3i + b^4*8i + a^2*b^2*4i))/(8*a^5) - (tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8))*(a^4*3i + b^4*8i + a^2*b^2*4i)*1i)/(8*a^5

))/((((((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*
a^13*b^3 + 4*a^14*b^2)/a^12 - (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*8i + a^2*b^
2*4i)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(16*a^13))*(a^4*3i + b^4*
8i + a^2*b^2*4i))/(8*a^5) + (tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9
*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 -
216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8))*(a^4*3i + b^
4*8i + a^2*b^2*4i))/(8*a^5) - (96*a*b^13 - 64*b^14 - 96*a^2*b^12 + 104*a^3*
b^11 - 104*a^4*b^10 + 88*a^5*b^9 - 48*a^6*b^8 + 33*a^7*b^7 - 18*a^8*b^6 + 9
*a^9*b^5)/a^12 + (((((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*
a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^
4*8i + a^2*b^2*4i)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(16*a^13))*(
a^4*3i + b^4*8i + a^2*b^2*4i))/(8*a^5) - (tan(c/2 + (d*x)/2)*(256*a*b^10 -
27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 2
56*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8))
*(a^4*3i + b^4*8i + a^2*b^2*4i))/(8*a^5)))*(a^4*3i + b^4*8i + a^2*b^2*4i)*1
i)/(4*a^5*d) - ((tan(c/2 + (d*x)/2)^7*(4*a*b^2 + 8*a^2*b + 5*a^3 + 8*b^3))/
(4*a^4) - (tan(c/2 + (d*x)/2)*(4*a*b^2 - 8*a^2*b + 5*a^3 - 8*b^3))/(4*a^4)
+ (tan(c/2 + (d*x)/2)^3*(40*a^2*b - 12*a*b^2 + 9*a^3 + 72*b^3))/(12*a^4) +
(tan(c/2 + (d*x)/2)^5*(12*a*b^2 + 40*a^2*b - 9*a^3 + 72*b^3))/(12*a^4))/(d*
(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 +
tan(c/2 + (d*x)/2)^8 + 1))

3.497 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=222

$$\frac{(6a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a^3(3a^2-4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d}-\frac{a(3a^2-2b^2)\tan(c+dx)}{b^3\left(a^2-b^2\right)d}+\frac{(}{}$$

```
[Out] 1/2*(6*a^2+b^2)*arctanh(sin(d*x+c))/b^4/d-2*a^3*(3*a^2-4*b^2)*arctanh((a-b)
^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^4/(a+b)^(3/2)/d-a*(3*a
^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)/d+1/2*(3*a^2-b^2)*sec(d*x+c)*tan(d*x+c)/
b^2/(a^2-b^2)/d-a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]

time = 0.43, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3930, 4177, 4167, 4083, 3855, 3916, 2738, 214}

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{(3a^2-b^2)\tan(c+dx)\sec(c+dx)}{2b^2d\left(a^2-b^2\right)}+\frac{(6a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{a(3a^2-2b^2)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}-\frac{2a^3(3a^2-4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a^3*(3*a^2 - 4*b^2)*Ar
cTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a +
b)^(3/2)*d) - (a*(3*a^2 - 2*b^2)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) + ((3*a^
2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*
x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{\sec^2(c+dx)\left(2a^2-ab\sec(c+dx)-(3a^2-b^2)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,d}{b\,(a^2-b^2)}$$

$$= \frac{(3a^2-b^2)\sec(c+dx)\tan(c+dx)}{2b^2\,(a^2-b^2)\,d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \sec(c+d}{}$$

$$= -\frac{a(3a^2-2b^2)\tan(c+dx)}{b^3\,(a^2-b^2)\,d} + \frac{(3a^2-b^2)\sec(c+dx)\tan(c+dx)}{2b^2\,(a^2-b^2)\,d} - \frac{a^2\sec^2(c+}{b\,(a^2-b^2)\,d}$$

$$= -\frac{a(3a^2-2b^2)\tan(c+dx)}{b^3\,(a^2-b^2)\,d} + \frac{(3a^2-b^2)\sec(c+dx)\tan(c+dx)}{2b^2\,(a^2-b^2)\,d} - \frac{a^2\sec^2(c+}{b\,(a^2-b^2)\,d}$$

$$= \frac{(6a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a(3a^2-2b^2)\tan(c+dx)}{b^3\,(a^2-b^2)\,d} + \frac{(3a^2-b^2)\sec(c+}{2b^2\,(a^2}$$

$$= \frac{(6a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a(3a^2-2b^2)\tan(c+dx)}{b^3\,(a^2-b^2)\,d} + \frac{(3a^2-b^2)\sec(c+}{2b^2\,(a^2}$$

$$= \frac{(6a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{2a^3(3a^2-4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d}$$

Mathematica [A]
time = 6.14, size = 357, normalized size = 1.61

$$\frac{2a^3(-3a^2+4b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\,(-a^2+b^2)\,d} + \frac{(-6a^2-b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2b^4d} + \frac{(6a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2b^4d} + \frac{1}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{2a\sin\left(\frac{1}{2}(c+dx)\right)}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{2a\sin\left(\frac{1}{2}(c+dx)\right)}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{a^4\sin(c+dx)}{b^3(-a+b)(a+b)d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^3*(-3*a^2 + 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(b^4*Sqrt[a^2 - b^2]*(-a^2 + b^2)*d) + ((-6*a^2 - b^2)*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/(2*b^4*d) + ((6*a^2 + b^2)*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]])/(2*b^4*d) + 1/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^2) - (2*a*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])) - 1/(4*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (2*a*Sin[(c + d
*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (a^4*Sin[c + d*x])/
(b^3*(-a + b)*(a + b)*d*(b + a*Cos[c + d*x]))
```

Maple [A]
time = 0.37, size = 277, normalized size = 1.25

method	result

derivativedivides	$\frac{-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(6a^2+b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^4}+\frac{2a^3\left(\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}\right.}{}}{}$
default	$\frac{-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(6a^2+b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^4}+\frac{2a^3\left(\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}\right.}{}}{}$
risch	$-\frac{i\left(-3a^3b\,e^{5i(dx+c)}+a\,b^3e^{5i(dx+c)}-6a^4e^{4i(dx+c)}+2a^2b^2e^{4i(dx+c)}+2b^4e^{4i(dx+c)}-12b\,a^3e^{3i(dx+c)}+8a\,b^3e^{3i(dx+c)}-12\right.}{d\,b^3\left(e^{2i(dx+c)}+1\right)^2\left(-a^2+b^2\right)\left(a\,e^{2i(dx+c)}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/b^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-4*a-b)/b^3/(tan(1/2*d*x+1/2*c)
+1)+1/2*(6*a^2+b^2)/b^4*ln(tan(1/2*d*x+1/2*c)+1)+2/b^4*a^3*(b*a/(a^2-b^2)*t
an(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(3*a^
2-4*b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/(
(a+b)*(a-b))^(1/2)))+1/2/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-4*a-b)/b^3/(tan
(1/2*d*x+1/2*c)-1)+1/2/b^4*(-6*a^2-b^2)*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(209) = 418$.
time = 5.97, size = 909, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((3*a^6 - 4*a^4*b^2)*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*x
 + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(
a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + ((6*a^7 - 11*a^5*b^2 + 4*
a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*
cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 +
a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*cos(d*x +
c)^2)*log(-sin(d*x + c) + 1) + 2*(a^4*b^3 - 2*a^2*b^5 + b^7 - 2*(3*a^6*b -
5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos
(d*x + c))*sin(d*x + c))/((a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 +
(a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2), -1/4*(4*((3*a^6 - 4*a^4*b^2)
*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (
(6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4
*b^3 + 4*a^2*b^5 + b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*a^7 - 1
1*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a
^2*b^5 + b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^4*b^3 - 2*a^2*b
^5 + b^7 - 2*(3*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 - 3*(a^5*b^2
- 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^4 - 2*a^3*b^6 + a*
b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.52, size = 299, normalized size = 1.35

$$\frac{\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^2b^3-b^5)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{4\,(3\,a^5-4\,a^3b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{(6\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{(6\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{2\left(4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^4*tan(1/2*d*x + 1/2*c)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 4*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/2*
```

```
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2))
+ (6*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - (6*a^2 + b^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(
1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

Mupad [B]
time = 8.01, size = 2500, normalized size = 11.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^2),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^5*(3*a*b^3 - 3*a^3*b + 6*a^4 + b^4 - 5*a^2*b^2))/((a
*b^3 - b^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(b^4 - 6*a^4 + 3*a^2*b^2))/(
b*(a*b^2 - b^3)*(a + b)) + (tan(c/2 + (d*x)/2)*(3*a^3*b - 3*a*b^3 + 6*a^4 +
 b^4 - 5*a^2*b^2))/(b^3*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^2*
(3*a + b) - tan(c/2 + (d*x)/2)^6*(a - b) + tan(c/2 + (d*x)/2)^4*(3*a - b)))
 - (atan((((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b
^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4
+ 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - ((6*a^2 +
 b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 +
6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (4*tan(c/2
+ (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10
 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))))/(2*b^4
))*1i)/(2*b^4) + ((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b
- 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a
^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + ((
6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*
b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (4*t
an(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a
^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))))
/(2*b^4))*1i)/(2*b^4))/((16*(108*a^11 - 54*a^10*b + 4*a^3*b^8 - 4*a^4*b^7 +
 41*a^5*b^6 - 9*a^6*b^5 + 63*a^7*b^4 + 81*a^8*b^3 - 216*a^9*b^2))/(a*b^11 +
 b^12 - a^2*b^10 - a^3*b^9) - ((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^1
0 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a
^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 -
a^3*b^6) - ((6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b
^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^
3*b^9) - (4*tan(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^
3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7
- a^3*b^6))))/(2*b^4)))/(2*b^4) + ((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72
*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 -
```

26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + ((6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (4*tan(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))))/(2*b^4)))/(2*b^4)))*(6*a^2 + b^2)*1i)/(b^4*d) - (a^3*atan(((a^3*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a^3*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (8*a^3*tan(c/2 + (d*x)/2)*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^3*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a^3*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^3*tan(c/2 + (d*x)/2)*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))/((16*(108*a^11 - 54*a^10*b + 4*a^3*b^8 - 4*a^4*b^7 + 41*a^5*b^6 - 9*a^6*b^5 + 63*a^7*b^4 + 81*a^8*b^3 - 216*a^9*b^2))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (a^3*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a^3*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (8*a^3*tan(c/2 + (d*x)/2)*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8...

3.498 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$-\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d}+\frac{2a^2(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d}+\frac{(2a^2-b^2)\tan(c+dx)}{b^2\,(a^2-b^2)\,d}-\frac{a^2\sec}{b\,(a^2-}$$

```
[Out] -2*a*arctanh(sin(d*x+c))/b^3/d+2*a^2*(2*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+(2*a^2-b^2)*tan(d
*x+c)/b^2/(a^2-b^2)/d-a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+
c))
```

Rubi [A]

time = 0.24, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3930, 4167, 4083, 3855, 3916, 2738, 214}

$$\frac{(2a^2-b^2)\tan(c+dx)}{b^2d\,(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\,(a^2-b^2)\,(a+b\sec(c+dx))}+\frac{2a^2(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*a^2*(2*a^2 - 3*b^2)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d)
 + ((2*a^2 - b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]*Tan
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)(a^2-ab\sec(c+dx)-(2a^2-b^2)\sec^2(c+dx))}{a+b\sec(c+dx)} a}{b\,(a^2-b^2)}$$

$$= \frac{(2a^2-b^2)\tan(c+dx)}{b^2\,(a^2-b^2)\,d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)(a^2b+2a(a^2}{a+b\sec(c}}{b^2\,(a^2-}$$

$$= \frac{(2a^2-b^2)\tan(c+dx)}{b^2\,(a^2-b^2)\,d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{(2a)\int\sec(c+dx)}{b^3}$$

$$= -\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{(2a^2-b^2)\tan(c+dx)}{b^2\,(a^2-b^2)\,d} - \frac{a^2\sec(c+dx)\tan(c+}{b\,(a^2-b^2)\,d(a+b\sec(c}$$

$$= -\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{(2a^2-b^2)\tan(c+dx)}{b^2\,(a^2-b^2)\,d} - \frac{a^2\sec(c+dx)\tan(c+}{b\,(a^2-b^2)\,d(a+b\sec(c}$$

$$= -\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{2a^2(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{(}{}$$

Mathematica [A]

time = 1.12, size = 162, normalized size = 0.99

$$\frac{-\frac{2a^2(2a^2-3b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + 2a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{a^3b\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))} + b\tan(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*a^2*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(3/2) + 2*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*a*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*Sin[c + d*x])/((a - b)*(a
 + b)*(b + a*Cos[c + d*x])) + b*Tan[c + d*x])/(b^3*d)
```

Maple [A]

time = 0.28, size = 209, normalized size = 1.27

method	result
derivativedivides	$\frac{-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} - \frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3} - \frac{2a^2\left(\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)} - \frac{(2a^2-3b^2)\arctan}{(a+b)(a-b)}\right.}{b^3}}{d}$

default	$\frac{-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}-\frac{2a^2\left(\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{(2a^2-3b^2)\operatorname{arctanh}\left(\cdots\right.}{(a+b)(a-b)\sqrt{\cdots}}\right.}{b^3}\cdots}{d}$
risch	$\frac{2i\left(-b\,a^2\mathrm{e}^{3i(dx+c)}-2a^3\mathrm{e}^{2i(dx+c)}+a\,b^2\mathrm{e}^{2i(dx+c)}-3a^2b\,\mathrm{e}^{i(dx+c)}+2b^3\mathrm{e}^{i(dx+c)}-2a^3+b^2a\right)}{(-a^2+b^2)d\,b^2\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{2a^4\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\cdots}{a\sqrt{a\cdots}}\right.}{\sqrt{a^2-b^2}\,(a+b)(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2/(tan(1/2*d*x+1/2*c)+1)-2*a/b^3*ln(tan(1/2*d*x+1/2*c)+1)-2/b^3*a
^2*(b*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+
1/2*c)^2-a-b)-(2*a^2-3*b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2*a/b^3*ln(tan(1/2*d*x+1/2*c)-1)-1/
b^2/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(155) = 310$.
time = 3.98, size = 760, normalized size = 4.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x +
 c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*((a^6 - 2*a^4*b^2 + a^2*b^4
```

```
)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x +
c) + 1) + 2*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^
3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b^2 - 2*a^2*b^4 +
b^6 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 -
 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*
x + c)), (((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*b^3)*cos(d
*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a
^2 - b^2)*sin(d*x + c))) - ((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a
^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((a^6 - 2*a
^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c)
)*log(-sin(d*x + c) + 1) + (a^4*b^2 - 2*a^2*b^4 + b^6 + (2*a^5*b - 3*a^3*b^
3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos
(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(155) = 310.
time = 0.49, size = 331, normalized size = 2.02

$$\frac{2\left(\frac{(2a^4-3a^2b^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{-a^2+b^2}}-\frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)(a^2b^2-b^4)}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3}+\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((2*a^4 - 3*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*
b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x +
 1/2*c)^3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2
*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^
4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2*b^2
 - b^4)) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^3)/d
```

Mupad [**B**]
time = 6.81, size = 3159, normalized size = 19.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - 2*a^3 - b^3))/(b^2*(a + b)*(a - b
)) - (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - 2*a^3 + b^3))/(b^2*(a + b)*(a -
 b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) - 2*a*tan(c/2 + (d*x)/2)^2))
 + (a*atan(((a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3
*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b
^4) - (2*a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 -
2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (64*a*tan(c/2 + (d*x)/2)*(2
*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3
*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4))))/b^3)*2i)/b^3 + (a*((32*tan(c/2 + (d*x
)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16
*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (2*a*((32*(2*a*b^11 - 3*a^2*
b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7
 - a^3*b^6) + (64*a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 +
 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)
)))/b^3)*2i)/b^3)/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4 + 6*a^5*b^3 - 20*a^6*b
^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (2*a*((32*tan(c/2 + (d*x)/2)*(8*a^
8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))
/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (2*a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a
^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6
) - (64*a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8
 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4))))/b^3))/
b^3 + (2*a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5
 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)
+ (2*a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^
6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (64*a*tan(c/2 + (d*x)/2)*(2*a*b
^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*
b^6 + b^7 - a^2*b^5 - a^3*b^4))))/b^3))/b^3))*4i)/(b^3*d) + (a^2*atan(((a^2
*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b
^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a^2*(2*
a^2 - 3*b^2)*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7
 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a^2*tan(c/2 + (d*x)/2
)*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^
3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b
^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2))/
(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)
^3)^(1/2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a^2*((32*tan(c/2 +
 (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3
 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a^2*(2*a^2 - 3*b^2)*((
32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(
a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a^2*tan(c/2 + (d*x)/2)*(2*a^2 - 3*b^
2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b
```

```
^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^
2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^
7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*1i)/(
b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4
+ 6*a^5*b^3 - 20*a^6*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a^2*((32*ta
n(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*
a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a^2*(2*a^2 - 3*
b^2)*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*
b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a^2*tan(c/2 + (d*x)/2)*(2*a^2
 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 +
4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9
 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3
*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2
))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) - (a^2*((32*tan(c/2 + (d*x)/2)*(
8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b
^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a^2*(2*a^2 - 3*b^2)*((32*(2*a*b^1
1 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9
 - a^2*b^7 - a^3*b^6) - (32*a^2*tan(c/2 + (d*x)/2)*(2*a^2 - 3*b^2)*((a + b)
^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*
b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a
^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b
^5 - a^6*b^3))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^
7 + 3*a^4*b^5 - a^6*b^3)))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/
(d*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

3.499 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=117

$$\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2a(a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d}-\frac{a^2\tan(c+dx)}{b(a^2-b^2)\,d(a+b\sec(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d-2*a*(a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d-a^2*tan(d*x+c)/b/(a^2-b^2)
/d/(a+b*sec(d*x+c))
```

Rubi [A]

time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3924, 4083, 3855, 3916, 2738, 214}

$$-\frac{2a(a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c+dx)}{bd\,(a^2-b^2)\,(a+b\sec(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) - (2*a*(a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - (a^2*Tan
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3924

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{a^2\tan(c+dx)}{b(a^2-b^2)\,d(a+b\sec(c+dx))}-\frac{\int \frac{\sec(c+dx)\left(-ab-(a^2-b^2)\sec(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{b(a^2-b^2)}\\
&= -\frac{a^2\tan(c+dx)}{b(a^2-b^2)\,d(a+b\sec(c+dx))}+\frac{\int \sec(c+dx)\,dx}{b^2}-\frac{\left(a(a^2-2b^2)\right)\int \frac{\sec(c+}{a+b\sec(c}}{b^2(a^2-b^2)}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{a^2\tan(c+dx)}{b(a^2-b^2)\,d(a+b\sec(c+dx))}-\frac{\left(a(a^2-2b^2)\right)\int \frac{}{1+\frac{a\,\mathrm{co}}{}}}{b^3(a^2-b^2)}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{a^2\tan(c+dx)}{b(a^2-b^2)\,d(a+b\sec(c+dx))}-\frac{\left(2a(a^2-2b^2)\right)\mathrm{Subst}}{}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2a(a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d}-\frac{}{b(a^2-b}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 146, normalized size = 1.25

$$\frac{\frac{2a(a^2-2b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{a^2b\sin(c+dx)}{(-a+b)(a+b)(b+a\cos(c+dx))}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a*(a^2 - 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(3/2) - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a
*Cos[c + d*x])))/(b^2*d)
```

Maple [A]
time = 0.25, size = 166, normalized size = 1.42

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a\left(\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{b^2}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a\left(\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{b^2}}{d}$
risch	$-\frac{2ia\left(b\,e^{i(dx+c)}+a\right)}{(a^2-b^2)db\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{a^3\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d\,b^2}-\frac{2a\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2}{a\sqrt{}}\right)}{\sqrt{a^2-b^2}\,(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2/b^2*a*(b*a/(a^2-b^2)*tan(1/2*d*x+1/2*
c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(a^2-2*b^2)/(a+b)/(a
-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
)))-1/b^2*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(108) = 216$.
time = 3.98, size = 596, normalized size = 5.09

$$\left[\frac{\left(a^3b-2\,ab^3+\left(a^4-2\,a^2b^2\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+\left(a^4b-2\,a^2b^3+b^5+\left(a^5-2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^4b-2\,a^2b^3+b^5+\left(a^5-2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^4b-a^2b^3\right)\sin\left(dx+c\right)}{2\left(\left(a^5b^2-2\,a^3b^4+ab^6\right)d\cos\left(dx+c\right)+\left(a^4b^3-2\,a^2b^5+b^7\right)d\right)},\ -\frac{2\left(a^3b-2\,ab^3+\left(a^4-2\,a^2b^2\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-\left(a^4b-2\,a^2b^3+b^5+\left(a^5-2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^4b-2\,a^2b^3+b^5+\left(a^5-2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a^4b-a^2b^3\right)\sin\left(dx+c\right)}{2\left(\left(a^5b^2-2\,a^3b^4+ab^6\right)d\cos\left(dx+c\right)+\left(a^4b^3-2\,a^2b^5+b^7\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*lo
g((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b
*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*
cos(d*x + c) + b^2)) + (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)
*cos(d*x + c))*log(sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*
a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^4*b - a^2*b^3)
*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2
*a^2*b^5 + b^7)*d), -1/2*(2*(a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*cos(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c))) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*
cos(d*x + c))*log(sin(d*x + c) + 1) + (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a
^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b - a^2*b^3)*
sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2*
a^2*b^5 + b^7)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.49, size = 203, normalized size = 1.74

$$\frac{\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^2b-b^3)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{2\,(a^3-2\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^2-b^4)\sqrt{-a^2+b^2}}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*tan(1/2*d*x + 1/2*c)/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*ta
n(1/2*d*x + 1/2*c)^2 - a - b)) - 2*(a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) + log(abs(
tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2)/d
```

Mupad [B]
time = 6.73, size = 2848, normalized size = 24.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^2),x)
```

```
[Out] - (atan((((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b
^6 - a^2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a
^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 -
a^3*b^2)))/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 +
3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*1i)/
b^2 - ((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6
- a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*
b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3
*b^2)))/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a
^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2
)/((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^
2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7
+ 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2
)))/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b
^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))/b^2 - (64*(
2*a*b^4 - a^4*b + a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a^
3*b^3) + (((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^
6 - a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^
3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a
^3*b^2)))/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3
*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))/b^2))
*2i)/(b^2*d) - (a*atan(((a*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*t
an(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3
- 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (a*(a^2 - 2*b^2)*((32*(2*
a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*
b^3) + (32*a*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(
2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b
^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*((a
 + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*1i)/(b^8
 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (a*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3
)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^
```

4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (a*(a^2 - 2*b^2)*((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*a*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))/((64*(2*a*b^4 - a^4*b + a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (a*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (a*(a^2 - 2*b^2)*((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*a*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (a*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (a*(a^2 - 2*b^2)*((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*a*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(d*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)) - (2*a^2*tan(c/2 + (d*x)/2))/(d*(a + b)*(a*b - b^2)*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))

3.500 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal. Leaf size=85

$$-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}+\frac{a\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}$$

```
[Out] -2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^
(3/2)/d+a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3921, 12, 3916, 2738, 214}

$$\frac{a\tan(c+dx)}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(
a + b)^(3/2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{a\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{\int \frac{b\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{-a^2+b^2} \\
&= \frac{a\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{b\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{a\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2-b^2} \\
&= \frac{a\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)\,d} \\
&= -\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 83, normalized size = 0.98

$$\frac{\frac{2b\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{a\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/
2) + (a*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

Maple [A]
time = 0.12, size = 118, normalized size = 1.39

method	result
derivativedivides	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{2b\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{2b\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{2i\left(b\,e^{i(dx+c)}+a\right)}{d\left(a^2-b^2\right)\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{b\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{b\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2 \ldots}}{a\sqrt{a^2-\ldots}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*
x+1/2*c)^2-a-b)-2*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.91, size = 329, normalized size = 3.87

$$\left[-\frac{\left(ab\cos\left(dx+c\right)+b^2\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-2\left(a^3-ab^2\right)\sin\left(dx+c\right)}{2\left(\left(a^5-2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b-2\,a^2b^3+b^5\right)d\right)},-\frac{\left(ab\cos\left(dx+c\right)+b^2\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-\left(a^3-ab^2\right)\sin\left(dx+c\right)}{\left(a^5-2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b-2\,a^2b^3+b^5\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*b*cos(d*x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (
a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*
```

```
x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*
(a^3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^
4*b - 2*a^2*b^3 + b^5)*d), -((a*b*cos(d*x + c) + b^2)*sqrt(-a^2 + b^2)*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^
3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b
 - 2*a^2*b^3 + b^5)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.48, size = 150, normalized size = 1.76

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)(a^2-b^2)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/((a^2 - b^2)*sq
rt(-a^2 + b^2)) + a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan
(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

Mupad [B]
time = 1.12, size = 92, normalized size = 1.08

$$\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,(a+b)\,(a-b)\,\left((b-a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^2),x)
```

```
[Out] (2*a*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b - tan(c/2 + (d*x)/2)^2*(
a - b))) - (2*b*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d
*(a + b)^(3/2)*(a - b)^(3/2))
```

3.501 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}$$

```
[Out] 2*a*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(
3/2)/d-b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3918, 12, 3916, 2738, 214}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
 + b)^(3/2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{b\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}-\frac{\int \frac{a\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{b\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}+\frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{b\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}+\frac{a\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b\,(a^2-b^2)}\\
&= -\frac{b\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}+\frac{(2a)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+\right.\right.}{b\,(a^2-b^2)\,d}\\
&= \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 83, normalized size = 0.97

$$\frac{-\frac{2a\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\sin(c+dx)}{(-a+b)(a+b)(b+a\cos(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3
/2) + (b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

Maple [A]
time = 0.12, size = 118, normalized size = 1.37

method	result
derivativedivides	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}+\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}+\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
risch	$-\frac{2ib\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{a\left(a^2-b^2\right)d\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b}{a\sqrt{a^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x
+1/2*c)^2-a-b)+2*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.91, size = 332, normalized size = 3.86

$$\left[-\frac{\left(a^2\cos\left(dx+c\right)+ab\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+2\left(a^2b-b^3\right)\sin\left(dx+c\right)}{2\left(\left(a^5-2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b-2\,a^2b^3+b^5\right)d\right)},\ \frac{\left(a^2\cos\left(dx+c\right)+ab\right)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-\left(a^2b-b^3\right)\sin\left(dx+c\right)}{\left(a^5-2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b-2\,a^2b^3+b^5\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (
a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*
```

```
x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*
(a^2*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^
4*b - 2*a^2*b^3 + b^5)*d), ((a^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*arcta
n(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^2
*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b
- 2*a^2*b^3 + b^5)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.47, size = 150, normalized size = 1.74

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)(a^2-b^2)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/((a^2 - b^2)*sqrt
(-a^2 + b^2)) - b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1
/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

Mupad [B]
time = 1.04, size = 92, normalized size = 1.07

$$\frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(b-a\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] (2*a*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d*(a + b)^(3
/2)*(a - b)^(3/2)) - (2*b*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b - t
an(c/2 + (d*x)/2)^2*(a - b)))
```

3.502 $\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{x}{a^2}-\frac{2b(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{b^2\tan(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))}$$

```
[Out] x/a^2-2*b*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a
^2/(a-b)^(3/2)/(a+b)^(3/2)/d+b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3870, 4004, 3916, 2738, 214}

$$-\frac{2b(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tan(c+dx)}{ad\,(a^2-b^2)\,(a+b\sec(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-2),x]
```

```
[Out] x/a^2 - (2*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tan[c + d*x])/(a*(a^2 - b^2
)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
```

```
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sec(c+dx))^2}\,dx &= \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-a^2+b^2+ab\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}\\
&= \frac{x}{a^2} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}\\
&= \frac{x}{a^2} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a^2-b^2\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2\left(a^2-b^2\right)}\\
&= \frac{x}{a^2} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx\right.}{a^2\left(a^2-b^2\right)d}\\
&= \frac{x}{a^2} - \frac{2b\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+}
\end{aligned}$$

Mathematica [A]
time = 0.50, size = 138, normalized size = 1.27

$$\frac{-\frac{2b\left(-2a^2+b^2\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{a\left(a^2-b^2\right)(c+dx)\cos(c+dx)+b\left(\left(a^2-b^2\right)(c+dx)+ab\sin(c+dx)\right)}{b+a\cos(c+dx)}}{a^2(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-2),x]
```

[Out] ((-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + b*((a^2 - b^2)*(
c + d*x) + a*b*Sin[c + d*x]))/(b + a*Cos[c + d*x]))/(a^2*(a - b)*(a + b)*d)

Maple [A]
time = 0.13, size = 151, normalized size = 1.39

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b\left(-\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b}}\right.}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right.}{a^2}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b\left(-\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b}}\right.}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right.}{a^2}}{d}$
risch	$\frac{x}{a^2}+\frac{2ib^2\left(b\,e^{i(dx+c)}+a\right)}{a^2(a^2-b^2)d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{2b\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{b^3\ln\left(e^{i(dx+c)}-\frac{ia^2-}{}\right.}{\sqrt{a^2-b^2}\,(}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2/a^2*arctan(tan(1/2*d*x+1/2*c))+2*b/a^2*(-b*a/(a^2-b^2)*tan(1/2*d*x+1
/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(2*a^2-b^2)/(a+b)
/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for
more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(100) = 200$.

time = 3.55, size = 484, normalized size = 4.44

$$\left[\frac{2\,(a^5-2\,a^3b^2+ab^4)dx\cos(dx+c)+2\,(a^4b-2\,a^2b^3+b^5)dx+(2\,a^2b^2-b^4+(2\,a^3b-ab^3)\cos(dx+c))\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+2\,(a^3b^2-ab^4)\sin(dx+c)}{2\,((a^7-2\,a^5b^2+a^3b^4)d\cos(dx+c)+(a^6b-2\,a^4b^3+a^2b^5)d)},\ \frac{(a^5-2\,a^3b^2+ab^4)dx\cos(dx+c)+(a^4b-2\,a^2b^3+b^5)dx-(2\,a^2b^2-b^4+(2\,a^3b-ab^3)\cos(dx+c))\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)+(a^3b^2-ab^4)\sin(dx+c)}{(a^7-2\,a^5b^2+a^3b^4)d\cos(dx+c)+(a^6b-2\,a^4b^3+a^2b^5)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3 +
 b^5)*d*x + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(a^2 - b
^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b
^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + b^2)) + 2*(a^3*b^2 - a*b^4)*sin(d*x + c))/((a^7 - 2*a^
5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), ((a^5 -
 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x - (2*a
^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^3*b^2 -
 a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b
- 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-2), x)
```

Giac [A]
time = 0.46, size = 179, normalized size = 1.64

$$-\frac{\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^3-ab^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{2\,(2\,a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-a^2b^2)\sqrt{-a^2+b^2}}-\frac{dx+c}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*b^2*tan(1/2*d*x + 1/2*c)/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c)^2 - a - b)) + 2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*
x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) - (d*x +
c)/a^2)/d
```

Mupad [B]
time = 6.67, size = 2886, normalized size = 26.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*atan((((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a
^6 - a^3*b^3 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*
b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^3 -
 a^3*b^2)))*1i)/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b
^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))
/a^2 - ((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6
 - a^3*b^3 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^
5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a
^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6
 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))/a
^2)/((64*(2*a^4*b - a*b^4 + b^5 - 3*a^2*b^3 + 2*a^3*b^2))/(a^5*b + a^6 - a^
3*b^3 - a^4*b^2) + (((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))
/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b
^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5
- a^2*b^3 - a^3*b^2)))*1i)/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*
a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3
- a^3*b^2))*1i)/a^2 + (((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^
2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^
4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a
^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b -
 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b
^3 - a^3*b^2))*1i)/a^2)))/(a^2*d) + (b*atan(((b*((32*tan(c/2 + (d*x)/2)*(a^
6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b
+ a^5 - a^2*b^3 - a^3*b^2) + (b*(2*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5
 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*b*tan(c/2
+ (d*x)/2)*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 +
 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 -
a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^(1
/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(2*a^2 - b^2)*((a + b)^3*(a -
 b)^3)^(1/2)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + (b*((32*tan(c/2
+ (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4
*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (b*(2*a^2 - b^2)*((32*(2*a^8*b -
 a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) +
(32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^9*b
 - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^
5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3
*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(2*a^2 - b^2)*(
```

```
(a + b)^3*(a - b)^3)^(1/2)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))/((6
4*(2*a^4*b - a*b^4 + b^5 - 3*a^2*b^3 + 2*a^3*b^2))/(a^5*b + a^6 - a^3*b^3 -
 a^4*b^2) + (b*((32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5
*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (b*(
2*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b +
 a^6 - a^3*b^3 - a^4*b^2) - (32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*((a + b)
^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^
3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b
^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 -
 3*a^6*b^2))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*
a^4*b^4 - 3*a^6*b^2) - (b*((32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5
+ 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*
b^2) - (b*(2*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2
))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^
2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4
 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^
6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 +
3*a^4*b^4 - 3*a^6*b^2))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a
^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)
*2i)/(d*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)) - (2*b^2*tan(c/2 + (d*x)/2
))/(d*(a + b)*(a*b - a^2)*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.503 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$-\frac{2bx}{a^3}+\frac{2b^2(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(a^2-2b^2)\sin(c+dx)}{a^2\left(a^2-b^2\right)d}+\frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

```
[Out] -2*b*x/a^3+2*b^2*(3*a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/a^3/(a-b)^(3/2)/(a+b)^(3/2)/d+(a^2-2*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/
d+b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.23, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3932, 4189, 4004, 3916, 2738, 214}

$$-\frac{2bx}{a^3}+\frac{(a^2-2b^2)\sin(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{2b^2(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*b*x)/a^3 + (2*b^2*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2 - 2*b^2)*Sin[c
+ d*x])/(a^2*(a^2 - b^2)*d) + (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Se
c[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-a^2+2b^2+ab\sec(c+dx)-b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-2b\left(a^2-b^2\right)+ab^2\sec(c+}{a+b\sec(c+dx)}}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2bx}{a^3} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(b^2(3a^2-2}{a}$$

$$= -\frac{2bx}{a^3} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(b(3a^2-2b}{a^3}$$

$$= -\frac{2bx}{a^3} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(2b(3a^2-2}{}$$

$$= -\frac{2bx}{a^3} + \frac{2b^2(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d}$$

Mathematica [A]
time = 0.79, size = 172, normalized size = 1.18

$$\frac{\frac{4b^2(-3a^2+2b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{-4ab(a^2-b^2)(c+dx)\cos(c+dx)+2ab(a^2-2b^2)\sin(c+dx)+(a^2-b^2)(-4b^2(c+dx)+a^2\sin(2(c+dx)))}{b+a\cos(c+dx)}}{2a^3(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b^2*(-3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + (-4*a*b*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + 2*a*b*(a
^2 - 2*b^2)*Sin[c + d*x] + (a^2 - b^2)*(-4*b^2*(c + d*x) + a^2*Sin[2*(c + d
*x)]))/(b + a*Cos[c + d*x]))/(2*a^3*(a - b)*(a + b)*d)
```

Maple [A]
time = 0.15, size = 184, normalized size = 1.26

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3} - \frac{2b^2\left(-\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)} - \frac{\left(3a^2-2b^2\right)\mathrm{arc}}{(a+b)(a-}\right.}{a^3}}{d}$

default	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3}-\frac{2b^2\left(-\frac{ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{(3a^2-2b^2)\,a}{(a+b)(}\right.}{a^3}}{d}$
risch	$-\frac{2bx}{a^3}-\frac{ie^{i(dx+c)}}{2a^2d}+\frac{ie^{-i(dx+c)}}{2a^2d}-\frac{2ib^3\left(b\,e^{i(dx+c)}+a\right)}{a^3(a^2-b^2)d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{3b^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2}}{a\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^3*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2*b*arctan(tan(
1/2*d*x+1/2*c)))-2*b^2/a^3*(-b*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*
x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(3*a^2-2*b^2)/(a+b)/(a-b)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.29, size = 565, normalized size = 3.87

$$\left[-\frac{4\,(a^5b-2\,a^3b^3+ab^5)dx\cos(dx+c)+4\,(a^4b^2-2\,a^2b^4+b^6)dx-(3\,a^2b^3-2\,b^5+(3\,a^3b^2-2\,ab^4)\cos(dx+c))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-2\,(a^5b-3\,a^3b^3+2\,ab^5+(a^6-2\,a^4b^2+a^2b^4)\cos(dx+c))\sin(dx+c)}{2\,((a^8-2\,a^6b^2+a^4b^4)d\cos(dx+c)+(a^7b-2\,a^5b^3+a^3b^5)d)},\ -\frac{2\,(a^5b-2\,a^3b^3+ab^5)dx\cos(dx+c)+2\,(a^4b^2-2\,a^2b^4+b^6)dx-(3\,a^2b^3-2\,b^5+(3\,a^3b^2-2\,ab^4)\cos(dx+c))\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^5b-3\,a^3b^3+2\,ab^5+(a^6-2\,a^4b^2+a^2b^4)\cos(dx+c))\sin(dx+c)}{(a^8-2\,a^6b^2+a^4b^4)d\cos(dx+c)+(a^7b-2\,a^5b^3+a^3b^5)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^4*b^2 - 2*a^2*
b^4 + b^6)*d*x - (3*a^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))*s
qrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*s
qrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + (a
^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a
```

```
^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d), -(2*(a^5*b - 2*a
^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - (3*a
^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arcta
n(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^5
*b - 3*a^3*b^3 + 2*a*b^5 + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 +
a^3*b^5)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 837 vs. 2(137) = 274.
time = 0.55, size = 837, normalized size = 5.73

$$-\frac{\frac{\left(2a^7b-5a^6b^2-4a^5b^3+9a^4b^4+2a^3b^5-4a^2b^6-2a^2b|-a^5+a^3b^2|-ab^2|-a^5+a^3b^2|+2b^3|-a^5+a^3b^2|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-\frac{a^4b-a^2b^3+\sqrt{(a^5+a^4b-a^3b^2-a^2b^3)(a^5-a^4b-a^3b^2+a^2b^3)+(a^4b-a^2b^3)^2}}{a^5-a^4b-a^3b^2+a^2b^3}}}\right)\right)}{a^4b|-a^5+a^3b^2|-a^2b^3|-a^5+a^3b^2|+(a^5-a^3b^2)^2}+\frac{\left((2a^2b+ab^2-2b^3)\sqrt{-a^2+b^2}|-a^5+a^3b^2||-a+b|+(2a^7b-5a^6b^2-4a^5b^3+9a^4b^4+2a^3b^5-4a^2b^6)\sqrt{-a^2+b^2}|-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-\frac{a^4b-a^2b^3-\sqrt{(a^5+a^4b-a^3b^2-a^2b^3)(a^5-a^4b-a^3b^2+a^2b^3)+(a^4b-a^2b^3)^2}}{a^5-a^4b-a^3b^2+a^2b^3}}}\right)\right)}{(a^5-a^3b^2)^2(a^2-2ab+b^2)-(a^6b-2a^5b^2+2a^3b^4-a^2b^5)|-a^5+a^3b^2|}-\frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)(a^4-a^2b^2)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((2*a^7*b - 5*a^6*b^2 - 4*a^5*b^3 + 9*a^4*b^4 + 2*a^3*b^5 - 4*a^2*b^6 - 2*
a^2*b*abs(-a^5 + a^3*b^2) - a*b^2*abs(-a^5 + a^3*b^2) + 2*b^3*abs(-a^5 + a^
3*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqr
t(-(a^4*b - a^2*b^3 + sqrt((a^5 + a^4*b - a^3*b^2 - a^2*b^3)*(a^5 - a^4*b -
 a^3*b^2 + a^2*b^3) + (a^4*b - a^2*b^3)^2))/(a^5 - a^4*b - a^3*b^2 + a^2*b^
3))))/(a^4*b*abs(-a^5 + a^3*b^2) - a^2*b^3*abs(-a^5 + a^3*b^2) + (a^5 - a^3
*b^2)^2) + ((2*a^2*b + a*b^2 - 2*b^3)*sqrt(-a^2 + b^2)*abs(-a^5 + a^3*b^2)*
abs(-a + b) + (2*a^7*b - 5*a^6*b^2 - 4*a^5*b^3 + 9*a^4*b^4 + 2*a^3*b^5 - 4*
a^2*b^6)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) +
arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^4*b - a^2*b^3 - sqrt((a^5 + a^4*b - a^
3*b^2 - a^2*b^3)*(a^5 - a^4*b - a^3*b^2 + a^2*b^3) + (a^4*b - a^2*b^3)^2))/
(a^5 - a^4*b - a^3*b^2 + a^2*b^3))))/((a^5 - a^3*b^2)^2*(a^2 - 2*a*b + b^2)
 - (a^6*b - 2*a^5*b^2 + 2*a^3*b^4 - a^2*b^5)*abs(-a^5 + a^3*b^2)) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x +
 1/2*c)^3 + 2*b^3*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - a^2*b
*tan(1/2*d*x + 1/2*c) + a*b^2*tan(1/2*d*x + 1/2*c) + 2*b^3*tan(1/2*d*x + 1/
2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d
*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)))/d
```

Mupad [B]
time = 7.01, size = 3169, normalized size = 21.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^2,x)
[Out] ((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - a^3 - 2*b^3))/(a^2*(a + b)*(a - b
)) - (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - a^3 + 2*b^3))/(a^2*(a + b)*(a -
 b)))/(d*(a + b - tan(c/2 + (d*x)/2)^4*(a - b) + 2*b*tan(c/2 + (d*x)/2)^2))
 - (4*b*atan(((2*b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 +
16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a
^5*b^2) + (b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 -
 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*tan(c/2 + (d*x)/2)*(2*
a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)*64i)/(
a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*2i)/a^3))/a^3 + (2*b*((32*tan(c/2 +
 (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^
3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b*((32*(2*a^11*b - 2*a
^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*
b^3 - a^7*b^2) + (b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 +
4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)*64i)/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*
b^2)))*2i)/a^3))/a^3)/((64*(8*b^8 - 4*a*b^7 - 20*a^2*b^6 + 6*a^3*b^5 + 12*a
^4*b^4))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*((32*tan(c/2 + (d*x)/2)*(8*
b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2
))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*
b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2
) - (b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4
*a^9*b^3 - 2*a^10*b^2)*64i)/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*2i)/a^
3)*2i)/a^3 + (b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*
a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*
b^2) - (b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*
a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b*tan(c/2 + (d*x)/2)*(2*a^1
1*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)*64i)/(a^3
*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*2i)/a^3)*2i)/a^3)))/(a^3*d) - (b^2*ata
n(((b^2*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5
+ 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (
b^2*(3*a^2 - 2*b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^
9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b^2*tan(c/2 +
(d*x)/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6
+ 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3
- a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^
(1/2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*((a + b)^3*
(a - b)^3)^(1/2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b^2*((32*ta
n(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8
```

*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b^2*(3*a^2 - 2*b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b^2*tan(c/2 + (d*x)/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))/((64*(8*b^8 - 4*a*b^7 - 20*a^2*b^6 + 6*a^3*b^5 + 12*a^4*b^4))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^2*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b^2*(3*a^2 - 2*b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b^2*tan(c/2 + (d*x)/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b^2*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b^2*(3*a^2 - 2*b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b^2*tan(c/2 + (d*x)/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(d*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))

3.504 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$\frac{(a^2+6b^2)\,x}{2a^4}-\frac{2b^3(4a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b(2a^2-3b^2)\sin(c+dx)}{a^3\,(a^2-b^2)\,d}+\frac{(a^2-3b^2)\cos(c+d}{2a^2\,(a^2-}$$

```
[Out] 1/2*(a^2+6*b^2)*x/a^4-2*b^3*(4*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/a^4/(a-b)^(3/2)/(a+b)^(3/2)/d-b*(2*a^2-3*b^2)*sin(d*x+c)
/a^3/(a^2-b^2)/d+1/2*(a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-b^2)/d+b^2*
cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.40, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3932, 4189, 4004, 3916, 2738, 214}

$$\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2a^2d\,(a^2-b^2)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\,(a^2-b^2)\,(a+b\sec(c+dx))}+\frac{x(a^2+6b^2)}{2a^4}-\frac{2b^3(4a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b(2a^2-3b^2)\sin(c+dx)}{a^3d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2 + 6*b^2)*x)/(2*a^4) - (2*b^3*(4*a^2 - 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(2*a^
2 - 3*b^2)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 3*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(a*(a
^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\cos(c+dx)\sin(c+dx)}{a(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^2(c+dx)(-a^2+3b^2+ab\sec(c+dx)-2b^2\sec^2(c+dx))}{a+b\sec(c+dx)}\,d}{a(a^2-b^2)}$$

$$= \frac{(a^2-3b^2)\cos(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)\,d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{a(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{\int \cos(c+}{}$$

$$= -\frac{b(2a^2-3b^2)\sin(c+dx)}{a^3(a^2-b^2)\,d} + \frac{(a^2-3b^2)\cos(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)\,d} + \frac{b^2\cos(c+}{a(a^2-b^2)\,a}$$

$$= \frac{(a^2+6b^2)\,x}{2a^4} - \frac{b(2a^2-3b^2)\sin(c+dx)}{a^3(a^2-b^2)\,d} + \frac{(a^2-3b^2)\cos(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)\,d} +$$

$$= \frac{(a^2+6b^2)\,x}{2a^4} - \frac{b(2a^2-3b^2)\sin(c+dx)}{a^3(a^2-b^2)\,d} + \frac{(a^2-3b^2)\cos(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)\,d} +$$

$$= \frac{(a^2+6b^2)\,x}{2a^4} - \frac{b(2a^2-3b^2)\sin(c+dx)}{a^3(a^2-b^2)\,d} + \frac{(a^2-3b^2)\cos(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)\,d} +$$

$$= \frac{(a^2+6b^2)\,x}{2a^4} - \frac{2b^3(4a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b(2a^2-3b^2)\,\mathrm{si}}{a^3(a^2-}$$

Mathematica [A]
time = 0.82, size = 144, normalized size = 0.69

$$\frac{2(a^2+6b^2)(c+dx) - \frac{8b^3(-4a^2+3b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - 8ab\sin(c+dx) + \frac{4ab^4\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))} + a^2\sin(2(c+dx))}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + 6*b^2)*(c + d*x) - (8*b^3*(-4*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - 8*a*b*Sin[c + d*x] + (4
*a*b^4*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*Sin[2*(c
+ d*x)])/(4*a^4*d)
```

Maple [A]
time = 0.19, size = 224, normalized size = 1.08

method	result

derivativedivides	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-2ba\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2-2ba\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2+6b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b^3\left(-\frac{ba\tan\left(\frac{d}{}\right.}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}\right.}{d}$
default	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-2ba\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2-2ba\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2+6b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b^3\left(-\frac{ba\tan\left(\frac{d}{}\right.}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}\right.}{d}$
risch	$\frac{x}{2a^2}+\frac{3x\,b^2}{a^4}-\frac{i\mathrm{e}^{2i(dx+c)}}{8a^2d}+\frac{ib\,\mathrm{e}^{i(dx+c)}}{a^3d}-\frac{ib\,\mathrm{e}^{-i(dx+c)}}{a^3d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^2d}+\frac{2ib^4\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{a^4(a^2-b^2)d\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^4*(((-1/2*a^2-2*b*a)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2-2*b*a)*tan(1/2*
d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+6*b^2)*arctan(tan(1/2*d*x+1
/2*c)))+2*b^3/a^4*(-b*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^
2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(4*a^2-3*b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.49, size = 660, normalized size = 3.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b +
4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*d*x + (4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3*a
*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a
^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (4*a^5*b^2 - 1
0*a^3*b^4 + 6*a*b^6 - (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b
 - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5
*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d), 1/2*((a^7 + 4*a^5*
b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b + 4*a^4*b^3 - 11*a^2*
b^5 + 6*b^7)*d*x - 2*(4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3*a*b^5)*cos(d*x + c
))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) - (4*a^5*b^2 - 10*a^3*b^4 + 6*a*b^6 - (a^7 - 2*a^5*b^2 +
 a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*si
n(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^
3 + a^4*b^5)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.49, size = 264, normalized size = 1.27

$$-\frac{\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^5-a^3b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{4\,(4\,a^2b^3-3\,b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-a^4b^2)\sqrt{-a^2+b^2}}-\frac{(a^2+6\,b^2)(dx+c)}{a^4}+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b^4*tan(1/2*d*x + 1/2*c)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 4*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/2
*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*
tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b^2))
 - (a^2 + 6*b^2)*(d*x + c)/a^4 + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*
d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d
```

Mupad [B]
time = 8.32, size = 2500, normalized size = 12.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^2,x)

[Out] (atan(((((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + ((a^2*1i + b^2*6i)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))*(a^2*1i + b^2*6i)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - ((a^2*1i + b^2*6i)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))*(a^2*1i + b^2*6i)*1i)/(2*a^4))/((16*(108*b^11 - 54*a*b^10 - 216*a^2*b^9 + 81*a^3*b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^5 - 4*a^7*b^4 + 4*a^8*b^3))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + ((a^2*1i + b^2*6i)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))*(a^2*1i + b^2*6i))/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - ((a^2*1i + b^2*6i)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))*(a^2*1i + b^2*6i))/(2*a^4)))*(a^2*1i + b^2*6i)*1i)/(a^4*d) - ((tan(c/2 + (d*x)/2)*(3*a*b^3 - 3*a^3*b + a^4 + 6*b^4 - 5*a^2*b^2))/((a^3*b - a^4)*(a + b)) + (tan(c/2 + (d*x)/2)^5*(3*a^3*b - 3*a*b^3 + a^4 + 6*b^4 - 5*a^2*b^2))/((a^3*b - a^4)*(a + b)) - (2*tan(c/2 + (d*x)/2)^3*(a^4 - 6*b^4 + 3*a^2*b^2))/(a*(a^2*b - a^3)*(a + b)))/(d*(a + b + tan(c/2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a - 3*b) - tan(c/2 + (d*x)/2)^6*(a - b))) + (b^3*atan(((b^3*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^3*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b

^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8*b^3*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + (b^3*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^3*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (8*b^3*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))/((16*(108*b^11 - 54*a*b^10 - 216*a^2*b^9 + 81*a^3*b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^5 - 4*a^7*b^4 + 4*a^8*b^3))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (b^3*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^3*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8*b^3*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3...

3.505 $\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=261

$$-\frac{b(a^2+4b^2)\,x}{a^5}+\frac{2b^4(5a^2-4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4\left(a^2-b^2\right)d}-\frac{b(a^2-2b^2)}{}$$

```
[Out] -b*(a^2+4*b^2)*x/a^5+2*b^4*(5*a^2-4*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/a^5/(a-b)^(3/2)/(a+b)^(3/2)/d+1/3*(2*a^4+7*a^2*b^2-12*b^4
)*sin(d*x+c)/a^4/(a^2-b^2)/d-b*(a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b
^2)/d+1/3*(a^2-4*b^2)*cos(d*x+c)^2*sin(d*x+c)/a^2/(a^2-b^2)/d+b^2*cos(d*x+c
)^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.58, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3932, 4189, 4004, 3916, 2738, 214}

$$\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3a^2d\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{bx(a^2+4b^2)}{a^5}+\frac{2b^4(5a^2-4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}+\frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4d\left(a^2-b^2\right)}-\frac{b(a^2-2b^2)\sin(c+dx)\cos(c+dx)}{a^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b*(a^2 + 4*b^2)*x)/a^5) + (2*b^4*(5*a^2 - 4*b^2)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((2*a^4
 + 7*a^2*b^2 - 12*b^4)*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) - (b*(a^2 - 2*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 4*b^2)*Cos[c +
d*x]^2*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Cos[c + d*x]^2*Sin[c + d*
x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\cos^2(c+dx)\sin(c+dx)}{a(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^3(c+dx)(-a^2+4b^2+ab\sec(c+dx)-3b^2\sec^2(c+dx))}{a+b\sec(c+dx)}\,dx}{a(a^2-b^2)}$$

$$= \frac{(a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)\,d} + \frac{b^2\cos^2(c+dx)\sin(c+dx)}{a(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{\int \frac{\cos^2(c\text{-}}{}}{}$$

$$= -\frac{b(a^2-2b^2)\cos(c+dx)\sin(c+dx)}{a^3(a^2-b^2)\,d} + \frac{(a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)\,d} + \frac{}{a}$$

$$= \frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4(a^2-b^2)\,d} - \frac{b(a^2-2b^2)\cos(c+dx)\sin(c+dx)}{a^3(a^2-b^2)\,d} + \frac{(a^2-}{}$$

$$= -\frac{b(a^2+4b^2)\,x}{a^5} + \frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4(a^2-b^2)\,d} - \frac{b(a^2-2b^2)\cos(c+dx)\sin}{a^3(a^2-b^2)\,d}$$

$$= -\frac{b(a^2+4b^2)\,x}{a^5} + \frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4(a^2-b^2)\,d} - \frac{b(a^2-2b^2)\cos(c+dx)\sin}{a^3(a^2-b^2)\,d}$$

$$= -\frac{b(a^2+4b^2)\,x}{a^5} + \frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4(a^2-b^2)\,d} - \frac{b(a^2-2b^2)\cos(c+dx)\sin}{a^3(a^2-b^2)\,d}$$

$$= -\frac{b(a^2+4b^2)\,x}{a^5} + \frac{2b^4(5a^2-4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} + \frac{(2a^4+7a^2b^2}{3a^4}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.14, size = 176, normalized size = 0.67

$$\frac{-12b(-ia+2b)(ia+2b)(c+dx) + \frac{24b^4(-5a^2+4b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + 9a(a^2+4b^2)\sin(c+dx) + \frac{12ab^5\sin(c+dx)}{(-a+b)(a+b)(b+a\cos(c+dx))} - 6a^2b\sin(2(c+dx)) + a^3\sin(3(c+dx))}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-12*b*((-I)*a + 2*b)*(I*a + 2*b)*(c + d*x) + (24*b^4*(-5*a^2 + 4*b^2)*ArcT
anh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 9*a*(
a^2 + 4*b^2)*Sin[c + d*x] + (12*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b +
a*Cos[c + d*x])) - 6*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*a^5
*d)
```

Maple [A]
time = 0.21, size = 263, normalized size = 1.01

method	result

derivativedivides	$-\frac{2\left(\frac{\left(-a^3-b\,a^2-3b^2a\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-6b^2a\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^3+b\,a^2-3b^2a\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+b\left(a^2+4b^2\right)\arctan\left(\tan\right.\right.}{a^5}$
default	$-\frac{2\left(\frac{\left(-a^3-b\,a^2-3b^2a\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-6b^2a\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^3+b\,a^2-3b^2a\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+b\left(a^2+4b^2\right)\arctan\left(\tan\right.\right.}{a^5}$
risch	$-\frac{bx}{a^3}-\frac{4b^3x}{a^5}+\frac{ib\,\mathrm{e}^{2i(dx+c)}}{4a^3d}-\frac{3i\mathrm{e}^{i(dx+c)}}{8a^2d}-\frac{3i\mathrm{e}^{i(dx+c)}b^2}{2a^4d}+\frac{3i\mathrm{e}^{-i(dx+c)}}{8a^2d}+\frac{3i\mathrm{e}^{-i(dx+c)}b^2}{2a^4d}-\frac{ib\,\mathrm{e}^{-2i(dx+c)}}{4a^3d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^5*(((-a^3-a^2*b-3*a*b^2)*tan(1/2*d*x+1/2*c)^5+(-2/3*a^3-6*b^2*a)*
tan(1/2*d*x+1/2*c)^3+(-a^3+a^2*b-3*a*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*
x+1/2*c)^2)^3+b*(a^2+4*b^2)*arctan(tan(1/2*d*x+1/2*c)))-2*b^4/a^5*(-b*a/(a^
2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-
b)-(5*a^2-4*b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.73, size = 757, normalized size = 2.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 6*(a^
6*b^2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^3*
b^4 - 4*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
 c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*
a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos
(d*x + c)^3 - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + 2*(a^8 + a^6
*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^
2 + a^6*b^4)*d*cos(d*x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d), -1/3*(3*(a^
7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*b^2 + 2*a^
4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^3*b^4 - 4*a*b^
6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a
*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^3 - 2*(a^7*b - 2*a^5*b^3 +
a^3*b^5)*cos(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x
 + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c) + (a^9*b
- 2*a^7*b^3 + a^5*b^5)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 335, normalized size = 1.28

$$\frac{\frac{6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-a^4b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{6\,(5\,a^2b^4-4\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^7-a^5b^2)\sqrt{-a^2+b^2}}-\frac{3\,(a^2b+4\,b^3)(dx+c)}{a^5}+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*b^5*tan(1/2*d*x + 1/2*c)/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 6*(5*a^2*b^4 - 4*b^6)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^2))
- 3*(a^2*b + 4*b^3)*(d*x + c)/a^5 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a*b
*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 18*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) - 3
```

```
*a*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2
*c)^2 + 1)^3*a^4))/d
```

Mupad [B]
time = 9.08, size = 2500, normalized size = 9.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
[Out] - ((2*tan(c/2 + (d*x)/2)^7*(a^5 - 2*a*b^4 + 4*b^5 - 3*a^2*b^3 + a^3*b^2))/(
a^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*a*b^4 - 8*a^4*b + a^5 + 3
6*b^5 - 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4*(a + b)*(a - b)) - (2*tan(c/2 + (d*
x)/2)^5*(6*a*b^4 + 8*a^4*b + a^5 - 36*b^5 + 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4
*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)*(a^5 - 2*a*b^4 - 4*b^5 + 3*a^2*b^
3 + a^3*b^2))/(a^4*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^8*(a -
b) + tan(c/2 + (d*x)/2)^2*(2*a + 4*b) - tan(c/2 + (d*x)/2)^6*(2*a - 4*b) +
6*b*tan(c/2 + (d*x)/2)^4)) - (2*b*atan(((b*(a^2 + 4*b^2)*((32*tan(c/2 + (d*
x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b
^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b +
a^11 - a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a^10*b^8 + 2*
a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15
 - a^12*b^3 - a^13*b^2) - (b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a^15*b - 2
*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2)*32i)/(a^5*(a
^10*b + a^11 - a^8*b^3 - a^9*b^2)))*1i)/a^5))/a^5 + (b*(a^2 + 4*b^2)*((32*t
an(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b
^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2)
)/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a
^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a
^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(
2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2)*
32i)/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*1i)/a^5))/a^5)/((64*(64*b^1
4 - 32*a*b^13 - 112*a^2*b^12 + 48*a^3*b^11 + 12*a^4*b^10 - 6*a^5*b^9 + 31*a
^6*b^8 - 5*a^7*b^7 + 5*a^8*b^6))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (b
*(a^2 + 4*b^2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 +
 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 -
2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^
2)*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^1
4*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (b*tan(c/2 + (d*
x)/2)*(a^2 + 4*b^2)*(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^
13*b^3 - 2*a^14*b^2)*32i)/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*1i)/a^
5)*1i)/a^5 + (b*(a^2 + 4*b^2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11
- 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5
 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) -
```

(b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2)*32i)/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*1i)/a^5)*1i)/a^5))*(a^2 + 4*b^2))/(a^5*d) - (b^4*atan(((b^4*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b^4*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (32*b^4*tan(c/2 + (d*x)/2)*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))*1i)/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2) + (b^4*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b^4*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (32*b^4*tan(c/2 + (d*x)/2)*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))*1i)/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))/((64*(64*b^14 - 32*a*b^13 - 112*a^2*b^12 + 48*a^3*b^11 + 12*a^4*b^10 - 6*a^5*b^9 + 31*a^6*b^8 - 5*a^7*b^7 + 5*a^8*b^6))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (b^4*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b^4*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (32*b^4*tan(c/2 + (d*x)/2)*(5*a^2 - 4*b^2)*((a + b)^3*(a ...

3.506 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=230

$$-\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{3a^2(2a^4-5a^2b^2+4b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d}+\frac{(3a^2-2b^2)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d}$$

```
[Out] -3*a*arctanh(sin(d*x+c))/b^4/d+3*a^2*(2*a^4-5*a^2*b^2+4*b^4)*arctanh((a-b)^
(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/d+1/2*(3*
a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)/d-1/2*a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^
2-b^2)/d/(a+b*sec(d*x+c))^2+3/2*a^3*(a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/
d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.49, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3930, 4175, 4167, 4083, 3855, 3916, 2738, 214}

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{(3a^2-2b^2)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}+\frac{3a^2(2a^4-5a^2b^2+4b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{3a^3(a^2-2b^2)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(b^4*d) + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a +
 b)^(5/2)*d) + ((3*a^2 - 2*b^2)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (a^2*
Sec[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (
3*a^3*(a^2 - 2*b^2)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x
]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4175

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b*
B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
```

```
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)\left(2a^2-2ab\sec(c+dx)-\left(3a^2-2b^2\right)\sec^2(c\text{-}}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \frac{\text{s}}{}}{}$$

$$= \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3a^3\left(a^2-2\right.}{2b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3a^3\left(a^2-2\right.}{2b^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c}{2b\left(a^2-b^2\right)d(a+b\sec(}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c}{2b\left(a^2-b^2\right)d(a+b\sec(}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a+b}}\right.}{(a-b)^{5/2}b^4(a+b)^{5/2}d}$$

Mathematica [A]
time = 4.87, size = 205, normalized size = 0.89

$$\frac{-\frac{6a^2\left(2a^4-5a^2b^2+4b^4\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + 6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{a^3b\left(5a^2b-8b^3+a\left(4a^2-7b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))^2} + 2b\tan(c+dx)}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*(5*a^2*b - 8*
b^3 + a*(4*a^2 - 7*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b
 + a*Cos[c + d*x])^2) + 2*b*Tan[c + d*x])/(2*b^4*d)
```

Maple [A]
time = 0.52, size = 296, normalized size = 1.29

method	result
derivativedivides	$\frac{-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2a^2\left(\frac{\frac{\left(4a^2-ba-8b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}-\frac{\left(4a^2+ba-8b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{3\left(2a^4-\right.}{}\right.}{b^4}}{d}$
default	$\frac{-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2a^2\left(\frac{\frac{\left(4a^2-ba-8b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}-\frac{\left(4a^2+ba-8b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{3\left(2a^4-\right.}{}\right.}{b^4}}{d}$
risch	$\frac{i\left(3a^5b\,e^{5i(dx+c)}-6a^3b^3e^{5i(dx+c)}+6a^6e^{4i(dx+c)}-3a^4b^2e^{4i(dx+c)}-12a^2b^4e^{4i(dx+c)}+24a^5b\,e^{3i(dx+c)}-44a^3b^3e^{3i(dx+c)}+8\right.}{\left(-a^2+b^2\right)^2db^3(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^3/(tan(1/2*d*x+1/2*c)+1)-3*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*a
^2*((1/2*(4*a^2-a*b-8*b^2)*b*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1
/2*(4*a^2+a*b-8*b^2)*b*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1
/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-3/2*(2*a^4-5*a^2*b^2+4*b^4)/(
a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2)))-1/b^3/(tan(1/2*d*x+1/2*c)-1)+3*a/b^4*ln(tan(1/2*d*x+1/2*
c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(215) = 430.
time = 4.20, size = 1354, normalized size = 5.89

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] [1/4*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^3 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 6*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(sin(d*x + c) + 1) + 6*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(2*a^6*b^3 - 6*a^4*b^5 + 6*a^2*b^7 - 2*b^9 + (6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)), 1/2*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^3 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*a^6*b^3 - 6*a^4*b^5 + 6*a^2*b^7 - 2*b^9 + (6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c))]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**3,x)

[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**3, x)

Giac [A]

time = 0.56, size = 383, normalized size = 1.67

$$-\frac{\frac{3\,(2\,a^6-5\,a^4b^2+4\,a^2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^4-2\,a^2b^6+b^8)\sqrt{-a^2+b^2}}+\frac{4\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^4b^3-2\,a^2b^5+b^7)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^2}+\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2
*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-
a^2 + b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(-a^2 + b^2)) + (4*a^6*tan(1/
2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*a^4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*tan(1/2*d*x + 1/2*c) -
 5*a^5*b*tan(1/2*d*x + 1/2*c) + 7*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*a^3*b^3*
tan(1/2*d*x + 1/2*c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*a*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/b^4 - 3*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*tan(1/2*d*x + 1
/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))/d
```

Mupad [B]
time = 9.29, size = 2500, normalized size = 10.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(2*a*b^4 - 3*a^4*b + 6*a^5 - 2*b^5 + 4*a^2*b^3 - 12*
a^3*b^2))/((a*b^3 - b^4)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*a*b^4 + 3*a^4*
b + 6*a^5 + 2*b^5 - 4*a^2*b^3 - 12*a^3*b^2))/((a + b)*(b^5 - 2*a*b^4 + a^2*
b^3)) - (2*tan(c/2 + (d*x)/2)^3*(6*a^6 - 2*b^6 + 6*a^2*b^4 - 13*a^4*b^2))/(
b*(a*b^2 - b^3)*(a + b)^2*(a - b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a*b
 + 3*a^2 - b^2) - tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - ta
n(c/2 + (d*x)/2)^4*(2*a*b - 3*a^2 + b^2))) + (a*atan(((a*((8*tan(c/2 + (d*x
)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5
*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288*a^10*b^2
))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b
^7 - a^7*b^6) - (3*a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15 + 26*a^4*b^1
4 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^
10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11
 - a^6*b^10 - a^7*b^9) - (24*a*tan(c/2 + (d*x)/2)*(8*a*b^17 - 8*a^2*b^16 -
32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^
8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*
b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))))/b^4)*3i)/b^4 + (a*((8*
tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4
*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3
```

- 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a
^5*b^8 - a^6*b^7 - a^7*b^6) + (3*a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^1
5 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*
a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^1
2 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (24*a*tan(c/2 + (d*x)/2)*(8*a*b^17 -
 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^
7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b^12 + b^13 - 3*a^2
*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))))/b^4)*3i)
/b^4)/((48*(36*a^12 - 18*a^11*b + 72*a^4*b^8 + 72*a^5*b^7 - 234*a^6*b^6 - 1
26*a^7*b^5 + 288*a^8*b^4 + 81*a^9*b^3 - 162*a^10*b^2))/(a*b^15 + b^16 - 3*a
^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (3*a
*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 3
6*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9
*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9
+ 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (3*a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^
3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10
 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^
4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (24*a*tan(c/2 + (d*x)/2)*(8*a*b
^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 -
32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b^12 + b^13 -
3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))))/b^4
))/b^4 + (3*a*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 7
2*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*
b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10
 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (3*a*((24*(4*a*b^17 - 8*a^2
*b^16 - 12*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11
+ 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^
3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (24*a*tan(c/2 + (d
x)/2)(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 4
8*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b
^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^
7*b^6))))/b^4))/b^4))*6i)/(b^4*d) + (a^2*atan(((a^2*((a + b)^5*(a - b)^5)^(
1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9
 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288
*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*
b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (3*a^2*((24*(4*a*b^17 - 8*a^2*b^16 -
 12*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^
8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13
+ 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (12*a^2*tan(c/2 + (d*x)/2
)*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2)*(8*a*b^17 - 8*a^2
*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11
 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*a^4*b^10
 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b
^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*((a + b)^5*(a - b)^5)^(1
/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a

```
^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + 4*b^4 - 5*a^2*b^2)*3i)/(2*(b^14 -
 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) + (a^2*((a
+ b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*...
```

3.507 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=188

$$\frac{\tanh^{-1}(\sin(c+dx))}{b^3d}-\frac{a(2a^4-5a^2b^2+6b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d}-\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx)}$$

[Out] arctanh(sin(d*x+c))/b^3/d-a*(2*a^4-5*a^2*b^2+6*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/2*a^2*(2*a^2-5*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))

Rubi [A]

time = 0.29, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3930, 4165, 4083, 3855, 3916, 2738, 214}

$$-\frac{a^2(2a^2-5b^2)\tan(c+dx)}{2b^2d\,(a^2-b^2)^2\,(a+b\sec(c+dx))}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\,(a^2-b^2)\,(a+b\sec(c+dx))^2}-\frac{a(2a^4-5a^2b^2+6b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\tanh^{-1}(\sin(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]

[Out] ArcTanh[Sin[c + d*x]]/(b^3*d) - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a^2*(2*a^2 - 5*b^2)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4165

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)),
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m +
 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(a^2-2ab\sec(c+dx)-2\left(a^2-b^2\right)\sec^2(c+dx\ldots\right.}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2(2a^2-5b^2)\tan(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \ldots}{\ldots}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2(2a^2-5b^2)\tan(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \mathrm{s}\ldots}{\ldots}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^3 d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2(2a^2-5b^2)\ldots}{2b^2\left(a^2-b^2\right)^2 d(a\ldots}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^3 d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2(2a^2-5b^2)\ldots}{2b^2\left(a^2-b^2\right)^2 d(a\ldots}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^3 d} - \frac{a(2a^4-5a^2b^2+6b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} - \ldots$$

Mathematica [A]

time = 1.39, size = 194, normalized size = 1.03

$$\frac{\frac{2a\left(2a^4-5a^2b^2+6b^4\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{a^2b\left(3b\left(a^2-2b^2\right)+a\left(2a^2-5b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))^2}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]])/(a^2 - b^2)^(5/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(3*b*(a^2 - 2*b^2) +
 a*(2*a^2 - 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*
Cos[c + d*x])^2))/(2*b^3*d)
```

Maple [A]

time = 0.44, size = 255, normalized size = 1.36

method	result

derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2a\left(\frac{\frac{\left(2a^2-ba-6b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}-\frac{\left(2a^2+ba-6b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{\left(2a^4-5b^2a^2+6b^4\right)\operatorname{arctanh}\left(\frac{(a-}{\sqrt{(a}}\right.}{2\left(a^4-2b^2a^2+b^4\right)\sqrt{(a+}}\right.}{b^3}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2a\left(\frac{\frac{\left(2a^2-ba-6b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}-\frac{\left(2a^2+ba-6b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{\left(2a^4-5b^2a^2+6b^4\right)\operatorname{arctanh}\left(\frac{(a-}{\sqrt{(a}}\right.}{2\left(a^4-2b^2a^2+b^4\right)\sqrt{(a+}}\right.}{b^3}}{d}$
risch	$\frac{ia\left(-b\,a^3 e^{3i(dx+c)}+4a\,b^3 e^{3i(dx+c)}-2a^4 e^{2i(dx+c)}+a^2b^2 e^{2i(dx+c)}+10b^4 e^{2i(dx+c)}-7a^3 b\,e^{i(dx+c)}+16b^3 a\,e^{i(dx+c)}-2a^4+5b\right.}{\left(-a^2+b^2\right)^2 d\,b^2\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*ln(tan(1/2*d*x+1/2*c)+1)+2/b^3*a*((1/2*(2*a^2-a*b-6*b^2)*b*a/(a-
b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(2*a^2+a*b-6*b^2)*b*a/(a+b)/(a^
2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*
c)^2-a-b)^2-1/2*(2*a^4-5*a^2*b^2+6*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(
1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-1/b^3*ln(tan(1/
2*d*x+1/2*c)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(175) = 350$.
time = 3.75, size = 1153, normalized size = 6.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)*co
s(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(a^2 -
 b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 -
 b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
+ 2*a*b*cos(d*x + c) + b^2)) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (
a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^
3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*(a^6*b^2 - 3
*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*
x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(-sin
(d*x + c) + 1) - 2*(3*a^6*b^2 - 9*a^4*b^4 + 6*a^2*b^6 + (2*a^7*b - 7*a^5*b^
3 + 5*a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^
7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10
)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d), -1/2*((2*a^
5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)*cos(d*x + c)^
2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))
*log(sin(d*x + c) + 1) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*
a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^
3*b^5 - a*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (3*a^6*b^2 - 9*a^4*b^
4 + 6*a^2*b^6 + (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*cos(d*x + c))*sin(d*x + c
))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b
^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7
+ 3*a^2*b^9 - b^11)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Giac [A]
time = 0.51, size = 347, normalized size = 1.85

$$\frac{\frac{(2a^5-5a^3b^2+6ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^3-2a^2b^5+b^7)\sqrt{-a^2+b^2}}+\frac{2a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^4b^2-2a^2b^4+b^6)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(-a^2 + b^2)) + (2*a^5*tan(1/2*d*x
 + 1/2*c)^3 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2*d*x + 1/2*
c)^3 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*a^5*tan(1/2*d*x + 1/2*c) - 3*a^
4*b*tan(1/2*d*x + 1/2*c) + 5*a^3*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1
/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^
3 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3)/d
```

Mupad [B]
time = 9.50, size = 2500, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^3),x)
```

```
[Out] - (atan((((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11
+ 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 +
b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)
 - (8*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12
 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10
*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6
- a^6*b^5 - a^7*b^4)))/b^3 - (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*
b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b
^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a
^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3 - ((((8*(12*a*b^14 - 4*b^1
5 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^
7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 +
3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (8*tan(c/2 + (d*x)/2)*(8*a*b^1
5 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32
*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3*a^
2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))/b^3 + (8*t
an(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^
3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a
*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^
7*b^4))*1i)/b^3)/((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a
^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a
*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 -
a^7*b^6) - (8*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*
a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7
- 8*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*
a^5*b^6 - a^6*b^5 - a^7*b^4)))/b^3 - (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*
b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 +
```

57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b ^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))/b^3 + (((8*(12*a*b^14 - 4* b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18 *a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (8*tan(c/2 + (d*x)/2)*(8*a* b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3 *a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))/b^3 + (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32 *a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2)) /(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))/b^3 - (16*(12*a*b^8 - 2*a^8*b + 4*a^9 + 24*a^2*b^7 - 34*a^3*b^6 - 26*a^4*b^5 + 36*a^5*b^4 + 13*a^6*b^3 - 18*a^7*b^2))/(a*b^12 + b^13 - 3*a^ 2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*2i)/(b^3 *d) - ((tan(c/2 + (d*x)/2)^3*(a^3*b - 2*a^4 + 6*a^2*b^2))/((a*b^2 - b^3)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(a^3*b + 2*a^4 - 6*a^2*b^2))/((a + b)*(b^4 - 2*a*b^3 + a^2*b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + ta n(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (a*atan(((a*((8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b ^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^ 10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b ^4) - (a*((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^ 13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (4*a*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2 *b^2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48 *a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a ^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*((a + b)^5*(a - b)^5)^(1/ 2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10* a^6*b^7 + 5*a^8*b^5 - a^10*b^3)) + (a*((8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^ 9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3 *b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) + (a*((8*(12*a*b^14 - 4*b ^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18* a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (4*a*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*...

3.508 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$\frac{(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{a^2\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2}+\frac{a(a^2-4b^2)\tan(c+dx)}{2b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))}$$

```
[Out] (a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)
/(a+b)^(5/2)/d-1/2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*a*(a
^2-4*b^2)*tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3924, 4088, 12, 3916, 2738, 214}

$$\frac{(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\tan(c+dx)}{2bd\,(a^2-b^2)\,(a+b\sec(c+dx))^2}+\frac{a(a^2-4b^2)\tan(c+dx)}{2bd\,(a^2-b^2)^2\,(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(5/2)*(a + b)^(5/2)*d) - (a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])^2) + (a*(a^2 - 4*b^2)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*
Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3924

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(-2ab-(a^2-2b^2)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2b(a^2-b^2)}$$

$$= -\frac{a^2\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(a^2-4b^2)\tan(c+dx)}{2b(a^2-b^2)^2\,d(a+b\sec(c+dx))} + \frac{\int b(\cdots}{\cdots}$$

$$= -\frac{a^2\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(a^2-4b^2)\tan(c+dx)}{2b(a^2-b^2)^2\,d(a+b\sec(c+dx))} + \frac{(a^2\cdots}{\cdots}$$

$$= -\frac{a^2\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(a^2-4b^2)\tan(c+dx)}{2b(a^2-b^2)^2\,d(a+b\sec(c+dx))} + \frac{(a^2\cdots}{\cdots}$$

$$= -\frac{a^2\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(a^2-4b^2)\tan(c+dx)}{2b(a^2-b^2)^2\,d(a+b\sec(c+dx))} + \frac{(a^2\cdots}{\cdots}$$

$$= \frac{(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{a^2\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} \cdots$$

Mathematica [A]
time = 0.45, size = 113, normalized size = 0.76

$$\frac{-\frac{2(a^2+2b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a(a^2-4b^2-3ab\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(5/2) + (a*(a^2 - 4*b^2 - 3*a*b*Cos[c + d*x])*Sin[c + d*x])/((a -
 b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.24, size = 184, normalized size = 1.23

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(4b+a)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}-\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}+\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^4-2b^2a^2+b^4\right)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(4b+a)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}-\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}+\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^4-2b^2a^2+b^4\right)\sqrt{(a+b)(a-b)}}}{d}$
risch	$-\frac{i\left(a^3 e^{3i(dx+c)}+2a\,b^2 e^{3i(dx+c)}+3a^2 b\,e^{2i(dx+c)}+6b^3 e^{2i(dx+c)}-a^3 e^{i(dx+c)}+10b^2 a\,e^{i(dx+c)}+3b\,a^2\right)}{d(-a^2+b^2)^2\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2}+\frac{a^2\ln\left(e^{i(dx+c)}+\right.}{2\sqrt{a^2-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(4*b+a)*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(a-4
*b)*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*t
an(1/2*d*x+1/2*c)^2-a-b)^2+(a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.58, size = 594, normalized size = 3.99

$$\left[\frac{\left(a^{2} b^{2}+2 b^{4}+\left(a^{4}+2 a^{2} b^{2}\right) \cos \left(d x+c\right)^{2}+2 \left(a^{3} b+2 a b^{3}\right) \cos \left(d x+c\right)\right) \sqrt{a^{2}-b^{2}} \log \left(\frac{2 a b \cos(dx+c)-(a^{2}-2 b^{2}) \cos(dx+c)^{2}+2 \sqrt{a^{2}-b^{2}} (b \cos(dx+c)+a) \sin(dx+c)+2 a^{2}-b^{2}}{a^{2} \cos(dx+c)^{2}+2 a b \cos(dx+c)+b^{2}}\right)+2 \left(a^{5}-5 a^{3} b^{2}+4 a b^{4}-3 \left(a^{4} b-a^{2} b^{3}\right) \cos \left(d x+c\right)\right) \sin \left(d x+c\right)}{4 \left(\left(a^{8}-3 a^{6} b^{2}+3 a^{4} b^{4}-a^{2} b^{6}\right) d \cos \left(d x+c\right)^{2}+2 \left(a^{7} b-3 a^{5} b^{3}+3 a^{3} b^{5}-a b^{7}\right) d \cos \left(d x+c\right)+\left(a^{6} b^{2}-3 a^{4} b^{4}+3 a^{2} b^{6}-b^{8}\right) d\right)}, \frac{\left(a^{2} b^{2}+2 b^{4}+\left(a^{4}+2 a^{2} b^{2}\right) \cos \left(d x+c\right)^{2}+2 \left(a^{3} b+2 a b^{3}\right) \cos \left(d x+c\right)\right) \sqrt{-a^{2}+b^{2}} \arctan \left(-\frac{\sqrt{-a^{2}+b^{2}} (b \cos(dx+c)+a)}{(a^{2}-b^{2}) \sin(dx+c)}\right)+\left(a^{5}-5 a^{3} b^{2}+4 a b^{4}-3 \left(a^{4} b-a^{2} b^{3}\right) \cos \left(d x+c\right)\right) \sin \left(d x+c\right)}{2 \left(\left(a^{8}-3 a^{6} b^{2}+3 a^{4} b^{4}-a^{2} b^{6}\right) d \cos \left(d x+c\right)^{2}+2 \left(a^{7} b-3 a^{5} b^{3}+3 a^{3} b^{5}-a b^{7}\right) d \cos \left(d x+c\right)+\left(a^{6} b^{2}-3 a^{4} b^{4}+3 a^{2} b^{6}-b^{8}\right) d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3*b + 2*a*
b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^5 - 5*a^3*
b^2 + 4*a*b^4 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)
, 1/2*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3*b + 2*a
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x +
c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^5 - 5*a^3*b^2 + 4*a*b^4 - 3*(a^4*b
 - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*
x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

Giac [A]
time = 0.51, size = 253, normalized size = 1.70

$$-\frac{\frac{\left(\pi \left\lfloor \frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)(a^{2}+2b^{2})}{(a^{4}-2a^{2}b^{2}+b^{4})\sqrt{-a^{2}+b^{2}}}-\frac{a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+3a^{2}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-4ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^{2}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^{4}-2a^{2}b^{2}+b^{4})\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a-b\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
+ 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(a^2 + 2*b^2)/((a^4 -
 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x
+ 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
- a - b)^2))/d
```

Mupad [B]
time = 3.24, size = 204, normalized size = 1.37

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(a^2+4\,b\,a\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a\,b-a^2\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)}+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^2-2\,a\,b+b^2\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{5/2}}\right)\,\left(a^2+2\,b^2\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(4*a*b + a^2))/((a + b)^2*(a - b)) - (tan(c/2 + (d*x
)/2)*(4*a*b - a^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 + (d
*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 +
 b^2)) + (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a
+ b)^(1/2)*(a - b)^(5/2)))*(a^2 + 2*b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.509 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=134

$$-\frac{3ab\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}+\frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+\frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}$$

```
[Out] -3*a*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b
)^(5/2)/d+1/2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*(a^2+2*b^2)*t
an(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3921, 4088, 12, 3916, 2738, 214}

$$\frac{\left(a^2+2b^2\right)\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3ab\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)
*(a + b)^(5/2)*d) + (a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) + ((a^2 + 2*b^2)*Tan[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)(-2b+a\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx}{2(a^2-b^2)}$$

$$= \frac{a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(a^2+2b^2)\tan(c+dx)}{2(a^2-b^2)^2\,d(a+b\sec(c+dx))} - \frac{\int \frac{3ab\sec(c}{a+b\sec(c}}{2(a^2-}$$

$$= \frac{a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(a^2+2b^2)\tan(c+dx)}{2(a^2-b^2)^2\,d(a+b\sec(c+dx))} - \frac{(3ab)\int \frac{}{a}}{2(a}$$

$$= \frac{a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(a^2+2b^2)\tan(c+dx)}{2(a^2-b^2)^2\,d(a+b\sec(c+dx))} - \frac{(3a)\int \frac{}{1+}}{2(a^2}$$

$$= \frac{a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(a^2+2b^2)\tan(c+dx)}{2(a^2-b^2)^2\,d(a+b\sec(c+dx))} - \frac{(3a)\mathrm{Subs}}{}$$

$$= -\frac{3ab\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{}{2(a^2}$$

Mathematica [A]

time = 0.41, size = 115, normalized size = 0.86

$$\frac{\frac{6ab\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{(b(a^2+2b^2)+a(2a^2+b^2)\cos(c+dx))\sin(c+dx)}{(b+a\cos(c+dx))^2}}{2(a-b)^2(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^
2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(b + a
*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

Maple [A]
time = 0.20, size = 195, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{-\frac{(2a^2+ba+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^2+2ba+b^2)}+\frac{(2a^2-ba+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^2-2ba+b^2)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{3ba\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a^4-2b^2a^2+b^4)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{\frac{-\frac{(2a^2+ba+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^2+2ba+b^2)}+\frac{(2a^2-ba+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^2-2ba+b^2)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{3ba\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a^4-2b^2a^2+b^4)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{i(3b\,a^3e^{3i(dx+c)}+2a^4e^{2i(dx+c)}+5a^2b^2e^{2i(dx+c)}+2b^4e^{2i(dx+c)}+5a^3b\,e^{i(dx+c)}+4b^3a\,e^{i(dx+c)}+2a^4+b^2a^2)}{a(-a^2+b^2)^2d(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a)^2}+\frac{3ab\ln\left(e^{i}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2*(2*a^2+a*b+2*b^2)/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1
/2*(2*a^2-a*b+2*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d
*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-3*b*a/(a^4-2*a^2*b^2+b^4)/((a+b)*
(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(121) = 242$.
time = 3.22, size = 565, normalized size = 4.22

$$\left[\frac{3\left(a^{3}b\cos\left(dx+c\right)^{2}+2\,a^{2}b^{2}\cos\left(dx+c\right)+ab^{3}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(dx+c)-(a^{2}-2\,b^{2})\cos(dx+c)^{2}-2\,\sqrt{a^{2}-b^{2}}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^{2}-b^{2}}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right)+2\left(a^{4}b+a^{2}b^{3}-2\,b^{5}+\left(2\,a^{5}-a^{3}b^{2}-ab^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\left(\left(a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{7}b-3\,a^{5}b^{3}+3\,a^{3}b^{5}-ab^{7}\right)d\cos\left(dx+c\right)+\left(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}\right)d\right)},-\frac{3\left(a^{3}b\cos\left(dx+c\right)^{2}+2\,a^{2}b^{2}\cos\left(dx+c\right)+ab^{3}\right)\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}(b\cos(dx+c)+a)}{(a^{2}-b^{2})\sin(dx+c)}\right)-\left(a^{4}b+a^{2}b^{3}-2\,b^{5}+\left(2\,a^{5}-a^{3}b^{2}-ab^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2\left(\left(a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{7}b-3\,a^{5}b^{3}+3\,a^{3}b^{5}-ab^{7}\right)d\cos\left(dx+c\right)+\left(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a^3*b*cos(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(a^2 -
b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 -
b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + b^2)) + 2*(a^4*b + a^2*b^3 - 2*b^5 + (2*a^5 - a^3*b^2
 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b
^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x
+ c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), -1/2*(3*(a^3*b*cos(d*x +
 c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4*b + a^2*b^3
- 2*b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(121) = 242$.
time = 0.53, size = 277, normalized size = 2.07

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)ab}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{-a^{2}+b^{2}}}-\frac{2\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a-b\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^4 - 2*a^2*b^
2 + b^4)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*
d*x + 1/2*c)^3 + a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2*c)^
3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*tan(1/2
*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

Mupad [B]
time = 3.37, size = 210, normalized size = 1.57

$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^2+a\,b+2\,b^2\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-a\,b+2\,b^2\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)}-\frac{3\,a\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^2-2\,a\,b+b^2\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{5/2}}\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^3),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^3*(a*b + 2*a^2 + 2*b^2))/((a + b)^2*(a - b)) - (tan(
c/2 + (d*x)/2)*(2*a^2 - a*b + 2*b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*
a*b - tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*
a*b + b^2) + a^2 + b^2)) - (3*a*b*atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^
2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(a - b)^(5/2))))/(d*(a + b)^(5/2)*(a - b
)^(5/2))
```

3.510 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}-\frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

```
[Out] (2*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)
/(a+b)^(5/2)/d-1/2*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-3/2*a*b*tan(
d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3918, 4088, 12, 3916, 2738, 214}

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(5/2)*(a + b)^(5/2)*d) - (b*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) - (3*a*b*Tan[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2a+b\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{(2a^2}{a\cdot}}{2}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{(2a^2 -}{}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{(2a^2 -}{}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{(2a^2 -}{}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} -$$

Mathematica [A]

time = 0.49, size = 115, normalized size = 0.86

$$\frac{-\frac{2(2a^2+b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{b(-3ab+(-4a^2+b^2)\cos(c+dx))\sin(c+dx)}{(b+a\cos(c+dx))^2}}{2(a-b)^2(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + (b*(-3*a*b + (-4*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(b
+ a*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

Maple [A]
time = 0.18, size = 186, normalized size = 1.40

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(4a+b)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{(4a-b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}+\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^4-2b^2a^2+b^4\right)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(4a+b)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{(4a-b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}+\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^4-2b^2a^2+b^4\right)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{ib\left(-5b\,a^3e^{3i(dx+c)}+2a\,b^3e^{3i(dx+c)}-4a^4e^{2i(dx+c)}-7a^2b^2e^{2i(dx+c)}+2b^4e^{2i(dx+c)}-11a^3b\,e^{i(dx+c)}+2b^3a\,e^{i(dx+c)}-4a^4+b\right.}{a^2\left(-a^2+b^2\right)^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(4*a
-b)*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*t
an(1/2*d*x+1/2*c)^2-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(120) = 240.
time = 2.97, size = 595, normalized size = 4.47

$$\left[\frac{\left(2\,a^{2}b^{2}+b^{4}+\left(2\,a^{4}+a^{2}b^{2}\right)\cos\left(dx+c\right)^{2}+2\,\left(2\,a^{3}b+ab^{3}\right)\cos\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(dx+c)-(a^{2}-2\,b^{2})\cos(dx+c)^{2}+2\,\sqrt{a^{2}-b^{2}}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^{2}-b^{2}}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right)-2\,\left(3\,a^{3}b^{2}-3\,ab^{4}+\left(4\,a^{4}b-5\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\,\left(\left(a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}\right)d\cos\left(dx+c\right)^{2}+2\,\left(a^{7}b-3\,a^{5}b^{3}+3\,a^{3}b^{5}-ab^{7}\right)d\cos\left(dx+c\right)+\left(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}\right)d\right)},\ \frac{\left(2\,a^{2}b^{2}+b^{4}+\left(2\,a^{4}+a^{2}b^{2}\right)\cos\left(dx+c\right)^{2}+2\,\left(2\,a^{3}b+ab^{3}\right)\cos\left(dx+c\right)\right)\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\,(b\cos(dx+c)+a)}{(a^{2}-b^{2})\sin(dx+c)}\right)-\left(3\,a^{3}b^{2}-3\,ab^{4}+\left(4\,a^{4}b-5\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2\,\left(\left(a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}\right)d\cos\left(dx+c\right)^{2}+2\,\left(a^{7}b-3\,a^{5}b^{3}+3\,a^{3}b^{5}-ab^{7}\right)d\cos\left(dx+c\right)+\left(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2*a^3*b + a*
b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(3*a^3*b^2 -
3*a*b^4 + (4*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3
*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3
*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
d), 1/2*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2*a^3*b +
 a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c))) - (3*a^3*b^2 - 3*a*b^4 + (4*a^4*b - 5
*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 -
 a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*co
s(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(120) = 240.
time = 0.53, size = 254, normalized size = 1.91

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a-2\,b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)\left(2\,a^{2}+b^{2}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{-a^{2}+b^{2}}}-\frac{4\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a-b\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
+ 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(2*a^2 + b^2)/((a^4 -
 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (4*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*a
*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*b*tan(1/2*
d*x + 1/2*c) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
- a - b)^2))/d
```

Mupad [B]
time = 3.18, size = 204, normalized size = 1.53

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(b^2+4\,a\,b\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a\,b-b^2\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)}+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^2-2\,a\,b+b^2\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{5/2}}\right)\,\left(2\,a^2+b^2\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(4*a*b + b^2))/((a + b)^2*(a - b)) - (tan(c/2 + (d*x
)/2)*(4*a*b - b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 + (d
*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 +
 b^2)) + (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a
+ b)^(1/2)*(a - b)^(5/2)))*(2*a^2 + b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.511 $\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=173

$$\frac{x}{a^3}-\frac{b(6a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}+\frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+\frac{b^2(5a^2-}{2a^2\left(a^2-b^2\right)^2}$$

```
[Out] x/a^3-b*(6*a^4-5*a^2*b^2+2*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+
b*sec(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec
(d*x+c))
```

Rubi [A]
time = 0.23, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3870, 4145, 4004, 3916, 2738, 214}

$$\frac{x}{a^3}+\frac{b^2(5a^2-2b^2)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{b(6a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-3),x]
```

```
[Out] x/a^3 - (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
]])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tan[c + d*x])/(
2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Tan[c + d*
x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
```

```
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2\tan(c+dx)}{2a(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2(a^2-b^2)+2ab\sec(c+dx)-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx}{2a(a^2-b^2)}$$

$$= \frac{b^2\tan(c+dx)}{2a(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tan(c+dx)}{2a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))} + \frac{\int \frac{2(}{}}{}$$

$$= \frac{x}{a^3} + \frac{b^2\tan(c+dx)}{2a(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tan(c+dx)}{2a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))} -$$

$$= \frac{x}{a^3} + \frac{b^2\tan(c+dx)}{2a(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tan(c+dx)}{2a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))} -$$

$$= \frac{x}{a^3} + \frac{b^2\tan(c+dx)}{2a(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tan(c+dx)}{2a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))} -$$

$$= \frac{x}{a^3} - \frac{b(6a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tan(c}{2a(a^2-b^2)\,d(a+}$$

Mathematica [A]
time = 0.81, size = 205, normalized size = 1.18

$$\frac{(b+a\cos(c+dx))\sec^3(c+dx)\left(2(c+dx)(b+a\cos(c+dx))^2+\frac{2b(6a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{(a^2-b^2)^{5/2}}+\frac{ab^3\sin(c+dx)}{(-a+b)(a+b)}+\frac{3ab^2(2a^2-b^2)(b+a\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2}\right)}{2a^3d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(2*(c + d*x)*(b + a*Cos[c + d*x])^2 +
(2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sin[c + d*x])
/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cos[c + d*x])*Sin[c + d
*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sec[c + d*x])^3)
```

Maple [A]
time = 0.18, size = 237, normalized size = 1.37

method	result

derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{2b\left(\frac{-\frac{\left(6a^2+ba-2b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{\left(6a^2-ba-2b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{\left(6a^4-5b^2a^2+2b^4\right)\operatorname{arctanh}\left(\ldots\right.}{2\left(a^4-2b^2a^2+b^4\right)\sqrt{(a\ldots}}\right.}{a^3}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{2b\left(\frac{-\frac{\left(6a^2+ba-2b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{\left(6a^2-ba-2b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{\left(6a^4-5b^2a^2+2b^4\right)\operatorname{arctanh}\left(\ldots\right.}{2\left(a^4-2b^2a^2+b^4\right)\sqrt{(a\ldots}}\right.}{a^3}}{d}$
risch	$\frac{x}{a^3}-\frac{ib^2\left(-7b\,a^3e^{3i(dx+c)}+4a\,b^3e^{3i(dx+c)}-6a^4e^{2i(dx+c)}-9a^2b^2e^{2i(dx+c)}+6b^4e^{2i(dx+c)}-17a^3b\,e^{i(dx+c)}+8b^3a\,e^{i(dx+c)}\ldots\right.}{a^3\left(-a^2+b^2\right)^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^3*arctan(tan(1/2*d*x+1/2*c))+2*b/a^3*((-1/2*(6*a^2+a*b-2*b^2)*b*a/
(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(6*a^2-a*b-2*b^2)*b*a/(a+b)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1
/2*c)^2-a-b)^2-1/2*(6*a^4-5*a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(160) = 320$.
time = 3.16, size = 919, normalized size = 5.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 + 8*(a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 4*(a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*x + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7 + (6*a^6*b - 5*a
^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*co
s(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(5*a^5*b^3 - 7*a^3*b
^5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x + c))*sin(d*x +
c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b
- 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 +
3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x
*cos(d*x + c)^2 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c
) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x - (6*a^4*b^3 - 5*a^2*b^5
+ 2*b^7 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 -
 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (5*a^5*b^3 - 7*a^3*b^
5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x + c))*sin(d*x + c
))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b -
 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3
*a^5*b^6 - a^3*b^8)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sec \left(c + dx\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-3), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(160) = 320.
time = 0.47, size = 322, normalized size = 1.86

$$\frac{\frac{\left(6\,a^4b-5\,a^2b^3+2\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{-a^2+b^2}}-\frac{6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^6-2\,a^4b^2+a^2b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^2}+\frac{dx+c}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b^2)) - (6*a^3*b^2*tan(1/2
```

```
*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*tan(1/2*d*x +
1/2*c)^3 + 2*b^5*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) -
5*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x + 1/2*c) + 2*b^5*tan(1
/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + (d*x + c)/a^3)/d
```

Mupad [B]
time = 9.22, size = 2500, normalized size = 14.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*atan((((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4
*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b +
a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)
 - (tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32
*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b
^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3
*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3 + (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b -
 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*
a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*
a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))/a^3 - ((((8*(12*a^14*b - 4*a^
15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*
b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8
*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (tan(c/2 + (d*x)/2)*(8*a^15*b
 - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11
*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^
4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3
- (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 +
 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^
2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3
- 3*a^9*b^2))/a^3)/((((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18
*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2)
)/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 -
 3*a^11*b^2) - (tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*
a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^
3 - 8*a^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3
*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3 + (8*tan(c/2 + (d*x)/2)*(4*a^10
 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a
^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 -
a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))*1i)/a^3 + (((((8*
(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a
^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7
```

- a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (tan(c/2 + (
d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a
^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)*8i)/(a^3*(a
^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^
9*b^2)))*1i)/a^3 - (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^
10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^
7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7
*b^4 - 3*a^8*b^3 - 3*a^9*b^2))*1i)/a^3 + (16*(12*a^8*b - 2*a*b^8 + 4*b^9 -
18*a^2*b^7 + 13*a^3*b^6 + 36*a^4*b^5 - 26*a^5*b^4 - 34*a^6*b^3 + 24*a^7*b^2
))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3
- 3*a^11*b^2))))/(a^3*d) + ((tan(c/2 + (d*x)/2)^3*(a*b^3 - 2*b^4 + 6*a^2*b^
2))/((a^2*b - a^3)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(a*b^3 + 2*b^4 - 6*a^2*
b^2))/((a + b)*(a^4 - 2*a^3*b + a^2*b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2
*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) +
(b*atan(((b*((8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 -
32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3
+ 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4
- 3*a^8*b^3 - 3*a^9*b^2) + (b*((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b
^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^
13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^1
0*b^3 - 3*a^11*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(
6*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8
- 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^
14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b
^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3
- 3*a^9*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*
(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*((a
+ b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*1i)/(2*(a^13 - a^3*b^10
+ 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)) + (b*((8*tan(c/2 + (d
x)/2)(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*
a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^1
1 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (b
((8(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 +
36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6
*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (4*b*ta
n(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(6...

3.512 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=223

$$-\frac{3bx}{a^4}+\frac{3b^2(4a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d}+\frac{(2a^4-11a^2b^2+6b^4)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d}+\frac{b}{2a\left(a^2-b^2\right.}$$

```
[Out] -3*b*x/a^4+3*b^2*(4*a^4-5*a^2*b^2+2*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*(2*a^4-11*a^2*b^2+6*b^4
)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*
x+c))^2+3/2*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.44, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3932, 4185, 4189, 4004, 3916, 2738, 214}

$$-\frac{3bx}{a^4}+\frac{3b^2(2a^2-b^2)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{3b^2(4a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{(2a^4-11a^2b^2+6b^4)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*b*x)/a^4 + (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((2*a^4 -
 11*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b^2*Sin[c + d
*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (3*b^2*(2*a^2 - b^2)*Sin[
c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2a^2+3b^2+2ab\sec(c+dx)-2b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3b^2(2a^2-b^2)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \cos(c}{}$$

$$= \frac{(2a^4-11a^2b^2+6b^4)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3b^2}{2a^2\left(a^2\right.}$$

$$= -\frac{3bx}{a^4} + \frac{(2a^4-11a^2b^2+6b^4)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= -\frac{3bx}{a^4} + \frac{(2a^4-11a^2b^2+6b^4)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= -\frac{3bx}{a^4} + \frac{(2a^4-11a^2b^2+6b^4)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= -\frac{3bx}{a^4} + \frac{3b^2(4a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d} + \frac{(2a^4-11a^2b^2}{2a^3\left(a\right.}$$

Mathematica [A]
time = 0.91, size = 229, normalized size = 1.03

$$\frac{(b+a\cos(c+dx))\sec^3(c+dx)\left(-6b(c+dx)(b+a\cos(c+dx))^2-\frac{6b^2(4a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{(a^2-b^2)^{5/2}}+\frac{ab^4\sin(c+dx)}{(a-b)(a+b)}-\frac{ab^3(8a^2-5b^2)(b+a\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2}+2a(b+a\cos(c+dx))^2\sin(c+dx)\right)}{2a^4d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(-6*b*(c + d*x)*(b + a*Cos[c + d*x])^2
 - (6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^4*Sin[c +
d*x])/((a - b)*(a + b)) - (a*b^3*(8*a^2 - 5*b^2)*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/((a - b)^2*(a + b)^2) + 2*a*(b + a*Cos[c + d*x])^2*Sin[c + d*x]))/
(2*a^4*d*(a + b*Sec[c + d*x])^3)
```

Maple [A]
time = 0.23, size = 270, normalized size = 1.21

method	result

derivativedivides	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+3b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^4}-\frac{2b^2\left(\frac{-\frac{\left(8a^2+ba-4b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{\left(8a^2-ba-4b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\right.}{a^4}}{d}$
default	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+3b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^4}-\frac{2b^2\left(\frac{-\frac{\left(8a^2+ba-4b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{\left(8a^2-ba-4b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\right.}{a^4}}{d}$
risch	$-\frac{3bx}{a^4}-\frac{ie^{i(dx+c)}}{2a^3d}+\frac{ie^{-i(dx+c)}}{2a^3d}+\frac{ib^3\left(-9b\,a^3e^{3i(dx+c)}+6a\,b^3e^{3i(dx+c)}-8a^4e^{2i(dx+c)}-11a^2b^2e^{2i(dx+c)}+10b^4e^{2i(\cdots\right.}{a^4(-a^2+b^2)^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^4*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+3*b*arctan(tan(
1/2*d*x+1/2*c)))-2*b^2/a^4*((-1/2*(8*a^2+a*b-4*b^2)*b*a/(a-b)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3+1/2*(8*a^2-a*b-4*b^2)*b*a/(a+b)/(a^2-2*a*b+b^2)*tan
(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-3/2*
(4*a^4-5*a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(208) = 416$.
time = 2.78, size = 1037, normalized size = 4.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 24
*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c) + 12*(a^6*b^3 -
 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (4*a
^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b^5 +
 2*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*
b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^7*b
^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8 + 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a
^3*b^6)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*co
s(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(
d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) +
(a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d), -1/2*(6*(a^8*b - 3*a^6*b^3
 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 12*(a^7*b^2 - 3*a^5*b^4 + 3*a^
3*b^6 - a*b^8)*d*x*cos(d*x + c) + 6*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)
*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^
6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(d*x + c))*sqrt(
-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(
d*x + c))) - (2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8 + 2*(a^9 - 3*a^
7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^
4*b^5 - 9*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*
b^4 - a^6*b^6)*d*cos(d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b
^7)*d*cos(d*x + c) + (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [A]
time = 0.55, size = 357, normalized size = 1.60

$$-\frac{\frac{3\,(4\,a^4b^2-5\,a^2b^4+2\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^8-2\,a^6b^2+a^4b^4)\sqrt{-a^2+b^2}}-\frac{8\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^7-2\,a^5b^2+a^3b^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^2}+\frac{3\,(dx+c)b}{a^4}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2
*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-
```

```
a^2 + b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(-a^2 + b^2)) - (8*a^3*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*a*b^5*tan(1/2*d
*x + 1/2*c)^3 + 4*b^6*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*b^3*tan(1/2*d*x + 1/2*
c) - 7*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*a*b^5*tan(1/2*d*x + 1/2*c) + 4*b^6*
tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*(d*x + c)*b/a^4 - 2*tan(1/2*d*
x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

Mupad [B]
time = 9.00, size = 2500, normalized size = 11.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(3*a*b^4 + 2*a^4*b + 2*a^5 + 6*b^5 - 12*a^2*b^3 - 4*a^
3*b^2))/((a + b)*(a^5 - 2*a^4*b + a^3*b^2)) - (tan(c/2 + (d*x)/2)^5*(3*a*b^
4 - 2*a^4*b + 2*a^5 - 6*b^5 + 12*a^2*b^3 - 4*a^3*b^2))/((a^3*b - a^4)*(a +
b)^2) + (2*tan(c/2 + (d*x)/2)^3*(2*a^6 - 6*b^6 + 13*a^2*b^4 - 6*a^4*b^2))/(
a*(a^2*b - a^3)*(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a*b
 - a^2 + 3*b^2) + tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - ta
n(c/2 + (d*x)/2)^4*(2*a*b + a^2 - 3*b^2))) - (6*b*atan(((3*b*((8*tan(c/2 +
(d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 -
432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10
*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*
b^3 - 3*a^11*b^2) + (b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^
8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*
a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 -
3*a^13*b^3 - 3*a^14*b^2) - (b*tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8
*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*
b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^
6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*3i)/a^4))/a^4 + (3*b
*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 +
 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^
9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^
9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b
^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 1
2*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5
+ 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (b*tan(c/2 + (d*x)/2)*(8*a^17*b -
 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13
*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*(a^12*b + a^13 - a
^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*3i)/a
^4))/a^4)/((48*(36*b^12 - 18*a*b^11 - 162*a^2*b^10 + 81*a^3*b^9 + 288*a^4*b
^8 - 126*a^5*b^7 - 234*a^6*b^6 + 72*a^7*b^5 + 72*a^8*b^4))/(a^15*b + a^16 -
```

a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (b*tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*3i)/a^4)*3i)/a^4 + (b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (b*tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*3i)/a^4)*3i)/a^4)))/(a^4*d) - (b^2*atan(((b^2*((a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (3*b^2*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (12*b^2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(4*a^4 + 2*b^4 - 5*a^2*b^2)*3i)/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) + (b^2*((a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d...

3.513 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=296

$$\frac{(a^2+12b^2)\,x}{2a^5}-\frac{b^3(20a^4-29a^2b^2+12b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d}-\frac{3b(2a^4-7a^2b^2+4b^4)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d}$$

```
[Out] 1/2*(a^2+12*b^2)*x/a^5-b^3*(20*a^4-29*a^2*b^2+12*b^4)*arctanh((a-b)^(1/2)*t
an(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)/d-3/2*b*(2*a^4-7
*a^2*b^2+4*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d+1/2*(a^4-10*a^2*b^2+6*b^4)*cos
(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/a/(a^2-b
^2)/d/(a+b*sec(d*x+c))^2+1/2*b^2*(7*a^2-4*b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a
^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]

time = 0.67, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3932, 4185, 4189, 4004, 3916, 2738, 214}

$$\frac{b^2(7a^2-4b^2)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{x(a^2+12b^2)}{2a^5}-\frac{3b(2a^4-7a^2b^2+4b^4)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)^2}-\frac{b^3(20a^4-29a^2b^2+12b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}+\frac{(a^4-10a^2b^2+6b^4)\sin(c+dx)\cos(c+dx)}{2a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 + 12*b^2)*x)/(2*a^5) - (b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh[(S
qrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)
*d) - (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^2*d
) + ((a^4 - 10*a^2*b^2 + 6*b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^
2)^2*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) + (b^2*(7*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-2a^2+4b^2+2ab\sec(c+dx)-3b^2\sec^2(c+d\cdots\right)}{(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2(7a^2-4b^2)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{\int \cdots}{\cdots}$$

$$= \frac{(a^4-10a^2b^2+6b^4)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx)\cdots}$$

$$= -\frac{3b(2a^4-7a^2b^2+4b^4)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{(a^4-10a^2b^2+6b^4)\cos(c+dx)\sin(c+\cdots}{2a^3\left(a^2-b^2\right)^2d}$$

$$= \frac{(a^2+12b^2)\,x}{2a^5} - \frac{3b(2a^4-7a^2b^2+4b^4)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{(a^4-10a^2b^2+6b^4)\cos(c\cdots}{2a^3\left(a^2-b\cdots\right.}$$

$$= \frac{(a^2+12b^2)\,x}{2a^5} - \frac{3b(2a^4-7a^2b^2+4b^4)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{(a^4-10a^2b^2+6b^4)\cos(c\cdots}{2a^3\left(a^2-b\cdots\right.}$$

$$= \frac{(a^2+12b^2)\,x}{2a^5} - \frac{3b(2a^4-7a^2b^2+4b^4)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{(a^4-10a^2b^2+6b^4)\cos(c\cdots}{2a^3\left(a^2-b\cdots\right.}$$

$$= \frac{(a^2+12b^2)\,x}{2a^5} - \frac{b^3(20a^4-29a^2b^2+12b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} - \frac{3b\cdots}{\cdots}$$

Mathematica [A]
time = 2.13, size = 199, normalized size = 0.67

$$\frac{2(a^2+12b^2)(c+dx) + \frac{4b^3(20a^4-29a^2b^2+12b^4)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - 12ab\sin(c+dx) + \frac{2ab^5\sin(c+dx)}{(-a+b)(a+b)(b+a\cos(c+dx))^2} + \frac{2ab^4(10a^2-7b^2)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))} + a^2\sin(2(c+dx))}{4a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(a^2 + 12*b^2)*(c + d*x) + (4*b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh
[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 12*a*b*S
in[c + d*x] + (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])
^2) + (2*a*b^4*(10*a^2 - 7*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*C
os[c + d*x])) + a^2*Sin[2*(c + d*x)])/(4*a^5*d)
```

Maple [A]
time = 0.25, size = 310, normalized size = 1.05

method	result
derivativedivides	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-3ba\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-3ba+\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2+12b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^5}+\frac{2b^3\left(-\frac{\frac{\left(10a^2+ba-6b^2\right)ba\left(\tan^3\right.}{2(a-b)\left(a^2+2ba+\right.}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}\right.}{d}$
default	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-3ba\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-3ba+\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2+12b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^5}+\frac{2b^3\left(-\frac{\frac{\left(10a^2+ba-6b^2\right)ba\left(\tan^3\right.}{2(a-b)\left(a^2+2ba+\right.}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}\right.}{d}$
risch	$\frac{x}{2a^3}+\frac{6xb^2}{a^5}-\frac{ie^{2i(dx+c)}}{8a^3d}+\frac{3ib\,e^{i(dx+c)}}{2d\,a^4}-\frac{3ib\,e^{-i(dx+c)}}{2d\,a^4}+\frac{ie^{-2i(dx+c)}}{8a^3d}-\frac{ib^4\left(-11b\,a^3e^{3i(dx+c)}+8a\,b^3e^{3i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^5*(((-1/2*a^2-3*b*a)*tan(1/2*d*x+1/2*c)^3+(-3*b*a+1/2*a^2)*tan(1/2
*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+12*b^2)*arctan(tan(1/2*d*x
+1/2*c)))+2*b^3/a^5*((-1/2*(10*a^2+a*b-6*b^2)*b*a/(a-b)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3+1/2*(10*a^2-a*b-6*b^2)*b*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*
d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-1/2*(20*a
^4-29*a^2*b^2+12*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 3.47, size = 1158, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] [1/4*(2*(a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*d*x*cos(d*x + c)^2 + 4*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*d*x*cos(d*x + c) + 2*(a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*d*x + (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*a^6*b^3 - 29*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(6*a^7*b^3 - 27*a^5*b^5 + 33*a^3*b^7 - 12*a*b^9 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*cos(d*x + c)^3 + 4*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c)^2 + (11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d), 1/2*((a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*d*x*cos(d*x + c)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*d*x*cos(d*x + c) + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*d*x - (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*a^6*b^3 - 29*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (6*a^7*b^3 - 27*a^5*b^5 + 33*a^3*b^7 - 12*a*b^9 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*cos(d*x + c)^3 + 4*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c)^2 + (11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**3,x)

[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1723 vs. 2(277) = 554.
time = 0.74, size = 1723, normalized size = 5.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] -1/2*(((a^6 - a^5*b + 10*a^4*b^2 + 10*a^3*b^3 - 23*a^2*b^4 - 6*a*b^5 + 12*b^6)*sqrt(-a^2 + b^2)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)*abs(-a + b) - (a^15 - a^14*b + 8*a^13*b^2 - 28*a^12*b^3 - 42*a^11*b^4 + 111*a^10*b^5 + 68*a^9*b^6 - 158*a^8*b^7 - 47*a^7*b^8 + 100*a^6*b^9 + 12*a^5*b^10 - 24*a^4*b^11)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^6*b^3 + a^4*b^5 + sqrt((a^9 + a^8*b - 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^8*b - 2*a^6*b^3 + a^4*b^5)^2))/(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5))))/((a^9 - 2*a^7*b^2 + a^5*b^4)^2*(a^2 - 2*a*b + b^2) + (a^10*b - 2*a^9*b^2 - a^8*b^3 + 4*a^7*b^4 - a^6*b^5 - 2*a^5*b^6 + a^4*b^7)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)) + (a^15 - a^14*b + 8*a^13*b^2 - 28*a^12*b^3 - 42*a^11*b^4 + 111*a^10*b^5 + 68*a^9*b^6 - 158*a^8*b^7 - 47*a^7*b^8 + 100*a^6*b^9 + 12*a^5*b^10 - 24*a^4*b^11 + a^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - a^5*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*a^4*b^2*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*a^3*b^3*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 23*a^2*b^4*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 6*a*b^5*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 12*b^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^6*b^3 + a^4*b^5 - sqrt((a^9 + a^8*b - 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^8*b - 2*a^6*b^3 + a^4*b^5)^2))/(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5))))/(a^8*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 2*a^6*b^3*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + a^4*b^5*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - (a^9 - 2*a^7*b^2 + a^5*b^4)^2) + 2*(a^7*tan(1/2*d*x + 1/2*c)^7 + 4*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 2*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 17*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 12*b^7*tan(1/2*d*x + 1/2*c)^7 - 3*a^7*tan(1/2*d*x + 1/2*c)^5 - 4*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 26*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 67*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 18*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*b^7*tan(1/2*d*x + 1/2*c)^5 + 3*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*b^7*tan(1/2*d*x + 1/2*c)^3 - a^7*tan(1/2*d*x + 1/2*c) + 4*a^6*b*tan(1/2*d*x + 1/2*c) + 13*a^5*b^2*tan(1/2*d*x + 1/2*c) - 2*a^4*b^3*tan(1/2*d*x + 1/2*c) - 33*a^3*b^4*tan(1/2*d*x + 1/2*c) - 17*a^2*b^5*tan(1/2*d*x + 1/2*c) + 18*a*b^6*tan(1/2*d*x + 1/2*c) + 12*b^7*tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d

Mupad [B]

time = 9.29, size = 2500, normalized size = 8.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (atan(((((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1
104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 +
 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^
12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a
^12*b^3 - 3*a^13*b^2) + ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^11 + 24
*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 27
6*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^
12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4
*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9
 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32
*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b
^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i + b^2*12i)*1
i)/(2*a^5) + (((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^
14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6
*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 +
 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4
 - 3*a^12*b^3 - 3*a^13*b^2) - ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^1
1 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^
6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^1
9 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2
) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8*a^
11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^
4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*
a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i + b^2*
12i)*1i)/(2*a^5))/((8*(1728*b^15 - 864*a*b^14 - 7344*a^2*b^13 + 3456*a^3*b^
12 + 11700*a^4*b^11 - 4770*a^5*b^10 - 7829*a^6*b^9 + 2326*a^7*b^8 + 1314*a^
8*b^7 - 11*a^9*b^6 + 411*a^10*b^5 - 20*a^11*b^4 + 20*a^12*b^3))/(a^18*b + a
^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b
^2) - (((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 11
04*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 +
872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^1
2*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^
12*b^3 - 3*a^13*b^2) + ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^11 + 24*
a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276
*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^1
2*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*
tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9
+ 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*
```

a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i + b^2*12i))/(2*a^5) + (((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i + b^2*12i))/(2*a^5)))*(a^2*1i + b^2*12i)*1i)/(a^5*d) - ((tan(c/2 + (d*x)/2)^3*(18*a*b^6 - 4*a^6*b + 3*a^7 + 36*b^7 - 67*a^2*b^5 - 29*a^3*b^4 + 26*a^4*b^3 + 5*a^5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) - (tan(c/2 + (d*x)/2)^5*(18*a*b^6 + 4*a^6*b + 3*a^7 - 36*b^7 + 67*a^2*b^5 - 29*a^3*b^4 - 26*a^4*b^3 + 5*a^5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) - (tan(c/2 + (d*x)/2)^7*(6*a*b^5 + 5*a^5*b + a^6 - 12*b^6 + 23*a^2*b^4 - 10*a^3*b^3 - 8*a^4*b^2))/((a^4*b - a^5)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(6*a*b^5 + 5*a^5*b - a^6 + 12*b^6 - 23*a^2*b^4 - 10*a^3*b^3 + 8*a^4*b^2))/((a + b)*(a^6 - 2*a^5*b + a^4*b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) + tan(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2) - tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (b^3*atan(((b^3*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3...

3.514 $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=316

$$-\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d}+\frac{a^2(8a^6-28a^4b^2+35a^2b^4-20b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^5(a+b)^{7/2}d}+\frac{(12a^4-23a^2b^2}{6b^4(}$$

```
[Out] -4*a*arctanh(sin(d*x+c))/b^5/d+a^2*(8*a^6-28*a^4*b^2+35*a^2*b^4-20*b^6)*arc
tanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7/2
)/d+1/6*(12*a^4-23*a^2*b^2+6*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^2/d-1/3*a^2*sec(
d*x+c)^3*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3-1/6*a^2*(4*a^2-9*b^2)*
sec(d*x+c)^2*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/2*a^3*(4*a^4
-11*a^2*b^2+12*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.76, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3930, 4183, 4175, 4167, 4083, 3855, 3916, 2738, 214}

$$-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{a^2\left(4a^2-9b^2\right)\tan(c+dx)\sec^2(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a^3\left(4a^4-11a^2b^2+12b^4\right)\tan(c+dx)}{2b^4d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-4*a*ArcTanh[Sin[c + d*x]])/(b^5*d) + (a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^
4 - 20*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(
7/2)*b^5*(a + b)^(7/2)*d) + ((12*a^4 - 23*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(6
*b^4*(a^2 - b^2)^2*d) - (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Sec[c + d*x])^3) - (a^2*(4*a^2 - 9*b^2)*Sec[c + d*x]^2*Tan[c + d*x
])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a^3*(4*a^4 - 11*a^2*b^
2 + 12*b^4)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4175

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b*
B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
```

```
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)\left(3a^2-3ab\sec(c+dx)-\left(4a^2-3b^2\right)\sec^2(c+\right.}{(a+b\sec(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\sec^2(c+dx)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= -\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\sec^2(c+dx)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}$$

$$= \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2(4}{6b^2}$$

$$= \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2(4}{6b^2}$$

$$= -\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{a^2\sec^3(c}{3b\left(a^2-b^2\right)}$$

$$= -\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{a^2\sec^3(c}{3b\left(a^2-b^2\right)}$$

$$= -\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-}}{\sqrt{}}\right.}{(a-b)^{7/2}b^5(a+b)^{7/2}d}$$

Mathematica [A]

time = 6.24, size = 416, normalized size = 1.32

$$-\frac{a^2(-8a^6+28a^4b^2-35a^2b^4+20b^6)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,(-a^2+b^2)^3\,d}+\frac{4a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^5d}-\frac{4a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^5d}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{a^3\sin(c+dx)}{3b^2(-a+b)(a+b)d(b+a\cos(c+dx))^3}+\frac{6a^5\sin(c+dx)-11a^3b^2\sin(c+dx)}{6b^3(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2}+\frac{-18a^7\sin(c+dx)+50a^5b^2\sin(c+dx)-47a^3b^4\sin(c+dx)}{6b^4(-a+b)^3(a+b)^3d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((a^2*(-8*a^6 + 28*a^4*b^2 - 35*a^2*b^4 + 20*b^6)*ArcTanh[((-a + b)*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)^3*d)) + (4*
a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(b^5*d) - (4*a*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]])/(b^5*d) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])) - (a^3*Sin[c + d*x])/(3*b^2*(-a + b)*(a + b)*d*(b + a*Cos[c +
 d*x])^3) + (6*a^5*Sin[c + d*x] - 11*a^3*b^2*Sin[c + d*x])/(6*b^3*(-a + b)^
2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) + (-18*a^7*Sin[c + d*x] + 50*a^5*b^2*
Sin[c + d*x] - 47*a^3*b^4*Sin[c + d*x])/(6*b^4*(-a + b)^3*(a + b)^3*d*(b +
a*Cos[c + d*x]))
```

Maple [A]

time = 0.43, size = 425, normalized size = 1.34

method	result
derivativedivides	$-\frac{1}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{4a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^5}-\frac{2a^2\left(\frac{\frac{\left(6a^4-2ba^3-18b^2a^2+5b^3a+20b^4\right)ba\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3ba^2+3b^2a+b^3\right)}-\frac{2\left(9a^4-29b^2a^2+30b^4\cdots\right.}{3\left(a^2+2ba+b^2\right)\cdots}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan\cdots\right.\right.}\cdots\right.}{\cdots}$
default	$-\frac{1}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{4a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^5}-\frac{2a^2\left(\frac{\frac{\left(6a^4-2ba^3-18b^2a^2+5b^3a+20b^4\right)ba\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3ba^2+3b^2a+b^3\right)}-\frac{2\left(9a^4-29b^2a^2+30b^4\cdots\right.}{3\left(a^2+2ba+b^2\right)\cdots}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan\cdots\right.\right.}\cdots\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4/(tan(1/2*d*x+1/2*c)+1)-4*a/b^5*ln(tan(1/2*d*x+1/2*c)+1)-2*a^2/b
^5*((1/2*(6*a^4-2*a^3*b-18*a^2*b^2+5*a*b^3+20*b^4)*b*a/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(9*a^4-29*a^2*b^2+30*b^4)*b*a/(a^2+2*a
*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(6*a^4+2*a^3*b-18*a^2*b^2-
5*a*b^3+20*b^4)*b*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*
tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^3-1/2*(8*a^6-28*a^4*b^2+35
```

```
*a^2*b^4-20*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-1/b^4/(tan(1/2*d*x+1/2*c)-1)
+4*a/b^5*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(299) = 598.
time = 7.76, size = 2058, normalized size = 6.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*((8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6)*cos(d*x + c)^4 +
3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c)^3 + 3*(8*a
^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + (8*a^8*b^3
- 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((
2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*co
s(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos
(d*x + c) + b^2)) - 24*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^
8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9
)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^
10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)
*cos(d*x + c))*log(sin(d*x + c) + 1) + 24*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 -
 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 -
4*a^5*b^7 + a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 -
 4*a^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 -
4*a^3*b^9 + a*b^11)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(6*a^8*b^4 - 2
4*a^6*b^6 + 36*a^4*b^8 - 24*a^2*b^10 + 6*b^12 + (24*a^11*b - 92*a^9*b^3 + 1
33*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9)*cos(d*x + c)^3 + 3*(20*a^10*b^2 - 77*a
^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x + c)^2 + (44*a^9*b^
3 - 169*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x + c))*sin(
d*x + c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d*cos
```

```
(d*x + c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)
*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^
15)*d*cos(d*x + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^
16)*d*cos(d*x + c)), 1/6*(3*((8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6
)*cos(d*x + c)^4 + 3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(
d*x + c)^3 + 3*(8*a^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x +
 c)^2 + (8*a^8*b^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c))*sq
rt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*s
in(d*x + c))) - 12*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*c
os(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*co
s(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*
cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cos
(d*x + c))*log(sin(d*x + c) + 1) + 12*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a
^6*b^6 + a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^
5*b^7 + a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a
^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^
3*b^9 + a*b^11)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (6*a^8*b^4 - 24*a^6*
b^6 + 36*a^4*b^8 - 24*a^2*b^10 + 6*b^12 + (24*a^11*b - 92*a^9*b^3 + 133*a^7
*b^5 - 71*a^5*b^7 + 6*a^3*b^9)*cos(d*x + c)^3 + 3*(20*a^10*b^2 - 77*a^8*b^4
 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x + c)^2 + (44*a^9*b^3 - 16
9*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x + c))*sin(d*x +
c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d*cos(d*x +
 c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*cos
(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*
cos(d*x + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*
cos(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*sec(c + d*x))**4, x)
```

Giac [**A**]
time = 0.55, size = 592, normalized size = 1.87

$$-\frac{\frac{3\,(8\,a^8-28\,a^6b^2+35\,a^4b^4-20\,a^2b^6)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6b^5-3\,a^4b^7+3\,a^2b^9-b^{11})\sqrt{-a^2+b^2}}+\frac{18\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-42\,a^8b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-24\,a^7b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+117\,a^6b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-24\,a^5b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,a^4b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+60\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-36\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+152\,a^7b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-236\,a^5b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+42\,a^8b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,a^7b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-117\,a^6b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,a^5b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,a^4b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+60\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6b^4-3\,a^4b^6+3\,a^2b^8-b^{10})\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^3}+\frac{12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^5}-\frac{12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^5}+\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 - 20*a^2*b^6)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*s
qrt(-a^2 + b^2)) + (18*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*a^8*b*tan(1/2*d*x +
1/2*c)^5 - 24*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*a^6*b^3*tan(1/2*d*x + 1/
2*c)^5 - 24*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*a^4*b^5*tan(1/2*d*x + 1/2*
c)^5 + 60*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*a^9*tan(1/2*d*x + 1/2*c)^3 +
152*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 236*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 1
20*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a^9*tan(1/2*d*x + 1/2*c) + 42*a^8*b*
tan(1/2*d*x + 1/2*c) - 24*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*a^6*b^3*tan(1/
2*d*x + 1/2*c) - 24*a^5*b^4*tan(1/2*d*x + 1/2*c) + 105*a^4*b^5*tan(1/2*d*x
+ 1/2*c) + 60*a^3*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b
^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3)
 + 12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5 - 12*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/b^5 + 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^
4))/d
```

Mupad [B]
time = 10.36, size = 2500, normalized size = 7.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(12*a^7*b - 72*a^8 - 18*b^8 + 72*a^2*b^6 + 60*a^3*b^
5 - 273*a^4*b^4 - 47*a^5*b^3 + 236*a^6*b^2))/(3*b^4*(a + b)^2*(a - b)^3) +
(tan(c/2 + (d*x)/2)^5*(12*a^7*b + 72*a^8 + 18*b^8 - 72*a^2*b^6 + 60*a^3*b^5
 + 273*a^4*b^4 - 47*a^5*b^3 - 236*a^6*b^2))/(3*b^4*(a + b)^3*(a - b)^2) - (
tan(c/2 + (d*x)/2)*(2*a*b^6 - 4*a^6*b - 8*a^7 + 2*b^7 - 6*a^2*b^5 - 26*a^3*
b^4 + 11*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)*(a - b)^3) + (tan(c/2 + (d*x)/
2)^7*(2*a*b^6 + 4*a^6*b - 8*a^7 - 2*b^7 + 6*a^2*b^5 - 26*a^3*b^4 - 11*a^4*b
^3 + 24*a^5*b^2))/(b^4*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2
+ (d*x)/2)^4*(6*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4*a^3 - 2*
b^3) - tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3 + tan(c/2
 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (a*atan(((a*((8*tan(c/2 +
 (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12
 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7
 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*
b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14
- 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8
) - (4*a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*
b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b
^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b
^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 1
```

0*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) - (32*a*tan(c
/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*
b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10
*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/(b^5*(a*b
^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b
^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8))))/b^5)*
4i)/b^5 + (a*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 -
128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2
025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4
+ 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 +
10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*
b^10 - a^10*b^9 - a^11*b^8) + (4*a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^
21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16
+ 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 -
8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*
a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13
- a^11*b^12) + (32*a*tan(c/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^2
1 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16
+ 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11
- 8*a^14*b^10))/(b^5*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^1
5 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^1
0*b^9 - a^11*b^8))))/b^5)*4i)/b^5)/((32*(128*a^16 - 64*a^15*b + 320*a^4*b^1
2 + 480*a^5*b^11 - 1520*a^6*b^10 - 1280*a^7*b^9 + 3088*a^8*b^8 + 1602*a^9*b
^7 - 3472*a^10*b^6 - 1088*a^11*b^5 + 2288*a^12*b^4 + 400*a^13*b^3 - 832*a^1
4*b^2))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^1
8 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*
b^12) - (4*a*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 -
128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2
025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4
+ 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 +
10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*
b^10 - a^10*b^9 - a^11*b^8) - (4*a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^
21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16
+ 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 -
8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*
a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13
- a^11*b^12) - (32*a*tan(c/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^2
1 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16
+ 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11
- 8*a^14*b^10))/(b^5*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^1
5 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^1
0*b^9 - a^11*b^8))))/b^5))/b^5 + (4*a*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 12
8*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^
6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*
a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5

```
*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b
^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11...
```

3.515 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=259

$$\frac{\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{a(2a^6-7a^4b^2+8a^2b^4-8b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d}-\frac{a^2\sec^2(c+dx)\tan(c}{3b\,(a^2-b^2)\,d(a+b\sec(c}$$

```
[Out] arctanh(sin(d*x+c))/b^4/d-a*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)*arctanh((a-b)
^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/2)/d-1/3*a^
2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*a^3*(3*a^2-8
*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*a^2*(9*a^4-28*a^2
*b^2+34*b^4)*tan(d*x+c)/b^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.52, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3930, 4175, 4165, 4083, 3855, 3916, 2738, 214}

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^3}-\frac{a^2(9a^4-28a^2b^2+34b^4)\tan(c+dx)}{6b^3d\,(a^2-b^2)^3\,(a+b\sec(c+dx))}+\frac{a^3(3a^2-8b^2)\tan(c+dx)}{6b^3d\,(a^2-b^2)^2\,(a+b\sec(c+dx))^2}-\frac{a(2a^6-7a^4b^2+8a^2b^4-8b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\tanh^{-1}(\sin(c+dx))}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^4*d) - (a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*
ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a
+ b)^(7/2)*d) - (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b
*Sec[c + d*x])^3) + (a^3*(3*a^2 - 8*b^2)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^2
*d*(a + b*Sec[c + d*x])^2) - (a^2*(9*a^4 - 28*a^2*b^2 + 34*b^4)*Tan[c + d*x
])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4165

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)),
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m +
 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2
, 0]
```

Rule 4175

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b*
B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
```

```
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx &= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec^2(c+dx)\left(2a^2-3ab\sec(c+dx)-3\left(a^2-b^2\right)\sec^2(c+dx\right.}{(a+b\sec(c+dx))^3}}{3b\left(a^2-b^2\right)} \\
&= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} - \frac{\int \text{se}}{} \\
&= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} - \frac{a^2(}{6b} \\
&= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} - \frac{a^2(}{6b} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\text{ta}}{6b^3\left(a^2-b^2\right)^2d(a+} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\text{ta}}{6b^3\left(a^2-b^2\right)^2d(a+} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a+b}}\right.}{(a-b)^{7/2}b^4(a+b)^{7/2}d}
\end{aligned}
$$

Mathematica [A]
time = 4.24, size = 250, normalized size = 0.97

$$
\frac{\frac{6a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} - 6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{a^2b\left(11a^4b^2-32a^2b^4+36b^6+15ab\left(a^4-3a^2b^2+4b^4\right)\cos(c+dx)+a^2\left(6a^4-17a^2b^2+26b^4\right)\cos^2(c+dx)\right)\sin(c+dx)}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6b^4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((6*a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[((-a + b)*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 6*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(11*a^4*
b^2 - 32*a^2*b^4 + 36*b^6 + 15*a*b*(a^4 - 3*a^2*b^2 + 4*b^4)*Cos[c + d*x] +
 a^2*(6*a^4 - 17*a^2*b^2 + 26*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^3
*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*b^4*d)
```

Maple [A]
time = 0.83, size = 383, normalized size = 1.48 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^4*ln(tan(1/2*d*x+1/2*c)+1)+2*a/b^4*((1/2*(2*a^4-a^3*b-6*a^2*b^2+4*
a*b^3+12*b^4)*b*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*
(3*a^4-11*a^2*b^2+18*b^4)*b*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1
/2*c)^3+1/2*(2*a^4+a^3*b-6*a^2*b^2-4*a*b^3+12*b^4)*b*a/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c
)^2-a-b)^3-1/2*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b
^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
)))-1/b^4*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(244) = 488$.
time = 7.20, size = 1822, normalized size = 7.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9 + (2*a^10 - 7*a^8*b^2
 + 8*a^6*b^4 - 8*a^4*b^6)*cos(d*x + c)^3 + 3*(2*a^9*b - 7*a^7*b^3 + 8*a^5*b
^5 - 8*a^3*b^7)*cos(d*x + c)^2 + 3*(2*a^8*b^2 - 7*a^6*b^4 + 8*a^4*b^6 - 8*a
^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^
2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 6*(a^8*b^3 -
 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
 4*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
4*a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
```

```
4*a^3*b^8 + a*b^10)*cos(d*x + c))*log(sin(d*x + c) + 1) - 6*(a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^
5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4
*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3
*b^8 + a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(11*a^8*b^3 - 43*a^
6*b^5 + 68*a^4*b^7 - 36*a^2*b^9 + (6*a^10*b - 23*a^8*b^3 + 43*a^6*b^5 - 26*
a^4*b^7)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3*b^8)*
cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10
 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4
*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 -
4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4
*a^2*b^13 + b^15)*d), -1/6*(3*(2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9
+ (2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6)*cos(d*x + c)^3 + 3*(2*a^9*b
- 7*a^7*b^3 + 8*a^5*b^5 - 8*a^3*b^7)*cos(d*x + c)^2 + 3*(2*a^8*b^2 - 7*a^6*
b^4 + 8*a^4*b^6 - 8*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*(a^8*b^3 - 4
*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4
*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*
a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*
a^3*b^8 + a*b^10)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3*(a^8*b^3 - 4*a^6*
b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*
b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b
^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b
^8 + a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (11*a^8*b^3 - 43*a^6*b^
5 + 68*a^4*b^7 - 36*a^2*b^9 + (6*a^10*b - 23*a^8*b^3 + 43*a^6*b^5 - 26*a^4*
b^7)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3*b^8)*cos(
d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a
^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^1
1 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^
3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2
*b^13 + b^15)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(244) = 488.

time = 0.55, size = 559, normalized size = 2.16

$$\frac{\frac{3\left(2a^{7}-7a^{5}b^{2}+8a^{3}b^{4}-8ab^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan(\frac{1}{2}dx+\frac{1}{2}c)-b\tan(\frac{1}{2}dx+\frac{1}{2}c)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{6}b^{4}-3a^{4}b^{6}+3a^{2}b^{8}-b^{10}\right)\sqrt{-a^{2}+b^{2}}}+\frac{6a^{8}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{5}-15a^{7}b\tan(\frac{1}{2}dx+\frac{1}{2}c)^{5}-6a^{6}b^{2}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{5}+45a^{5}b^{3}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{5}-6a^{4}b^{4}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{5}-60a^{3}b^{5}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{5}+36a^{2}b^{6}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{5}-12a^{8}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{3}+56a^{6}b^{2}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{3}-116a^{4}b^{4}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{3}+72a^{2}b^{6}\tan(\frac{1}{2}dx+\frac{1}{2}c)^{3}+6a^{8}\tan(\frac{1}{2}dx+\frac{1}{2}c)+15a^{7}b\tan(\frac{1}{2}dx+\frac{1}{2}c)-6a^{6}b^{2}\tan(\frac{1}{2}dx+\frac{1}{2}c)-45a^{5}b^{3}\tan(\frac{1}{2}dx+\frac{1}{2}c)-6a^{4}b^{4}\tan(\frac{1}{2}dx+\frac{1}{2}c)+60a^{3}b^{5}\tan(\frac{1}{2}dx+\frac{1}{2}c)+36a^{2}b^{6}\tan(\frac{1}{2}dx+\frac{1}{2}c)}{\left(a^{6}b^{3}-3a^{4}b^{5}+3a^{2}b^{7}-b^{9}\right)\left(a\tan(\frac{1}{2}dx+\frac{1}{2}c)^{2}-b\tan(\frac{1}{2}dx+\frac{1}{2}c)^{2}-a-b\right)^{3}}+\frac{3\log\left(\left|\tan(\frac{1}{2}dx+\frac{1}{2}c)+1\right|\right)}{b^{4}}-\frac{3\log\left(\left|\tan(\frac{1}{2}dx+\frac{1}{2}c)-1\right|\right)}{b^{4}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^7 - 7*a^5*b^2 + 8*a^3*b^4 - 8*a*b^6)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(-a
^2 + b^2)) + (6*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^
5 - 6*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 45*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 -
6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 60*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a
^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 12*a^8*tan(1/2*d*x + 1/2*c)^3 + 56*a^6*b^2*
tan(1/2*d*x + 1/2*c)^3 - 116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 72*a^2*b^6*ta
n(1/2*d*x + 1/2*c)^3 + 6*a^8*tan(1/2*d*x + 1/2*c) + 15*a^7*b*tan(1/2*d*x +
1/2*c) - 6*a^6*b^2*tan(1/2*d*x + 1/2*c) - 45*a^5*b^3*tan(1/2*d*x + 1/2*c) -
 6*a^4*b^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b^5*tan(1/2*d*x + 1/2*c) + 36*a^2*
b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*log(abs(tan(1/
2*d*x + 1/2*c) + 1))/b^4 - 3*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4)/d
```

Mupad [B]
time = 12.45, size = 2500, normalized size = 9.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^4),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^5*(2*a^6 - a^5*b + 12*a^2*b^4 + 4*a^3*b^3 - 6*a^4*b^
2))/((a*b^3 - b^4)*(a + b)^3) - (4*tan(c/2 + (d*x)/2)^3*(3*a^6 + 18*a^2*b^4
 - 11*a^4*b^2))/(3*(a + b)^2*(b^5 - 2*a*b^4 + a^2*b^3)) + (tan(c/2 + (d*x)/
2)*(a^5*b + 2*a^6 + 12*a^2*b^4 - 4*a^3*b^3 - 6*a^4*b^2))/((a + b)*(3*a*b^5
- b^6 - 3*a^2*b^4 + a^3*b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b
- 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3)
 + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b
+ a^3 - b^3))) - (atan((((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^
18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13
+ 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*
b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*
b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (8*t
an(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*
a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*
a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/(b^4*(a
```

*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6
*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))/b^4 -
(8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12
+ 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164
*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16
+ b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11
- 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))*1i)/b^4 - ((
((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5
*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^
11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 -
5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8
*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (8*tan(c/2 + (d*x)/2)*(8*a*b^2
1 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 -
160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 +
48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15
- 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^
8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))/b^4 + (8*tan(c/2 + (d*x)/2)*(8*a
^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^1
0 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 1
17*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a
^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9
+ 5*a^9*b^8 - a^10*b^7 - a^11*b^6))*1i)/b^4)/((((8*(16*a*b^20 - 4*b^21 + 1
2*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a
^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*
b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 +
10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b
^10 - a^11*b^9) - (8*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^1
9 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14
+ 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9
- 8*a^14*b^8))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13
+ 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^
7 - a^11*b^6)))/b^4 - (8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 +
4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b
^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 -
48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*
a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a
^11*b^6))/b^4 + (((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a
^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*
b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^
20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10
*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (8*tan(c/2 +
(d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17
- 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12
- 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/(b^4*(a*b^16 + b
^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 1

```
0*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))/b^4 + (8*tan(c/
2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*
b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6
- 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 -
 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7
*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))/b^4 - (16*(16*a*b^12
- 2*a^12*b + 4*a^13 + 48*a^2*b^11 - 64*a^3*b^10...
```

3.516 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4} dx$

Optimal. Leaf size=222

$$-\frac{b(3a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{a^2\sec(c+dx)\tan(c+dx)}{3b(a^2-b^2)\,d(a+b\sec(c+dx))^3}-\frac{a^2(2a^2-7b^2)\tan(c+ \ldots}{6b^2(a^2-b^2)^2\,d(a+b\sec(c \ldots}$$

```
[Out] -b*(3*a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^
(7/2)/(a+b)^(7/2)/d-1/3*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*
x+c))^3-1/6*a^2*(2*a^2-7*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
^2+1/6*a*(2*a^4-5*a^2*b^2+18*b^4)*tan(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*sec(d*x
+c))
```

Rubi [A]
time = 0.30, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3930, 4165, 4088, 12, 3916, 2738, 214}

$$-\frac{b(3a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2(2a^2-7b^2)\tan(c+dx)}{6b^2d\,(a^2-b^2)^2\,(a+b\sec(c+dx))^2}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^3}+\frac{a(2a^4-5a^2b^2+18b^4)\tan(c+dx)}{6b^2d\,(a^2-b^2)^3\,(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((b*(3*a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
(a - b)^(7/2)*(a + b)^(7/2)*d)) - (a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^3) - (a^2*(2*a^2 - 7*b^2)*Tan[c + d*x])/(6*b
^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4
)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4165

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)),
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m +
 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)\left(a^2-3ab\sec(c+dx)-(2a^2-3b^2)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}}{3b\,(a^2-b^2)}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{a^2(2a^2-7b^2)\tan(c+dx)}{6b^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{\int \text{se}}{}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{a^2(2a^2-7b^2)\tan(c+dx)}{6b^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2}{6b^2}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{a^2(2a^2-7b^2)\tan(c+dx)}{6b^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2}{6b^2}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{a^2(2a^2-7b^2)\tan(c+dx)}{6b^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2}{6b^2}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{a^2(2a^2-7b^2)\tan(c+dx)}{6b^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2}{6b^2}$$

$$= -\frac{b(3a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx)}$$

Mathematica [A]
time = 1.06, size = 158, normalized size = 0.71

$$\frac{\frac{6b(3a^2+2b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{a\left(2a^4-5a^2b^2+18b^4+3ab(a^2+9b^2)\cos(c+dx)+a^2(4a^2+11b^2)\cos^2(c+dx)\right)\sin(c+dx)}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((6*b*(3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(a^2 - b^2)^(7/2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4 + 3*a*b*(a^2 + 9*b^2)*C
os[c + d*x] + a^2*(4*a^2 + 11*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^3
*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

Maple [A]
time = 0.40, size = 285, normalized size = 1.28

method	result

derivativedivides	$\frac{\frac{-\frac{(2a^2+3ba+6b^2)a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{4(a^2+9b^2)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ba+b^2)(a^2-2ba+b^2)}-\frac{(2a^2-3ba+6b^2)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^3-3b\,a^2+3b^2a-b^3)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}-\frac{b(3a^2+2b^2)\operatorname{arctanh}\left(\frac{}{\sqrt{(}}\right.}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{}}}{d}$
default	$\frac{\frac{-\frac{(2a^2+3ba+6b^2)a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{4(a^2+9b^2)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ba+b^2)(a^2-2ba+b^2)}-\frac{(2a^2-3ba+6b^2)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^3-3b\,a^2+3b^2a-b^3)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}-\frac{b(3a^2+2b^2)\operatorname{arctanh}\left(\frac{}{\sqrt{(}}\right.}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{}}}{d}$
risch	$-\frac{i(9a^4b\,e^{5i(dx+c)}+6a^2b^3e^{5i(dx+c)}+45a^3b^2e^{4i(dx+c)}+30a\,b^4e^{4i(dx+c)}+24a^4b\,e^{3i(dx+c)}+82a^2b^3e^{3i(dx+c)}+44b^5e^{3i(dx-}}{3(-a^2+b^2)^3d(a\,e^{2i(dx+c)}+2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2*(2*a^2+3*a*b+6*b^2)*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5+2/3*(a^2+9*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3-1/2*(2*a^2-3*a*b+6*b^2)*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^3-b*(3*a^2+
2*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(207) = 414$.
time = 3.46, size = 903, normalized size = 4.07

$$\left[-\frac{3\left(3a^2b^4+2b^6+(3a^5b+2a^3b^3)\cos(dx+c)^3+3(3a^4b^2+2a^2b^4)\cos(dx+c)^2+3(3a^3b^3+2ab^5)\cos(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)-2\left(2a^7-7a^5b^2+23a^3b^4-18ab^6+(4a^7+7a^5b^2-11a^3b^4)\cos(dx+c)^2+3(a^6b+8a^4b^3-9a^2b^5)\cos(dx+c)\right)\sin(dx+c)}{12\left((a^{11}-4a^9b^2+6a^7b^4-4a^5b^6+a^3b^8)d\cos(dx+c)^3+3(a^{10}b-4a^8b^3+6a^6b^5-4a^4b^7+a^2b^9)d\cos(dx+c)^2+3(a^9b^2-4a^7b^4+6a^5b^6-4a^3b^8+ab^{10})d\cos(dx+c)+(a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11})d\right)},\ -\frac{3\left(3a^2b^4+2b^6+(3a^5b+2a^3b^3)\cos(dx+c)^3+3(3a^4b^2+2a^2b^4)\cos(dx+c)^2+3(3a^3b^3+2ab^5)\cos(dx+c)\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-\left(2a^7-7a^5b^2+23a^3b^4-18ab^6+(4a^7+7a^5b^2-11a^3b^4)\cos(dx+c)^2+3(a^6b+8a^4b^3-9a^2b^5)\cos(dx+c)\right)\sin(dx+c)}{6\left((a^{11}-4a^9b^2+6a^7b^4-4a^5b^6+a^3b^8)d\cos(dx+c)^3+3(a^{10}b-4a^8b^3+6a^6b^5-4a^4b^7+a^2b^9)d\cos(dx+c)^2+3(a^9b^2-4a^7b^4+6a^5b^6-4a^3b^8+ab^{10})d\cos(dx+c)+(a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11})d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a^2*b^4 + 2*b^6 + (3*a^5*b + 2*a^3*b^3)*cos(d*x + c)^3 + 3*(3*
a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2
*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^7 - 7*a^5*b^2 + 23*a^3*b^4
 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b +
8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a
^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*
a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a
^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(3*a^2*b^4 + 2*b^6 + (3*a^5*b + 2*a^
3*b^3)*cos(d*x + c)^3 + 3*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 3*(3*a^3
*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*
cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7 - 7*a^5*b^2 + 23*a^3
*b^4 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*
b + 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 +
 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3
+ 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^
4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 +
 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**4, x)
```

Giac [A]
time = 0.52, size = 403, normalized size = 1.82

$$-\frac{\frac{3\,(3\,a^2b+2\,b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-32\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+27\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(3*a^2*b + 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan(1/
2*d*x + 1/2*c)^5 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b^2*tan(1/2*d*x +
 1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*
```

```
c)^5 - 4*a^5*tan(1/2*d*x + 1/2*c)^3 - 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 3
6*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*d*x + 1/2*c) + 3*a^4*b*tan(1
/2*d*x + 1/2*c) + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*a^2*b^3*tan(1/2*d*x +
 1/2*c) + 18*a*b^4*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

Mupad [B]
time = 4.41, size = 378, normalized size = 1.70

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^3+3\,a^2\,b+6\,a\,b^2\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}-\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(a^3+9\,a\,b^2\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3-3\,a^2\,b+6\,a\,b^2\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a^2\,b+a^3+b^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}-\frac{b\,\mathrm{atanh}\left(\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2+2\,b^2\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\left(3\,a^2\,b+2\,b^3\right)\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(3\,a^2+2\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 3*a^2*b + 2*a^3))/((a + b)^3*(a - b)) - (
4*tan(c/2 + (d*x)/2)^3*(9*a*b^2 + a^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) +
 (tan(c/2 + (d*x)/2)*(6*a*b^2 - 3*a^2*b + 2*a^3))/((a + b)*(3*a*b^2 - 3*a^2
*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b
^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 +
3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))
) - (b*atanh((b*tan(c/2 + (d*x)/2)*(3*a^2 + 2*b^2)*(2*a - 2*b)*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))/(2*(3*a^2*b + 2*b^3)*(a + b)^(1/2)*(a - b)^(7/2)))*(3*
a^2 + 2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.517 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=206

$$\frac{a(a^2+4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3}+\frac{a(a^2-6b^2)\tan(c+dx)}{6b\,(a^2-b^2)^2\,d(a+b\sec(c+ ...}$$

```
[Out] a*(a^2+4*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/
2)/(a+b)^(7/2)/d-1/3*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*a*
(a^2-6*b^2)*tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*(a^4-10*a^2*b
^2-6*b^4)*tan(d*x+c)/b/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.24, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3924, 4088, 12, 3916, 2738, 214}

$$\frac{a(a^2+4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\tan(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^3}+\frac{a(a^2-6b^2)\tan(c+dx)}{6bd\,(a^2-b^2)^2\,(a+b\sec(c+dx))^2}+\frac{(a^4-10a^2b^2-6b^4)\tan(c+dx)}{6bd\,(a^2-b^2)^3\,(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*(a^2 + 4*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a -
 b)^(7/2)*(a + b)^(7/2)*d) - (a^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*S
ec[c + d*x])^3) + (a*(a^2 - 6*b^2)*Tan[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a +
b*Sec[c + d*x])^2) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Tan[c + d*x])/(6*b*(a^2 -
b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3924

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)\left(-3ab-(a^2-3b^2)\sec(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3b\,(a^2-b^2)}$$

$$= -\frac{a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2-6b^2)\tan(c+dx)}{6b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \int \sec$$

$$= -\frac{a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2-6b^2)\tan(c+dx)}{6b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{(a^4}{6b\,(a}$$

$$= -\frac{a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2-6b^2)\tan(c+dx)}{6b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{(a^4}{6b\,(a}$$

$$= -\frac{a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2-6b^2)\tan(c+dx)}{6b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{(a^4}{6b\,(a}$$

$$= -\frac{a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2-6b^2)\tan(c+dx)}{6b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{(a^4}{6b\,(a}$$

$$= \frac{a(a^2+4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3}$$

Mathematica [A]
time = 1.13, size = 165, normalized size = 0.80

$$\frac{-\frac{6a(a^2+4b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{(b(a^4-10a^2b^2-6b^4)+3a(a^4-9a^2b^2-2b^4)\cos(c+dx)-a^2b(13a^2+2b^2)\cos^2(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-6*a*(a^2 + 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(7/2) + ((b*(a^4 - 10*a^2*b^2 - 6*b^4) + 3*a*(a^4 - 9*a^2*b^2 -
 2*b^4)*Cos[c + d*x] - a^2*b*(13*a^2 + 2*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])
/((a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

Maple [A]
time = 0.37, size = 294, normalized size = 1.43

method	result

derivativedivides	$\frac{-\frac{2\left(-\frac{(a^3+6b\,a^2+2b^2a+2b^3)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{2(7a^2+3b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ba+b^2)(a^2-2ba+b^2)}+\frac{(a^3-6b\,a^2+2b^2a-2b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3b\,a^2+3b^2a-b^3)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}+\frac{a(a^2+4}{(a^6-3a}}{d}$
default	$\frac{-\frac{2\left(-\frac{(a^3+6b\,a^2+2b^2a+2b^3)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{2(7a^2+3b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ba+b^2)(a^2-2ba+b^2)}+\frac{(a^3-6b\,a^2+2b^2a-2b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3b\,a^2+3b^2a-b^3)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}+\frac{a(a^2+4}{(a^6-3a}}{d}$
risch	$\frac{i(3a^6e^{5i(dx+c)}+12a^4b^2e^{5i(dx+c)}+15a^5b\,e^{4i(dx+c)}+60a^3b^3e^{4i(dx+c)}+78a^4b^2e^{3i(dx+c)}+64a^2b^4e^{3i(dx+c)}+8b^6e^{3i(dx+c)}}{3a(-a^2+b^2)^3d(a\,e^{2i(dx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(a^3+6*a^2*b+2*a*b^2+2*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5+2/3*(7*a^2+3*b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3+1/2*(a^3-6*a^2*b+2*a*b^2-2*b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a
-b)^3+a*(a^2+4*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(191) = 382$.
time = 2.82, size = 902, normalized size = 4.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^3*b^3 + 4*a*b^5 + (a^6 + 4*a^4*b^2)*cos(d*x + c)^3 + 3*(a^5*b
+ 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c))*sqrt(a^
2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^
2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)
^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6*b^7
 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 + 3*(a^7 - 10*a^5*b^2
 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*
a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6
*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*
a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(a^3*b^3 + 4*a*b^5 + (a^6 + 4*a^4*b^
2)*cos(d*x + c)^3 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a^4*b^2 + 4*a
^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6
*b^7 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 + 3*(a^7 - 10*a^5
*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2
+ 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3
 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b
^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5
+ 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(191) = 382.
time = 0.58, size = 431, normalized size = 2.09

$$-\frac{\frac{3\,(a^3+4\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}-\frac{3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-28\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+27\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 4*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^5*tan(1/2*d*
x + 1/2*c)^5 + 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1
```

```
/2*c)^5 + 12*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^
5 + 6*b^5*tan(1/2*d*x + 1/2*c)^5 - 28*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*a^2
*b^3*tan(1/2*d*x + 1/2*c)^3 + 12*b^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^5*tan(1/2
*d*x + 1/2*c) + 12*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 12*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^4*tan(1/2*d*x + 1/2*c) + 6*b
^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

Mupad [B]
time = 4.38, size = 380, normalized size = 1.84

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(7\,a^2\,b+3\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(a^3+6\,a^2\,b+2\,a\,b^2+2\,b^3\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^3-6\,a^2\,b+2\,a\,b^2-2\,b^3\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a^2\,b+a^3+b^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}+\frac{a\,\mathrm{atanh}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+4\,b^2\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}\,\left(a^3+4\,a\,b^2\right)}\right)\,\left(a^2+4\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^3*(7*a^2*b + 3*b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2
)) - (tan(c/2 + (d*x)/2)^5*(2*a*b^2 + 6*a^2*b + a^3 + 2*b^3))/((a + b)^3*(a
 - b)) + (tan(c/2 + (d*x)/2)*(2*a*b^2 - 6*a^2*b + a^3 - 2*b^3))/((a + b)*(3
*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b
 - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3
) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b
 + a^3 - b^3))) + (a*atanh((a*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a - 2*b)*
(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*a*b^2 +
a^3)))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.518 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=192

$$-\frac{b(4a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}+\frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}+\frac{(2a^2+3b^2)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+d}$$

```
[Out] -b*(4*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7
/2)/(a+b)^(7/2)/d+1/3*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*(2*a^
2+3*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*a*(2*a^2+13*b^2)*t
an(d*x+c)/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3921, 4088, 12, 3916, 2738, 214}

$$-\frac{b(4a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a(2a^2+13b^2)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{(2a^2+3b^2)\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((b*(4*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(7/2)*(a + b)^(7/2)*d)) + (a*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Se
c[c + d*x])^3) + ((2*a^2 + 3*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*S
ec[c + d*x])^2) + (a*(2*a^2 + 13*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
 b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{a\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)(-3b+2a\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx}{3(a^2-b^2)}$$

$$= \frac{a\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2+3b^2)\tan(c+dx)}{6(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} - \frac{\int \sec(c+d}{}$$

$$= \frac{a\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2+3b^2)\tan(c+dx)}{6(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2a^2}{6(a^2-b}$$

$$= \frac{a\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2+3b^2)\tan(c+dx)}{6(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2a^2}{6(a^2-b}$$

$$= \frac{a\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2+3b^2)\tan(c+dx)}{6(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2a^2}{6(a^2-b}$$

$$= \frac{a\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2+3b^2)\tan(c+dx)}{6(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{a(2a^2}{6(a^2-b}$$

$$= -\frac{b(4a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3}$$

Mathematica [A]
time = 1.26, size = 164, normalized size = 0.85

$$\frac{\frac{6b(4a^2+b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{(2a^3b^2+13ab^4-3b(-2a^4-9a^2b^2+b^4)\cos(c+dx)+a(6a^4+10a^2b^2-b^4)\cos^2(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((6*b*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + ((2*a^3*b^2 + 13*a*b^4 - 3*b*(-2*a^4 - 9*a^2*b^2 + b^4)*
Cos[c + d*x] + a*(6*a^4 + 10*a^2*b^2 - b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/(
(a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

Maple [A]
time = 0.30, size = 297, normalized size = 1.55

method	result

derivativedivides	$\frac{\frac{-\frac{(2a^3+2b\,a^2+6b^2a+b^3)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{4(3a^2+7b^2)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ba+b^2)(a^2-2ba+b^2)}-\frac{(2a^3-2b\,a^2+6b^2a-b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^3-3b\,a^2+3b^2a-b^3)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}-\frac{b(4a^2+b^2)\,\mathrm{arc}}{(a^6-3a^4b^2+3a}}{d}$
default	$\frac{\frac{-\frac{(2a^3+2b\,a^2+6b^2a+b^3)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{4(3a^2+7b^2)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ba+b^2)(a^2-2ba+b^2)}-\frac{(2a^3-2b\,a^2+6b^2a-b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^3-3b\,a^2+3b^2a-b^3)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}-\frac{b(4a^2+b^2)\,\mathrm{arc}}{(a^6-3a^4b^2+3a}}{d}$
risch	$\frac{i(-12a^6b\,e^{5i(dx+c)}-3a^4b^3e^{5i(dx+c)}-6a^7e^{4i(dx+c)}-42a^5b^2e^{4i(dx+c)}-33a^3b^4e^{4i(dx+c)}+6a\,b^6e^{4i(dx+c)}-36a^6b\,e^{3i(dx+c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2*(2*a^3+2*a^2*b+6*a*b^2+b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
n(1/2*d*x+1/2*c)^5+2/3*(3*a^2+7*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3-1/2*(2*a^3-2*a^2*b+6*a*b^2-b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-
b)^3-b*(4*a^2+b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(177) = 354$.
time = 2.91, size = 901, normalized size = 4.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(4*a^2*b^4 + b^6 + (4*a^5*b + a^3*b^3)*cos(d*x + c)^3 + 3*(4*a^4*
b^2 + a^2*b^4)*cos(d*x + c)^2 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c))*sqrt(a^
2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^
2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)
^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6 + (6
*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7*a^4*
b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*
a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6
*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*
a^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(4*a^2*b^4 + b^6 + (4*a^5*b + a^3*b
^3)*cos(d*x + c)^3 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + 3*(4*a^3*b^3
+ a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
 + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6
 + (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7
*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2
 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^
3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*
b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5
 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(177) = 354.
time = 0.53, size = 431, normalized size = 2.24

$$-\frac{\frac{3\,(4\,a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+28\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+27\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(4*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan(1/2*
d*x + 1/2*c)^5 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x +
1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^4*tan(1/2*d*x + 1/2*c
```

```
)^5 + 3*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*a^5*tan(1/2*d*x + 1/2*c)^3 - 16*a^3
*b^2*tan(1/2*d*x + 1/2*c)^3 + 28*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1
/2*d*x + 1/2*c) + 6*a^4*b*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1
/2*c) + 27*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a*b^4*tan(1/2*d*x + 1/2*c) - 3
*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*
d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

Mupad [B]
time = 4.34, size = 382, normalized size = 1.99

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^3+2\,a^2\,b+6\,a\,b^2+b^3\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}-\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,a^3+7\,a\,b^2\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3-2\,a^2\,b+6\,a\,b^2-b^3\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a^2\,b+a^3+b^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}-\frac{b\,\mathrm{atanh}\left(\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a^2+b^2\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}\,\left(4\,a^2\,b+b^3\right)}\right)\,\left(4\,a^2+b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 2*a^2*b + 2*a^3 + b^3))/((a + b)^3*(a - b
)) - (4*tan(c/2 + (d*x)/2)^3*(7*a*b^2 + 3*a^3))/(3*(a + b)^2*(a^2 - 2*a*b +
 b^2)) + (tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^2*b + 2*a^3 - b^3))/((a + b)*(3
*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b
 - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3
) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b
 + a^3 - b^3))) - (b*atanh((b*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a - 2*b)*
(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*a^2*b +
b^3)))*(4*a^2 + b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.519 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=184

$$\frac{a(2a^2+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}-\frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\sec(c+d$$

```
[Out] a*(2*a^2+3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(
7/2)/(a+b)^(7/2)/d-1/3*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^3-5/6*a*b*
tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*tan(d*x+c)
/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,
Rules used = {3918, 4088, 12, 3916, 2738, 214}

$$\frac{a(2a^2+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b(11a^2+4b^2)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{5ab\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(7/2)*(a + b)^(7/2)*d) - (b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])^3) - (5*a*b*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]
)^2) - (b*(11*a^2 + 4*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Sec[c +
d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{b\tan(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3a+2b\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx}{3\,(a^2-b^2)}$$

$$= -\frac{b\tan(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c\text{-}}{}}{}$$

$$= -\frac{b\tan(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} - \frac{b(1}{6\,(a^2\,-}$$

$$= -\frac{b\tan(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} - \frac{b(1}{6\,(a^2\,-}$$

$$= -\frac{b\tan(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} - \frac{b(1}{6\,(a^2\,-}$$

$$= -\frac{b\tan(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} - \frac{b(1}{6\,(a^2\,-}$$

$$= \frac{a(2a^2+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\tan(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^3}$$

Mathematica [A]
time = 1.20, size = 163, normalized size = 0.89

$$-\frac{\frac{12a(2a^2+3b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b(18a^4+17a^2b^2+10b^4+6ab(9a^2+b^2)\cos(c+dx)+(18a^4-5a^2b^2+2b^4)\cos(2(c+dx)))\sin(c+dx)}{(b+a\cos(c+dx))^3}}{12(a-b)^3(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -1/12*((12*a*(2*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/Sqrt[a^2 - b^2] + (b*(18*a^4 + 17*a^2*b^2 + 10*b^4 + 6*a*b*(9*a^2 +
 b^2)*Cos[c + d*x] + (18*a^4 - 5*a^2*b^2 + 2*b^4)*Cos[2*(c + d*x)])*Sin[c +
 d*x])/(b + a*Cos[c + d*x])^3)/((a - b)^3*(a + b)^3*d)
```

Maple [A]
time = 0.28, size = 284, normalized size = 1.54

method	result

derivativedivides	$\frac{-\frac{2\left(-\frac{(6a^2+3ba+2b^2)b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{2(9a^2+b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ba+b^2)(a^2-2ba+b^2)}-\frac{(6a^2-3ba+2b^2)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3b\,a^2+3b^2a-b^3)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}+\frac{a(2a^2+3b^2)\operatorname{arctanh}}{(a^6-3a^4b^2+3a^2b^4-}}{d}$
default	$\frac{-\frac{2\left(-\frac{(6a^2+3ba+2b^2)b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{2(9a^2+b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ba+b^2)(a^2-2ba+b^2)}-\frac{(6a^2-3ba+2b^2)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3b\,a^2+3b^2a-b^3)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}+\frac{a(2a^2+3b^2)\operatorname{arctanh}}{(a^6-3a^4b^2+3a^2b^4-}}{d}$
risch	$\frac{ib(27a^6b\,e^{5i(dx+c)}-18a^4b^3e^{5i(dx+c)}+6a^2b^5e^{5i(dx+c)}+18a^7e^{4i(dx+c)}+81a^5b^2e^{4i(dx+c)}-36a^3b^4e^{4i(dx+c)}+12a\,b^6e^{4i(dx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^3+a*(2*a^2
+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(169) = 338.
time = 3.10, size = 905, normalized size = 4.92

$$\left[-\frac{3\left(2a^3b^3+3ab^5+(2a^6+3a^4b^2)\cos(dx+c)^3+3(2a^5b+3a^3b^3)\cos(dx+c)^2+3(2a^4b^2+3a^2b^4)\cos(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)+2\left(11a^4b^3-7a^2b^5-4b^7+(18a^6b-23a^4b^3+7a^2b^5-2b^7)\cos(dx+c)^2+3(9a^5b^2-8a^3b^4-ab^6)\cos(dx+c)\right)\sin(dx+c)}{12\left((a^{11}-4a^9b^2+6a^7b^4-4a^5b^6+a^3b^8)d\cos(dx+c)^3+3(a^{10}b-4a^8b^3+6a^6b^5-4a^4b^7+a^2b^9)d\cos(dx+c)^2+3(a^9b^2-4a^7b^4+6a^5b^6-4a^3b^8+ab^{10})d\cos(dx+c)+(a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11})d\right)},\ \frac{3\left(2a^3b^3+3ab^5+(2a^6+3a^4b^2)\cos(dx+c)^3+3(2a^5b+3a^3b^3)\cos(dx+c)^2+3(2a^4b^2+3a^2b^4)\cos(dx+c)\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-\left(11a^4b^3-7a^2b^5-4b^7+(18a^6b-23a^4b^3+7a^2b^5-2b^7)\cos(dx+c)^2+3(9a^5b^2-8a^3b^4-ab^6)\cos(dx+c)\right)\sin(dx+c)}{6\left((a^{11}-4a^9b^2+6a^7b^4-4a^5b^6+a^3b^8)d\cos(dx+c)^3+3(a^{10}b-4a^8b^3+6a^6b^5-4a^4b^7+a^2b^9)d\cos(dx+c)^2+3(a^9b^2-4a^7b^4+6a^5b^6-4a^3b^8+ab^{10})d\cos(dx+c)+(a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11})d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^3*b^3 + 3*a*b^5 + (2*a^6 + 3*a^4*b^2)*cos(d*x + c)^3 + 3*(2*
a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2
*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7
 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*(9*a^5*b^
2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a
^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*
a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a
^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(2*a^3*b^3 + 3*a*b^5 + (2*a^6 + 3*a^4
*b^2)*cos(d*x + c)^3 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 3*(2*a^4*b^
2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (11*a^4*b^3 - 7*a^2*b^5 - 4*
b^7 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*(9*a^5
*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 +
6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 +
 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4
 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 +
6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(169) = 338.
time = 0.52, size = 403, normalized size = 2.19

$$-\frac{\frac{3\,(2\,a^3+3\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}-\frac{18\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-36\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+32\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+27\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
+ arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (18*a^4*b*tan(1
/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^3*tan(1/2*d
```

```
*x + 1/2*c)^5 - 3*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*b^5*tan(1/2*d*x + 1/2*c)
^5 - 36*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 32*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 +
4*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*t
an(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x
+ 1/2*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

Mupad [B]
time = 4.32, size = 378, normalized size = 2.05

$$\frac{a\,\mathrm{atanh}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+3\,b^2\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\left(2\,a^3+3\,a\,b^2\right)\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(2\,a^2+3\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(6\,a^2\,b+3\,a\,b^2+2\,b^3\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(9\,a^2\,b+b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^2\,b-3\,a\,b^2+2\,b^3\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a^2\,b+a^3+b^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^4),x)
```

```
[Out] (a*atanh((a*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2
*b + a^3 - b^3))/(2*(3*a*b^2 + 2*a^3)*(a + b)^(1/2)*(a - b)^(7/2)))*(2*a^2
+ 3*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((tan(c/2 + (d*x)/2)^5*(3*a*b^2
 + 6*a^2*b + 2*b^3))/((a + b)^3*(a - b)) - (4*tan(c/2 + (d*x)/2)^3*(9*a^2*b
 + b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)*(6*a^2*b -
 3*a*b^2 + 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 +
 (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a
*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 +
 (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```

3.520 $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=242

$$\frac{x}{a^4}-\frac{b(8a^6-8a^4b^2+7a^2b^4-2b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d}+\frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}+\frac{b^2}{6a^2\left(a^2\right.}$$

```
[Out] x/a^4-b*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+1/3*b^2*tan(d*x+c)/a/(a^2-
b^2)/d/(a+b*sec(d*x+c))^3+1/6*b^2*(8*a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/
d/(a+b*sec(d*x+c))^2+1/6*b^2*(26*a^4-17*a^2*b^2+6*b^4)*tan(d*x+c)/a^3/(a^2-
b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.40, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3870, 4145, 4004, 3916, 2738, 214}

$$\frac{x}{a^4}+\frac{b^2(8a^2-3b^2)\tan(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{b(8a^6-8a^4b^2+7a^2b^4-2b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2(26a^4-17a^2b^2+6b^4)\tan(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-4),x]
```

```
[Out] x/a^4 - (b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d) + (b^2*Tan
[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (b^2*(8*a^2 - 3*b^2
)*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (b^2*(26*a
^4 - 17*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c
+ d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \frac{b^2\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{\int \frac{-3(a^2-b^2)+3ab\sec(c+dx)-2b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx}{3a\,(a^2-b^2)}$$

$$= \frac{b^2\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b^2(8a^2-3b^2)\tan(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{\int 6(a}{}$$

$$= \frac{b^2\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b^2(8a^2-3b^2)\tan(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{b^2(26}{6a^3}$$

$$= \frac{x}{a^4} + \frac{b^2\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b^2(8a^2-3b^2)\tan(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} +$$

$$= \frac{x}{a^4} + \frac{b^2\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b^2(8a^2-3b^2)\tan(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} +$$

$$= \frac{x}{a^4} + \frac{b^2\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b^2(8a^2-3b^2)\tan(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} +$$

$$= \frac{x}{a^4} - \frac{b(8a^6-8a^4b^2+7a^2b^4-2b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d} + \frac{b^2}{3a\,(a^2-b^2}$$

Mathematica [A]
time = 1.50, size = 268, normalized size = 1.11

$$\frac{(b+a\cos(c+dx))\sec^4(c+dx)\left(6(c+dx)(b+a\cos(c+dx))^3 - \frac{6b(-8a^6+8a^4b^2-7a^2b^4+2b^6)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^3}{(a^2-b^2)^{7/2}} + \frac{2ab^4\sin(c+dx)}{(a-b)(a+b)} - \frac{ab^3(12a^2-7b^2)(b+a\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2} + \frac{ab^2(36a^4-32a^2b^2+11b^4)(b+a\cos(c+dx))^2\sin(c+dx)}{(a-b)^3(a+b)^3}\right)}{6a^4d(a+b\sec(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-4),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(6*(c + d*x)*(b + a*Cos[c + d*x])^3 -
(6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTanh[((-a + b)*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2) + (2*a*b^
4*Sin[c + d*x])/((a - b)*(a + b)) - (a*b^3*(12*a^2 - 7*b^2)*(b + a*Cos[c +
d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) + (a*b^2*(36*a^4 - 32*a^2*b^2 + 1
1*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a + b)^3)))/(6*a^4*
d*(a + b*Sec[c + d*x])^4)
```

Maple [A]
time = 0.24, size = 365, normalized size = 1.51 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^4*arctan(tan(1/2*d*x+1/2*c))+2*b/a^4*((-1/2*(12*a^4+4*a^3*b-6*a^2*
b^2-a*b^3+2*b^4)*b*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2
/3*(18*a^4-11*a^2*b^2+3*b^4)*b*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3-1/2*(12*a^4-4*a^3*b-6*a^2*b^2+a*b^3+2*b^4)*b*a/(a+b)/(a^3-3*a^2*
b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/
2*c)^2-a-b)^3-1/2*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(227) = 454$.
time = 3.10, size = 1456, normalized size = 6.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos(d*x
 + c)^3 + 36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(
d*x + c)^2 + 36*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*
cos(d*x + c) + 12*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*x
 + 3*(8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10 + (8*a^9*b - 8*a^7*b^3 + 7*
a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^3 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6
 - 2*a^2*b^8)*cos(d*x + c)^2 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*a^3*b^7 - 2*a*b^
9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(26*a^7*b^4 - 4
3*a^5*b^6 + 23*a^3*b^8 - 6*a*b^10 + (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 -
 11*a^3*b^8)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b
^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^
```

```
6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8
*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4
*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 -
4*a^6*b^9 + a^4*b^11)*d), 1/6*(6*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6
+ a^3*b^8)*d*x*cos(d*x + c)^3 + 18*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*
b^7 + a^2*b^9)*d*x*cos(d*x + c)^2 + 18*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4
*a^3*b^8 + a*b^10)*d*x*cos(d*x + c) + 6*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 -
4*a^2*b^9 + b^11)*d*x - 3*(8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10 + (8*
a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^3 + 3*(8*a^8*b^2 -
8*a^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c)^2 + 3*(8*a^7*b^3 - 8*a^5*b^
5 + 7*a^3*b^7 - 2*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
 b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (26*a^7*b^4 - 43*a
^5*b^6 + 23*a^3*b^8 - 6*a*b^10 + (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 - 11
*a^3*b^8)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)
*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 +
 a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^
7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^
7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a
^6*b^9 + a^4*b^11)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(227) = 454.
time = 0.47, size = 532, normalized size = 2.20

$$\frac{\frac{3\left(8a^6b-8a^4b^3+7a^2b^5-2b^7\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^{10}-3a^8b^2+3a^6b^4-a^4b^6\right)\sqrt{-a^2+b^2}}-\frac{36a^6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-60a^5b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^4b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+45a^3b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^2b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15ab^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6b^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-72a^6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+116a^4b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-56a^2b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12b^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+36a^6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+60a^5b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6a^4b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-45a^3b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6a^2b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+15ab^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6b^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a^9-3a^7b^2+3a^5b^4-a^3b^6\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)^3}+\frac{3(dx+c)}{a^4}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a
^2 + b^2)) - (36*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*b^3*tan(1/2*d*x +
1/2*c)^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*b^5*tan(1/2*d*x + 1/2*
c)^5 - 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a*b^7*tan(1/2*d*x + 1/2*c)^5 +
```

```
6*b^8*tan(1/2*d*x + 1/2*c)^5 - 72*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 116*a^4
*b^4*tan(1/2*d*x + 1/2*c)^3 - 56*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 12*b^8*ta
n(1/2*d*x + 1/2*c)^3 + 36*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*a^5*b^3*tan(1/2
*d*x + 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*a^3*b^5*tan(1/2*d*x + 1
/2*c) - 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*a*b^7*tan(1/2*d*x + 1/2*c) + 6*
b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*(d*x + c)/a^4)
/d
```

Mupad [B]
time = 12.79, size = 2500, normalized size = 10.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^4,x)
```

```
[Out] (2*atan((((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11
 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 +
110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a
^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 +
 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (tan(c/2 + (d*x)/2)
*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^
12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a
^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b + a^17 -
 a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 +
10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4 + (8*tan(c/2
 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b
^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6
- 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 -
a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 1
0*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))/a^4 - ((((8*(16*a^20*b
 - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12
*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^
4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a
^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^
4 - 5*a^17*b^3 - 5*a^18*b^2) + (tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 +
 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 +
160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 4
8*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*
a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4
 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4 - (8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a
^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*
a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b
^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a
```

^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))/a^4)/((((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4 + (8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))*1i)/a^4 + (((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4 - (8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))*1i)/a^4 + (16*(16*a^12*b - 2*a*b^12 + 4*b^13 - 26*a^2*b^11 + 11*a^3*b^10 + 70*a^4*b^9 - 34*a^5*b^8 - 110*a^6*b^7 + 66*a^7*b^6 + 110*a^8*b^5 - 64*a^9*b^4 - 64*a^10*b^3 + 48*a^11*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2))))/(a^4*d) - ((tan(c/2 + (d*x)/2)^5*(2*b^6 - a*b^5 - 6*a^2*b^4 + 4*a^3*b^3 + 12*a^4*b^2))/((a^3*b - a^4)*(a + b)^3) + (4*tan(c/2 + (d*x)/2)^3*(3*b^6 - 11*a^2*b^4 + 18*a^4*b^2))/(3*(a + b)^2*(a^5 - 2*a^4*b + a^3*b^2)) + (tan(c/2 + (d*x)/2)*(a*b^5 + 2*b^6 - 6*...

3.521 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=299

$$-\frac{4bx}{a^5}+\frac{b^2(20a^6-35a^4b^2+28a^2b^4-8b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d}+\frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\,\mathrm{si}}{6a^4\left(a^2-b^2\right)^3 d}$$

```
[Out] -4*b*x/a^5+b^2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)*arctanh((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(7/2)/(a+b)^(7/2)/d+1/6*(6*a^6-65*a^
4*b^2+68*a^2*b^4-24*b^6)*sin(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b^2*sin(d*x+c)/a/
(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*b^2*(9*a^2-4*b^2)*sin(d*x+c)/a^2/(a^2-b^
2)^2/d/(a+b*sec(d*x+c))^2+1/2*b^2*(12*a^4-11*a^2*b^2+4*b^4)*sin(d*x+c)/a^3/
(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.73, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3932, 4185, 4189, 4004, 3916, 2738, 214}

$$-\frac{4bx}{a^5}+\frac{b^2(9a^2-4b^2)\sin(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\sin(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b^2(12a^4-11a^2b^2+4b^4)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{b^2(20a^6-35a^4b^2+28a^2b^4-8b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-4*b*x)/a^5 + (b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d
) + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 - 24*b^6)*Sin[c + d*x])/(6*a^4*(a^2 -
 b^2)^3*d) + (b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3)
+ (b^2*(9*a^2 - 4*b^2)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c +
d*x])^2) + (b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(2*a^3*(a^2 - b
^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{b^2\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(-3a^2+4b^2+3ab\sec(c+dx)-3b^2\sec^2(c+dx}{(a+b\sec(c+dx))^3}}{3a(a^2-b^2)}$$

$$= \frac{b^2\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b^2(9a^2-4b^2)\sin(c+dx)}{6a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{\int c}{}$$

$$= \frac{b^2\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b^2(9a^2-4b^2)\sin(c+dx)}{6a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{b^2(}{2a}$$

$$= \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\sin(c+dx)}{6a^4(a^2-b^2)^3\,d} + \frac{b^2\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))}$$

$$= -\frac{4bx}{a^5} + \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\sin(c+dx)}{6a^4(a^2-b^2)^3\,d} + \frac{b^2\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec}$$

$$= -\frac{4bx}{a^5} + \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\sin(c+dx)}{6a^4(a^2-b^2)^3\,d} + \frac{b^2\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec}$$

$$= -\frac{4bx}{a^5} + \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\sin(c+dx)}{6a^4(a^2-b^2)^3\,d} + \frac{b^2\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec}$$

$$= -\frac{4bx}{a^5} + \frac{b^2(20a^6-35a^4b^2+28a^2b^4-8b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d} + \frac{(6}{}$$

Mathematica [A]
time = 1.68, size = 293, normalized size = 0.98

$$\frac{(b+a\cos(c+dx))\sec^4(c+dx)\left(-24b(c+dx)(b+a\cos(c+dx))^3+\frac{6b^2(-20a^6+35a^4b^2-28a^2b^4+8b^6)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^3}{(a^2-b^2)^{7/2}}+\frac{2ab^5\sin(c+dx)}{(-a+b)(a+b)}+\frac{5ab^4(3a^2-2b^2)(b+a\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2}-\frac{ab^3(60a^4-71a^2b^2+26b^4)(b+a\cos(c+dx))^2\sin(c+dx)}{(a-b)^3(a+b)^3}+6a(b+a\cos(c+dx))^3\sin(c+dx)\right)}{6a^5d(a+b\sec(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(-24*b*(c + d*x)*(b + a*Cos[c + d*x])^
3 + (6*b^2*(-20*a^6 + 35*a^4*b^2 - 28*a^2*b^4 + 8*b^6)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2)
+ (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)) + (5*a*b^4*(3*a^2 - 2*b^2)*(b +
 a*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) - (a*b^3*(60*a^4 - 71*
a^2*b^2 + 26*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a + b)^3
) + 6*a*(b + a*Cos[c + d*x])^3*Sin[c + d*x]))/(6*a^5*d*(a + b*Sec[c + d*x])
^4)
```

Maple [A]
time = 0.32, size = 399, normalized size = 1.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-2/a^5*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+4*b*arctan(tan(1/2*d*x+1/2*c)))-2*b^2/a^5*((-1/2*(20*a^4+5*a^3*b-18*a^2*b^2-2*a*b^3+6*b^4)*b*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(30*a^4-29*a^2*b^2+9*b^4)*b*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(20*a^4-5*a^3*b-18*a^2*b^2+2*a*b^3+6*b^4)*b*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^3-1/2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 773 vs. 2(282) = 564.
time = 3.31, size = 1603, normalized size = 5.36

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")

[Out] [-1/12*(48*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(d*x + c)^3 + 144*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d*x*cos(d*x + c)^2 + 144*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*d*x*cos(d*x + c) + 48*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*d*x - 3*(20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b^11 + (20*a^9*b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c)^3 + 3*(20*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + 3*(20*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^11 + 6

```
*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^3 + (18
*a^11*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c
)^2 + 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^10)*
cos(d*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 +
 a^8*b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^
7 + a^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a
^8*b^8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*
a^7*b^9 + a^5*b^11)*d), -1/6*(24*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^
7 + a^3*b^9)*d*x*cos(d*x + c)^3 + 72*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*
a^4*b^8 + a^2*b^10)*d*x*cos(d*x + c)^2 + 72*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^
7 - 4*a^3*b^9 + a*b^11)*d*x*cos(d*x + c) + 24*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*
b^8 - 4*a^2*b^10 + b^12)*d*x - 3*(20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*
b^11 + (20*a^9*b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c)^3 +
3*(20*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + 3*(20
*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c))*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
) - (6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^11 + 6*(a^1
2 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^3 + (18*a^11
*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c)^2 +
 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^10)*cos(d
*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*
b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a
^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^
8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b
^9 + a^5*b^11)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

Giac [A]
time = 0.49, size = 564, normalized size = 1.89

$$-\frac{\frac{3\,(20\,a^6b^2-35\,a^4b^4+28\,a^2b^6-8\,b^8)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^{11}-3\,a^9b^2+3\,a^7b^4-a^5b^6)\sqrt{-a^2+b^2}}-\frac{60\,a^6b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,a^5b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-24\,a^4b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+117\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-24\,a^2b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-42\,ab^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,b^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-120\,a^6b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+236\,a^4b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-152\,a^2b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,b^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+60\,a^6b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,a^5b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,a^4b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-117\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,a^2b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+42\,ab^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,b^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^3}+\frac{12\,(dx+c)b}{a^5}-\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(20*a^6*b^2 - 35*a^4*b^4 + 28*a^2*b^6 - 8*b^8)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
```

```
*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*s
qrt(-a^2 + b^2)) - (60*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*a^5*b^4*tan(1/2
*d*x + 1/2*c)^5 - 24*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*a^3*b^6*tan(1/2*d
*x + 1/2*c)^5 - 24*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*a*b^8*tan(1/2*d*x +
1/2*c)^5 + 18*b^9*tan(1/2*d*x + 1/2*c)^5 - 120*a^6*b^3*tan(1/2*d*x + 1/2*c)
^3 + 236*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 - 152*a^2*b^7*tan(1/2*d*x + 1/2*c)^
3 + 36*b^9*tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^3*tan(1/2*d*x + 1/2*c) + 105*a
^5*b^4*tan(1/2*d*x + 1/2*c) - 24*a^4*b^5*tan(1/2*d*x + 1/2*c) - 117*a^3*b^6
*tan(1/2*d*x + 1/2*c) - 24*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*a*b^8*tan(1/2*
d*x + 1/2*c) + 18*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4
- a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3)
 + 12*(d*x + c)*b/a^5 - 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1
)*a^4))/d
```

Mupad [B]
time = 10.38, size = 2500, normalized size = 8.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^4,x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^7*(4*a*b^6 + 2*a^6*b - 2*a^7 - 8*b^7 + 24*a^2*b^5 -
11*a^3*b^4 - 26*a^4*b^3 + 6*a^5*b^2))/((a^4*b - a^5)*(a + b)^3) + (tan(c/2
+ (d*x)/2)^3*(12*a*b^7 - 18*a^8 - 72*b^8 + 236*a^2*b^6 - 47*a^3*b^5 - 273*a
^4*b^4 + 60*a^5*b^3 + 72*a^6*b^2))/(3*(a + b)^2*(3*a^6*b - a^7 + a^4*b^3 -
3*a^5*b^2)) - (tan(c/2 + (d*x)/2)*(4*a*b^6 - 2*a^6*b - 2*a^7 + 8*b^7 - 24*a
^2*b^5 - 11*a^3*b^4 + 26*a^4*b^3 + 6*a^5*b^2))/((a + b)*(3*a^6*b - a^7 + a^
4*b^3 - 3*a^5*b^2)) + (tan(c/2 + (d*x)/2)^5*(12*a*b^7 + 18*a^8 + 72*b^8 - 2
36*a^2*b^6 - 47*a^3*b^5 + 273*a^4*b^4 + 60*a^5*b^3 - 72*a^6*b^2))/(3*(a^4*b
 - a^5)*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x)/2)^4*(6
*a^2*b - 6*b^3) + tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 2*a^3 + 4*b^3) + tan(c/2
+ (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3) + a^3 + b^3 - tan(c/2 + (d*x)/2)^8*(
3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (8*b*atan(((4*b*((8*tan(c/2 + (d*x)/2)*(
128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*
a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824
*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18
*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 1
0*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (b*((16
*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*
a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^
19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^
11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a
^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (b*tan(c/2 + (d*x)/2)*(8
*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^1
```

4*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)*32i)/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*4i)/a^5))/a^5 + (4*b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + (b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)*32i)/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*4i)/a^5))/a^5)/((32*(128*b^16 - 64*a*b^15 - 832*a^2*b^14 + 400*a^3*b^13 + 2288*a^4*b^12 - 1088*a^5*b^11 - 3472*a^6*b^10 + 1602*a^7*b^9 + 3088*a^8*b^8 - 1280*a^9*b^7 - 1520*a^10*b^6 + 480*a^11*b^5 + 320*a^12*b^4))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)*32i)/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*4i)/a^5)*4i)/a^5 + (b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^...

3.522 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=387

$$\frac{(a^2+20b^2)\,x}{2a^6}-\frac{b^3(40a^6-84a^4b^2+69a^2b^4-20b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6(a-b)^{7/2}(a+b)^{7/2}d}-\frac{b(24a^6-146a^4b^2+167a^2}{6a^5\,(a^2-}$$

```
[Out] 1/2*(a^2+20*b^2)*x/a^6-b^3*(40*a^6-84*a^4*b^2+69*a^2*b^4-20*b^6)*arctanh((a
-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^6/(a-b)^(7/2)/(a+b)^(7/2)/d-1/6
*b*(24*a^6-146*a^4*b^2+167*a^2*b^4-60*b^6)*sin(d*x+c)/a^5/(a^2-b^2)^3/d+1/2
*(a^6-23*a^4*b^2+27*a^2*b^4-10*b^6)*cos(d*x+c)*sin(d*x+c)/a^4/(a^2-b^2)^3/d
+1/3*b^2*cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+5/6*b^2*(2*
a^2-b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*b^2
*(48*a^4-53*a^2*b^2+20*b^4)*cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*se
c(d*x+c))
```

Rubi [A]

time = 0.98, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3932, 4185, 4189, 4004, 3916, 2738, 214}

$$\frac{5b^2(2a^2-b^2)\sin(c+dx)\cos(c+dx)}{6a^2d\,(a^2-b^2)^2\,(a+b\sec(c+dx))^2}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\,(a^2-b^2)\,(a+b\sec(c+dx))^3}+\frac{x(a^2+20b^2)}{2a^6}+\frac{(a^6-23a^4b^2+27a^2b^4-10b^6)\sin(c+dx)\cos(c+dx)}{2a^4d\,(a^2-b^2)^3}-\frac{b^3(40a^6-84a^4b^2+69a^2b^4-20b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2(48a^4-53a^2b^2+20b^4)\sin(c+dx)\cos(c+dx)}{6a^3d\,(a^2-b^2)^3\,(a+b\sec(c+dx))}-\frac{b(24a^6-146a^4b^2+167a^2b^4-60b^6)\sin(c+dx)}{6a^5d\,(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((a^2 + 20*b^2)*x)/(2*a^6) - (b^3*(40*a^6 - 84*a^4*b^2 + 69*a^2*b^4 - 20*b^
6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*(a - b)^(7/2)*
(a + b)^(7/2)*d) - (b*(24*a^6 - 146*a^4*b^2 + 167*a^2*b^4 - 60*b^6)*Sin[c +
 d*x])/(6*a^5*(a^2 - b^2)^3*d) + ((a^6 - 23*a^4*b^2 + 27*a^2*b^4 - 10*b^6)*
Cos[c + d*x]*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + (b^2*Cos[c + d*x]*Sin[
c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (5*b^2*(2*a^2 - b^2)
*Cos[c + d*x]*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2)
+ (b^2*(48*a^4 - 53*a^2*b^2 + 20*b^4)*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^
2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
```

```
e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{b^2\cos(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(-3a^2+5b^2+3ab\sec(c+dx)-4b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^3}}{3a(a^2-b^2)}$$

$$= \frac{b^2\cos(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{5b^2(2a^2-b^2)\cos(c+dx)\sin(c+dx)}{6a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \int$$

$$= \frac{b^2\cos(c+dx)\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{5b^2(2a^2-b^2)\cos(c+dx)\sin(c+dx)}{6a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \frac{b^2(}{}$$

$$= \frac{(a^6-23a^4b^2+27a^2b^4-10b^6)\cos(c+dx)\sin(c+dx)}{2a^4(a^2-b^2)^3\,d} + \frac{b^2\cos(c+dx)\sin(}{3a(a^2-b^2)\,d(a+b\sec}$$

$$= -\frac{b(24a^6-146a^4b^2+167a^2b^4-60b^6)\sin(c+dx)}{6a^5(a^2-b^2)^3\,d} + \frac{(a^6-23a^4b^2+27a^2b^4-10}{2a^4(a^2}$$

$$= \frac{(a^2+20b^2)\,x}{2a^6} - \frac{b(24a^6-146a^4b^2+167a^2b^4-60b^6)\sin(c+dx)}{6a^5(a^2-b^2)^3\,d} + \frac{(a^6-23a^4b^2}{}$$

$$= \frac{(a^2+20b^2)\,x}{2a^6} - \frac{b(24a^6-146a^4b^2+167a^2b^4-60b^6)\sin(c+dx)}{6a^5(a^2-b^2)^3\,d} + \frac{(a^6-23a^4b^2}{}$$

$$= \frac{(a^2+20b^2)\,x}{2a^6} - \frac{b(24a^6-146a^4b^2+167a^2b^4-60b^6)\sin(c+dx)}{6a^5(a^2-b^2)^3\,d} + \frac{(a^6-23a^4b^2}{}$$

$$= \frac{(a^2+20b^2)\,x}{2a^6} - \frac{b^3(40a^6-84a^4b^2+69a^2b^4-20b^6)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{a^6(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A]
time = 6.38, size = 326, normalized size = 0.84

$$\frac{(a^2+20b^2)(c+dx)}{2a^6d} + \frac{b^3(-40a^6+84a^4b^2-69a^2b^4+20b^6)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,(-a^2+b^2)^3\,d} - \frac{4b\sin(c+dx)}{a^5d} - \frac{b^6\sin(c+dx)}{3a^5(-a+b)(a+b)d(b+a\cos(c+dx))^3} + \frac{-18a^2b^5\sin(c+dx)+13b^7\sin(c+dx)}{6a^5(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2} + \frac{-90a^4b^4\sin(c+dx)+122a^2b^6\sin(c+dx)-47b^8\sin(c+dx)}{6a^5(-a+b)^3(a+b)^3d(b+a\cos(c+dx))} + \frac{\sin(2(c+dx))}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((a^2 + 20*b^2)*(c + d*x))/(2*a^6*d) + (b^3*(-40*a^6 + 84*a^4*b^2 - 69*a^2*
b^4 + 20*b^6)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*Sq
rt[a^2 - b^2]*(-a^2 + b^2)^3*d) - (4*b*Sin[c + d*x])/(a^5*d) - (b^6*Sin[c +
```

```
 d*x])/(3*a^5*(-a + b)*(a + b)*d*(b + a*Cos[c + d*x])^3) + (-18*a^2*b^5*Sin
[c + d*x] + 13*b^7*Sin[c + d*x])/(6*a^5*(-a + b)^2*(a + b)^2*d*(b + a*Cos[c
 + d*x])^2) + (-90*a^4*b^4*Sin[c + d*x] + 122*a^2*b^6*Sin[c + d*x] - 47*b^8
*Sin[c + d*x])/(6*a^5*(-a + b)^3*(a + b)^3*d*(b + a*Cos[c + d*x])) + Sin[2*
(c + d*x)]/(4*a^4*d)
```

Maple [A]
time = 0.36, size = 439, normalized size = 1.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^6*(((-1/2*a^2-4*b*a)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2-4*b*a)*tan(1/2*
d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+20*b^2)*arctan(tan(1/2*d*x+
1/2*c)))+2*b^3/a^6*((-1/2*(30*a^4+6*a^3*b-34*a^2*b^2-3*a*b^3+12*b^4)*b*a/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(45*a^4-53*a^2*b^2+1
8*b^4)*b*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(30*a^4
-6*a^3*b-34*a^2*b^2+3*a*b^3+12*b^4)*b*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan
(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^3-1/2*
(40*a^6-84*a^4*b^2+69*a^2*b^4-20*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*
(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(366) = 732$.
time = 3.67, size = 1767, normalized size = 4.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(6*(a^13 + 16*a^11*b^2 - 74*a^9*b^4 + 116*a^7*b^6 - 79*a^5*b^8 + 20*a
^3*b^10)*d*x*cos(d*x + c)^3 + 18*(a^12*b + 16*a^10*b^3 - 74*a^8*b^5 + 116*a
```

```
^6*b^7 - 79*a^4*b^9 + 20*a^2*b^11)*d*x*cos(d*x + c)^2 + 18*(a^11*b^2 + 16*a
^9*b^4 - 74*a^7*b^6 + 116*a^5*b^8 - 79*a^3*b^10 + 20*a*b^12)*d*x*cos(d*x +
c) + 6*(a^10*b^3 + 16*a^8*b^5 - 74*a^6*b^7 + 116*a^4*b^9 - 79*a^2*b^11 + 20
*b^13)*d*x + 3*(40*a^6*b^6 - 84*a^4*b^8 + 69*a^2*b^10 - 20*b^12 + (40*a^9*b
^3 - 84*a^7*b^5 + 69*a^5*b^7 - 20*a^3*b^9)*cos(d*x + c)^3 + 3*(40*a^8*b^4 -
 84*a^6*b^6 + 69*a^4*b^8 - 20*a^2*b^10)*cos(d*x + c)^2 + 3*(40*a^7*b^5 - 84
*a^5*b^7 + 69*a^3*b^9 - 20*a*b^11)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) - 2*(24*a^9*b^4 - 170*a^7*b^6 + 313*a^5*b^8 - 227*a^3*b^10 + 6
0*a*b^12 - 3*(a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^7*b^6 + a^5*b^8)*cos(d*x
+ c)^4 + 15*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7 + a^4*b^9)*cos(d*x
 + c)^3 + (63*a^11*b^2 - 342*a^9*b^4 + 590*a^7*b^6 - 421*a^5*b^8 + 110*a^3*
b^10)*cos(d*x + c)^2 + 3*(23*a^10*b^3 - 146*a^8*b^5 + 263*a^6*b^7 - 190*a^4
*b^9 + 50*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4*a^15*b^2 + 6*a^1
3*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*b^3 + 6
*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4*a^13*b
^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 - 4*a^12
*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d), 1/6*(3*(a^13 + 16*a^11*b^2 -
74*a^9*b^4 + 116*a^7*b^6 - 79*a^5*b^8 + 20*a^3*b^10)*d*x*cos(d*x + c)^3 + 9
*(a^12*b + 16*a^10*b^3 - 74*a^8*b^5 + 116*a^6*b^7 - 79*a^4*b^9 + 20*a^2*b^1
1)*d*x*cos(d*x + c)^2 + 9*(a^11*b^2 + 16*a^9*b^4 - 74*a^7*b^6 + 116*a^5*b^8
 - 79*a^3*b^10 + 20*a*b^12)*d*x*cos(d*x + c) + 3*(a^10*b^3 + 16*a^8*b^5 - 7
4*a^6*b^7 + 116*a^4*b^9 - 79*a^2*b^11 + 20*b^13)*d*x - 3*(40*a^6*b^6 - 84*a
^4*b^8 + 69*a^2*b^10 - 20*b^12 + (40*a^9*b^3 - 84*a^7*b^5 + 69*a^5*b^7 - 20
*a^3*b^9)*cos(d*x + c)^3 + 3*(40*a^8*b^4 - 84*a^6*b^6 + 69*a^4*b^8 - 20*a^2
*b^10)*cos(d*x + c)^2 + 3*(40*a^7*b^5 - 84*a^5*b^7 + 69*a^3*b^9 - 20*a*b^11
)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
 a)/((a^2 - b^2)*sin(d*x + c))) - (24*a^9*b^4 - 170*a^7*b^6 + 313*a^5*b^8 -
 227*a^3*b^10 + 60*a*b^12 - 3*(a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^7*b^6 +
a^5*b^8)*cos(d*x + c)^4 + 15*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7 +
 a^4*b^9)*cos(d*x + c)^3 + (63*a^11*b^2 - 342*a^9*b^4 + 590*a^7*b^6 - 421*a
^5*b^8 + 110*a^3*b^10)*cos(d*x + c)^2 + 3*(23*a^10*b^3 - 146*a^8*b^5 + 263*
a^6*b^7 - 190*a^4*b^9 + 50*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4
*a^15*b^2 + 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b
 - 4*a^14*b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^
15*b^2 - 4*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (
a^14*b^3 - 4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

Giac [A]
time = 0.50, size = 615, normalized size = 1.59

$$\frac{\frac{6\left(40a^{6}b^{3}-84a^{4}b^{5}+69a^{2}b^{7}-20b^{9}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2a-2b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{12}-3a^{10}b^{2}+3a^{8}b^{4}-a^{6}b^{6}\right)\sqrt{-a^{2}+b^{2}}}-\frac{2\left(90a^{6}b^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-162a^{5}b^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-48a^{4}b^{6}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+213a^{3}b^{7}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-48a^{2}b^{8}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-81ab^{9}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+36b^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-180a^{6}b^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+392a^{4}b^{6}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-284a^{2}b^{8}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+72b^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+90a^{6}b^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+162a^{5}b^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-48a^{4}b^{6}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-213a^{3}b^{7}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-48a^{2}b^{8}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+81ab^{9}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+36b^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(a^{11}-3a^{9}b^{2}+3a^{7}b^{4}-a^{5}b^{6}\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a-b\right)^{3}}+\frac{3\left(a^{2}+20b^{2}\right)\left(dx+c\right)}{a^{6}}-\frac{6\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+8b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{2}a^{5}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(6*(40*a^6*b^3 - 84*a^4*b^5 + 69*a^2*b^7 - 20*b^9)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*
sqrt(-a^2 + b^2)) - 2*(90*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 162*a^5*b^5*tan(
1/2*d*x + 1/2*c)^5 - 48*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 213*a^3*b^7*tan(1/
2*d*x + 1/2*c)^5 - 48*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 81*a*b^9*tan(1/2*d*x
 + 1/2*c)^5 + 36*b^10*tan(1/2*d*x + 1/2*c)^5 - 180*a^6*b^4*tan(1/2*d*x + 1/
2*c)^3 + 392*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 284*a^2*b^8*tan(1/2*d*x + 1/2
*c)^3 + 72*b^10*tan(1/2*d*x + 1/2*c)^3 + 90*a^6*b^4*tan(1/2*d*x + 1/2*c) +
162*a^5*b^5*tan(1/2*d*x + 1/2*c) - 48*a^4*b^6*tan(1/2*d*x + 1/2*c) - 213*a^
3*b^7*tan(1/2*d*x + 1/2*c) - 48*a^2*b^8*tan(1/2*d*x + 1/2*c) + 81*a*b^9*tan
(1/2*d*x + 1/2*c) + 36*b^10*tan(1/2*d*x + 1/2*c))/((a^11 - 3*a^9*b^2 + 3*a^
7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a -
 b)^3) + 3*(a^2 + 20*b^2)*(d*x + c)/a^6 - 6*(a*tan(1/2*d*x + 1/2*c)^3 + 8*b
*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 8*b*tan(1/2*d*x + 1/2*c)
)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5))/d
```

Mupad [B]
time = 10.80, size = 2500, normalized size = 6.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^4,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^9*(7*a^7*b - 10*a*b^7 + a^8 + 20*b^8 - 59*a^2*b^6 + 27
*a^3*b^5 + 57*a^4*b^4 - 21*a^5*b^3 - 11*a^6*b^2))/(a^5*(a + b)^3*(a - b)) +
 (2*tan(c/2 + (d*x)/2)^3*(30*a*b^8 + 21*a^8*b - 6*a^9 + 120*b^9 - 364*a^2*b
^7 - 71*a^3*b^6 + 369*a^4*b^5 + 45*a^5*b^4 - 111*a^6*b^3 - 3*a^7*b^2))/(3*a
^5*(a + b)^2*(a - b)^3) - (2*tan(c/2 + (d*x)/2)^7*(21*a^8*b - 30*a*b^8 + 6*
a^9 + 120*b^9 - 364*a^2*b^7 + 71*a^3*b^6 + 369*a^4*b^5 - 45*a^5*b^4 - 111*a
^6*b^3 + 3*a^7*b^2))/(3*a^5*(a + b)^3*(a - b)^2) + (2*tan(c/2 + (d*x)/2)^5*
(9*a^10 + 180*b^10 - 611*a^2*b^8 + 740*a^4*b^6 - 324*a^6*b^4 + 36*a^8*b^2))
/(3*a^5*(a + b)^3*(a - b)^3) + (tan(c/2 + (d*x)/2)*(10*a*b^7 - 7*a^7*b + a^
```

8 + 20*b^8 - 59*a^2*b^6 - 27*a^3*b^5 + 57*a^4*b^4 + 21*a^5*b^3 - 11*a^6*b^2
))/(a^5*(a + b)*(a - b)^3))/(d*(tan(c/2 + (d*x)/2)^2*(9*a*b^2 + 3*a^2*b - a
^3 + 5*b^3) + tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^2*b - 2*a^3 + 10*b^3) - t
an(c/2 + (d*x)/2)^6*(6*a*b^2 + 6*a^2*b - 2*a^3 - 10*b^3) + 3*a*b^2 + 3*a^2*
b + a^3 + b^3 - tan(c/2 + (d*x)/2)^10*(3*a*b^2 - 3*a^2*b + a^3 - b^3) + tan
(c/2 + (d*x)/2)^8*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))) - (atan(((((((4*(4*a^
27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*a^15*b^12 - 1404*a^1
6*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 - 1764*a^20*b^7 + 724
*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3 + 52*a^25*b^2))/(a^2
5*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7
- 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*t
an(c/2 + (d*x)/2)*(a^2*1i + b^2*20i)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13
+ 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8
- 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 -
8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*
a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b
^3 - 5*a^19*b^2)))*(a^2*1i + b^2*20i))/(2*a^6) - (8*tan(c/2 + (d*x)/2)*(800
*a*b^17 + 2*a^17*b - a^18 - 800*b^18 + 4720*a^2*b^16 - 4720*a^3*b^15 - 1152
2*a^4*b^14 + 11522*a^5*b^13 + 14837*a^6*b^12 - 14812*a^7*b^11 - 10385*a^8*b
^10 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 45*a^12*b^6 + 350*a^1
3*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^20*b + a^21 - a^10*b^
11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a
^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2))*(a^2*1i + b^2*20i)*1i)/(2
a^6) - (((((4(4*a^27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*
a^15*b^12 - 1404*a^16*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 -
1764*a^20*b^7 + 724*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3
+ 52*a^25*b^2))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^1
8*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3
- 5*a^24*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*20i)*(8*a^25*b - 8*a^12
*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^1
7*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22
*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^
10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10
*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(a^2*1i + b^2*20i))/(2*a^6) + (8*tan
(c/2 + (d*x)/2)*(800*a*b^17 + 2*a^17*b - a^18 - 800*b^18 + 4720*a^2*b^16 -
4720*a^3*b^15 - 11522*a^4*b^14 + 11522*a^5*b^13 + 14837*a^6*b^12 - 14812*a^
7*b^11 - 10385*a^8*b^10 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 4
5*a^12*b^6 + 350*a^13*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^2
0*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7
- 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2))*(a^2*
1i + b^2*20i)*1i)/(2*a^6))/((((((4*(4*a^27 - 80*a^12*b^15 + 40*a^13*b^14 +
516*a^14*b^13 - 248*a^15*b^12 - 1404*a^16*b^11 + 640*a^17*b^10 + 2076*a^18*
b^9 - 896*a^19*b^8 - 1764*a^20*b^7 + 724*a^21*b^6 + 816*a^22*b^5 - 316*a^23
*b^4 - 160*a^24*b^3 + 52*a^25*b^2))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10
+ 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^

22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*20i)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120 *a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 12 0*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^ 6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(a^2*1i + b^2*20 i))/(2*a^6) - (8*tan(c/2 + (d*x)/2)*(800*a*b^17 + 2*a^17*b - a^18 - 800*b^1 8 + 4720*a^2*b^16 - 4720*a^3*b^15 - 11522*a^4*b^14 + 11522*a^5*b^13 + 14837 *a^6*b^12 - 14812*a^7*b^11 - 10385*a^8*b^10 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 45*a^12*b^6 + 350*a^13*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^1 3*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2))*(a^2*1i + b^2*20i))/(2*a^6) - (8...

3.523 $\int \frac{1}{3+5\sec(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{x}{12}+\frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{6d}$$

[Out] -1/12*x+5/6*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3868, 2736}

$$\frac{5\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{6d}-\frac{x}{12}$$

Antiderivative was successfully verified.

[In] Int[(3 + 5*Sec[c + d*x])^(-1),x]

[Out] -1/12*x + (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(6*d)

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+5\sec(c+dx)}\,dx &= \frac{x}{3}-\frac{1}{3}\int \frac{1}{1+\frac{3}{5}\cos(c+dx)}\,dx\\ &= -\frac{x}{12}+\frac{5\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{6d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 30, normalized size = 0.97

$$\frac{2(c+dx)+5\text{ArcTan}\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-1),x]
[Out] (2*(c + d*x) + 5*ArcTan[2*Cot[(c + d*x)/2]])/(6*d)
```

Maple [A]
time = 0.06, size = 32, normalized size = 1.03

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{6}}{d}$	32
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{6}}{d}$	32
risch	$\frac{x}{3}-\frac{5i\ln\left(e^{i(dx+c)}+3\right)}{12d}+\frac{5i\ln\left(e^{i(dx+c)}+\frac{1}{3}\right)}{12d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(2/3*arctan(tan(1/2*d*x+1/2*c))-5/6*arctan(1/2*tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.46, size = 47, normalized size = 1.52

$$\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)-5\arctan\left(\frac{\sin(dx+c)}{2\left(\cos(dx+c)+1\right)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/6*(4*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 5*arctan(1/2*sin(d*x + c)/
(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.79, size = 33, normalized size = 1.06

$$\frac{4dx+5\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*d*x + 5*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{5\sec(c+dx)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x)
```

```
[Out] Integral(1/(5*sec(c + d*x) + 3), x)
```

Giac [A]
time = 0.42, size = 30, normalized size = 0.97

$$-\frac{dx+c-10\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(d*x + c - 10*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B]
time = 0.82, size = 21, normalized size = 0.68

$$\frac{x}{3}-\frac{5\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5/cos(c + d*x) + 3),x)
```

```
[Out] x/3 - (5*atan(tan(c/2 + (d*x)/2)/2))/(6*d)
```

3.524 $\int \frac{1}{(3+5\sec(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$\frac{29x}{576}+\frac{35\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{288d}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}$$

```
[Out] 29/576*x+35/288*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/48*tan(d*x+c)/d/(3+5
*sec(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3870, 4004, 3916, 2736}

$$\frac{35\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{288d}-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}+\frac{29x}{576}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-2),x]
```

```
[Out] (29*x)/576 + (35*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(288*d) - (25*Tan
[c + d*x])/(48*d*(3 + 5*Sec[c + d*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3+5\sec(c+dx))^2}\,dx &= -\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}+\frac{1}{48}\int \frac{16+15\sec(c+dx)}{3+5\sec(c+dx)}\,dx\\ &= \frac{x}{9}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}-\frac{35}{144}\int \frac{\sec(c+dx)}{3+5\sec(c+dx)}\,dx\\ &= \frac{x}{9}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}-\frac{7}{144}\int \frac{1}{1+\frac{3}{5}\cos(c+dx)}\,dx\\ &= \frac{29x}{576}+\frac{35\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{288d}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 73, normalized size = 1.30

$$\frac{160(c+dx)+96(c+dx)\cos(c+dx)+35\text{ArcTan}\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+dx))-150\sin(c+dx)}{288d(5+3\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-2),x]
```

```
[Out] (160*(c + d*x) + 96*(c + d*x)*Cos[c + d*x] + 35*ArcTan[2*Cot[(c + d*x)/2]]*
(5 + 3*Cos[c + d*x]) - 150*Sin[c + d*x])/(288*d*(5 + 3*Cos[c + d*x]))
```

Maple [A]
time = 0.07, size = 58, normalized size = 1.04

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{48\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}-\frac{35\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{288}}{d}$	58
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{48\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}-\frac{35\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{288}}{d}$	58

risch	$\frac{x}{9}-\frac{25i\left(5\,\mathrm{e}^{i(dx+c)}+3\right)}{72d\left(3\,\mathrm{e}^{2i(dx+c)}+10\,\mathrm{e}^{i(dx+c)}+3\right)}+\frac{35i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{1}{3}\right)}{576d}-\frac{35i\ln\left(\mathrm{e}^{i(dx+c)}+3\right)}{576d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/9*arctan(tan(1/2*d*x+1/2*c))-25/48*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1
/2*c)^2+4)-35/288*arctan(1/2*tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.46, size = 88, normalized size = 1.57

$$-\frac{\frac{150\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+4\right)(\cos(dx+c)+1)}-64\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+35\,\arctan\left(\frac{\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)}{288\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/288*(150*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)*(cos(d*
x + c) + 1)) - 64*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 35*arctan(1/2*s
in(d*x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.58, size = 73, normalized size = 1.30

$$\frac{192\,dx\cos\left(dx+c\right)+320\,dx+35\,\left(3\,\cos\left(dx+c\right)+5\right)\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)-300\,\sin\left(dx+c\right)}{576\,\left(3\,d\cos\left(dx+c\right)+5\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/576*(192*d*x*cos(d*x + c) + 320*d*x + 35*(3*cos(d*x + c) + 5)*arctan(1/4*
(5*cos(d*x + c) + 3)/sin(d*x + c)) - 300*sin(d*x + c))/(3*d*cos(d*x + c) +
5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(5\sec\left(c+dx\right)+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))**2,x)
```

```
[Out] Integral((5*sec(c + d*x) + 3)**(-2), x)
```

Giac [A]
time = 0.42, size = 59, normalized size = 1.05

$$\frac{29\,dx+29\,c-\frac{300\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4}+70\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+3}\right)}{576\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/576*(29*d*x + 29*c - 300*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 4
) + 70*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B]
time = 0.86, size = 52, normalized size = 0.93

$$\frac{x}{9}-\frac{\frac{35\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{288}+\frac{25\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{48\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+4\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5/cos(c + d*x) + 3)^2,x)
```

```
[Out] x/9 - ((35*atan(tan(c/2 + (d*x)/2)/2))/288 + (25*tan(c/2 + (d*x)/2))/(48*(t
an(c/2 + (d*x)/2)^2 + 4)))/d
```

3.525 $\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$

Optimal. Leaf size=81

$$-\frac{1007x}{55296}+\frac{3055\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{27648d}-\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}$$

```
[Out] -1007/55296*x+3055/27648*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/96*tan(d*x+
c)/d/(3+5*sec(d*x+c))^2-125/4608*tan(d*x+c)/d/(3+5*sec(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3870, 4145, 4004, 3916, 2736}

$$\frac{3055\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{27648d}-\frac{125\tan(c+dx)}{4608d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}-\frac{1007x}{55296}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-3),x]
```

```
[Out] (-1007*x)/55296 + (3055*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(27648*d)
- (25*Tan[c + d*x])/(96*d*(3 + 5*Sec[c + d*x])^2) - (125*Tan[c + d*x])/(460
8*d*(3 + 5*Sec[c + d*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5\sec(c+dx))^3}\,dx &= -\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}+\frac{1}{96}\int \frac{32+30\sec(c+dx)-25\sec^2(c+dx)}{(3+5\sec(c+dx))^2}\,dx \\
&= -\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}+\frac{\int \frac{512-165\sec(c+dx)}{3+5\sec(c+dx)}\,dx}{4608} \\
&= \frac{x}{27}-\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}-\frac{3055\int \frac{\sec(c+dx)}{3+5\sec(c+dx)}}{13824} \\
&= \frac{x}{27}-\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}-\frac{611\int \frac{1}{1+\frac{3}{5}\cos(c+dx)}}{13824} \\
&= -\frac{1007x}{55296}+\frac{3055\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{27648d}-\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c}{4608d(3+5\,\mathrm{s}}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 108, normalized size = 1.33

$$\frac{30208c+30208dx+30720(c+dx)\cos(c+dx)+3055\text{ArcTan}\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+dx))^2+4608c\cos(2(c+dx))+4608dx\cos(2(c+dx))-3750\sin(c+dx)-4725\sin(2(c+dx))}{27648d(5+3\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-3),x]
```

[Out] (30208*c + 30208*d*x + 30720*(c + d*x)*Cos[c + d*x] + 3055*ArcTan[2*Cot[(c + d*x)/2]]*(5 + 3*Cos[c + d*x])^2 + 4608*c*Cos[2*(c + d*x)] + 4608*d*x*Cos[2*(c + d*x)] - 3750*Sin[c + d*x] - 4725*Sin[2*(c + d*x)])/(27648*d*(5 + 3*Cos[c + d*x])^2)

Maple [A]
time = 0.08, size = 74, normalized size = 0.91

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{27}-\frac{5\left(-\frac{285\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{165\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{108\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{2}}-\frac{3055\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{27648}}{d}$	74
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{27}-\frac{5\left(-\frac{285\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{165\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{108\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{2}}-\frac{3055\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{27648}}{d}$	74
risch	$\frac{x}{27}-\frac{25i\left(185\,\mathrm{e}^{3i(dx+c)}+413\,\mathrm{e}^{2i(dx+c)}+235\,\mathrm{e}^{i(dx+c)}+63\right)}{2304d\left(3\,\mathrm{e}^{2i(dx+c)}+10\,\mathrm{e}^{i(dx+c)}+3\right)^{2}}+\frac{3055i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{1}{3}\right)}{55296d}-\frac{3055i\ln\left(\mathrm{e}^{i(dx+c)}+3\right)}{55296d}$	108

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3+5*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(2/27*arctan(tan(1/2*d*x+1/2*c))-5/108*(-285/128*tan(1/2*d*x+1/2*c)^3+165/32*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2+4)^2-3055/27648*arctan(1/2*tan(1/2*d*x+1/2*c)))

Maxima [A]
time = 0.46, size = 131, normalized size = 1.62

$$-\frac{\frac{150\left(\frac{44\sin(dx+c)}{\cos(dx+c)+1}-\frac{19\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{\frac{8\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+16}-2048\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+3055\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{27648\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="maxima")

[Out] -1/27648*(150*(44*sin(d*x + c)/(cos(d*x + c) + 1) - 19*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 16) - 2048*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 3055*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d

Fricas [A]
time = 3.63, size = 116, normalized size = 1.43

$$\frac{18432\,dx\cos(dx+c)^{2}+61440\,dx\cos(dx+c)+51200\,dx+3055\left(9\cos(dx+c)^{2}+30\cos(dx+c)+25\right)\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)-300\left(63\cos(dx+c)+25\right)\sin(dx+c)}{55296\left(9\,d\cos(dx+c)^{2}+30\,d\cos(dx+c)+25\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(18432*d*x*cos(d*x + c)^2 + 61440*d*x*cos(d*x + c) + 51200*d*x + 30
55*(9*cos(d*x + c)^2 + 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x + c) + 3
)/sin(d*x + c)) - 300*(63*cos(d*x + c) + 25)*sin(d*x + c))/(9*d*cos(d*x + c
)^2 + 30*d*cos(d*x + c) + 25*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\sec(c+dx)+3)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))**3,x)
```

```
[Out] Integral((5*sec(c + d*x) + 3)**(-3), x)
```

Giac [A]
time = 0.46, size = 75, normalized size = 0.93

$$-\frac{1007\,dx+1007\,c-\frac{300\left(19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-44\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^2}-6110\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{55296\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/55296*(1007*d*x + 1007*c - 300*(19*tan(1/2*d*x + 1/2*c)^3 - 44*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^2 - 6110*arctan(sin(d*x + c)/(cos
(d*x + c) + 3)))/d
```

Mupad [B]
time = 0.91, size = 79, normalized size = 0.98

$$\frac{x}{27}-\frac{3055\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{27648\,d}-\frac{\frac{275\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1152}-\frac{475\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4608}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5/cos(c + d*x) + 3)^3,x)
```

```
[Out] x/27 - (3055*atan(tan(c/2 + (d*x)/2)/2))/(27648*d) - ((275*tan(c/2 + (d*x)/
2))/1152 - (475*tan(c/2 + (d*x)/2)^3)/4608)/(d*(8*tan(c/2 + (d*x)/2)^2 + ta
n(c/2 + (d*x)/2)^4 + 16))
```

3.526 $\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$

Optimal. Leaf size=106

$$\frac{21553x}{2654208}+\frac{11215\text{ArcTan}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1327104d}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+dx)}{221184d(3+5\sec(c+dx))}$$

```
[Out] 21553/2654208*x+11215/1327104*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/144*ta
n(d*x+c)/d/(3+5*sec(d*x+c))^3-25/4608*tan(d*x+c)/d/(3+5*sec(d*x+c))^2-16925
/221184*tan(d*x+c)/d/(3+5*sec(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3870, 4145, 4004, 3916, 2736}

$$\frac{11215\text{ArcTan}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{1327104d}-\frac{16925\tan(c+dx)}{221184d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{4608d(5\sec(c+dx)+3)^2}-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}+\frac{21553x}{2654208}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-4),x]
```

```
[Out] (21553*x)/2654208 + (11215*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(132710
4*d) - (25*Tan[c + d*x])/(144*d*(3 + 5*Sec[c + d*x])^3) - (25*Tan[c + d*x])
/(4608*d*(3 + 5*Sec[c + d*x])^2) - (16925*Tan[c + d*x])/(221184*d*(3 + 5*Se
c[c + d*x]))
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5\sec(c+dx))^4}\,dx &= -\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}+\frac{1}{144}\int\frac{48+45\sec(c+dx)-50\sec^2(c+dx)}{(3+5\sec(c+dx))^3}\,dx \\
&= -\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}+\frac{\int\frac{1536-870\sec(c+dx)-75}{(3+5\sec(c+dx)}}{13824} \\
&= -\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+d}{221184d(3+5\sec(c} \\
&= \frac{x}{81}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c}{221184d(3+5\mathrm{se}} \\
&= \frac{x}{81}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c}{221184d(3+5\mathrm{se}} \\
&= \frac{21553x}{2654208}+\frac{11215\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1327104d}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\,\mathrm{tan}}{4608d(3+}
\end{aligned}$$

Mathematica [A]
time = 0.56, size = 141, normalized size = 1.33

$$\frac{6307840c+6307840dx+8036352(c+dx)\cos(c+dx)+22430\mathrm{ArcTan}\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+dx))^3+2211840c\cos(2(c+dx))+2211840dx\cos(2(c+dx))+221184c\cos(3(c+dx))+221184dx\cos(3(c+dx))-5660475\sin(c+dx)-3082500\sin(2(c+dx))-582975\sin(3(c+dx))}{2654208d(5+3\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-4),x]
```

```
[Out] (6307840*c + 6307840*d*x + 8036352*(c + d*x)*Cos[c + d*x] + 22430*ArcTan[2*
Cot[(c + d*x)/2]]*(5 + 3*Cos[c + d*x])^3 + 2211840*c*Cos[2*(c + d*x)] + 221
1840*d*x*Cos[2*(c + d*x)] + 221184*c*Cos[3*(c + d*x)] + 221184*d*x*Cos[3*(c
 + d*x)] - 5660475*Sin[c + d*x] - 3082500*Sin[2*(c + d*x)] - 582975*Sin[3*(
c + d*x)])/(2654208*d*(5 + 3*Cos[c + d*x])^3)
```

Maple [A]
time = 0.09, size = 87, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{81}+\frac{-\frac{25925\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{221184}-\frac{3575\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6912}-\frac{17675\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{13824}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^3}-\frac{11215\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1327104}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{81}+\frac{-\frac{25925\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{221184}-\frac{3575\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6912}-\frac{17675\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{13824}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^3}-\frac{11215\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1327104}}{d}$
risch	$\frac{x}{81}-\frac{25i\left(164835\,e^{5i(dx+c)}+931257\,e^{4i(dx+c)}+1995070\,e^{3i(dx+c)}+1610514\,e^{2i(dx+c)}+534735\,e^{i(dx+c)}+69957\right)}{995328d\left(3\,e^{2i(dx+c)}+10\,e^{i(dx+c)}+3\right)^3}+\frac{11}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/81*arctan(tan(1/2*d*x+1/2*c))+5/648*(-15555/1024*tan(1/2*d*x+1/2*c)^
5-2145/32*tan(1/2*d*x+1/2*c)^3-10605/64*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/
2*c)^2+4)^3-11215/1327104*arctan(1/2*tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.49, size = 171, normalized size = 1.61

$$-\frac{\frac{150\left(\frac{11312\sin(dx+c)}{\cos(dx+c)+1}+\frac{4576\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1037\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{12\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+64}-32768\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+11215\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{1327104\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/1327104*(150*(11312*sin(d*x + c)/(cos(d*x + c) + 1) + 4576*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 1037*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 64) - 32768*arctan(sin(d*x + c)/(cos(
d*x + c) + 1)) + 11215*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 3.72, size = 159, normalized size = 1.50

$$\frac{884736\,dx\cos(dx+c)^3+4423680\,dx\cos(dx+c)^2+7372800\,dx\cos(dx+c)+4096000\,dx+11215\left(27\cos(dx+c)^3+135\cos(dx+c)^2+225\cos(dx+c)+125\right)\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)-300\left(7773\cos(dx+c)^2+20550\cos(dx+c)+16925\right)\sin(dx+c)}{2654208\left(27\,d\cos(dx+c)^3+135\,d\cos(dx+c)^2+225\,d\cos(dx+c)+125\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2654208*(884736*d*x*cos(d*x + c)^3 + 4423680*d*x*cos(d*x + c)^2 + 7372800
*d*x*cos(d*x + c) + 4096000*d*x + 11215*(27*cos(d*x + c)^3 + 135*cos(d*x +
c)^2 + 225*cos(d*x + c) + 125)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)
) - 300*(7773*cos(d*x + c)^2 + 20550*cos(d*x + c) + 16925)*sin(d*x + c))/(2
7*d*cos(d*x + c)^3 + 135*d*cos(d*x + c)^2 + 225*d*cos(d*x + c) + 125*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\sec(c+dx)+3)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))**4,x)
```

```
[Out] Integral((5*sec(c + d*x) + 3)**(-4), x)
```

Giac [A]
time = 0.44, size = 88, normalized size = 0.83

$$\frac{21553\,dx + 21553\,c - \frac{300\left(1037\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4576\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+11312\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^3} + 22430\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{2654208\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2654208*(21553*d*x + 21553*c - 300*(1037*tan(1/2*d*x + 1/2*c)^5 + 4576*ta
n(1/2*d*x + 1/2*c)^3 + 11312*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
+ 4)^3 + 22430*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

Mupad [B]
time = 1.09, size = 105, normalized size = 0.99

$$\frac{x}{81} - \frac{11215\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{1327104\,d} - \frac{\frac{25925\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{221184} + \frac{3575\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6912} + \frac{17675\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{13824}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5/cos(c + d*x) + 3)^4,x)
```

```
[Out] x/81 - (11215*atan(tan(c/2 + (d*x)/2)/2))/(1327104*d) - ((17675*tan(c/2 + (
d*x)/2))/13824 + (3575*tan(c/2 + (d*x)/2)^3)/6912 + (25925*tan(c/2 + (d*x)/
2)^5)/221184)/(d*(48*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + tan(c
/2 + (d*x)/2)^6 + 64))
```

3.527 $\int \frac{1}{5+3\sec(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{x}{5}+\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

```
[Out] 1/5*x+3/20*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-3/20*ln(2*cos(1/2*
d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3868, 2738, 212}

$$\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20d}+\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-1),x]
```

```
[Out] x/5 + (3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(20*d) - (3*Log[2*Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{x}{5} - \frac{1}{5}\int \frac{1}{1+\frac{5}{3}\cos(c+dx)}\,dx$$
$$= \frac{x}{5} - \frac{2\text{Subst}\left(\int \frac{1}{\frac{8}{3}-\frac{2x^2}{3}}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{5d}$$
$$= \frac{x}{5} + \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d} - \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.}{20d}$$

Mathematica [A]
time = 0.06, size = 69, normalized size = 0.99

$$\frac{4(c+dx)+3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-1),x]
[Out] (4*(c + d*x) + 3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[2*Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d)
```

Maple [A]
time = 0.07, size = 46, normalized size = 0.66

method	result	size
norman	$\frac{x}{5}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20d}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20d}$	39
risch	$\frac{x}{5}+\frac{3\ln\left(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{20d}-\frac{3\ln\left(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{20d}$	43
derivativedivides	$\frac{\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}}{d}$	46
default	$\frac{\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}}{d}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(3/20*ln(tan(1/2*d*x+1/2*c)-2)-3/20*ln(tan(1/2*d*x+1/2*c)+2)+2/5*arctan
(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.46, size = 70, normalized size = 1.00

$$\frac{8\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)-3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)+3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/20*(8*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 3*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 2) + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Fricas [A]
time = 2.96, size = 52, normalized size = 0.74

$$\frac{8\,dx-3\,\log\left(\frac{3}{2}\,\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)+\frac{5}{2}\right)+3\,\log\left(\frac{3}{2}\,\cos\left(dx+c\right)-2\,\sin\left(dx+c\right)+\frac{5}{2}\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*d*x - 3*log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) + 3*log(3/2*co
s(d*x + c) - 2*sin(d*x + c) + 5/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{3\sec\left(c+dx\right)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x)
```

```
[Out] Integral(1/(3*sec(c + d*x) + 5), x)
```

Giac [A]
time = 0.44, size = 43, normalized size = 0.61

$$\frac{4\,dx+4\,c-3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/20*(4*d*x + 4*c - 3*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 3*log(abs(tan(1/
2*d*x + 1/2*c) - 2)))/d
```

Mupad [B]
time = 0.88, size = 21, normalized size = 0.30

$$\frac{x}{5}-\frac{3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3/cos(c + d*x) + 5),x)
```

```
[Out] x/5 - (3*atanh(tan(c/2 + (d*x)/2)/2))/(10*d)
```

3.528 $\int \frac{1}{(5+3\sec(c+dx))^2} dx$

Optimal. Leaf size=95

$$\frac{x}{25}+\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}-\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}+\frac{9\tan(}{80d(5+3}$$

```
[Out] 1/25*x+123/1600*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-123/1600*ln(2
*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+9/80*tan(d*x+c)/d/(5+3*sec(d*x+c)
)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3870, 4004, 3916, 2738, 212}

$$\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}+\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}-\frac{123\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}+\frac{x}{25}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-2),x]
```

```
[Out] x/25 + (123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(1600*d) - (123*Log
[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(1600*d) + (9*Tan[c + d*x])/(80*d*
(5 + 3*Sec[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
```

```
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5+3\sec(c+dx))^2}\,dx &= \frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}-\frac{1}{80}\int \frac{-16+15\sec(c+dx)}{5+3\sec(c+dx)}\,dx\\
&= \frac{x}{25}+\frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}-\frac{123}{400}\int \frac{\sec(c+dx)}{5+3\sec(c+dx)}\,dx\\
&= \frac{x}{25}+\frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}-\frac{41}{400}\int \frac{1}{1+\frac{5}{3}\cos(c+dx)}\,dx\\
&= \frac{x}{25}+\frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}-\frac{41\text{Subst}\left(\int \frac{1}{\frac{8}{3}-\frac{2x^2}{3}}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{200d}\\
&= \frac{x}{25}+\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}-\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx\right.\right.}{1600}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 162, normalized size = 1.71

$$\frac{5\cos(c+dx)\left(64(c+dx)+123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+3\left(64c+64dx+123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\sin(c+dx)\right)}{1600d(3+5\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-2),x]
```

```
[Out] (5*Cos[c + d*x]*(64*(c + d*x) + 123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] - 123*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*(64*c + 64*d*x +
123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 123*Log[2*Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + 60*Sin[c + d*x]))/(1600*d*(3 + 5*Cos[c + d*x]))
```

Maple [A]
time = 0.07, size = 76, normalized size = 0.80

method	result	siz
derivativedivides	$\frac{-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600}-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{25}}{d}$	76
default	$\frac{-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600}-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{25}}{d}$	76
norman	$\frac{-\frac{4x}{25}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80d}+\frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{25}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-4}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600d}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600d}$	84
risch	$\frac{x}{25}+\frac{9i\left(3\,e^{i(dx+c)}+5\right)}{200d\left(5\,e^{2i(dx+c)}+6\,e^{i(dx+c)}+5\right)}+\frac{123\ln\left(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{1600d}-\frac{123\ln\left(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{1600d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-9/160/(tan(1/2*d*x+1/2*c)-2)+123/1600*ln(tan(1/2*d*x+1/2*c)-2)-9/160/
(tan(1/2*d*x+1/2*c)+2)-123/1600*ln(tan(1/2*d*x+1/2*c)+2)+2/25*arctan(tan(1/
2*d*x+1/2*c)))
```

Maxima [A]
time = 0.46, size = 111, normalized size = 1.17

$$-\frac{\frac{180\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-4\right)(\cos(dx+c)+1)}-128\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+123\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-123\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{1600\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/1600*(180*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4)*(cos(d
*x + c) + 1)) - 128*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 123*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 2) - 123*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 2))/d
```

Fricas [A]
time = 2.62, size = 102, normalized size = 1.07

$$\frac{640\,dx\cos(dx+c)+384\,dx-123\,(5\,\cos(dx+c)+3)\log\left(\frac{3}{2}\,\cos(dx+c)+2\,\sin(dx+c)+\frac{5}{2}\right)+123\,(5\,\cos(dx+c)+3)\log\left(\frac{3}{2}\,\cos(dx+c)-2\,\sin(dx+c)+\frac{5}{2}\right)+360\,\sin(dx+c)}{3200\,(5\,d\cos(dx+c)+3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3200*(640*d*x*cos(d*x + c) + 384*d*x - 123*(5*cos(d*x + c) + 3)*log(3/2*c
os(d*x + c) + 2*sin(d*x + c) + 5/2) + 123*(5*cos(d*x + c) + 3)*log(3/2*cos(
```

```
d*x + c) - 2*sin(d*x + c) + 5/2) + 360*sin(d*x + c))/(5*d*cos(d*x + c) + 3*
d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sec(c+dx)+5)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))**2,x)
[Out] Integral((3*sec(c + d*x) + 5)**(-2), x)
```

Giac [A]
time = 0.43, size = 69, normalized size = 0.73

$$\frac{64\,dx+64\,c-\frac{180\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4}-123\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+123\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{1600\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="giac")
[Out] 1/1600*(64*d*x + 64*c - 180*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
4) - 123*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 123*log(abs(tan(1/2*d*x + 1/2
*c) - 2)))/d
```

Mupad [B]
time = 0.88, size = 52, normalized size = 0.55

$$\frac{x}{25}-\frac{\frac{123\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{800}+\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{80\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3/cos(c + d*x) + 5)^2,x)
[Out] x/25 - ((123*atanh(tan(c/2 + (d*x)/2)/2))/800 + (9*tan(c/2 + (d*x)/2))/(80*
(tan(c/2 + (d*x)/2)^2 - 4)))/d
```

3.529 $\int \frac{1}{(5+3\sec(c+dx))^3} \, dx$

Optimal. Leaf size=120

$$\frac{x}{125}+\frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d}-\frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d}+\frac{9}{160d(5}$$

```
[Out] 1/125*x+8361/256000*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-8361/2560
00*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+9/160*tan(d*x+c)/d/(5+3*se
c(d*x+c))^2+963/12800*tan(d*x+c)/d/(5+3*sec(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3870, 4145, 4004, 3916, 2738, 212}

$$\frac{963\tan(c+dx)}{12800d(3\sec(c+dx)+5)}+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}+\frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d}-\frac{8361\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{256000d}+\frac{x}{125}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-3),x]
```

```
[Out] x/125 + (8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(256000*d) - (836
1*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(256000*d) + (9*Tan[c + d*x])
/(160*d*(5 + 3*Sec[c + d*x])^2) + (963*Tan[c + d*x])/(12800*d*(5 + 3*Sec[c
+ d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
```

```
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} - \frac{1}{160}\int \frac{-32+30\sec(c+dx)-9\sec^2(c+dx)}{(5+3\sec(c+dx))^2}\,dx$$

$$= \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} + \frac{\int \frac{512-1365\sec(c+dx)}{5+3\sec(c+dx)}\,dx}{12800}$$

$$= \frac{x}{125} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} - \frac{8361\int \frac{\sec(c+}{5+3\sec(}}{64000}$$

$$= \frac{x}{125} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} - \frac{2787\int \frac{1}{1+\frac{5}{3}\cos(}}{64000}$$

$$= \frac{x}{125} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} - \frac{2787\text{Subst}\Big(\int}{}$$

$$= \frac{x}{125} + \frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d} - \frac{8361\log\left(2\cos\left(\frac{1}{2}(c+\right.\right.}{256}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(120) = 240.
time = 0.33, size = 241, normalized size = 2.01

$$\frac{88064c+88064dx+359523\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\cos(c+dx)\left(2048(c+dx)+8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+25\cos(2(c+dx))\left(2048(c+dx)+8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-359523\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+115560\sin(c+dx)+110700\sin(2(c+dx))}{512000d(3+5\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-3),x]
```

```
[Out] (88064*c + 88064*d*x + 359523*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
60*Cos[c + d*x]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - 8361*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 25*Cos[2*(c + d
*x)]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 83
61*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 359523*Log[2*Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]] + 115560*Sin[c + d*x] + 110700*Sin[2*(c + d*x)])/(5
12000*d*(3 + 5*Cos[c + d*x])^2)
```

Maple [A]
time = 0.07, size = 106, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000}+\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}}{d}$
default	$\frac{-\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000}+\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}}{d}$
risch	$\frac{x}{125}+\frac{27i\left(695\,\mathrm{e}^{3i(dx+c)}+1763\,\mathrm{e}^{2i(dx+c)}+1765\,\mathrm{e}^{i(dx+c)}+1025\right)}{32000d\left(5\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+5\right)^2}+\frac{8361\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{256000d}-\frac{8361\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}+\right.}{256000d}$
norman	$\frac{\frac{16x}{125}+\frac{1053\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3200d}-\frac{1323\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12800d}-\frac{8x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{125}+\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{125}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-4\right)^2}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000d}-8361$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-27/2560/(tan(1/2*d*x+1/2*c)-2)^2-1323/25600/(tan(1/2*d*x+1/2*c)-2)+83
61/256000*ln(tan(1/2*d*x+1/2*c)-2)+27/2560/(tan(1/2*d*x+1/2*c)+2)^2-1323/25
600/(tan(1/2*d*x+1/2*c)+2)-8361/256000*ln(tan(1/2*d*x+1/2*c)+2)+2/125*arcta
n(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.48, size = 155, normalized size = 1.29

$$-\frac{\frac{540\left(\frac{156\sin(dx+c)}{\cos(dx+c)+1}-\frac{49\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-16}-4096\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+8361\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-8361\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{256000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/256000*(540*(156*sin(d*x + c)/(cos(d*x + c) + 1) - 49*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - 16) - 4096*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) +
 8361*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 8361*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 2))/d
```

Fricas [A]
time = 2.82, size = 155, normalized size = 1.29

$$\frac{102400\,dx\cos\left(dx+c\right)^{2}+122880\,dx\cos\left(dx+c\right)+36864\,dx-8361\,\left(25\,\cos\left(dx+c\right)^{2}+30\,\cos\left(dx+c\right)+9\right)\log\left(\frac{3}{2}\,\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)+\frac{5}{2}\right)+8361\,\left(25\,\cos\left(dx+c\right)^{2}+30\,\cos\left(dx+c\right)+9\right)\log\left(\frac{3}{2}\,\cos\left(dx+c\right)-2\,\sin\left(dx+c\right)+\frac{5}{2}\right)+1080\,\left(205\,\cos\left(dx+c\right)+107\right)\sin\left(dx+c\right)}{512000\,\left(25\,d\cos\left(dx+c\right)^{2}+30\,d\cos\left(dx+c\right)+9\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/512000*(102400*d*x*cos(d*x + c)^2 + 122880*d*x*cos(d*x + c) + 36864*d*x -
 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*cos(d*x + c) + 2*si
n(d*x + c) + 5/2) + 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*
cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(205*cos(d*x + c) + 107)*sin(d*
x + c))/(25*d*cos(d*x + c)^2 + 30*d*cos(d*x + c) + 9*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\sec\left(c+dx\right)+5\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))**3,x)
```

```
[Out] Integral((3*sec(c + d*x) + 5)**(-3), x)
```

Giac [A]
time = 0.45, size = 85, normalized size = 0.71

$$\frac{2048\,dx+2048\,c-\frac{540\,\left(49\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-156\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\right)^{2}}-8361\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+8361\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{256000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/256000*(2048*d*x + 2048*c - 540*(49*tan(1/2*d*x + 1/2*c)^3 - 156*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 4)^2 - 8361*log(abs(tan(1/2*d*x + 1
/2*c) + 2)) + 8361*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B]
time = 0.95, size = 78, normalized size = 0.65

$$\frac{x}{125}-\frac{8361\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{128000\,d}+\frac{\frac{1053\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3200}-\frac{1323\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12800}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3/cos(c + d*x) + 5)^3,x)
```

```
[Out] x/125 - (8361*atanh(tan(c/2 + (d*x)/2)/2))/(128000*d) + ((1053*tan(c/2 + (d
*x)/2))/3200 - (1323*tan(c/2 + (d*x)/2)^3)/12800)/(d*(tan(c/2 + (d*x)/2)^4
- 8*tan(c/2 + (d*x)/2)^2 + 16))
```

3.530 $\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$\frac{x}{625}+\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d}-\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d}+$$

```
[Out] 1/625*x+278151/20480000*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-27815
1/20480000*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+3/80*tan(d*x+c)/d/
(5+3*sec(d*x+c))^3+519/12800*tan(d*x+c)/d/(5+3*sec(d*x+c))^2+38733/1024000*
tan(d*x+c)/d/(5+3*sec(d*x+c))
```

Rubi [A]

time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3870, 4145, 4004, 3916, 2738, 212}

$$\frac{38733\tan(c+dx)}{1024000d(3\sec(c+dx)+5)}+\frac{519\tan(c+dx)}{12800d(3\sec(c+dx)+5)^2}+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}+\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d}-\frac{278151\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d}+\frac{x}{625}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-4),x]
```

```
[Out] x/625 + (278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(20480000*d) -
(278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(20480000*d) + (3*Tan[c
 + d*x])/(80*d*(5 + 3*Sec[c + d*x])^3) + (519*Tan[c + d*x])/(12800*d*(5 + 3
*Sec[c + d*x])^2) + (38733*Tan[c + d*x])/(1024000*d*(5 + 3*Sec[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
```

```
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} - \frac{1}{240}\int \frac{-48+45\sec(c+dx)-18\sec^2(c+dx)}{(5+3\sec(c+dx))^3}\,dx$$

$$= \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{\int \frac{1536-4230\sec(c+dx)+15}{(5+3\sec(c+dx}}{38400}$$

$$= \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan(c+d}{1024000d(5+3\sec(c}$$

$$= \frac{x}{625} + \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan}{1024000d(5+}$$

$$= \frac{x}{625} + \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan}{1024000d(5+}$$

$$= \frac{x}{625} + \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan}{1024000d(5+}$$

$$= \frac{x}{625} + \frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d} - \frac{278151\log\left(2\cos\left(\frac{1}{2}\right.\right.}{2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 344 vs. 2(145) = 290.
time = 0.53, size = 344, normalized size = 2.37

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-4),x]
```

```
[Out] (18284544*c + 18284544*d*x + 4096000*c*Cos[3*(c + d*x)] + 4096000*d*x*Cos[3
*(c + d*x)] + 155208258*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 347688
75*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 915*Cos[c
+ d*x]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - 278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 450*Cos[2*(c + d*x)
]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 27
8151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 155208258*Log[2*Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] - 34768875*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + 52174260*Sin[c + d*x] + 51462000*Sin[2*(c + d*x)
] + 24286500*Sin[3*(c + d*x)])/(81920000*d*(3 + 5*Cos[c + d*x])^3)
```

Maple [A]
time = 0.08, size = 136, normalized size = 0.94

method	result
risch	$\frac{x}{625}+\frac{27i(166525\,\mathrm{e}^{5i(dx+c)}+581495\,\mathrm{e}^{4i(dx+c)}+1003842\,\mathrm{e}^{3i(dx+c)}+1064590\,\mathrm{e}^{2i(dx+c)}+643025\,\mathrm{e}^{i(dx+c)}+224875)}{2560000d(5\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+5)^{3}}+$
derivativedivides	$\frac{-\frac{27}{10240\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{3}}-\frac{1431}{102400\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}-\frac{69093}{2048000\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{278151\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20480000}-\frac{27}{10240\left(\tan\left(\frac{dx}{2}+\right.\right.}}{d}$
default	$\frac{-\frac{27}{10240\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{3}}-\frac{1431}{102400\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}-\frac{69093}{2048000\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{278151\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20480000}-\frac{27}{10240\left(\tan\left(\frac{dx}{2}+\right.\right.}}{d}$
norman	$\frac{-\frac{64x}{625}-\frac{44523\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64000d}+\frac{13527\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32000d}-\frac{69093\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024000d}+\frac{48x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{625}-\frac{12x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{625}+\frac{x\left(\tan^{6}\left(\right.\right.}{62}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-4\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-27/10240/(tan(1/2*d*x+1/2*c)-2)^3-1431/102400/(tan(1/2*d*x+1/2*c)-2)^
2-69093/2048000/(tan(1/2*d*x+1/2*c)-2)+278151/20480000*ln(tan(1/2*d*x+1/2*c
)-2)-27/10240/(tan(1/2*d*x+1/2*c)+2)^3+1431/102400/(tan(1/2*d*x+1/2*c)+2)^2
-69093/2048000/(tan(1/2*d*x+1/2*c)+2)-278151/20480000*ln(tan(1/2*d*x+1/2*c)
+2)+2/625*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.46, size = 194, normalized size = 1.34

$$-\frac{\frac{540\left(\frac{26384\sin(dx+c)}{\cos(dx+c)+1}-\frac{16032\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2559\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{\frac{48\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{12\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-64}-65536\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+278151\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-278151\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{20480000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/20480000*(540*(26384*sin(d*x + c)/(cos(d*x + c) + 1) - 16032*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 2559*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 - 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 64) - 65536*arctan(sin(d*x + c)/(co
s(d*x + c) + 1)) + 278151*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 278151
*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Fricas [A]
time = 2.66, size = 208, normalized size = 1.43

$$\frac{8192000\,dx\cos(dx+c)^{3}+14745600\,dx\cos(dx+c)^{2}+8847360\,dx\cos(dx+c)+1769472\,dx-278151\,(125\cos(dx+c)^{3}+225\cos(dx+c)^{2}+135\cos(dx+c)+27)\log\left(\frac{3}{2}\cos(dx+c)+2\sin(dx+c)+\frac{5}{2}\right)+278151\,(125\cos(dx+c)^{3}+225\cos(dx+c)^{2}+135\cos(dx+c)+27)\log\left(\frac{3}{2}\cos(dx+c)-2\sin(dx+c)+\frac{5}{2}\right)+1080\,(44975\cos(dx+c)^{2}+47650\cos(dx+c)+12911)\sin(dx+c)}{40960000\,(125\,d\cos(dx+c)^{3}+225\,d\cos(dx+c)^{2}+135\,d\cos(dx+c)+27\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/40960000*(8192000*d*x*cos(d*x + c)^3 + 14745600*d*x*cos(d*x + c)^2 + 8847
360*d*x*cos(d*x + c) + 1769472*d*x - 278151*(125*cos(d*x + c)^3 + 225*cos(d
*x + c)^2 + 135*cos(d*x + c) + 27)*log(3/2*cos(d*x + c) + 2*sin(d*x + c) +
5/2) + 278151*(125*cos(d*x + c)^3 + 225*cos(d*x + c)^2 + 135*cos(d*x + c) +
 27)*log(3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(44975*cos(d*x + c
)^2 + 47650*cos(d*x + c) + 12911)*sin(d*x + c))/(125*d*cos(d*x + c)^3 + 225
*d*cos(d*x + c)^2 + 135*d*cos(d*x + c) + 27*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sec(c+dx)+5)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))**4,x)
```

```
[Out] Integral((3*sec(c + d*x) + 5)**(-4), x)
```

Giac [A]
time = 0.44, size = 98, normalized size = 0.68

$$\frac{32768\,dx+32768\,c-\frac{540\left(2559\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-16032\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+26384\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\right)^3}-278151\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+278151\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{20480000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/20480000*(32768*d*x + 32768*c - 540*(2559*tan(1/2*d*x + 1/2*c)^5 - 16032*
tan(1/2*d*x + 1/2*c)^3 + 26384*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^
2 - 4)^3 - 278151*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 278151*log(abs(tan(1
/2*d*x + 1/2*c) - 2)))/d
```

Mupad [B]
time = 1.10, size = 105, normalized size = 0.72

$$\frac{x}{625}-\frac{278151\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{10240000\,d}-\frac{\frac{69093\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{1024000}-\frac{13527\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32000}+\frac{44523\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64000}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3/cos(c + d*x) + 5)^4,x)
```

```
[Out] x/625 - (278151*atanh(tan(c/2 + (d*x)/2)/2))/(10240000*d) - ((44523*tan(c/2
 + (d*x)/2))/64000 - (13527*tan(c/2 + (d*x)/2)^3)/32000 + (69093*tan(c/2 +
(d*x)/2)^5)/1024000)/(d*(48*tan(c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^4
+ tan(c/2 + (d*x)/2)^6 - 64))
```

3.531 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=292

$$\frac{2(a-b)\sqrt{a+b}\,(2a^2-9b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{15b^3d}$$

[Out] 2/15*(a-b)*(2*a^2-9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/15*(a-b)*(2*a+9*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/5*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/b/d-4/15*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d

Rubi [A]

time = 0.30, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = $\{3925, 4087, 4090, 3917, 4089\}$

$$\frac{2(a-b)\sqrt{a+b}\,(2a^2-9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+\frac{2(a-b)\sqrt{a+b}\,(2a+9b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^2d}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]

[Out] (2*(a - b)*Sqrt[a + b]*(2*a^2 - 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) + (2*(a - b)*Sqrt[a + b]*(2*a + 9*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*d) - (4*a*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b*d) + (2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
```

```
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] &&  !LtQ[m, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \frac{2\int \sec(c+dx)\left(\frac{3b}{2}-a\sec(c\right.}{}$$

$$= -\frac{4a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15bd} + \frac{2(a+b\sec(c+dx))^{3/2}\tan}{5bd}$$

$$= -\frac{4a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15bd} + \frac{2(a+b\sec(c+dx))^{3/2}\tan}{5bd}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(2a^2-9b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{15b^3d}$$

Mathematica [A]
time = 30.01, size = 401, normalized size = 1.37

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(2(2a^3+2a^2b-9ab^2-9b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2b(-2a^2+7ab+9b^2)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+(2a^2-9b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{15b^2d(b+a\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}}+\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2(-2a^2+9b^2)\sin(c+dx)}{15b^2}+\frac{2a\tan(c+dx)}{15b}+\frac{2}{5}\sec(c+dx)\tan(c+dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(2*a^3
 + 2*a^2*b - 9*a*b^2 - 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + 2*b*(-2*a^2 + 7*a*b + 9*b^2)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (2*a^2 - 9*b^2)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b
^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]) + (S
qrt[a + b*Sec[c + d*x]]*((2*(-2*a^2 + 9*b^2)*Sin[c + d*x])/(15*b^2) + (2*a*
Tan[c + d*x])/(15*b) + (2*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1583 vs. 2(262) = 524.
time = 0.92, size = 1584, normalized size = 5.42

method	result	size
default	Expression too large to display	1584

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(3*b^3+9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*b^3-cos(d*x+c)^4*a^2*b+2*cos(d*x+c)^4*a^3-2*cos(d*x
+c)^3*a^3-9*cos(d*x+c)^3*b^3+6*cos(d*x+c)^2*b^3-9*cos(d*x+c)^4*a*b^2+2*cos(
d*x+c)^3*a^2*b+5*cos(d*x+c)^3*a*b^2-cos(d*x+c)^2*a^2*b+4*cos(d*x+c)*a*b^2+2
*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^2*b-7*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+9*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*
b^2+2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
```

```
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^2*b-7*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*cos(d*x+c)^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+9*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b^2-9*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*b^3-2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+9*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-9*cos(d*x+c)
^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*b^3-2*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{\left(c + dx \right)}} \sec^{3}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.532 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=241

$$-\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec\cdots}{a\cdots}}}{3b^2d}$$

```
[Out] -2/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/b^2/d-2/3*(a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/3*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/
d
```

Rubi [A]
time = 0.19, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3920, 4090, 3917, 4089}

$$-\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) - (2*(a - b)*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(3*b*d) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[
e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
```

```
0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{1}{3}\int \frac{\sec(c+dx)(b+a\sec(c+}{\sqrt{a+b\sec(c+dx)}}\\ &= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{1}{3}a\int \frac{\sec(c+dx)(1+\sec(c+}{\sqrt{a+b\sec(c+dx)}}\\ &= -\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a}{a}\right.}{3b^2d}\end{aligned}$$

Mathematica [A]
time = 14.44, size = 293, normalized size = 1.22

$$-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-2b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+a\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3bd(b+a\cos(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3b}+\frac{2}{3}\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(2*a*(a + b)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
```

```
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*
d*(b + a*Cos[c + d*x])) + (Sqrt[a + b*Sec[c + d*x]]*((2*a*Sin[c + d*x])/(3*
b) + (2*Tan[c + d*x])/3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(215) = 430$.
time = 0.22, size = 914, normalized size = 3.79

method	result	size
default	Expression too large to display	914

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-cos(d
*x+c)^3*a^2-cos(d*x+c)^3*a*b+cos(d*x+c)^2*a^2-cos(d*x+c)^2*a*b-cos(d*x+c)^2
*b^2+2*cos(d*x+c)*a*b+b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c
))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{(c + dx)}} \sec^2{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.533 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=209

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

```
[Out] -2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a
-b))^(1/2)/b/d+2*(a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3914, 3917, 4089}

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(b*d)
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = (a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + b\int \frac{\sec(c+dx)(1+\sec(c+dx}{\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{bd}$$

Mathematica [A]
time = 9.90, size = 232, normalized size = 1.11

$$\frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d}-\frac{2\sqrt{a+b\sec(c+dx)}\left(\frac{(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}+(b+a\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d(b+a\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d - (2*Sqrt[a + b*Sec[c + d*x]]*(
((a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b + a*Co
s[c + d*x])*Tan[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2
]^2]*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(191) = 382$.
time = 0.18, size = 814, normalized size = 3.89

method	result

default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sin(dx+c)\,\mathrm{Ellip}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2
*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a+cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+cos(d*x+c)^2*a-a*cos(
d*x+c)+cos(d*x+c)*b-b)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.534 $\int \sqrt{a+b\sec(c+dx)}\, dx$

Optimal. Leaf size=125

$$-\frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\text{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}}{\sqrt{a+b}\,d}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-b)/
(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)*(
b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3865}

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\text{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
+ d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x
]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x])
)/(Sqrt[a + b]*d)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\, dx = -\frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+}{a+b\sec(c+}}}{\sqrt{a+b}\,d}$$

Mathematica [A]
time = 1.75, size = 151, normalized size = 1.21

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((-a+b)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2a\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sqrt{a+b\sec(c+dx)}}{d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

Maple [A]
time = 0.25, size = 215, normalized size = 1.72

method	result
default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}(1+\cos(dx+c))^2\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)a-\text{Ellipti}\right.}{d(b+a\cos(dx+c))\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2), x)
```

3.535 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=330

$$\frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

[Out] (a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A]
time = 0.22, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3942, 4144, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{b\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]

[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(2*d*n), Int[(d*Csc[e + f*x])^(n + 1)*(Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*Cs
c[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{1}{2}\int \frac{b-b\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{1}{2}\int \frac{b+b\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx-\frac{1}{2}$$
$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$
$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 18.36, size = 2713, normalized size = 8.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
]*(I*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] +
(2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))
/(Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(
c + d*x)/2]^4]*((Sec[(c + d*x)/2]^2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*T
an[(c + d*x)/2]*(I*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + (2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b
)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2]))/(Sqrt[(-a +
 b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]
) + (a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(I*(a - b)*Ellipt
icE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*I)*b*EllipticPi[
-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
+ b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - Sqr
```

```
t[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Co
s[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))/(2*Sqrt[(-a + b)/(
a + b)]*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]) -
 (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(I*(a - b)*EllipticE[I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*I)*b*EllipticPi[-((a + b)/(a - b))
, I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)
/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c
 + d*x)/2])*(-(Sec[(c + d*x)/2]^4*Sin[c + d*x]) + 2*Cos[c + d*x]*Sec[(c + d
*x)/2]^4*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))/(2*Sqrt[(-a + b)/(a +
 b)]*Sqrt[b + a*Cos[c + d*x]]*(Cos[c + d*x]*Sec[(c + d*x)/2]^4)^(3/2)) + (S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*(-((Sqrt[(-a
 + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/Sqrt[2]) + Sqrt[2]*a*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sin[c + d*x]*Tan[(c + d*x)/2] - (Sqrt[(-a + b)/(a
 + b)]*(b + a*Cos[c + d*x])*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])
^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Tan[(c + d*x)/2])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]) + ((I/2)*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sec[(c + d*x)/2]^2*
Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
+ (I*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a +
 b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))
/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (b*Sqrt[(-a + b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)])/((1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b))*Sqrt[1 + ((-a + b)*
Tan[(c + d*x)/2]^2)/(a - b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)
]) - ((a - b)*Sqrt[(-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)
/(a - b)])/(2*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])))/(Sqrt[(-a
+ b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4
]) + ((I*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)
] + (2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]
*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]
^2)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2
*Sec[c + d*x]*Tan[c + d*x]))/(2*Sqrt[(-a + b)/(a + b)]*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]])))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 828 vs.

$2(301) = 602.$
time = 0.26, size = 829, normalized size = 2.51

method	result
default	$\frac{(-1+\cos(dx+c))^2 \left(2\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sin(dx+c)\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \sqrt{\frac{a-b}{a+b}}\right)b-\cos(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2
))*b*sin(d*x+c)-cos(d*x+c)^3*a+cos(d*x+c)^2*a-cos(d*x+c)^2*b+cos(d*x+c)*b)*
(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d
*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec (c + dx)} \cos (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c + d\,x)\,\sqrt{a + \frac{b}{\cos (c + d\,x)}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2), x)
```

3.536 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=396

$$\frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4ad}$$

```
[Out] 1/4*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/a/d+1/4*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+
b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-
b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/4*(4*a^2-b^2)*cot(d*x+c)*EllipticPi((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b
*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1/4*b*si
n(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/2*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/d
```

Rubi [A]

time = 0.37, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3942, 4189, 4143, 4006, 3869, 3917, 4089}

$$-\frac{\sqrt{a+b}\,(4a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}+\frac{\sqrt{a+b}\,(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+\frac{(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4ad}+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(2*a + b)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(4*a*d) - (Sqrt[a + b]*(4*a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^
2*d) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (Cos[c + d*x]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(2*d*n), Int[(d*Csc[e + f*x])^(n + 1)*(Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*Cs
c[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
```

```
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{4}\int \frac{\cos(c+dx)\,(b+}{\sqrt{}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4ad} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4ad} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.34, size = 1173, normalized size = 2.96

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d*x
]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]^5 - (8*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSi
nh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)
```

```
*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2
*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
 I*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(2*
a^2 - a*b - b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a*
Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1 +
Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]
*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(355) = 710$.
time = 0.21, size = 1254, normalized size = 3.17

method	result	size
default	Expression too large to display	1254

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(8*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))-2*cos(d*x+c)*b^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x
+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))-4*cos(d*x
+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+2*cos(d*x+c)^4*a^2+8*a^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x
+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^
2*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
```

```
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^2*sin(d*x+c)+3*cos(d*x+c)^3*a*b-2*cos(d*x+c)^2*a^2-cos(d*x+
c)^2*a*b+cos(d*x+c)^2*b^2-2*cos(d*x+c)*a*b-cos(d*x+c)*b^2)*(1+cos(d*x+c))^2
*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec{(c+dx)}}\cos^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

3.537 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=405

$$-\frac{2(a-b)\sqrt{a+b}\,(8a^4+33a^2b^2+147b^4)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+}}}{315b^4d}$$

```
[Out] -2/315*(a-b)*(8*a^4+33*a^2*b^2+147*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/315*(a-b)*(8*a^3+6*a^2*b+
39*a*b^2-147*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/b^3/d+2/315*(8*a^2+49*b^2)*(a+b*sec(d*x+c))^(3/2)*tan(d
*x+c)/b^2/d-8/63*a*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b^2/d+2/9*sec(d*x+c)*(
a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d+2/315*a*(8*a^2+39*b^2)*(a+b*sec(d*x+c)
)^(1/2)*tan(d*x+c)/b^2/d
```

Rubi [A]

time = 0.56, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3950, 4167, 4087, 4090, 3917, 4089}

$$\frac{2(8a^2+49b^2)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315b^2d}+\frac{2a(8a^2+39b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^2d}-\frac{2(a-b)\sqrt{a+b}\,(8a^4+33a^2b^2+147b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^4d}-\frac{2(a-b)\sqrt{a+b}\,(8a^3+6a^2b+39ab^2-147b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^3d}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{63b^2d}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b
^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^3 + 6*a^2*b + 39*a*b^2 - 147*b^3)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(315*b^3*d) + (2*a*(8*a^2 + 39*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c
 + d*x])/(315*b^2*d) + (2*(8*a^2 + 49*b^2)*(a + b*Sec[c + d*x])^(3/2)*Tan[c
 + d*x])/(315*b^2*d) - (8*a*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b^
2*d) + (2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3950

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Dist[d^3/(b*(m + n -
1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b
*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{9bd} + \frac{2\int \sec(c+dx)(\cdots}{\cdots}$$

$$= -\frac{8a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63b^2d} + \frac{2\sec(c+dx)(a+b\sec(c\cdots}{9bd\cdots}$$

$$= \frac{2(8a^2+49b^2)\left(a+b\sec(c+dx)\right)^{3/2}\tan(c+dx)}{315b^2d} - \frac{8a(a+b\sec(c\cdots}{\cdots}$$

$$= \frac{2a(8a^2+39b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{315b^2d} + \frac{2(8a^2+49b^2)\,(\cdots}{\cdots}$$

$$= \frac{2a(8a^2+39b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{315b^2d} + \frac{2(8a^2+49b^2)\,(\cdots}{\cdots}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(8a^4+33a^2b^2+147b^4)\cot(c+dx)E\Big(\sin^{-1}\Big(\frac{\sqrt{\cdots}}{\cdots}}{\cdots}$$

Mathematica [**A**]
time = 17.96, size = 550, normalized size = 1.36

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\left(2(8a^5+8a^4b+33a^3b^2+33a^2b^3+147ab^4+147b^5)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)-2b(8a^4+2a^3b+33a^2b^2+186ab^3+147b^4)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+(8a^4+33a^2b^2+147b^4)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{315b^3d(b+a\cos(c+dx))^2\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\,\sec^{\frac{3}{2}}(c+dx)}+\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2(8a^4+33a^2b^2+147b^4)\sin(c+dx)}{315b^3}+\frac{2\sec^2(c+dx)(3a^2\sin(c+dx)+49b^2\sin(c+dx))}{315b}+\frac{8\sec(c+dx)(-a^3\sin(c+dx)+22ab^2\sin(c+dx))}{315b^2}+\frac{20}{63}a\sec^2(c+dx)\tan(c+dx)+\frac{2}{9}b\sec^3(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(8*
a^5 + 8*a^4*b + 33*a^3*b^2 + 33*a^2*b^3 + 147*a*b^4 + 147*b^5)*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^4 +
2*a^3*b + 33*a^2*b^2 + 186*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 + 33*a^2*b^2 + 147*b^4)*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315
*b^3*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2))
+ (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(8*a^4 + 33*a^2*b^2 + 147*b^
4)*Sin[c + d*x])/(315*b^3) + (2*Sec[c + d*x]^2*(3*a^2*Sin[c + d*x] + 49*b^2
*Sin[c + d*x]))/(315*b) + (8*Sec[c + d*x]*(-(a^3*Sin[c + d*x]) + 22*a*b^2*S
in[c + d*x]))/(315*b^2) + (20*a*Sec[c + d*x]^2*Tan[c + d*x])/63 + (2*b*Sec[
c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2521 vs. $2(367) = 734$.
time = 0.68, size = 2522, normalized size = 6.23

method	result	size
default	Expression too large to display	2522

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/315/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(-4*cos(d*x+c)^6*a^4*b+33*cos(d*x+c)^6*a^3*b^2+88*cos(d*x+c)^6*a^2*b^3
+147*cos(d*x+c)^6*a*b^4+8*cos(d*x+c)^5*a^4*b-34*cos(d*x+c)^5*a^3*b^2+33*cos
(d*x+c)^5*a^2*b^3-10*cos(d*x+c)^5*a*b^4-4*cos(d*x+c)^4*a^4*b-68*cos(d*x+c)^
4*a^2*b^3+cos(d*x+c)^3*a^3*b^2-52*cos(d*x+c)^3*a*b^4-53*cos(d*x+c)^2*a^2*b^
3-85*cos(d*x+c)*a*b^4+147*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5-8*cos(d*x+c)^5*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5-147*cos(d*x+c)^
5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*b^5+147*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*b^5-8*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5-147*cos(d*x+c)^4*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5+2*cos(d*x+c)
^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a^3*b^2+33*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a^2*b^3+186*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4-8*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b-
33*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^3*b^2-33*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3-147*cos(d*x+c)^4*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4-35*b^5+
8*cos(d*x+c)^6*a^5-8*cos(d*x+c)^5*a^5+147*cos(d*x+c)^5*b^5-98*cos(d*x+c)^4*
b^5-14*cos(d*x+c)^2*b^5+8*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((

```
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b+2*cos(d*x+c)^5*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2+33*cos(d*
x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*a^2*b^3+186*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4-8*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b-33*cos(d*x+c)^5*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*
b^2-33*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*sin(d*x+c)*a^2*b^3-147*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4+8*cos(d*x+c)^4*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b)/(b+a*
cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))^{\frac{3}{2}} \sec^4 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

3.538 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=342

$$\frac{4a(a-b)\sqrt{a+b}\,(3a^2-41b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\cdot}}{105b^3d}$$

```
[Out] 4/105*a*(a-b)*(3*a^2-41*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a
+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/105*(a-b)*(6*a^2+57*a*b-25*b^2)*cot(
d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a
+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^
2/d-4/35*a*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/b/d+2/7*(a+b*sec(d*x+c))^(5/2)
*tan(d*x+c)/b/d-2/105*(6*a^2-25*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.41, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3925, 4087, 4090, 3917, 4089}

$$\frac{2(a-b)\sqrt{a+b}\,(6a^2+57ab-25b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^2d}+\frac{4a(a-b)\sqrt{a+b}\,(3a^2-41b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^3d}-\frac{2(6a^2-25b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105bd}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{35bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*a*(a - b)*Sqrt[a + b]*(3*a^2 - 41*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(a
 - b)*Sqrt[a + b]*(6*a^2 + 57*a*b - 25*b^2)*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) - (2*
(6*a^2 - 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) - (4*a*(a
 + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*(a + b*Sec[c + d*x])^(
5/2)*Tan[c + d*x])/(7*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] &&  !LtQ[m, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd} + \frac{2\int \sec(c+dx)\left(\frac{5b}{2}-a\sec(c\right.}{}$$

$$= -\frac{4a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35bd} + \frac{2(a+b\sec(c+dx))^{5/2}\,\mathrm{ta}}{7bd}$$

$$= -\frac{2(6a^2-25b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{105bd} - \frac{4a(a+b\sec(c}{}$$

$$= -\frac{2(6a^2-25b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{105bd} - \frac{4a(a+b\sec(c}{}$$

$$= \frac{4a(a-b)\sqrt{a+b}\,(3a^2-41b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c}}{\sqrt{a+b}}\right.\right.}{105b^3}$$

Mathematica [**A**]
time = 14.22, size = 471, normalized size = 1.38

$$\frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\left(2a(3a^3+3a^2b-41ab^2-41b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+b(-6a^3+51a^2b+82ab^2+25b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+a(3a^2-41b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{105b^2d(b+a\cos(c+dx))^2\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(-\frac{4a(3a^2-41b^2)\sin(c+dx)}{105b^2}+\frac{2\sec(c+dx)(3a^2\sin(c+dx)+25b^2\sin(c+dx))}{105b}+\frac{16}{35}a\sec(c+dx)\tan(c+dx)+\frac{2}{7}b\sec^2(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*a*(3
*a^3 + 3*a^2*b - 41*a*b^2 - 41*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] + b*(-6*a^3 + 51*a^2*b + 82*a*b^2 + 25*b^3
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 a*(3*a^2 - 41*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]))/(105*b^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]
*Sec[c + d*x]^(3/2)) + (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-4*a*(3*a
^2 - 41*b^2)*Sin[c + d*x])/(105*b^2) + (2*Sec[c + d*x]*(3*a^2*Sin[c + d*x]
+ 25*b^2*Sin[c + d*x]))/(105*b) + (16*a*Sec[c + d*x]*Tan[c + d*x])/35 + (2*
b*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1851 vs. $2(308) = 616$.
time = 0.36, size = 1852, normalized size = 5.42

method	result	size
default	Expression too large to display	1852

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/105/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(6*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*cos(d*x+c)^4*b^4-10*cos(d*x+c)^2*b^4-6*cos(d*x+c)^5*a^4+6*cos(d*x+c)^4*a^4-15*b^4+3*cos(d*x+c)^5*a^3*b+82*cos(d*x+c)^5*a^2*b^2+25*cos(d*x+c)^5*a*b^3-6*cos(d*x+c)^4*a^3*b-55*cos(d*x+c)^4*a^2*b^2+82*cos(d*x+c)^4*a*b^3+3*cos(d*x+c)^3*a^3*b-68*cos(d*x+c)^3*a*b^3-27*cos(d*x+c)^2*a^2*b^2-39*cos(d*x+c)*a*b^3+25*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+6*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+6*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-82*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-82*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+51*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+82*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+6*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-82*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-82*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+51*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+82*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))

```
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^3/si
n(d*x+c)^5/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{3}{2}}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

3.539 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=282

$$-\frac{2(a-b)\sqrt{a+b}\,(a^2+3b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{5b^2d}$$

```
[Out] -2/5*(a-b)*(a^2+3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/5*(a-3*b)*(a-b)*cot(d*x+c)*EllipticF((a+b*
sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*
x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/5*(a+b*sec(d*x+c))
^(3/2)*tan(d*x+c)/d+2/5*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.28, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3920, 4087, 4090, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,(a^2+3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5b^2d}-\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5bd}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}+\frac{2a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^2*d) - (2*(a - 3*b
)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b*d) + (2*a*Sqrt[a + b*Sec[c + d*x]]
*Tan[c + d*x])/(5*d) + (2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{3}{5}\int \sec(c+dx)(b+a\sec($$

$$=\frac{2a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{5d}+\frac{2(a+b\sec(c+dx))^{3/2}\tan$$

$$=\frac{2a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{5d}+\frac{2(a+b\sec(c+dx))^{3/2}\tan$$

$$=-\frac{2(a-b)\sqrt{a+b}\,(a^2+3b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c}}{\sqrt{a+b}}\right.\right.}{5b^{}}$$

Mathematica [**A**]
time = 13.39, size = 408, normalized size = 1.45

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\left(2(a^3+a^2b+3ab^2+3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)-2b(a^2+4ab+3b^2)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+(a^2+3b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{5bd(b+a\cos(c+dx))^2\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sec^{\frac{3}{2}}(c+dx)}+\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2(a^2+3b^2)\sin(c+dx)}{5b}+\frac{4}{5}a\tan(c+dx)+\frac{2}{5}b\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(a^
3 + a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Cos[c +
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*b*d*(b +
 a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)) + (Cos[c +
d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(a^2 + 3*b^2)*Sin[c + d*x])/(5*b) + (4*
a*Tan[c + d*x])/5 + (2*b*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d
*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1565 vs. $2(252) = 504$.
time = 0.28, size = 1566, normalized size = 5.55

method	result	size
default	Expression too large to display	1566

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a^2*b+4*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+3*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-cos(d*x+c)^3*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3
-cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^2*b-3*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-3*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
```

```
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+
4*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*sin(d*x+c)*a*b^2+3*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-3*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
sin(d*x+c)*a*b^2-3*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+cos(d*x+c)^4*a^3+2*cos(d*x+c)^4*a^2*b+3*
cos(d*x+c)^4*a*b^2-cos(d*x+c)^3*a^3+cos(d*x+c)^3*a^2*b+3*cos(d*x+c)^3*b^3-3
*cos(d*x+c)^2*a^2*b-2*cos(d*x+c)^2*b^3-3*cos(d*x+c)*a*b^2-b^3)/(b+a*cos(d*x
+c))/cos(d*x+c)^2/sin(d*x+c)^5/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^{\frac{3}{2}} \sec^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

3.540 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=249

$$-\frac{8a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{a}}}{3bd}$$

```
[Out] -8/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/b/d+2/3*(a-b)*(3*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^
(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/3*b*(a+b*sec(d*x+c))^(1/2)*tan
(d*x+c)/d
```

Rubi [A]

time = 0.20, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3915, 4090, 3917, 4089}

$$\frac{2(a-b)(3a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}-\frac{8a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-8*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*(a - b)*(3*a - b)*Sqrt
[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(3*b*d) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(3*d)
```

Rule 3915

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^
2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
```

```
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{\sec(c+dx)\left(\frac{3a^2}{2}+\frac{b^2}{2}+\right.}{\sqrt{a+b\sec(c}}$$

$$= \frac{2b\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{1}{3}((a-b)(3a-b))\int \frac{\sec}{\sqrt{a+}}$$

$$= -\frac{8a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\right.}{3bd}$$

Mathematica [A]
time = 10.26, size = 304, normalized size = 1.22

$$-\frac{2\sqrt{a+b\sec(c+dx)}\left(8a(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-2(3a^2+4ab+b^2)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-5ab\sin(c+dx)-2a^2\sin(2(c+dx))+4ab\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)+4a^2\cos^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)-b^2\tan(c+dx)\right)}{3d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*Sec[c + d*x]]*(8*a*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*(3*a^2 + 4*a*
```

```
b + b^2)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] - 5*a*b*Sin[c + d*x] - 2*a^2*Sin[2*(c + d*x)] + 4*
a*b*Cos[c + d*x]*Tan[(c + d*x)/2] + 4*a^2*Cos[c + d*x]^2*Tan[(c + d*x)/2] -
 b^2*Tan[c + d*x]))/(3*d*(b + a*Cos[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(223) = 446$.

time = 0.19, size = 1106, normalized size = 4.44

method	result	size
default	Expression too large to display	1106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))^2*(3*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+4*sin(d*x+c)*cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b
^2-4*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^2-4*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+4*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^2-4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2-4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+4*cos(d*x+c)^3*a^2+cos(d*x+c)^3*a
*b-4*cos(d*x+c)^2*a^2+4*cos(d*x+c)^2*a*b+cos(d*x+c)^2*b^2-5*cos(d*x+c)*a*b-
b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/
cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^{\frac{3}{2}} \sec (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)}^{3/2}}{\cos \left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.541 $\int (a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=309

$$-\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$

```
[Out] -2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a
-b))^(1/2)/d+2*(2*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/d-2*a*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A]

time = 0.15, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3866, 4006, 3869, 3917, 4089}

$$\frac{2(2a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2a\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(2*a - b)*Sqrt[a + b]*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/d - (2*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rule 3866

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Dist[b^2, Int[Csc[c
 + d*x]*((1 + Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]]), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}\,dx = b^2\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{a^2+(2a-b)b\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.54, size = 882, normalized size = 2.85

[illegible]

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[c + d*x])^(3/2),x]

[Out] (2*b*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])) + (2*(a + b*Sec[c + d*x])^(3/2)*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)^2*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. $2(282) = 564$.
time = 0.18, size = 1199, normalized size = 3.88

method	result	size
default	Expression too large to display	1199

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1

```
/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*
cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b*sin(d*x+c)-EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-2*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)-cos(d*x+c)^2*a*b+cos(d*x+c)*a*b
-cos(d*x+c)*b^2+b^2)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2), x)
```

3.542 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=334

$$\frac{a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

```
[Out] a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/b/d+(a+2*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+s
ec(d*x+c))/(a-b))^(1/2)/d-3*b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b
))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+a*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/d
```

Rubi [A]

time = 0.24, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3949, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{3b\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(a + 2*b)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/d - (3*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (a*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/d
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3949

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)/S
qrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e
 + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}\int \frac{-3ab-2b^2\sec(c+dx)+}{\sqrt{a+b\sec(c+}}$$

$$= \frac{a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}\int \frac{-3ab+(-ab-2b^2)\sec(c}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{bd}$$

$$= \frac{a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{bd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.38, size = 439, normalized size = 1.31

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(-2ia(a-b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(i\sinh^{-1}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a+b}{a-b}\right)+4i(a-b)b\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(i\sinh^{-1}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a+b}{a-b}\right)-12iab\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\Pi\left(-\frac{a+b}{a-b};i\sinh^{-1}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a+b}{a-b}\right)+a\sqrt{\frac{-a+b}{a+b}}\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\frac{-a+b}{a+b}}\,d(b+a\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-2*I)*a*(a -
b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)] + (4*I)*(a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*
ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - (12*I)
*a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + a*Sqrt[(-a + b)/(a + b)]
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(S
qrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(305) = 610$.
time = 0.20, size = 1029, normalized size = 3.08

method	result	size
default	Expression too large to display	1029

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-6*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b-
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-cos(d*x+c)^3*a^2+cos(d
*x+c)^2*a^2-cos(d*x+c)^2*a*b+cos(d*x+c)*a*b)*(1+cos(d*x+c))^2*((b+a*cos(d*x
+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec{(c + dx)})^{\frac{3}{2}} \cos{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos{(c + d\,x)}\, \left(a + \frac{b}{\cos{(c + d\,x)}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2), x)
```

3.543 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=390

$$\frac{5(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-}}}{4d}$$

[Out] 5/4*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+1/4*(2*a+5*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(4*a^2+3*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+5/4*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/2*a*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A]

time = 0.38, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3949, 4189, 4143, 4006, 3869, 3917, 4089}

$$-\frac{\sqrt{a+b}\,(4a^2+3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+\frac{\sqrt{a+b}\,(2a+5b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}+\frac{5(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}+\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]

[Out] (5*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a + 5*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (5*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3949

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)/S
qrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e
 + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
```

```
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{a\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d} - \frac{1}{4}\int \frac{\cos(c+dx)}{\ldots}$$

$$= \frac{5b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{a\cos(c+dx)\sqrt{a+b\sec(c\ldots}}{2d}$$

$$= \frac{5b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{a\cos(c+dx)\sqrt{a+b\sec(c\ldots}}{2d}$$

$$= \frac{5(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a\ldots}{a\ldots}\right.}{4d}$$

$$= \frac{5(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a\ldots}{a\ldots}\right.}{4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.03, size = 1159, normalized size = 2.97

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[2*(c + d*x)])/(4*d*(b + a*Co
s[c + d*x])) - ((a + b*Sec[c + d*x])^(3/2)*(5*a*b*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2] + 5*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 10*a*b*Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 5*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2]^5 - 5*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (8*I)*a^2
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*b^2*EllipticPi[-((a
 + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
```

```
b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^2*EllipticPi[-((a + b)/(a - b)),
I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*b^2*EllipticPi[-((a + b)/(a - b)
), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (5*I)*(a - b)*b*EllipticE[I*ArcSinh
[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(2*a^2 - a*b - b^2)*EllipticF[I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*Sqrt[(-a + b)/(a + b)]*d*(b + a*Co
s[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-
1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1439 vs. $2(349) = 698$.
time = 0.18, size = 1440, normalized size = 3.69

method	result	size
default	Expression too large to display	1440

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(5*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+5*cos(d*x+c)*b^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+8*cos(d*x+c)*a^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2
))+6*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,((a-b)/(a+b))^(1/2))-4*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-8*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+2*co
s(d*x+c)^4*a^2+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
```

```
/2))*a*b*sin(d*x+c)+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^2*sin(d*x+c)+8*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,((a-b)/(a+b))^(1/2))*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-8*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+7*cos(d*x+c)^3*a*
b-2*cos(d*x+c)^2*a^2-5*cos(d*x+c)^2*a*b+5*cos(d*x+c)^2*b^2-2*cos(d*x+c)*a*b
-5*cos(d*x+c)*b^2)*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+
a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2), x)
```

3.544 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=463

$$-\frac{2a(a-b)\sqrt{a+b}\,(8a^4+51a^2b^2+741b^4)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec\cdots}{a\cdots}}}{693b^4d}$$

```
[Out] -2/693*a*(a-b)*(8*a^4+51*a^2*b^2+741*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(
a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/693*(a-b)*(8*a^4+6*a^3*
b+57*a^2*b^2-606*a*b^3+135*b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/693*a*(8*a^2+67*b^2)*(a+b*sec(d*x
+c))^(3/2)*tan(d*x+c)/b^2/d+2/693*(8*a^2+81*b^2)*(a+b*sec(d*x+c))^(5/2)*tan
(d*x+c)/b^2/d-8/99*a*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b^2/d+2/11*sec(d*x+c
)*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b/d+2/693*(8*a^4+57*a^2*b^2+135*b^4)*(a
+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d
```

Rubi [A]

time = 0.71, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3950, 4167, 4087, 4090, 3917, 4089}

$$\frac{2(8a^2+81b^2)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{693b^2d}+\frac{2a(8a^2+67b^2)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{693b^2d}-\frac{2a(a-b)\sqrt{a+b}\,(8a^4+51a^2b^2+741b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{693b^4d}+\frac{2(8a^4+57a^2b^2+135b^4)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{693b^2d}-\frac{2(a-b)\sqrt{a+b}\,(8a^4+6a^3b+57a^2b^2-606ab^3+135b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{693b^3d}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{99b^2d}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(693
*b^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^4 + 6*a^3*b + 57*a^2*b^2 - 606*a*b^3
+ 135*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(693*b^3*d) + (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(693*b^2*d) + (2*a*(8*a^2 + 67*b^2)*(a
 + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(693*b^2*d) + (2*(8*a^2 + 81*b^2)*(a
 + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(693*b^2*d) - (8*a*(a + b*Sec[c + d*
x])^(7/2)*Tan[c + d*x])/(99*b^2*d) + (2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(
7/2)*Tan[c + d*x])/(11*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
```

```
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3950

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Dist[d^3/(b*(m + n -
1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b
*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
```

```
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{11bd} + \frac{2\int \sec(c+dx)}{}$$

$$= -\frac{8a(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{99b^2d} + \frac{2\sec(c+dx)(a+b\sec(}{11}$$

$$= \frac{2(8a^2+81b^2)\left(a+b\sec(c+dx)\right)^{5/2}\tan(c+dx)}{693b^2d} - \frac{8a(a+b\sec(c}{}$$

$$= \frac{2a(8a^2+67b^2)\left(a+b\sec(c+dx)\right)^{3/2}\tan(c+dx)}{693b^2d} + \frac{2(8a^2+81b^2}{}$$

$$= \frac{2(8a^4+57a^2b^2+135b^4)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{693b^2d} + \frac{2a(8}{}$$

$$= \frac{2(8a^4+57a^2b^2+135b^4)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{693b^2d} + \frac{2a(8}{}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\,(8a^4+51a^2b^2+741b^4)\cot(c+dx)E\Big(\sin^{-1}\Big(}{}$$

Mathematica [A]
time = 17.21, size = 615, normalized size = 1.33

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*(
8*a^5 + 8*a^4*b + 51*a^3*b^2 + 51*a^2*b^3 + 741*a*b^4 + 741*b^5)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^5
+ 2*a^4*b + 51*a^3*b^2 + 663*a^2*b^3 + 741*a*b^4 + 135*b^5)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4 + 51*a^
2*b^2 + 741*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(693*b^3*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Se
c[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*a*(8*a^4
```

```
 + 51*a^2*b^2 + 741*b^4)*Sin[c + d*x])/(693*b^3) + (2*Sec[c + d*x]^3*(113*a
^2*Sin[c + d*x] + 81*b^2*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(3*a^3*Sin[
c + d*x] + 229*a*b^2*Sin[c + d*x]))/(693*b) + (2*Sec[c + d*x]*(-4*a^4*Sin[c
 + d*x] + 205*a^2*b^2*Sin[c + d*x] + 135*b^4*Sin[c + d*x]))/(693*b^2) + (46
*a*b*Sec[c + d*x]^3*Tan[c + d*x])/99 + (2*b^2*Sec[c + d*x]^4*Tan[c + d*x])/
11))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2805 vs. $2(421) = 842$.

time = 0.88, size = 2806, normalized size = 6.06

method	result	size
default	Expression too large to display	2806

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/693/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(8*cos(d*x+c)^7*a^6-8*cos(d*x+c)^6*a^6+135*cos(d*x+c)^6*b^6-54*cos(d*x
+c)^4*b^6-18*cos(d*x+c)^2*b^6-86*cos(d*x+c)^3*a*b^5-274*cos(d*x+c)^2*a^2*b^
4-224*cos(d*x+c)*a*b^5+205*cos(d*x+c)^7*a^3*b^3+741*cos(d*x+c)^7*a^2*b^4+13
5*cos(d*x+c)^7*a*b^5+8*cos(d*x+c)^6*a^5*b-52*cos(d*x+c)^6*a^4*b^2+51*cos(d*
x+c)^6*a^3*b^3-307*cos(d*x+c)^6*a^2*b^4+741*cos(d*x+c)^6*a*b^5-4*cos(d*x+c)
^5*a^5*b-140*cos(d*x+c)^5*a^3*b^3-566*cos(d*x+c)^5*a*b^5+cos(d*x+c)^4*a^4*b
^2-160*cos(d*x+c)^4*a^2*b^4-116*cos(d*x+c)^3*a^3*b^3-4*cos(d*x+c)^7*a^5*b+5
1*cos(d*x+c)^7*a^4*b^2-8*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6+135*sin(d*x+c)*cos(d*x+c)^5*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^6-8*sin(d*x+c)*c
os(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^6+135*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^6-51*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^3-741*sin(d*x+c)*cos(d*x+c)^5*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^4-741*
sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b^5+8*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b+2*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
```

```
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2+51*sin(d*x+c)*c
os(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^3*b^3+663*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b^4+741*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^5-8*sin(d*x+c)*cos(d*x+
c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b-5
1*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^4*b^2-51*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^3-741*sin(d*x+c)*cos(d*x+c)^6*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^4-741*si
n(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a*b^5+8*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b+2*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2+51*sin(d*x+c)*cos
(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
3*b^3+663*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b^4+741*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^5-8*sin(d*x+c)*cos(d*x+c)
^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b-51*
sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^4*b^2-63*b^6)/(b+a*cos(d*x+c))/cos(d*x+c)^5/sin(d*x+c)^5/b^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^6 + 2*a*b*sec(d*x + c)^5 + a^2*sec(d*x + c)^4)*s
qrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

3.545 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=399

$$\frac{2(a-b)\sqrt{a+b}\,(10a^4-279a^2b^2-147b^4)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{a-}}}{315b^3d}$$

```
[Out] 2/315*(a-b)*(10*a^4-279*a^2*b^2-147*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+
c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a
+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/315*(a-b)*(10*a^3+165*a^
2*b-114*a*b^2+147*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/315*(10*a^2-49*b^2)*(a+b*sec(d*x+c))^(3/2)
*tan(d*x+c)/b/d-4/63*a*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d+2/9*(a+b*sec(d
*x+c))^(7/2)*tan(d*x+c)/b/d-4/315*a*(5*a^2-57*b^2)*(a+b*sec(d*x+c))^(1/2)*t
an(d*x+c)/b/d
```

Rubi [A]
time = 0.52, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3925, 4087, 4090, 3917, 4089}

$$-\frac{2(10a^2-49b^2)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315bd}-\frac{4a(5a^2-57b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315bd}+\frac{2(a-b)\sqrt{a+b}\,(10a^4-279a^2b^2-147b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^3d}+\frac{2(a-b)\sqrt{a+b}\,(10a^3+165a^2b-114ab^2+147b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^2d}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{63bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a^4 - 279*a^2*b^2 - 147*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*
b^3*d) + (2*(a - b)*Sqrt[a + b]*(10*a^3 + 165*a^2*b - 114*a*b^2 + 147*b^3)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(315*b^2*d) - (4*a*(5*a^2 - 57*b^2)*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(315*b*d) - (2*(10*a^2 - 49*b^2)*(a + b*Sec[c + d*x])^(3/2)*T
an[c + d*x])/(315*b*d) - (4*a*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*
b*d) + (2*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] &&  !LtQ[m, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd} + \frac{2\int \sec(c+dx)\left(\frac{7b}{2} - a\sec\right.}{}$$

$$= -\frac{4a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63bd} + \frac{2(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$= -\frac{2(10a^2-49b^2)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{315bd} - \frac{4a(a+b\,\mathrm{se}}{}$$

$$= -\frac{4a(5a^2-57b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{315bd} - \frac{2(10a^2-4}{}$$

$$= -\frac{4a(5a^2-57b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{315bd} - \frac{2(10a^2-4}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(10a^4-279a^2b^2-147b^4)\cot(c+dx)E\left(\sin^{-1}\left(\right.\right.}{}$$

Mathematica [A]
time = 16.56, size = 552, normalized size = 1.38

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(10*
a^5 + 10*a^4*b - 279*a^3*b^2 - 279*a^2*b^3 - 147*a*b^4 - 147*b^5)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(-10*a
^4 + 155*a^3*b + 279*a^2*b^2 + 261*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (10*a^4 - 279*a^2*b^2 -
147*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(315*b^2*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*
x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(-10*a^4 + 279*a
^2*b^2 + 147*b^4)*Sin[c + d*x])/(315*b^2) + (2*Sec[c + d*x]^2*(75*a^2*Sin[c
 + d*x] + 49*b^2*Sin[c + d*x]))/315 + (2*Sec[c + d*x]*(5*a^3*Sin[c + d*x] +
 163*a*b^2*Sin[c + d*x]))/(315*b) + (38*a*b*Sec[c + d*x]^2*Tan[c + d*x])/63
 + (2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2522 vs. $2(361) = 722$.
time = 0.53, size = 2523, normalized size = 6.32

method	result	size
default	Expression too large to display	2523

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(5*cos(d*x+c)^6*a^4*b+279*cos(d*x+c)^6*a^3*b^2+163*cos(d*x+c)^6*a^2*b^
3+147*cos(d*x+c)^6*a*b^4-10*cos(d*x+c)^5*a^4*b-199*cos(d*x+c)^5*a^3*b^2+279
*cos(d*x+c)^5*a^2*b^3+65*cos(d*x+c)^5*a*b^4+5*cos(d*x+c)^4*a^4*b-272*cos(d*
x+c)^4*a^2*b^3-80*cos(d*x+c)^3*a^3*b^2-82*cos(d*x+c)^3*a*b^4-170*cos(d*x+c)
^2*a^2*b^3-130*cos(d*x+c)*a*b^4+147*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5+10*cos(d*x+c)^5*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5-147*
cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*b^5+147*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5+10*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5-147*cos(d*x+c)^4*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5+
155*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a^3*b^2+279*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3+261*cos(d*x+c)^4*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4+10*co
s(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^4*b-279*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2-279*cos(d*x+c)^4*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3-147*cos(d*x
+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a*b^4-35*b^5-10*cos(d*x+c)^6*a^5+10*cos(d*x+c)^5*a^5+147*cos(d*x+c)^5*
b^5-98*cos(d*x+c)^4*b^5-14*cos(d*x+c)^2*b^5-10*cos(d*x+c)^5*(cos(d*x+c)/(1+
```

```
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b+155*cos(d*
x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*a^3*b^2+279*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3+261*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4+10*cos(d*x+c)^5
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^4*b-279*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^3*b^2-279*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3-147*cos(d*x+c)^5*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4
-10*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a^4*b)/(b+a*cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^5 + 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3)*s
qrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

3.546 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=333

$$-\frac{2a(a-b)\sqrt{a+b}\,(3a^2+29b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{21b^2d}$$

```
[Out] -2/21*a*(a-b)*(3*a^2+29*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a
+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/21*(a-b)*(3*a^2-24*a*b+5*b^2)*cot(d*
x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b
)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+
2/7*a*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+b*sec(d*x+c))^(5/2)*tan(d*
x+c)/d+2/21*(3*a^2+5*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.39, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3920, 4087, 4090, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,(3a^2-24ab+5b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{21bd}-\frac{2a(a-b)\sqrt{a+b}\,(3a^2+29b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{21b^2d}+\frac{2(3a^2+5b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{21d}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}+\frac{2a\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(3*a^2 + 29*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b^2*d) - (2*(a
 - b)*Sqrt[a + b]*(3*a^2 - 24*a*b + 5*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b*d) + (2*(3*a
^2 + 5*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*(a + b*Sec
[c + d*x])^(3/2)*Tan[c + d*x])/(7*d) + (2*(a + b*Sec[c + d*x])^(5/2)*Tan[c
+ d*x])/(7*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[
e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{5}{7}\int \sec(c+dx)(b+a\sec($$

$$= \frac{2a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{7d} + \frac{2(a+b\sec(c+dx))^{5/2}\text{ta}}{7d}$$

$$= \frac{2(3a^2+5b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{21d} + \frac{2a(a+b\sec(c+}{}$$

$$= \frac{2(3a^2+5b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{21d} + \frac{2a(a+b\sec(c+}{}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\,(3a^2+29b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\text{se}}}{\sqrt{a}}\right.\right.}{2}$$

Mathematica [A]
time = 13.93, size = 474, normalized size = 1.42

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\left(2a(3a^3+3a^2b+29ab^2+29b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)-2b(3a^3+27a^2b+29ab^2+5b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+a(3a^2+29b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{21bd(b+a\cos(c+dx))^3\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sec^{\frac{5}{2}}(c+dx)}+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2a(3a^2+29b^2)\sin(c+dx)}{21b}+\frac{2}{21}\sec(c+dx)\left(9a^2\sin(c+dx)+5b^2\sin(c+dx)\right)+\frac{6}{7}ab\sec(c+dx)\tan(c+dx)+\frac{2}{7}b^2\sec^2(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*(
3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(3*a^3 + 27*a^2*b + 29*a*b^2 + 5*b^
3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
+ a*(3*a^2 + 29*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(21*b*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*S
ec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*a*(3*a^
2 + 29*b^2)*Sin[c + d*x])/(21*b) + (2*Sec[c + d*x]*(9*a^2*Sin[c + d*x] + 5*
b^2*Sin[c + d*x]))/21 + (6*a*b*Sec[c + d*x]*Tan[c + d*x])/7 + (2*b^2*Sec[c
+ d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1851 vs. $2(299) = 598$.
time = 0.35, size = 1852, normalized size = 5.56

method	result	size
default	Expression too large to display	1852

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/21/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(-3*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+5*cos(d*x+c)^4*b^4-2*cos(d*x+c)^2*b^4+3*cos(d*x+c)^5*a^4-3*cos(d*x+c)^4*a^4-3*b^4+9*cos(d*x+c)^5*a^3*b+29*cos(d*x+c)^5*a^2*b^2+5*cos(d*x+c)^5*a*b^3+3*cos(d*x+c)^4*a^3*b-11*cos(d*x+c)^4*a^2*b^2+29*cos(d*x+c)^4*a*b^3-12*cos(d*x+c)^3*a^3*b-22*cos(d*x+c)^3*a*b^3-18*cos(d*x+c)^2*a^2*b^2-12*cos(d*x+c)*a*b^3+5*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-3*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+5*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-3*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-29*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-29*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+3*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+27*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+29*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-3*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-29*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-29*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+3*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+27*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+29*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2

```
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x
+c)^5/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^{\frac{5}{2}} \sec^2 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

3.547 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=296

$$-\frac{2(a-b)\sqrt{a+b}\,(23a^2+9b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{15bd}$$

```
[Out] -2/15*(a-b)*(23*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/15*(a-b)*(15*a^2-8*a*b+9*b^2)*cot(d*x+c)
*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1
/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/5*
b*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+16/15*a*b*(a+b*sec(d*x+c))^(1/2)*tan(
d*x+c)/d
```

Rubi [A]
time = 0.31, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3915, 4087, 4090, 3917, 4089}

$$\frac{2(a-b)\sqrt{a+b}\,(15a^2-8ab+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bd}-\frac{2(a-b)\sqrt{a+b}\,(23a^2+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bd}+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}+\frac{16ab\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(23*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) + (2*(a - b
)*Sqrt[a + b]*(15*a^2 - 8*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) + (16*a*b*Sqr
t[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*b*(a + b*Sec[c + d*x])^(3/2
)*Tan[c + d*x])/(5*d)
```

Rule 3915

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^
2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
```

```
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2b(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}\int \sec(c+dx)\sqrt{a+b\sec(}$$

$$= \frac{16ab\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d} + \frac{2b(a+b\sec(c+dx))^{3/2}\tan}{5d}$$

$$= \frac{16ab\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d} + \frac{2b(a+b\sec(c+dx))^{3/2}\tan}{5d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(23a^2+9b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c}}{\sqrt{a+b}}\right.\right.}{15bd}$$

Mathematica [**A**]
time = 16.45, size = 440, normalized size = 1.49

$$-\frac{2(a+b\sec(c+dx))^{5/2}\left(-2(23a^3+23a^2b+9ab^2+9b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2(15a^3+23a^2b+17ab^2+9b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-(23a^2+9b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{15d(b+a\cos(c+dx))^3\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2}{15}(23a^2+9b^2)\sin(c+dx)+\frac{22}{15}ab\tan(c+dx)+\frac{2}{5}b^2\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a + b*Sec[c + d*x])^(5/2)*(-2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*(
15*a^3 + 23*a^2*b + 17*a*b^2 + 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] - (23*a^2 + 9*b^2)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*d*(b + a*Cos[c + d*x
])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x
])^(5/2)*((2*(23*a^2 + 9*b^2)*Sin[c + d*x])/15 + (22*a*b*Tan[c + d*x])/15 +
 (2*b^2*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1774 vs. $2(266) = 532$.
time = 0.28, size = 1775, normalized size = 6.00

method	result	size
default	Expression too large to display	1775

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(15*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^3-3*b^3-9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+15*cos(d*x+c)^3*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+11*cos(d
*x+c)^4*a^2*b+23*cos(d*x+c)^4*a^3-23*cos(d*x+c)^3*a^3+9*cos(d*x+c)^3*b^3-6*
cos(d*x+c)^2*b^3+9*cos(d*x+c)^4*a*b^2+23*cos(d*x+c)^3*a^2*b+5*cos(d*x+c)^3*
a*b^2-34*cos(d*x+c)^2*a^2*b-14*cos(d*x+c)*a*b^2+23*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+17*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
```

```
n(d*x+c)*a*b^2-23*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+23*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*
b+17*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*a*b^2-23*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-9*cos(d*x+c)^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+9*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*b^3-23*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^3-9*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+9*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-23*cos(d*x+c)
^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec{(c + dx)})^{\frac{5}{2}} \sec{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(5/2)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

3.548 $\int (a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=352

$$-\frac{14a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{se}}{a}}}{3d}$$

```
[Out] -14/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/d+2/3*(9*a^2-7*a*b+b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*a^2*cot(d*x+c)*EllipticPi((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b
*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/3*b^2*(a+b
*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.23, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3867, 4143, 4006, 3869, 3917, 4089}

$$\frac{2\sqrt{a+b}\,(9a^2-7ab+b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3d}-\frac{2a^2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{14a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3d}+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-14*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) + (2*Sqrt[a + b]*(9*a^2 - 7*
a*b + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(3*d) - (2*a^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/d + (2*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/2}\,dx = \frac{2b^2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3a^3}{2}+\frac{1}{2}b(9a^2+b^2)\sec(c+dx)+\frac{7}{2}}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3a^3}{2}+\left(-\frac{7ab^2}{2}+\frac{1}{2}b(9a^2+b^2)\right)\sec(}{\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{14a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1}{}}}{3d}$$

$$= -\frac{14a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1}{}}}{3d}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 17.73, size = 713, normalized size = 2.03

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(5/2)*((-7*I)*a*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] - I*(3*a^3 - 9*a^2*b + 7*a*b^2 - b^3)*EllipticF[I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*a^3*EllipticPi[-((a + b)/(a
 - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 7*a*b*Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]*(b - b*Tan[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^
2)^2)))/(3*Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]
^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + T
an[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^
(5/2)*((14*a*b*Sin[c + d*x])/3 + (2*b^2*Tan[c + d*x])/3))/(d*(b + a*Cos[c +
 d*x])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1513 vs. $2(317) = 634$.
time = 0.20, size = 1514, normalized size = 4.30

method	result	size
default	Expression too large to display	1514

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*cos(d*x+c)^2*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-7*cos(d*x+c)
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a*b^2-cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*b^3-6*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3+7*cos(d*x+c)^2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+7*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a*b^2+3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3-9*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-7*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
3-6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))*a^3+7*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+7*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-7*cos(d*x+c)^3*a^
2*b-cos(d*x+c)^3*a*b^2+7*cos(d*x+c)^2*a^2*b-7*cos(d*x+c)^2*a*b^2-cos(d*x+c)
^2*b^3+8*cos(d*x+c)*a*b^2+b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d
*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2),x)
[Out] Integral((a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{b}{\cos (c + d\,x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2), x)
```

3.549 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=353

$$\frac{(a-b)\sqrt{a+b}\,(a^2-2b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{bd}$$

[Out] (a-b)*(a^2-2*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+(a^2+6*a*b-2*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-5*a*b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+a^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A]
time = 0.23, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3926, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(a^2+6ab-2b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+b}\,(a^2-2b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}+\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{5ab\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]

[Out] ((a - b)*Sqrt[a + b]*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(a^2 + 6*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (5*a*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \int \frac{\frac{5a^2b}{2}+3ab^2\sec(c+dx)-}{\sqrt{a+b\mathrm{se}}}$$

$$= \frac{a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}\left(b\left(a^2-2b^2\right)\right)\int \frac{\sec(c+d}{\sqrt{a-}}$$

$$= \frac{(a-b)\sqrt{a+b}\,(a^2-2b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+a}}{\sqrt{a+b}}\right.\right.}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}\,(a^2-2b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+a}}{\sqrt{a+b}}\right.\right.}{bd}$$

Mathematica [A]
time = 16.36, size = 454, normalized size = 1.29

$$\frac{2b^2\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d(b+a\cos(c+dx))^2} + \frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\left(2(a^3+a^2b-2ab^2-2b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+4b(-3a^2+3ab+b^2)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+20a^2b\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\Pi\left(-1;\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+\frac{1}{2}(a^2-2b^2)(b+a\cos(c+dx))\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{d(b+a\cos(c+dx))^3\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sec^{\frac{5}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*(b + a*Co
s[c + d*x])^2) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x]
)^(5/2)*(2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*b*(-3*a^2 + 3*a*b + b^2)*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 20*a^2*
b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] + ((a^2 - 2*b^2)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(-Sin[(c + d*x
)/2] + Sin[(3*(c + d*x))/2]))/2))/(d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d
*x)/2]^2]*Sec[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1639 vs. $2(324) = 648$.
time = 0.22, size = 1640, normalized size = 4.65

method	result	size
default	Expression too large to display	1640

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(-1+cos(d*x+c))^2*(10*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b-2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+cos(d*x+c)^3*a^3-cos(d*x+c)^2*a^3+cos(d*x+c)^2*a^2*b+2*cos(d*x+c)^2*a*b^2-cos(d*x+c)*a^2*b-2*cos(d*x+c)*a*b^2+2*cos(d*x+c)*b^3-2*b^3)*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c)
 + a^2*cos(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2), x)
```

3.550 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=399

$$\frac{9a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-}}}{4d}$$

```
[Out] 9/4*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/d+1/4*(2*a^2+9*a*b+8*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(4*a^2+15*b^2)*cot(d*x+c)*E
llipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(
a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d
+9/4*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)*
(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.42, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3926, 4189, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(2a^2+9ab+8b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\,(4a^2+15b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}+\frac{9a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}+\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (9*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a^2 + 9*a*b
+ 8*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2 + 15*b^2)*Cot[c + d*x]*E
llipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(4*d) + (9*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (
a^2*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\cos(c+dx)}{}$$

$$= \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c}{2d}$$

$$= \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c}{2d}$$

$$= \frac{9a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}}{4d}$$

$$= \frac{9a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}}{4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.88, size = 4588, normalized size = 11.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[2*(c + d*x)])/(4*d*(b +
a*Cos[c + d*x])^2) + ((a^3/(2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
+ (3*a*b^2)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (11*a^2*b*Sqrt[
Sec[c + d*x]])/(8*Sqrt[b + a*Cos[c + d*x]]) + (b^3*Sqrt[Sec[c + d*x]])/Sqrt
[b + a*Cos[c + d*x]] + (9*a^2*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(8*Sqr
t[b + a*Cos[c + d*x]]))*(a + b*Sec[c + d*x])^(5/2)*((18*I)*a*(a - b)*b*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]]
, (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d
```

*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)
/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a -
 b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]))/(4*Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c +
 d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*(-1/4*(Sqrt[Sec[(c + d*x)/2]^2]*T
an[(c + d*x)/2]*((18*I)*a*(a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - (4*I)*(2*a^3 -
 a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(S
qrt[(-a + b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)^2) + (a*Sin[c + d*x]*((18*I)*a*(a - b)*
b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
 + b)/(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((
a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(8*Sqrt[(-a + b)/(a + b)]*(b + a*Cos
[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
x]](-1 + Tan[(c + d*x)/2]^2)) - (Tan[(c + d*x)/2]*((18*I)*a*(a - b)*b*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b
)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
- b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(8*Sqrt[(-a + b)/(a + b)]*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-
1 + Tan[(c + d*x)/2]^2)) + ((-9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b

```
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((9*I)*a*(a - b)*b*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*((Cos[c + d*x]*Sin[c + d*x])/
(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])] - ((2*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*((Cos[c + d*x]*Sin[c + d*x]
)/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])] + ((2*I)*a*(4*a^2 + 15*b^2)*Sqrt[(b + a*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*((Cos[c + d*x]*Sin[
c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])] + ((9*I)*a*(a - b...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1645 vs. $2(358) = 716$.
time = 0.18, size = 1646, normalized size = 4.13

method	result	size
default	Expression too large to display	1646

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(-4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-24*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^2+8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^3+8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3+30*b^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a+9*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^2*b+9*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)^4*a^3-4*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c
```

```
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-24*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+8*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+8*a^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2
))+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*
b^2*sin(d*x+c)+9*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*sin(d*x+c)*b+9*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*sin(d*x+c)*a+11*cos(d*x+c)^3*a^2*b-2*cos(d*x+c)^2*a^3-9*cos
(d*x+c)^2*a^2*b+9*cos(d*x+c)^2*a*b^2-2*cos(d*x+c)*a^2*b-9*cos(d*x+c)*a*b^2)
*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(
d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x
+ c) + a^2*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2), x)
```

3.551 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=460

$$\frac{(a-b)\sqrt{a+b}\,(16a^2+33b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{24bd}$$

```
[Out] 1/24*(a-b)*(16*a^2+33*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/24*(16*a^2+26*a*b+33*b^2)*cot(d*x+c)*Ell
ipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*
(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-5/8*b*(4*a
^2+b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((
a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*
x+c))/(a-b))^(1/2)/a/d+1/24*(16*a^2+33*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/
2)/d+13/12*a*b*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/3*a^2*cos(d
*x+c)^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.61, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3926, 4189, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(16a^2+26ab+33b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24d}+\frac{(a-b)\sqrt{a+b}\,(16a^2+33b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24bd}-\frac{5b\sqrt{a+b}\,(4a^2+b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{8ad}+\frac{(16a^2+33b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}+\frac{13ab\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2 + 33*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*b*d) + (Sqrt[a + b
]*(16*a^2 + 26*a*b + 33*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*d) - (5*b*Sqrt[a + b]*(4*a^2
 + b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((16*a^2 + 33*b^2)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(24*d) + (13*a*b*Cos[c + d*x]*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(12*d) + (a^2*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(3*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\cos^2(c+ \ldots}{\ldots}$$

$$= \frac{13ab\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d} + \frac{a^2\cos^2(c+d \ldots}{\ldots}$$

$$= \frac{(16a^2+33b^2)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d} + \frac{13ab\cos(c+dx \ldots}{\ldots}$$

$$= \frac{(16a^2+33b^2)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d} + \frac{13ab\cos(c+dx \ldots}{\ldots}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2+33b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(\ldots}}{\sqrt{a+\ldots}}\right.\right.\ldots}{24b\ldots}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2+33b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(\ldots}}{\sqrt{a+\ldots}}\right.\right.\ldots}{24b\ldots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1018 vs. $2(460) = 920$.
time = 14.65, size = 1018, normalized size = 2.21

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((a^2*Sin[c + d*x])/12 + (13*a*b
*Sin[2*(c + d*x)])/24 + (a^2*Sin[3*(c + d*x)])/12))/(d*(b + a*Cos[c + d*x])
```

```
^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(16*a
^3*Tan[(c + d*x)/2] + 16*a^2*b*Tan[(c + d*x)/2] + 33*a*b^2*Tan[(c + d*x)/2]
 + 33*b^3*Tan[(c + d*x)/2] - 32*a^3*Tan[(c + d*x)/2]^3 - 66*a*b^2*Tan[(c +
d*x)/2]^3 + 16*a^3*Tan[(c + d*x)/2]^5 - 16*a^2*b*Tan[(c + d*x)/2]^5 + 33*a*
b^2*Tan[(c + d*x)/2]^5 - 33*b^3*Tan[(c + d*x)/2]^5 + 120*a^2*b*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^
3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 120*a^2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^3*EllipticPi[-1, ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (16*a^3 + 16*a^2*b + 33*a*b^2 + 33*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*(
38*a^2 - 13*a*b + 24*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*d*(b + a*Cos[c + d
*x])^(5/2)*Sec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1880 vs. $2(415) = 830$.
time = 0.22, size = 1881, normalized size = 4.09

method	result	size
default	Expression too large to display	1881

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^2*(8*cos(d*x+c)^5*a^3-16*cos(d*x+c)^2*a^3-33*cos(d*
x+c)*b^3+30*cos(d*x+c)*b^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))+16*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+120*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b+34*c
os(d*x+c)^4*a^2*b+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^3*sin(d*x+c)+33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
```

```
/(a+b))^(1/2))*b^3*sin(d*x+c)-48*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-33*cos(d*x+c)^2*a*b^2+8*cos(d
*x+c)^3*a^3+33*cos(d*x+c)^2*b^3-48*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+33*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+120*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x
+c)+16*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*s
in(d*x+c)*b+33*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*sin(d*x+c)*a-76*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*b*sin(d*x+c)+26*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-76*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+26*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+1
6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^2*b+33*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+59*cos(d*x+c)^3*a*b^2-18*cos(d*x+c)^2*
a^2*b-26*cos(d*x+c)*a*b^2-16*cos(d*x+c)*a^2*b)*(1+cos(d*x+c))^2*((b+a*cos(d
*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^3*sec(d*x
+ c) + a^2*cos(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2), x)
```

3.552 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=530

$$\frac{(a-b)\sqrt{a+b}\,(284a^2+15b^2)\cot(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{}}{192ad}$$

```
[Out] 1/192*(a-b)*(284*a^2+15*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a
+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/192*(72*a^3+284*a^2*b+118*a*b^2+15*b^3
)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/a/d-1/64*(48*a^4+120*a^2*b^2-5*b^4)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+
c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*
x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1/192*b*(284*a^2+1
5*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/96*(36*a^2+59*b^2)*cos(d*x+c
)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+17/24*a*b*cos(d*x+c)^2*sin(d*x+c)*(a+
b*sec(d*x+c))^(1/2)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2
)/d
```

Rubi [A]
time = 0.86, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3926, 4189, 4143, 4006, 3869, 3917, 4089}

$$\frac{(a-b)\sqrt{a+b}\,(284a^2+15b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{192ad}+\frac{b(284a^2+15b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192ad}+\frac{(36a^2+59b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{96d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}-\frac{\sqrt{a+b}\,(48a^4+120a^2b^2-5b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\operatorname{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{64a^2d}+\frac{\sqrt{a+b}\,(72a^3+284a^2b+118ab^2+15b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{192ad}+\frac{17ab\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(284*a^2 + 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) + (Sqrt[a +
 b]*(72*a^3 + 284*a^2*b + 118*a*b^2 + 15*b^3)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) - (Sq
rt[a + b]*(48*a^4 + 120*a^2*b^2 - 5*b^4)*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^2*
d) + (b*(284*a^2 + 15*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d)
 + ((36*a^2 + 59*b^2)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
96*d) + (17*a*b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d
) + (a^2*Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
```

```
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{1}{4}\int \frac{\cos^3(c+ \ldots}{\ldots}$$

$$= \frac{17ab\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d} + \frac{a^2\cos^3(c+ \ldots}{\ldots}$$

$$= \frac{(36a^2+59b^2)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{96d} + \frac{17a \ldots}{\ldots}$$

$$= \frac{b(284a^2+15b^2)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{192ad} + \frac{(36a^2+59b^2 \ldots}{\ldots}$$

$$= \frac{b(284a^2+15b^2)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{192ad} + \frac{(36a^2+59b^2 \ldots}{\ldots}$$

$$= \frac{(a-b)\sqrt{a+b}\,(284a^2+15b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec \ldots}}{\sqrt{a+ \ldots}}\right.\right.}{192 \ldots}$$

$$= \frac{(a-b)\sqrt{a+b}\,(284a^2+15b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec \ldots}}{\sqrt{a+ \ldots}}\right.\right.}{192 \ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.07, size = 1688, normalized size = 3.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((17*a*b*Sin[c + d*x])/96 + ((48
*a^2 + 59*b^2)*Sin[2*(c + d*x)])/192 + (17*a*b*Sin[3*(c + d*x)])/96 + (a^2*
Sin[4*(c + d*x)])/32))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(
5/2)*(-284*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 284*a^2*b^2*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 15*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2] - 15*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 568*a^3*b*Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 30*a*b^3*Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]^3 - 284*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 2
84*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*a*b^3*Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2]^5 + 15*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]^5 + (288*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (720*
I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*b^4*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (288*I)*a^4*EllipticPi[-((a +
b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
 - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (720*I)*a^2*b^2*EllipticP
i[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (
a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*b^4*Ellip
ticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*b*(284*a^3
 - 284*a^2*b + 15*a*b^2 - 15*b^3)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] - (2*I)*(72*a^4 - 36*a^3*b + 38*a^2*b^2 - 59*a*b^3 - 15*b^4)*Elli
pticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(192*a*Sqrt[(-a + b)/(a +
b)]*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(1 - Tan[(c + d*x)
/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2329 vs. $2(481) = 962$.
time = 0.26, size = 2330, normalized size = 4.40

method	result	size
default	Expression too large to display	2330

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/192/d*(-1+cos(d*x+c))^2*(-72*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+644*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-118*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-284*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-284*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-15*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-720*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2-48*cos(d*x+c)^6*a^4+72*cos(d*x+c)^2*a^4+15*cos(d*x+c)*b^4-288*a^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)-15*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-15*cos(d*x+c)^2*b^4-24*cos(d*x+c)^4*a^4-184*cos(d*x+c)^5*a^3*b-254*cos(d*x+c)^4*a^2*b^2-172*cos(d*x+c)^3*a^3*b-133*cos(d*x+c)^3*a*b^3-30*cos(d*x+c)^2*a^2*b^2+118*cos(d*x+c)*a*b^3+144*a^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)-15*b^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)-288*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4+30*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4-72*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+644*(cos(d

```
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)
-118*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*s
in(d*x+c)-284*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*b-284*a^2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)-15*b^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a-720*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+144*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+284*
cos(d*x+c)^2*a^3*b+15*cos(d*x+c)^2*a*b^3+72*cos(d*x+c)*a^3*b+284*cos(d*x+c)
*a^2*b^2)*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x
+c))/sin(d*x+c)^5/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
```

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2), x)
```

3.553 $\int (a+b\sec(c+dx))^{7/2}\,dx$

Optimal. Leaf size=403

$$-\frac{2(a-b)\sqrt{a+b}\,(58a^2+9b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{}}{15d}$$

```
[Out] -2/15*(a-b)*(58*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/15*(60*a^3-58*a^2*b+22*a*b^2-9*b^3)*cot(d*
x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b
)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*
a^3*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/d+2/5*b^2*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+26/15*a*b^2*(a
+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.35, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3867, 4141, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2a^3\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)\sqrt{a+b}\,(58a^2+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15d}+\frac{2\sqrt{a+b}\,(60a^3-58a^2b+22ab^2-9b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15d}+\frac{26ab^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15d}+\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(58*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d) + (2*Sqrt[a +
 b]*(60*a^3 - 58*a^2*b + 22*a*b^2 - 9*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d) - (2*a^3*Sq
rt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (26*a*b^2*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(15*d) + (2*b^2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d
)
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
```

```
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{2}{5}\int\sqrt{a+b\sec(c+dx)}\left(\frac{5a^3}{2}+\frac{3}{2}b(5$$

$$=\frac{26ab^2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

$$=\frac{26ab^2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

$$=-\frac{2(a-b)\sqrt{a+b}\,(58a^2+9b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a-}{a-}\right.}{15d}$$

$$=-\frac{2(a-b)\sqrt{a+b}\,(58a^2+9b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a-}{a-}\right.}{15d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.38, size = 1150, normalized size = 2.85

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(7/2)*(58*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2] + 58*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 9*a*b^3*Sqrt[(-
a + b)/(a + b)]*Tan[(c + d*x)/2] + 9*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2] - 116*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 18*a*b^3*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 58*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]^5 - 58*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 9*
a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 9*b^4*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]^5 + (30*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSin
h[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] + (30*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
```

```
2)/(a + b)] + I*b*(-58*a^3 + 58*a^2*b - 9*a*b^2 + 9*b^3)*EllipticE[I*ArcSin
h[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(15*a^4 - 60*a^3*b + 58*a^2*b^2 - 22*a
*b^3 + 9*b^4)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(15*Sqrt
[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(1
- Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]
^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + T
an[(c + d*x)/2]^2)]) + (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(7/2)*((2*b*(58
*a^2 + 9*b^2)*Sin[c + d*x])/15 + (32*a*b^2*Tan[c + d*x])/15 + (2*b^3*Sec[c
+ d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2184 vs. $2(364) = 728$.
time = 0.30, size = 2185, normalized size = 5.42

method	result	size
default	Expression too large to display	2185

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(-60*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^3*b-58*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-22*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+58*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^3*b+58*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+9*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+6*cos(d*x+c)^2*b
^4+3*b^4-58*cos(d*x+c)^4*a^3*b-16*cos(d*x+c)^4*a^2*b^2-9*cos(d*x+c)^4*a*b^3
+58*cos(d*x+c)^3*a^3*b-10*cos(d*x+c)^3*a*b^3+74*cos(d*x+c)^2*a^2*b^2+19*cos
(d*x+c)*a*b^3-58*cos(d*x+c)^3*a^2*b^2-9*cos(d*x+c)^3*b^4-9*sin(d*x+c)*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+
15*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
```

```
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^4+9*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-30*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4+15*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^4-9*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^4+9*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-30*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4+58*sin(d*x+c
)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*b+58*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b^2+9*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-60*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-58
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*b^2-22*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/si
n(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(7/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(7/2), x)
```

3.554 $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=359

$$\frac{8a(a-b)\sqrt{a+b}\,(12a^2+11b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{}}{105b^5d}$$

[Out] 8/105*a*(a-b)*(12*a^2+11*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^5/d+2/105*(48*a^3-12*a^2*b+44*a*b^2+25*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d+2/105*(24*a^2+25*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^3/d-12/35*a*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/7*sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d

Rubi [A]
time = 0.46, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {3945, 4177, 4167, 4090, 3917, 4089}

$$\frac{8a(a-b)\sqrt{a+b}\,(12a^2+11b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^5d}+\frac{2(24a^2+25b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105b^3d}+\frac{2\sqrt{a+b}\,(48a^3-12a^2b+44ab^2+25b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^4d}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{35b^2d}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]

[Out] (8*a*(a - b)*Sqrt[a + b]*(12*a^2 + 11*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^5*d) + (2*Sqrt[a + b]*(48*a^3 - 12*a^2*b + 44*a*b^2 + 25*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^4*d) + (2*(24*a^2 + 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^3*d) - (12*a*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b^2*d) + (2*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d)

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{7bd} + \frac{\int \frac{\sec^2(c+dx)(4a+5b\sec(c+dx)-6a\sec}{\sqrt{a+b\sec(c+dx)}}}{7b}$$

$$= -\frac{12a\sec(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{35b^2d} + \frac{2\sec^2(c+dx)\sqrt{a+b\sec(}{7bd}$$

$$= \frac{2(24a^2+25b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{105b^3d} - \frac{12a\sec(c+dx)\sqrt{a+b\sec(}{35b^2d}$$

$$= \frac{2(24a^2+25b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{105b^3d} - \frac{12a\sec(c+dx)\sqrt{a+b\sec(}{35b^2d}$$

$$= \frac{8a(a-b)\sqrt{a+b}\,(12a^2+11b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a}{a}\right.}{105b^5d}$$

Mathematica [A]
time = 11.41, size = 463, normalized size = 1.29

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(4*a*(12*a^3 +
12*a^2*b + 11*a*b^2 + 11*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + b*(-48*a^3 - 12*a^2*b - 44*a*b^2 + 25*b^3)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*
(12*a^2 + 11*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(105*b^4*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]
) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-8*a*(12*a^2 + 11*b^2)*Sin[c + d*x
])/(105*b^4) + (2*Sec[c + d*x]*(24*a^2*Sin[c + d*x] + 25*b^2*Sin[c + d*x]))
/(105*b^3) - (12*a*Sec[c + d*x]*Tan[c + d*x])/(35*b^2) + (2*Sec[c + d*x]^2*
Tan[c + d*x])/(7*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1851 vs. 2(325) = 650.
time = 0.38, size = 1852, normalized size = 5.16

method	result	size
default	Expression too large to display	1852

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/105/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(48*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^4+25*cos(d*x+c)^4*b^4-10*cos(d*x+c)^2*b^4-48*cos(d*x
+c)^5*a^4+48*cos(d*x+c)^4*a^4-15*b^4+24*cos(d*x+c)^5*a^3*b-44*cos(d*x+c)^5*
a^2*b^2+25*cos(d*x+c)^5*a*b^3-48*cos(d*x+c)^4*a^3*b+50*cos(d*x+c)^4*a^2*b^2
-44*cos(d*x+c)^4*a*b^3+24*cos(d*x+c)^3*a^3*b+16*cos(d*x+c)^3*a*b^3-6*cos(d*
x+c)^2*a^2*b^2+3*cos(d*x+c)*a*b^3+25*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+48*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*
sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b^4+48*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+44*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+44*sin(d*x+c)*c
os(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b^3-48*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^3*b-12*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-44*sin(d*x+c)*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+48*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3*b+44*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+44*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-48*sin(d*x+c)*
cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))

```
*a^3*b-12*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b^2-44*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3)/(b+a*cos(d*x+c))/cos(d*
x+c)^3/sin(d*x+c)^5/b^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(1/2)), x)
```

3.555 $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=301

$$-\frac{2(a-b)\sqrt{a+b}\,(8a^2+9b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{15b^4d}$$

```
[Out] -2/15*(a-b)*(8*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*
(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/15*(8*a^2-2*a*b+9*b^2)*cot(d*x+c)*Ellip
ticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b
*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d-8/15*a*(
a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/5*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2
)*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.29, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3945, 4167, 4090, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,(8a^2+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}-\frac{2\sqrt{a+b}\,(8a^2-2ab+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15b^2d}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4*d) - (2*Sqrt[
a + b]*(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) - (8*a*Sqrt[a + b*S
ec[c + d*x]]*Tan[c + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c +
d*x]]*Tan[c + d*x])/(5*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{5bd} + \frac{\int \frac{\sec(c+dx)(2a+3b\sec(c+dx)-4a\sec^2(\cdots}{\sqrt{a+b\sec(c+dx)}}}{5b}$$

$$= -\frac{8a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15b^2d} + \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(\cdots}{5bd}$$

$$= -\frac{8a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15b^2d} + \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(\cdots}{5bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(8a^2+9b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\cdots\right.}{15b^4d}$$

Mathematica [A]
time = 8.93, size = 365, normalized size = 1.21

$$\frac{2\sqrt{\sec(c+dx)}\left(-\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(8a^3+8a^2b+9ab^2+9b^3)E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}-2b(8a^2+2ab+9b^2)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}+(8a^2+9b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}+(b+a\cos(c+dx))\sqrt{\sec(c+dx)}\left((8a^2+9b^2)\sin(c+dx)+b(-4a+3b\sec(c+dx))\tan(c+dx)\right)\right)}{15b^3d\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(8*a^3 +
 8*a^2*b + 9*a*b^2 + 9*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 +
 Sec[c + d*x]))] - 2*b*(8*a^2 + 2*a*b + 9*b^2)*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c +
d*x])/((a + b)*(1 + Sec[c + d*x]))] + (8*a^2 + 9*b^2)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2]
) + (b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((8*a^2 + 9*b^2)*Sin[c + d*x] +
 b*(-4*a + 3*b*Sec[c + d*x])*Tan[c + d*x])))/(15*b^3*d*Sqrt[a + b*Sec[c + d
*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1582 vs. 2(271) = 542.
time = 0.31, size = 1583, normalized size = 5.26

method	result	size
default	Expression too large to display	1583

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-3*b^3-9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*b^3-4*cos(d*x+c)^4*a^2*b+8*cos(d*x+c)^4*a^3-8*cos
(d*x+c)^3*a^3+9*cos(d*x+c)^3*b^3-6*cos(d*x+c)^2*b^3+9*cos(d*x+c)^4*a*b^2+8*
cos(d*x+c)^3*a^2*b-10*cos(d*x+c)^3*a*b^2-4*cos(d*x+c)^2*a^2*b+cos(d*x+c)*a*
b^2+8*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^2*b+2*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-8*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-9*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b^2+8*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a^2*b+2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-8*cos(d*x+c)^2*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-9*cos(
d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a*b^2+9*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*b^3-8*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-9*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+9*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*b^3-8*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5/b^
3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/2)), x)
```

3.556 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{4a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{3b^3d}$$

```
[Out] 4/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/b^3/d+2/3*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/3*(a+b*sec(d*x+c))^(1/2)*tan(d*x+
c)/b/d
```

Rubi [A]
time = 0.19, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3925, 4090, 3917, 4089}

$$\frac{4a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^3d}+\frac{2\sqrt{a+b}\,(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) + (2*Sqrt[a + b]*(2*a + b)
*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(3*b^2*d) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d
)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
```

```
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] &&  !LtQ[m, -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3bd} + \frac{2\int \frac{\sec(c+dx)\left(\frac{b}{2}-a\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{3b} \\
&= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3bd} - \frac{(2a)\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{3b} + \frac{(2a+b)}{} \\
&= \frac{4a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-s}{}}}{3b^3d}
\end{aligned}$$

Mathematica [A]
time = 10.45, size = 341, normalized size = 1.40

$$\frac{4\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\tfrac{1}{2}(c+dx)\right)\right)|\tfrac{a-b}{a+b}\right)-(2a-b)b\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\tfrac{1}{2}(c+dx)\right)\right)|\tfrac{a-b}{a+b}\right)+a\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\tfrac{1}{2}(c+dx)\right)\tan\left(\tfrac{1}{2}(c+dx)\right)\right)}{3b^2d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a+b\sec(c+dx)}}+\frac{(b+a\cos(c+dx))\sec(c+dx)\left(-\frac{4a\sin(c+dx)}{3b^2}+\frac{2\tan(c+dx)}{3b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*a*(a + b)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (2*
a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2]))/(3*b^2*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) + ((b +
a*Cos[c + d*x])*Sec[c + d*x]*((-4*a*Sin[c + d*x])/(3*b^2) + (2*Tan[c + d*x]
)/(3*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(218) = 436$.
time = 0.23, size = 919, normalized size = 3.77

method	result	size
default	Expression too large to display	919

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2-2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-2*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b+2*cos(d*x+c)^3*a^2-cos(d*x+c)^3*a*b-2*cos(d*x+c)^2*a^2+2*cos(d*x+c)^
2*a*b-cos(d*x+c)^2*b^2-cos(d*x+c)*a*b+b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1
/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)), x)
```

3.557 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=204

$$-\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a}}}{b^2d}$$

```
[Out] -2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a
-b))^(1/2)/b^2/d-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(
d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A]

time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3922, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3922

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[Csc[e + f*
x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f},
```

```
x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= -\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx\\ &= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{b^2d}\end{aligned}$$

Mathematica [A]
time = 11.24, size = 238, normalized size = 1.17

$$\frac{(1+\cos(c+dx))\left(-2(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sec(c+dx)\sqrt{\frac{1}{1+\sec(c+dx)}}+2bF\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sec(c+dx)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}-(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)+2(b+a\cos(c+dx))\tan(c+dx)}{bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((1 + Cos[c + d*x])*(-2*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d
*x]*Sqrt[(1 + Sec[c + d*x])^(-1)] + 2*b*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Se
c[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]) + 2*(b + a*Cos[c + d*x])*Tan[c + d*x])/(b*d*Sqrt
[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(186) = 372$.
time = 0.19, size = 639, normalized size = 3.13

method	result

default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sin(dx+c)\,\mathrm{Ellip}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2 *(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x +c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a +b))^(1/2))*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+ c),((a-b)/(a+b))^(1/2))*a-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+ a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+ c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b +a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x +c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d* x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d *x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+cos(d*x+c)^2*a-a*cos(d*x+c)+cos(d* x+c)*b-b)/sin(d*x+c)^5/(b+a*cos(d*x+c))/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)), x)
```

3.558 $\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{a+b}\cot(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

```
[Out] 2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3917}

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{bd}$$

Mathematica [A]
time = 0.73, size = 93, normalized size = 0.94

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\operatorname{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A]
time = 0.18, size = 143, normalized size = 1.44

method	result
default	$\frac{2\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(-1+\cos(dx+c))\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,(1+\cos(dx+}{d(b+a\cos(dx+c))\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-1+cos(
d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c
))/sin(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.559 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+\ldots}{a-b}}}{ad}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{3869\}$

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec\ldots}{a\ldots}}}{ad}$$

Mathematica [A]
time = 0.92, size = 138, normalized size = 1.30

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-2\Pi\left(-1;\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sec(c+dx)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A]
time = 0.18, size = 178, normalized size = 1.68

method	result
default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}(1+\cos(dx+c))^2\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)-2\,\mathrm{Elli}\right.}{d(b+a\cos(dx+c))\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin(d
*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos{(c+d\,x)}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(1/2), x)
```

3.560 $\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=338

$$\frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{abd}$$

```
[Out] (a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b/d+cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/a/d+b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/
d
```

Rubi [A]
time = 0.19, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3946, 4144, 4006, 3869, 3917, 4089}

$$\frac{b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}+\frac{(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abd}+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*
d) + (b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3946

```
Int[1/(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]),
 x_Symbol] :> Simp[(-Cos[e + f*x])*(Sqrt[a + b*Csc[e + f*x]]/(a*f)), x] - D
ist[b/(2*a), Int[(1 + Csc[e + f*x]^2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{b\int \frac{1+\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{b\int \frac{1-\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{2a} - \frac{b\int \frac{\sec(c+dx)}{\sqrt{a+b}}}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{se}}{a}}}{abd}$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{se}}{a}}}{abd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 19.58, size = 5060, normalized size = 14.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(309) = 618$.
time = 0.21, size = 652, normalized size = 1.93

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(2\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\ \text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{\frac{a-b}{a+b}}\right)b-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*
```

```
x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-c
os(d*x+c)^3*a+cos(d*x+c)^2*a-cos(d*x+c)^2*b+cos(d*x+c)*b)*(1+cos(d*x+c))^2*
((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)/(a + b/cos(c + d*x))^(1/2), x)
```

3.561 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=401

$$-\frac{3(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^2d}$$

```
[Out] -3/4*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/a^2/d+1/4*(2*a-3*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-1/4*(4*a^2+3*b^2)*cot(d*x+c)*Ellip
ticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)
^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
-3/4*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)*(a
+b*sec(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.35, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3948, 4189, 4143, 4006, 3869, 3917, 4089}

$$\frac{(2a-3b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}-\frac{3(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}-\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4a^2d}-\frac{\sqrt{a+b}\,(4a^2+3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d}+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-3*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((2*a - 3*b)*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*Ellipti
cPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(4*a^3*d) - (3*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*d) + (
Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3948

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(3b-2a\sec(c+dx)-b\sec^2(c+d\right.}{\sqrt{a+b\sec(c+dx)}}}{4a}$$

$$= -\frac{3b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4a^2d} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+}{2ad}$$

$$= -\frac{3b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4a^2d} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+}{2ad}$$

$$= -\frac{3(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{4a^2d}$$

$$= -\frac{3(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{4a^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.93, size = 1195, normalized size = 2.98

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)])/(4*a*d*Sqrt[a + b*Sec[
c + d*x]]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(3*a*b*
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 3*b^2*Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2] - 6*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 3*a*b*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 3*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]^5 + (8*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (
```

```
6*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a^2*Ellipti
cPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*b^2*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (3*I)*(a -
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*(2*a^2 - a*
b + 3*b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a^2*Sqrt
[(-a + b)/(a + b)]*d*Sqrt[a + b*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*Sqr
t[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)
/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1258 vs. $2(360) = 720$.
time = 0.18, size = 1259, normalized size = 3.14

method	result	size
default	Expression too large to display	1259

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(8*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))+6*cos(d*x+c)*b^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x
+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))-4*cos(d*x
+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b-3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+8*a^2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)+6*(cos(d*x+
```

```
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-4*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+
c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin
(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b^2*sin(d*x+c)+2*cos(d*x+c)^4*a^2-cos(d*x+c)^3*a*b-2*cos(d*x+c)^2*a^2+3*
cos(d*x+c)^2*a*b-3*cos(d*x+c)^2*b^2-2*cos(d*x+c)*a*b+3*cos(d*x+c)*b^2)*(1+c
os(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c
)^5/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

3.562 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=399

$$-\frac{2(16a^4-8a^2b^2-3b^4)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\ldots}{}}}{5b^5\sqrt{a+b}\,d}$$

```
[Out] -2/5*(16*a^4-8*a^2*b^2-3*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/b^5/d/(a+b)^(1/2)-2/5*(4*a+3*b)*(4*a^2+b^2)*cot(d*x+c)*
EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec
(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d/(a+b)^(1/2)-2*a
^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2/5*a*(8*a^
2-3*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^3/(a^2-b^2)/d+2/5*(6*a^2-b^2)*
sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

Rubi [A]

time = 0.53, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3930, 4177, 4167, 4090, 3917, 4089}

$$-\frac{2(4a+3b)(4a^2+b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5b^4d\sqrt{a+b}}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2(6a^2-b^2)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5b^2d(a^2-b^2)}-\frac{2a(8a^2-3b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{5b^3d(a^2-b^2)}-\frac{2(16a^4-8a^2b^2-3b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5b^5d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^5*Sqrt[a + b]*d) - (2*(
4*a + 3*b)*(4*a^2 + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^4*Sqrt[a + b]*d) - (2*a^2*Sec[c
 + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*a*(
8*a^2 - 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^3*(a^2 - b^2)*d)
 + (2*(6*a^2 - b^2)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*
b^2*(a^2 - b^2)*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec^2(c+dx)\left(2a^2-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(6a^2-b^2\right)\sec^2}{\sqrt{a+b\sec(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2(6a^2-b^2)\sec(c+dx)\sqrt{a+b\sec(c+d}}{5b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a(8a^2-3b^2)\sqrt{a+b\sec(c+dx)}\tan(c}{5b^3\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a(8a^2-3b^2)\sqrt{a+b\sec(c+dx)}\tan(c}{5b^3\left(a^2-b^2\right)d}$$

$$= -\frac{2(16a^4-8a^2b^2-3b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{5b^5\sqrt{a+b}\,d}$$

Mathematica [A]
time = 10.20, size = 455, normalized size = 1.14

$$\frac{(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(\frac{2(4a^2+b^2)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(4a^3+4a^2b-3ab^2-3b^3)E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}+2b(-4a^2-ab+3b^2)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}+(4a^2-3b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^4(-a^2+b^2)\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}}+\frac{2\sqrt{\sec(c+dx)}\left((-8a^4b+5a^2b^3+3b^5)\sin(c+dx)+\left(-8a^5+4a^3b^2+\frac{3ab^4}{2}\right)\sin(2(c+dx))+b^2(-a^2+b^2)(b-2a\cos(c+dx))\sec(c+dx)\tan(c+dx)\right)}{-a^2b^4+b^6}\right)}{5d(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((2*(4*a^2 + b^2)*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]*(2*(4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(
a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*b*(-4*a^2 - a*b + 3*b
^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c +
d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + (4*a^
2 - 3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]))/(b^4*(-a^2 + b^2)*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Sec[c + d*x]]
*((-8*a^4*b + 5*a^2*b^3 + 3*b^5)*Sin[c + d*x] + (-8*a^5 + 4*a^3*b^2 + (3*a*
b^4)/2)*Sin[2*(c + d*x)] + b^2*(-a^2 + b^2)*(b - 2*a*Cos[c + d*x])*Sec[c +
d*x]*Tan[c + d*x]))/(-(a^2*b^4) + b^6)))/(5*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2476 vs. 2(367) = 734.
time = 0.54, size = 2477, normalized size = 6.21

method	result	size
default	Expression too large to display	2477

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-3*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*b^5-8*co
s(d*x+c)^4*a^4*b+3*cos(d*x+c)^4*a^2*b^3+6*cos(d*x+c)^3*a^3*b^2+5*cos(d*x+c)
^3*a*b^4+6*cos(d*x+c)^2*a^2*b^3-2*cos(d*x+c)*a*b^4+b^5-16*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a^5+3
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^
3*sin(d*x+c)*b^5-3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^5-16*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^5+3*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^5-8*c
os(d*x+c)^4*a^3*b^2-3*cos(d*x+c)^4*a*b^4+16*cos(d*x+c)^3*a^4*b-8*cos(d*x+c)
^3*a^2*b^3-8*cos(d*x+c)^2*a^4*b+2*cos(d*x+c)*a^3*b^2+2*cos(d*x+c)^2*b^5+16*
cos(d*x+c)^4*a^5-16*cos(d*x+c)^3*a^5-3*cos(d*x+c)^3*b^5+EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a*b^4-1
6*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)
^3*sin(d*x+c)*a^4*b+8*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a^3*b^2+8*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a^2*b^3+3*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+
c)*a*b^4+16*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
cos(d*x+c)^2*sin(d*x+c)*a^4*b+4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2-8*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^2*b^3+E
```

```
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*
sin(d*x+c)*a*b^4-16*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^4*b+8*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2+8*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a
^2*b^3+3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos
(d*x+c)^2*sin(d*x+c)*a*b^4+16*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a^4*b+4*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a^3*b^2-8*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*si
n(d*x+c)*a^2*b^3-a^2*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)/(a-b)/(a
+b)/b^4
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(3/2)), x)
```

3.563 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=325

$$\frac{2a(8a^2-5b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^4\sqrt{a+b}\,d}$$

```
[Out] 2/3*a*(8*a^2-5*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/b^4/d/(a+b)^(1/2)+2/3*(2*a+b)*(4*a+b)*cot(d*x+c)*EllipticF((a+b*s
ec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^
(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/(a+b)^(1/2)-2*a^2*sec(d*x+c)*ta
n(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*sec(d*x+
c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

Rubi [A]
time = 0.34, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3930, 4167, 4090, 3917, 4089}

$$\frac{2a(8a^2-5b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^4d\sqrt{a+b}}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2(4a^2-b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^2d(a^2-b^2)}+\frac{2(2a+b)(4a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) + (2*(2*a + b)*(4*
a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) - (2*a^2*Sec[c + d*x]*Tan[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a +
b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(a^2-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}(4a^2-b^2)\sec^2(c\cdots\right.}{\sqrt{a+b\sec(c+dx)}}}{b\,(a^2-b^2)}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+\cdots}{3b^2\,(a^2-b^2)\,d}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\,\tan(c+\cdots}{3b^2\,(a^2-b^2)\,d}$$

$$= \frac{2a(8a^2-5b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec\cdots}{a+\cdots}}}{3b^4\sqrt{a+b}\,d}$$

Mathematica [**A**]
time = 11.08, size = 416, normalized size = 1.28

$$\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2a(8a^3+8a^2b-5ab^2-5b^3)E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}-2b(8a^3+2a^2b-5ab^2+b^3)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}+a(8a^2-5b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)-\frac{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(a^2(8a^2-5b^2)\sin(c+dx)+b(-a^2+b^2)(b-4a\cos(c+dx))\sec(c+dx)\tan(c+dx)\right)}{\sqrt{\sec(c+dx)}}\right)}{3b^3(a^2-b^2)\,d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\,(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*(2*a*(8*a^3 + 8*a^2*b - 5*a*b^2 - 5*b^3)*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c +
 d*x])/((a + b)*(1 + Sec[c + d*x]))] - 2*b*(8*a^3 + 2*a^2*b - 5*a*b^2 + b^3
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*
x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + a*(8*a^
2 - 5*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]) - (Sqrt[Sec[(c + d*x)/2]^2]*(a^2*(8*a^2 - 5*b^2)*Sin[c + d*x] + b*(-
a^2 + b^2)*(b - 4*a*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x]))/Sqrt[Sec[c +
d*x]]))/(3*b^3*(a^2 - b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^
(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1791 vs. 2(297) = 594.
time = 0.28, size = 1792, normalized size = 5.51

method	result	size
default	Expression too large to display	1792

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/3/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-8*cos(d*x+c)^3*a^4+8*cos(d*x+c)^2*a^4-8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+5*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-5*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-5*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-cos(d*x+c)^2*b^4+b^4+4*cos(d*x+c)^3*a^3*b-cos(d*x+c)^3*a*b^3-4*cos(d*x+c)^2*a^2*b^2-4*cos(d*x+c)*a*b^3+5*cos(d*x+c)^3*a^2*b^2-sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-8*cos(d*

```
x+c)^2*a^3*b+5*cos(d*x+c)^2*a*b^3+4*cos(d*x+c)*a^3*b-b^2*a^2)/(b+a*cos(d*x+
c))/sin(d*x+c)/cos(d*x+c)/(a-b)/(a+b)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(3/2)), x)
```

3.564 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{2(2a^2-b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+d$$

```
[Out] -2*(2*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(
1/2)/b^3/d/(a+b)^(1/2)-2*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+se
c(d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)-2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+
b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3924, 4090, 3917, 4089}

$$-\frac{2(2a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^3d\sqrt{a+b}}-\frac{2a^2\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) - (2*(2*a + b)*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(b^2*Sqrt[a + b]*d) - (2*a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3924

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
```

```
 + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= -\frac{2a^2\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{2\int\frac{\sec(c+dx)\left(-\frac{ab}{2}-\frac{1}{2}\left(2a^2-b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)} \\
&= -\frac{2a^2\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{(2a+b)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b(a+b)}+ \cdots \\
&= -\frac{2\left(2a^2-b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec \cdots}{a \cdots}}\cdots}{b^3\sqrt{a+b}\,d}
\end{aligned}$$

Mathematica [A]
time = 8.96, size = 395, normalized size = 1.54

$$-\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(\left(-a^2b+b^3+\left(-2a^3+ab^2\right)\cos(c+dx)\right)\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}\sin(c+dx)+\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2\left(2a^3+2a^2b-ab^2-b^3\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2b\left(-2a^2-ab+b^2\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+\left(2a^2-b^2\right)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{b^2\left(a^2-b^2\right)d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((-(a^2*b) + b^3 + (-2*a^3 + a*
b^2)*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
 + Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
2*b*(-2*a^2 - a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)] + (2*a^2 - b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])))/(b^2*(a^2 - b^2)*d*Sqrt[Sec[(c + d*x)/2
]^2]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1450 vs. $2(237) = 474$.
time = 0.22, size = 1451, normalized size = 5.65

method	result	size
default	Expression too large to display	1451

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-2*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*sin(d*x+c)-2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*sin(d*x+c)*b+EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
```

```
-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+2*cos(d*x+c)^2*a^3-cos(d*x+
c)^2*a^2*b-cos(d*x+c)^2*a*b^2-2*cos(d*x+c)*a^3+2*cos(d*x+c)*a^2*b+cos(d*x+c
)*a*b^2-cos(d*x+c)*b^3-b*a^2+b^3)/(b+a*cos(d*x+c))/sin(d*x+c)/b^2/(a+b)/(a-
b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)), x)
```

3.565 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2a\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}\,d}+\frac{2\,c}{}$$

```
[Out] 2*a*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/
(a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/b/d/(a+b)^(1/2)+2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3921, 4090, 3917, 4089}

$$\frac{2a\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2a\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}+\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d)
 + (2*a*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
```

```
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{b}{2}-\frac{1}{2}a\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a+b} - \frac{a\int \frac{\sec(c+dx)(1+\cdots}{\sqrt{a+b\sec\cdots}}}{a^2-\cdots}$$

$$= \frac{2a\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\ \sqrt{\cdots}}{b^2\sqrt{a+b}\,d}$$

Mathematica [A]
time = 5.76, size = 249, normalized size = 1.05

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4a(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}-4b(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}+a(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]*(4*a*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
```

```
2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*b*(a + b)*Cos[(c +
d*x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] +
 a*(a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(b*(a^2 - b^2)*d*Sqr
t[a + b*Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 836 vs. $2(217) = 434$.
time = 0.19, size = 837, normalized size = 3.53

method	result
default	$\frac{\sqrt{4}\,\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\left(\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+cos(d*x+c)^2*a^2-cos(d*x+c)^2*a*b
-cos(d*x+c)*a^2+cos(d*x+c)*a*b)/(b+a*cos(d*x+c))/sin(d*x+c)/b/(a+b)/(a-b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2 (c + dx)}{(a + b \sec (c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos (c + d\,x)^2\,\left(a + \frac{b}{\cos (c + d\,x)}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)), x)
```

3.566 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{2\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}\,d}+\frac{2c}{\cdots}$$

```
[Out] -2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/(a+
b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)
/b/d/(a+b)^(1/2)-2*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3918, 21, 3914, 3917, 4089}

$$-\frac{2b\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}-\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(b*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d) -
(2*b*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= -\frac{2b\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2} \\
&= -\frac{2b\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx}{a^2-b^2} \\
&= -\frac{2b\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a+b} + \frac{b\int \frac{\sec(c+dx)(}{\sqrt{a+b\,\mathrm{s}}}}{a^2} \\
&= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{b\sqrt{a+b}\,d}
\end{aligned}$$

Mathematica [A]
time = 6.07, size = 244, normalized size = 1.03

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}-4(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}+(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((Sec[(c + d*x)/2]*Sec[c + d*x]*(4*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*(a + b)*Cos[(c + d*
x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + (
a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/((a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(216) = 432$.
time = 0.19, size = 817, normalized size = 3.46

method	result
default	$-\frac{\sqrt{4}\,\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sin(dx+c)\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b+cos(d*x+c)^2*a-cos(d*x+c)^2*b-a*cos(d*x+c)+cos(d
*x+c)*b)/(b+a*cos(d*x+c))/sin(d*x+c)/(a+b)/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b*
sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.567 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$\frac{2\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d}-\frac{2\,\mathrm{co}}{}$$

```
[Out] 2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b
)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/
a/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(
d*x+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3870, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}+\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-a^2+b^2)+\frac{1}{2}ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-a^2+b^2)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int}{}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a\sqrt{a+b}\,d}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a\sqrt{a+b}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.84, size = 1249, normalized size = 3.60

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Se
c[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*
b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^5 - (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*Ell
ipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a -
```

```
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a^2 + a*b - 2*
b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/
(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)
*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c +
d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1208 vs. $2(318) = 636$.
time = 0.19, size = 1209, normalized size = 3.48

method	result	size
default	Expression too large to display	1209

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*a^2*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b-cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))-2*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))+2*cos(d*x+c)*b^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin
(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))+(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)
-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x
+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(
d*x+c)-2*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1
/2))*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
```

```
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2-cos(d*x+c)*a*b+c
os(d*x+c)*b^2)/(b+a*cos(d*x+c))/sin(d*x+c)/a/(a+b)/(a-b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
+ a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(1/(a + b/cos(c + d*x))^(3/2), x)
```

3.568 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=396

$$\frac{(a^2-3b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{a^2b\sqrt{a+b}\,d}$$

```
[Out] (a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2
)/a^2/b/d/(a+b)^(1/2)+(a+3*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)+3*b*cot(d*x+c)*EllipticPi((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(
d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+sin(d*x+c)/a/d/(
a+b*sec(d*x+c))^(1/2)+b*(a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sec(d*x
+c))^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3931, 4146, 4143, 4006, 3869, 3917, 4089}

$$\frac{3b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^3d}+\frac{(a^2-3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2bd\sqrt{a+b}}+\frac{(a+3b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}}+\frac{b(a^2-3b^2)\tan(c+dx)}{a^2d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + ((a + 3*b)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a^2*Sqrt[a + b]*d) + (3*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a
, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*d)
 + Sin[c + d*x]/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(a^2 - 3*b^2)*Tan[c + d
*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3931

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
```

```
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{-\frac{3b}{2}+\frac{1}{2}b\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b(a^2-3b^2)\tan(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{3}{4}b(a^2-b^2)-}{}$$

$$= \frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b(a^2-3b^2)\tan(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{4}b(a^2-b^2)+}{\sqrt{}}}{}$$

$$= \frac{(a^2-3b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{a^2b\sqrt{a+b}\,d}$$

$$= \frac{(a^2-3b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{a^2b\sqrt{a+b}\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1069 vs. $2(396) = 792$.
time = 13.01, size = 1069, normalized size = 2.70

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*b^2*Sin[c + d*x])/(a^2*(-a^2 +
b^2)) - (2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a
 + b*Sec[c + d*x])^(3/2)) - ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a^3*Tan[(c + d*x)/2] + a^2*b
*Tan[(c + d*x)/2] - 3*a*b^2*Tan[(c + d*x)/2] - 3*b^3*Tan[(c + d*x)/2] - 2*a
^3*Tan[(c + d*x)/2]^3 + 6*a*b^2*Tan[(c + d*x)/2]^3 + a^3*Tan[(c + d*x)/2]^5
```

```
 - a^2*b*Tan[(c + d*x)/2]^5 - 3*a*b^2*Tan[(c + d*x)/2]^5 + 3*b^3*Tan[(c + d
*x)/2]^5 - 6*a^2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)] + 6*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*b*EllipticPi[-1, ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*
b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)
/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*T
an[(c + d*x)/2]^2)/(a + b)] + (a^3 + a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + T
an[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + 2*a*b*(a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a^2*(a^2 - b^2)*d*(
a + b*Sec[c + d*x])^(3/2)*Sqrt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*
x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1661 vs. $2(365) = 730$.

time = 0.19, size = 1662, normalized size = 4.20

method	result	size
default	Expression too large to display	1662

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-cos(d*x+c)^2*a^3+3*cos
(d*x+c)*b^3+6*cos(d*x+c)*b^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-6*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b+(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)
+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*s
in(d*x+c)+3*cos(d*x+c)^2*a*b^2+cos(d*x+c)^3*a^3-3*cos(d*x+c)^2*b^3-3*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
```

```
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^3-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*a^2*b*sin(d*x+c)+EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*sin(d*x+c)*b-3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*a+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^2*b-3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-cos(d*x+c)^3*a*b^2+cos(d*x+c)^2*a^2
*b-2*cos(d*x+c)*a*b^2-cos(d*x+c)*a^2*b)/(b+a*cos(d*x+c))/sin(d*x+c)/a^2/(a+
b)/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b*
sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)/(a + b/cos(c + d*x))^(3/2), x)
```

3.569 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=470

$$-\frac{(7a^2-15b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}{a-b}}}{4a^3\sqrt{a+b}\,d}$$

```
[Out] -1/4*(7*a^2-15*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a^3/d/(a+b)^(1/2)+1/4*(2*a^2-5*a*b-15*b^2)*cot(d*x+c)*EllipticF((
a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a
+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)-1/4*(4*a^2+15*
b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/a^4/d-5/4*b*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(1/2)+1/2*cos
(d*x+c)*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(1/2)-1/4*b^2*(7*a^2-15*b^2)*tan(d*
x+c)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3931, 4189, 4145, 4143, 4006, 3869, 3917, 4089}

$$-\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b}\,(4a^2+15b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^4d}+\frac{(2a^2-5ab-15b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d\sqrt{a+b}}-\frac{(7a^2-15b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d\sqrt{a+b}}-\frac{b^2(7a^2-15b^2)\tan(c+dx)}{4a^3d\,(a^2-b^2)\,\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/4*((7*a^2 - 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*Sqrt[a + b]*d) + ((2*a^2 - 5*a*b
 - 15*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b
]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(4*a^2 + 15*b^
2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(4*a^4*d) - (5*b*Sin[c + d*x])/(4*a^2*d*Sqrt[a
 + b*Sec[c + d*x]]) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a + b*Sec[c +
 d*x]]) - (b^2*(7*a^2 - 15*b^2)*Tan[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3931

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{5b}{2}+a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{2a}$$

$$= -\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\frac{1}{4}\left(-4a^2-15b^2\right)-\frac{3}{2}ab\,\mathrm{s}\ldots}{(a+b\sec\ldots}}{\ldots}$$

$$= -\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b^2(7a^2-15b^2)\ldots}{4a^3\left(a^2-b^2\right)d\sqrt{a\ldots}}$$

$$= -\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b^2(7a^2-15b^2)\ldots}{4a^3\left(a^2-b^2\right)d\sqrt{a\ldots}}$$

$$= -\frac{(7a^2-15b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{s}\ldots}{a\ldots}}}{4a^3\sqrt{a+b}\,d}$$

$$= -\frac{(7a^2-15b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{s}\ldots}{a\ldots}}}{4a^3\sqrt{a+b}\,d}$$

Mathematica [C] Result contains complex when optimal does not.

time = 12.82, size = 1745, normalized size = 3.71

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b^3*Sin[c + d*x])/(a^3*(-a^2 + b
^2)) + (2*b^4*Sin[c + d*x])/(a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])) + Sin[2*
(c + d*x)]/(4*a^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-7*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2] - 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*b^3*S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*b^4*Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2] + 14*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 30*a*b^
3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 7*a^3*b*Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]^5 + 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5
+ 15*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*b^4*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]^5 - (8*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - (22*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + (30*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^4*E
llipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (22*I)*a
^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
(30*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + I*b*(7*a^3 - 7*a^2*b - 15*a*b^2 + 15*b^3)*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + (2*I)*(2*a^4 - a^3*b + 9*a^2*b^2 + 5*a*b^3 - 15*b^4
)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
- b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a^3*Sqrt[(-a + b)
/(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2
```

```
)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c +
 d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2297 vs. $2(425) = 850$.
time = 0.22, size = 2298, normalized size = 4.89

method	result	size
default	Expression too large to display	2298

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+4*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^2*b^2+10*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+7*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-15*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a*b^3-22*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2+2*cos(d*x+c)^2*a^4+15*cos(d*x+c)*b
^4-8*a^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*sin(d*x+c)+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*b^4*sin(d*x+c)-15*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-15*cos(d*x+c)^2*b^4-2*cos(d*x+c)^4*a
^4+2*cos(d*x+c)^4*a^2*b^2+5*cos(d*x+c)^3*a^3*b-5*cos(d*x+c)^3*a*b^3+5*cos(d
*x+c)^2*a^2*b^2-10*cos(d*x+c)*a*b^3+4*a^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)-15*b^4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)-8*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4+30*cos(d
```

```
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*b^4-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*b*sin(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*b^2*sin(d*x+c)+10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+7*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b+7*a^2*b^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)-15*b^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a-22*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+
4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^4-7*cos(d*x+c)^2*a^3*b+15*cos(d*x+c)^2*a*b^3+2*cos(d*x+c)*a^3
*b-7*cos(d*x+c)*a^2*b^2)/(b+a*cos(d*x+c))/sin(d*x+c)/a^3/(a+b)/(a-b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

3.570 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=427

$$\frac{8a(4a^4-7a^2b^2+2b^4)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{s}}{}}}{3(a-b)b^5(a+b)^{3/2}d}$$

```
[Out] 8/3*a*(4*a^4-7*a^2*b^2+2*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/(a-b)/b^5/(a+b)^(3/2)/d+2/3*(16*a^4+12*a^3*b-16*a^2*b^2
-9*a*b^3-b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/(a-b)/b^4/(a+b)^(3/2)/d-2/3*a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(
a+b*sec(d*x+c))^(3/2)+4/3*a^3*(3*a^2-5*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/d/(a
+b*sec(d*x+c))^(1/2)+2/3*(2*a^2-b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^3/
(a^2-b^2)/d
```

Rubi [A]

time = 0.63, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3930, 4175, 4167, 4090, 3917, 4089}

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{2(2a^2-b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^3d(a^2-b^2)}+\frac{8a(4a^4-7a^2b^2+2b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^5d(a-b)(a+b)^{3/2}}+\frac{4a^3(3a^2-5b^2)\tan(c+dx)}{3b^3d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{2(16a^4+12a^3b-16a^2b^2-9ab^3-b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^4d(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (8*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^5*(a + b)^(3/2)
*d) + (2*(16*a^4 + 12*a^3*b - 16*a^2*b^2 - 9*a*b^3 - b^4)*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*
(a - b)*b^4*(a + b)^(3/2)*d) - (2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^
2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a^3*(3*a^2 - 5*b^2)*Tan[c + d*x
])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(2*a^2 - b^2)*Sqrt
[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4175

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b*
```

```
B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec^2(c+dx)\left(2a^2-\frac{3}{2}ab\sec(c+dx)-\frac{3}{2}\left(2a^2-b^2\right)\text{s}\cdots}{(a+b\sec(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \cdots$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= \frac{8a\left(4a^4-7a^2b^2+2b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{b\cdots}}{3(a-b)b^5(a+b)^{3/2}d}$$

Mathematica [A]
time = 13.02, size = 578, normalized size = 1.35

$$\frac{4(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(4a\left(4a^5+4a^4b-7a^3b^2-7a^2b^3+2ab^4+2b^5\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+b\left(-16a^5-4a^4b+28a^3b^2+7a^2b^3-8ab^4+b^5\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+2a\left(4a^4-7a^2b^2+2b^4\right)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^4\left(a^2-b^2\right)^2 d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}(a+b\sec(c+dx))^{5/2}}+\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(-\frac{8a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{3b^4\left(-a^2+b^2\right)^2}-\frac{2a^3\sin(c+dx)}{3b^2\left(-a^2+b^2\right)(b+a\cos(c+dx))^2}-\frac{2\left(-7a^5\sin(c+dx)+11a^3b^2\sin(c+dx)\right)}{3b^3\left(-a^2+b^2\right)^2(b+a\cos(c+dx))}+\frac{2\tan(c+dx)}{3b^3}\right)}{d(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (4*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*(4*a*(4*a^5 + 4*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4 + 2*b^5)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(
-16*a^5 - 4*a^4*b + 28*a^3*b^2 + 7*a^2*b^3 - 8*a*b^4 + b^5)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*(4*a^4 - 7*a
^2*b^2 + 2*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c +
 d*x])^(5/2)) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-8*a*(4*a^4 - 7*a^
2*b^2 + 2*b^4)*Sin[c + d*x])/(3*b^4*(-a^2 + b^2)^2) - (2*a^3*Sin[c + d*x])/
```

```
(3*b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) - (2*(-7*a^5*Sin[c + d*x] + 11*
a^3*b^2*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) + (2*Tan
[c + d*x])/(3*b^3)))/(d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4175 vs. $2(393) = 786$.

time = 0.51, size = 4176, normalized size = 9.78

method	result	size
default	Expression too large to display	4176

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*4^(1/2)*(-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^7-16*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^6*b+28*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^
3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*
b^2+28*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*sin(d*x+c)*a^4*b^3-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^4-8*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^5+16*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^6*b+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b^2-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^3-7*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*
x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^3*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b^5-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^6-32*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6*b+12*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
```

```
2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^5*b^2+56*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^3+20*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^4-16*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos
(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b^5-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b^6+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6*b+20*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b^2
-24*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^4*b^3-35*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^4+(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^5+7*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*c
os(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^6-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^6*b-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b^2+28*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(
d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^3+28
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3*b^4-28*cos(d*x+c)^4*a^5*b^2+13*cos(d*x+c)^4*a^4*b^3+8*cos(d*
x+c)^4*a^3*b^4-cos(d*x+c)^4*a^2*b^5+32*cos(d*x+c)^3*a^6*b+18*cos(d*x+c)^3*a
^5*b^2-56*cos(d*x+c)^3*a^4*b^3+8*cos(d*x+c)^3*a...
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(5/2)), x)
```

3.571 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=362

$$-\frac{2(8a^4-15a^2b^2+3b^4)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{3(a-b)b^4(a+b)^{3/2}d}$$

```
[Out] -2/3*(8*a^4-15*a^2*b^2+3*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/(a-b)/b^4/(a+b)^(3/2)/d-2/3*(8*a^3+6*a^2*b-9*a*b^2-3*b^
3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b
^3/(a+b)^(3/2)/d-2/3*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c
))^(3/2)-8/3*a^2*(a^2-2*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^
(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3930, 4165, 4090, 3917, 4089}

$$-\frac{8a^2(a^2-2b^2)\tan(c+dx)}{3b^2d\,(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^{3/2}}-\frac{2(8a^4-15a^2b^2+3b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^4d(a-b)(a+b)^{3/2}}-\frac{2(8a^3+6a^2b-9ab^2-3b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^3d(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^4*(a + b)^(3/2)
*d) - (2*(8*a^3 + 6*a^2*b - 9*a*b^2 - 3*b^3)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(
a + b)^(3/2)*d) - (2*a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a +
 b*Sec[c + d*x])^(3/2)) - (8*a^2*(a^2 - 2*b^2)*Tan[c + d*x])/(3*b^2*(a^2 -
b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4165

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)),
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m +
 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(a^2-\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}(4a^2-3b^2)\operatorname{se}\ldots\right.}{(a+b\sec(c+dx))^{3/2}}}{3b\,(a^2-b^2)}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{8a^2(a^2-2b^2)\tan(c+dx)}{3b^2\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}} + \ldots$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{8a^2(a^2-2b^2)\tan(c+dx)}{3b^2\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}} - \ldots$$

$$= -\frac{2(8a^4-15a^2b^2+3b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\ldots}}{3(a-b)b^4(a+b)^{3/2}d}$$

Mathematica [A]
time = 13.33, size = 556, normalized size = 1.54

$$\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(\frac{2(8a^4-15a^2b^2+3b^4)\sin(c+dx)}{3b^3(-a^2+b^2)^2}+\frac{2a^2\sin(c+dx)}{3b(-a^2+b^2)(b+a\cos(c+dx))^2}+\frac{8(-a^4\sin(c+dx)+2a^2b^2\sin(c+dx))}{3b^2(-a^2+b^2)^2(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{5/2}} - \frac{2(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(8a^5+8a^4b-15a^3b^2-15a^2b^3+3ab^4+3b^5)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-2b(8a^4+2a^3b-15a^2b^2-6ab^3+3b^4)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+(8a^4-15a^2b^2+3b^4)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^3\left(a^2-b^2\right)^2d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\,(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Sin
[c + d*x])/(3*b^3*(-a^2 + b^2)^2) + (2*a^2*Sin[c + d*x])/(3*b*(-a^2 + b^2)*
(b + a*Cos[c + d*x])^2) + (8*(-(a^4*Sin[c + d*x]) + 2*a^2*b^2*Sin[c + d*x])
)/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/
2)) - (2*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]]*(2*(8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4 + 3*b
^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 - 2*b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 - 15*a^2*b^2
+ 3*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^
(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3673 vs. 2(332) = 664.
time = 0.34, size = 3674, normalized size = 10.15

method	result	size

default	Expression too large to display	3674

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3/d*4^(1/2)*(6*cos(d*x+c)^2*a^2*b^4-6*cos(d*x+c)*a*b^5+8*cos(d*x+c)^3*a^3*b^3-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6-4*cos(d*x+c)^3*a^5*b-15*cos(d*x+c)^3*a^4*b^2+3*cos(d*x+c)^3*a^2*b^4+16*cos(d*x+c)^2*a^5*b+10*cos(d*x+c)^2*a^4*b^2-30*cos(d*x+c)^2*a^3*b^3+6*cos(d*x+c)^2*a*b^5-12*cos(d*x+c)*a^5*b+8*cos(d*x+c)*a^4*b^2+22*cos(d*x+c)*a^3*b^3-15*cos(d*x+c)*a^2*b^4+8*cos(d*x+c)^3*a^6-8*cos(d*x+c)^2*a^6+3*cos(d*x+c)*b^6+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^6-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^6+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^6-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^6+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^3-15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^4-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^5-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^3+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^4-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^5-3*b^6-3*a^4*b^2+6*a^2*b^4-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b

```
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b+30*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^3-
6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*a*b^5+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2-15*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x
+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^3*b^3-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b^4+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^5-8*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5*b+15*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^4*b^2+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^4-3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)), x)
```

3.572 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=337

$$\frac{4a(a^2-3b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{3(a-b)b^3(a+b)^{3/2}d}$$

```
[Out] 4/3*a*(a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d+2/3*(2*a^2+3*a*b-3*b^2)*cot(d*x+c)*Elliptic
F((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))
/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-2/3*a
^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*a*(a^2-3*b^2)*tan(d*
x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3924, 4088, 4090, 3917, 4089}

$$\frac{2(2a^2+3ab-3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d(a-b)(a+b)^{3/2}}+\frac{4a(a^2-3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^3d(a-b)(a+b)^{3/2}}-\frac{2a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{4a(a^2-3b^2)\tan(c+dx)}{3bd(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (4*a*(a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(a + b)^(3/2)*d) + (2*(2*a^
2 + 3*a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*d) - (2*a^2*T
an[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a*(a^2 - 3
*b^2)*Tan[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3924

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3ab}{2}-\frac{1}{2}(2a^2-3b^2)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3b\,(a^2-b^2)}$$

$$= -\frac{2a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4a(a^2-3b^2)\tan(c+dx)}{3b\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= -\frac{2a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4a(a^2-3b^2)\tan(c+dx)}{3b\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= \frac{4a(a^2-3b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(\cdots}{a+\cdots}} \cdots}{3(a-b)b^3(a+b)^{3/2}d}$$

Mathematica [A]
time = 11.09, size = 503, normalized size = 1.49

$$\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(\frac{4a(-a^2+3b^2)\sin(c+dx)}{3b^2(-a^2+b^2)^2}-\frac{2a\sin(c+dx)}{3(-a^2+b^2)(b+a\cos(c+dx))^2}-\frac{2(-a^3\sin(c+dx)+5ab^2\sin(c+dx))}{3b(-a^2+b^2)^2(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{5/2}}+\frac{4(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2a(a^3+a^2b-3ab^2-3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+b(-2a^3+a^2b+6ab^2+3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+a(a^2-3b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3(-a^2b+b^3)^2d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((4*a*(-a^2 + 3*b^2)*Sin[c + d*x])/(
3*b^2*(-a^2 + b^2)^2) - (2*a*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c + d
*x])^2) - (2*(-(a^3*Sin[c + d*x]) + 5*a*b^2*Sin[c + d*x]))/(3*b*(-a^2 + b^2
)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (4*(b + a*Cos[
c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*a*(
a^3 + a^2*b - 3*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)] + b*(-2*a^3 + a^2*b + 6*a*b^2 + 3*b^3)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(a^2 -
3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(3*(-(a^2*b) + b^3)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(
5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. $2(307) = 614$.
time = 0.19, size = 2733, normalized size = 8.11

method	result	size

default	Expression too large to display	2733

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/3/d*4^(1/2)*(-6*cos(d*x+c)^3*a^3*b^2-12*cos(d*x+c)^2*a^2*b^3-6*cos(d*x+c)*a*b^4-2*cos(d*x+c)^2*a^5+4*cos(d*x+c)^2*a^3*b^2+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-7*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-9*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+12*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+6*cos(d*x+c)^2*a*b^4-3*cos(d*x+c)*a^4*b+7*cos(d*x+c)*a^2*b^3-3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^5-cos(d*x+c)^3*a^4*b+5*cos(d*x+c)^3*a^2*b^3+4*cos(d*x+c)^2*a^4*b+2*cos(d*x+c)*a^3*b^2+2*cos(d*x+c)^3*a^5-3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+

```
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-2
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^3*b^2+6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^3+6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^4+2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^4*b-EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2-6*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+
c)*a^2*b^3-3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*cos(d*x+c)^2*sin(d*x+c)*a*b^4-2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^4*b+6*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2+6*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2
*sin(d*x+c)*a^2*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*co
s(d*x+c))^2/(a-b)^2/(a+b)^2/b^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)), x)
```

3.573 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$\frac{2(a^2+3b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx)}{a-b}}}{3(a-b)b^2(a+b)^{3/2}d}$$

```
[Out] 2/3*(a^2+3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a
+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/(a-b)/b^2/(a+b)^(3/2)/d+2/3*(a-3*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*
(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b/(a+b)^(3/2)/d+2/3*a*tan(d*x+c)/(a^2
-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2/3*(a^2+3*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+
b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3921, 4088, 4090, 3917, 4089}

$$\frac{2(a^2+3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d(a-b)(a+b)^{3/2}}+\frac{2(a^2+3b^2)\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}+\frac{2(a-3b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a - 3*
b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2)*d) + (2*a*Tan[c + d*x])/(3*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(a^2 + 3*b^2)*Tan[c + d*x])/(3*(
a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{3b}{2}+\frac{1}{2}a\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2(a^2+3b^2)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{4\int \ldots}{\ldots}$$

$$= \frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2(a^2+3b^2)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{(a-\ldots}{\ldots}$$

$$= \frac{2(a^2+3b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c\ldots}{a+b\ldots}}\ldots}{3(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [**A**]
time = 10.64, size = 486, normalized size = 1.53

$$\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(-\frac{2(a^2+3b^2)\sin(c+dx)}{3b(-a^2+b^2)^2}+\frac{2b\sin(c+dx)}{3(-a^2+b^2)(b+a\cos(c+dx))^2}+\frac{4(a^2\sin(c+dx)+b^2\sin(c+dx))}{3(-a^2+b^2)^2(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{5/2}}+\frac{2(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(a^3+a^2b+3ab^2+3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-2b(a^2+4ab+3b^2)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+(a^2+3b^2)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b\left(a^2-b^2\right)^2d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(a^2 + 3*b^2)*Sin[c + d*x])/(3*
b*(-a^2 + b^2)^2) + (2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x])
^2) + (4*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b + a*Co
s[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*S
ec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a^3 + a^2*b + 3
*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b^2)^2*d*Sqr
t[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2409 vs. $2(287) = 574$.
time = 0.18, size = 2410, normalized size = 7.60

method	result	size
default	Expression too large to display	2410

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*4^(1/2)*(-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^3*b-5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-7*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+2*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^3*b+4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-cos(d*x+c)^3*a^4+cos(d*
x+c)^2*a^4-3*cos(d*x+c)*b^4-sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-4*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-3*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b^3+sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^3*b+3*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+3*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^4+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^4+3*cos(d*x+c)^2*b^4+2*cos(d*x+c)^3*a^3*b+2*cos
(d*x+c)^3*a*b^3+4*cos(d*x+c)^2*a^2*b^2+4*cos(d*x+c)*a*b^3-3*cos(d*x+c)^3*a^
2*b^2+3*b^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
sin(d*x+c)-3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*b^4-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
```

```
2))*a^2*b^2*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a*b^3*sin(d*x+c)+a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*b+a^2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)+3*b^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d
*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b^4-2*cos(d*x+c)^2*a^3*b-6*cos(d*x+c)^2*a*b^3-cos(d*x+c)*a^2*b^2)*((b+a*cos
(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)^2/(a+b)^2/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)), x)
```

3.574 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=304

$$-\frac{8a\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b(a+b)^{3/2}d}+\frac{2(3\cdots}{}$$

```
[Out] -8/3*a*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-
b)/b/(a+b)^(3/2)/d+2/3*(3*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(
d*x+c))/(a-b))^(1/2)/(a-b)/b/(a+b)^(3/2)/d-2/3*b*tan(d*x+c)/(a^2-b^2)/d/(a+
b*sec(d*x+c))^(3/2)-8/3*a*b*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3918, 4088, 4090, 3917, 4089}

$$-\frac{8ab\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2(3a-b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd(a-b)(a+b)^{3/2}}-\frac{8a\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-8*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2)*d) + (2*(3*a - b)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(3*(a - b)*b*(a + b)^(3/2)*d) - (2*b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^(3/2)) - (8*a*b*Tan[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
```

```
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8ab\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4 \ldots}{\ldots}$$

$$= -\frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8ab\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{(3 \ldots}{\ldots}$$

$$= -\frac{8a\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}} \ldots}{3(a-b)b(a+b)^{3/2}d}$$

Mathematica [A]
time = 5.03, size = 360, normalized size = 1.18

$$-\frac{2(b+a\cos(c+dx))\sec^3(c+dx)\left(b^2\left(-a^2+b^2\right)\sin(c+dx)-b\left(-5a^2+b^2\right)(b+a\cos(c+dx))\sin(c+dx)-4a^2(b+a\cos(c+dx))^2\sin(c+dx)+2a\cos^2\left(\frac{1}{2}(c+dx)\right)(b+a\cos(c+dx))\left(4a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-\left(3a^2+4ab+b^2\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2a\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{3a\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^3*(b^2*(-a^2 + b^2)*Sin[c + d*x] - b*
(-5*a^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] - 4*a^2*(b + a*Cos[c + d*x
])^2*Sin[c + d*x] + 2*a*Cos[(c + d*x)/2]^2*(b + a*Cos[c + d*x])*(4*a*(a + b
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] -
 (3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] + 2*a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2])))/(3*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. 2(274) = 548.
time = 0.16, size = 1781, normalized size = 5.86

method	result	size
default	Expression too large to display	1781

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*4^(1/2)*(-4*cos(d*x+c)^2*a^3-cos(d*x+c)*b^3+3*cos(d*x+c)^2*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-4*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
3*sin(d*x+c)-4*cos(d*x+c)^2*a*b^2+4*cos(d*x+c)^3*a^3+cos(d*x+c)^3*b^3+3*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^3+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3-4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)*b-4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+7*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+5*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^2-8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*b-4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-5*cos(d*x+c)^3*a^2*b+8*co
s(d*x+c)^2*a^2*b+4*cos(d*x+c)*a*b^2+4*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-4
*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^2*b-4*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-3*cos(d*x+c)*a^2*b)*((b+a*cos(d*x+c))/
cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)^2/(a+b)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b^
2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (c+dx)}{(a+b\sec (c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos (c+d\,x)\,{\left(a+\frac{b}{\cos (c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.575 $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=448

$$\frac{2(7a^2-3b^2)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d}$$

```
[Out] 2/3*(7*a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d-2/3*(6*a^2-a*b-3*b^2)*cot(d*x+c)*EllipticF(
(a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(
a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d-2*cot(d
*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/a^3/d+2/3*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2/3*b^2
*(7*a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3870, 4145, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^3d}-\frac{2(6a^2-ab-3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^2d(a-b)(a+b)^{3/2}}+\frac{2(7a^2-3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^2d(a-b)(a+b)^{3/2}}+\frac{2b^2(7a^2-3b^2)\tan(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-5/2),x]
```

```
[Out] (2*(7*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*(a + b)^(3/2)*d) - (2*(6*a^
2 - a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*(a + b)^(3/2)*d) - (2*Sqrt[a
+ b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (2*b^2*Tan[c + d*x])/(3*a*(a^2 - b
^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(7*a^2 - 3*b^2)*Tan[c + d*x])/(3
*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
```

```
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(a^2-b^2\right)+\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,d}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2(7a^2-3b^2)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= \frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2(7a^2-3b^2)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= \frac{2(7a^2-3b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{a-}}}{3a^2(a-b)(a+b)^{3/2}d}$$

$$= \frac{2(7a^2-3b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{a-}}}{3a^2(a-b)(a+b)^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.61, size = 1798, normalized size = 4.01

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b*(-7*a^2 + 3*b^2)*Sin[c + d*x])
/(3*a^2*(-a^2 + b^2)^2) - (2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(b + a*Co
s[c + d*x])^2) - (8*(-2*a^2*b^2*Sin[c + d*x] + b^4*Sin[c + d*x]))/(3*a^2*(a
^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2*(b
+ a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(7*a^3*b*Sqrt[(-a + b)
```

```
/(a + b)]*Tan[(c + d*x)/2] + 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2] - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*b^4*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2] - 14*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]^3 + 6*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 7*a^3*b*Sqrt[(-a
 + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]^5 - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*b^4*Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (6*I)*a^4*EllipticPi[-((a + b)/(a
 - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)] + (12*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - (6*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (
6*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
+ (12*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] - (6*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] - I*b*(7*a^3 - 7*a^2*b - 3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + I*(3*a^4 + 6*a^3*b - 13*a^2*b^2 - 2*a*b^3 +
6*b^4)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*a^2*Sqrt[(-a
 + b)/(a + b)]*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*(-1 + Tan[(c + d*
x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + T
an[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3888 vs. $2(409) = 818$.

time = 0.18, size = 3889, normalized size = 8.68

method	result	size
default	Expression too large to display	3889

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

[Out] -1/3/d*4^(1/2)*(6*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5+8*cos(d*x+c)^3*a^3*b^2-4*cos(d*x+c)^3*a*b^4+4*cos(d*x+c)^2*a^2*b^3-2*cos(d*x+c)*a*b^4-14*cos(d*x+c)^2*a^3*b^2+6*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b-12*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^2-12*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^3+6*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4-12*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^2+6*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4+6*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5-3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5-9*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-7*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+14*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+7*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+6*cos(d*x+c)^2*a*b^4-7*cos(d*x+c)*a^2*b^3+3*cos(d*x+c)*b^5-3*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5+6*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1

```
+cos(d*x+c))/(a+b))^(1/2)*a^5+6*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5-3*sin(d*x+c)*cos(d*x+c)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5-3*sin(d*x
+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*a^5+6*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*a^4*b-12*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*a^2*b^3-3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b-7*cos(d*x+c)^3*a^4*b+3*cos(d*x+c)^3*
a^2*b^3+7*cos(d*x+c)^2*a^4*b+6*cos(d*x+c)*a^3*b^2-3*cos(d*x+c)^2*b^5-6*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b
^2-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b^3+2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b^4+7*sin(d*x+c)*(cos(d*x+c)/(1+co...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c
)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(5/2), x)
```

3.576 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=510

$$\frac{(3a^4-26a^2b^2+15b^4)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{3a^3(a-b)b(a+b)^{3/2}d}$$

```
[Out] sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(3/2)+1/3*(3*a^4-26*a^2*b^2+15*b^4)*cot(d*x
+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/b/(a+b)
^(3/2)/d+1/3*(3*a^3+21*a^2*b-5*a*b^2-15*b^3)*cot(d*x+c)*EllipticF((a+b*sec(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d+5*b*cot(d*x+c)*E
llipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(
a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a
^4/d+1/3*b*(3*a^2-5*b^2)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+
1/3*b*(3*a^4-26*a^2*b^2+15*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3931, 4146, 4145, 4143, 4006, 3869, 3917, 4089}

$$\frac{5b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^4d}+\frac{b(3a^2-5b^2)\tan(c+dx)}{3a^2d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{(3a^3+21a^2b-5ab^2-15b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^3d(a-b)(a+b)^{3/2}}+\frac{(3a^4-26a^2b^2+15b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^3bd(a-b)(a+b)^{3/2}}+\frac{b(3a^4-26a^2b^2+15b^4)\tan(c+dx)}{3a^3d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((3*a^4 - 26*a^2*b^2 + 15*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*(a - b)*b*(a + b)^(3/2)
*d) + ((3*a^3 + 21*a^2*b - 5*a*b^2 - 15*b^3)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*(a - b)*(
a + b)^(3/2)*d) + (5*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSi
n[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*d) + Sin[
c + d*x]/(a*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^2 - 5*b^2)*Tan[c + d*x]
)/(3*a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^4 - 26*a^2*b^2
 + 15*b^4)*Tan[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3931

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{-\frac{5b}{2}+\frac{3}{2}b\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^2-5b^2)\tan(c+dx)}{3a^2(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{15}{4}b(a^2-b^2 …}{…}}{…}$$

$$= \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^2-5b^2)\tan(c+dx)}{3a^2(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^4-26a …}{3a^3(a^2-b^2 …}$$

$$= \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^2-5b^2)\tan(c+dx)}{3a^2(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^4-26a …}{3a^3(a^2-b^2 …}$$

$$= \frac{(3a^4-26a^2b^2+15b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b …}{…}}}{3a^3(a-b)b(a+b)^{3/2}d}$$

$$= \frac{(3a^4-26a^2b^2+15b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b …}{…}}}{3a^3(a-b)b(a+b)^{3/2}d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1481 vs. $2(510) = 1020$.
time = 14.20, size = 1481, normalized size = 2.90

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-4*b^2*(-5*a^2 + 3*b^2)*Sin[c + d*
x])/(3*a^3*(-a^2 + b^2)^2) + (2*b^4*Sin[c + d*x])/(3*a^3*(a^2 - b^2)*(b + a
*Cos[c + d*x])^2) + (2*(-11*a^2*b^3*Sin[c + d*x] + 7*b^5*Sin[c + d*x]))/(3*
a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) -
((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2
)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan
[(c + d*x)/2]^2)]*(3*a^5*Tan[(c + d*x)/2] + 3*a^4*b*Tan[(c + d*x)/2] - 26*a
^3*b^2*Tan[(c + d*x)/2] - 26*a^2*b^3*Tan[(c + d*x)/2] + 15*a*b^4*Tan[(c + d
*x)/2] + 15*b^5*Tan[(c + d*x)/2] - 6*a^5*Tan[(c + d*x)/2]^3 + 52*a^3*b^2*Ta
n[(c + d*x)/2]^3 - 30*a*b^4*Tan[(c + d*x)/2]^3 + 3*a^5*Tan[(c + d*x)/2]^5 -
 3*a^4*b*Tan[(c + d*x)/2]^5 - 26*a^3*b^2*Tan[(c + d*x)/2]^5 + 26*a^2*b^3*Ta
n[(c + d*x)/2]^5 + 15*a*b^4*Tan[(c + d*x)/2]^5 - 15*b^5*Tan[(c + d*x)/2]^5
- 30*a^4*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] + 60*a^2*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b^5*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4*b*EllipticPi[-1, A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] + 60*a^2*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b^5*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] + (3*a^5 + 3*a^4*b - 26*a^3*b^2 - 26*a^2*b^3 + 15*a*b^4 + 15*b^5)*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] + 2*a*b*(6*a^3 + 9*a^2*b - 2*a*b^2 - 5*b^3)*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)]))/(3*a*(a^3 - a*b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*Sqrt[1 + T
an[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2
)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4579 vs. $2(469) = 938$.
time = 0.26, size = 4580, normalized size = 8.98

method	result	size
default	Expression too large to display	4580

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*4^(1/2)*(15*cos(d*x+c)^2*b^6+20*cos(d*x+c)^3*a*b^5-14*cos(d*x+c)^2*a
^2*b^4+10*cos(d*x+c)*a*b^5-6*cos(d*x+c)^4*a^4*b^2+3*cos(d*x+c)^4*a^2*b^4-34
*cos(d*x+c)^3*a^3*b^3-30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^6+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6+6*cos(d*x+c)^3
*a^5*b+26*cos(d*x+c)^3*a^4*b^2-15*cos(d*x+c)^3*a^2*b^4-6*cos(d*x+c)^2*a^5*b
-17*cos(d*x+c)^2*a^4*b^2+52*cos(d*x+c)^2*a^3*b^3-30*cos(d*x+c)^2*a*b^5-3*co
s(d*x+c)*a^4*b^2-18*cos(d*x+c)*a^3*b^3+26*cos(d*x+c)*a^2*b^4-3*cos(d*x+c)^3
*a^6-15*cos(d*x+c)*b^6+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b^6+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6+15*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^6+12*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d
*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2+18*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^3*b^3-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b^4-10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^5+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^5*b+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2-26*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^3-26*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^4+15*(cos(d*x+c)/
```

```
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^5-30*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a
^4*b^2+60*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a^2*b^4-30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*b^6+3*cos(d*x+c)^4*a^6+6*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b-52*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d
*x+c)*a^3*b^3+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*a*b^5+12*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b+18*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^
2-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^3*b^3-10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^4+3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5*b-26*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^4*b^2-26*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^3+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*c...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b^
2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)/(a + b/cos(c + d*x))^(5/2), x)
```

3.577 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=562

$$-\frac{(33a^4-170a^2b^2+105b^4)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{12a^4(a-b)(a+b)^{3/2}d}$$

[Out] -7/4*b*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(3/2)+1/2*cos(d*x+c)*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(3/2)-1/12*(33*a^4-170*a^2*b^2+105*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d+1/12*(a+3*b)*(6*a^3-45*a^2*b+35*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d-1/4*(4*a^2+35*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/d-1/12*b^2*(27*a^2-35*b^2)*tan(d*x+c)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-1/12*b^2*(33*a^4-170*a^2*b^2+105*b^4)*tan(d*x+c)/a^4/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)

Rubi [A]
time = 0.75, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3931, 4189, 4145, 4143, 4006, 3869, 3917, 4089}

$$-\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}}-\frac{\sqrt{a+b}\,(4a^2+35b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^5d}-\frac{(33a^4-170a^2b^2+105b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{12a^4d(a-b)(a+b)^{3/2}}-\frac{b^2(33a^4-170a^2b^2+105b^4)\tan(c+dx)}{12a^4d\,(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}-\frac{b^2(27a^2-35b^2)\tan(c+dx)}{12a^3d\,(a^2-b^2)\,(a+b\sec(c+dx))^{3/2}}+\frac{(a+3b)\,(6a^3-45a^2b+35b^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{12a^4d(a-b)(a+b)^{3/2}}+\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]

[Out] -1/12*((33*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*(a - b)*(a + b)^(3/2)*d) + ((a + 3*b)*(6*a^3 - 45*a^2*b + 35*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*a^4*(a - b)*(a + b)^(3/2)*d) - (Sqrt[a + b]*(4*a^2 + 35*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^5*d) - (7*b*Sin[c + d*x])/(4*a^2*d*(a + b*Sec[c + d*x])^(3/2)) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x])^(3/2)) - (b^2*(27*a^2 - 35*b^2)*Tan[c + d*x])/(12*a^3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)

```
) - (b^2*(33*a^4 - 170*a^2*b^2 + 105*b^4)*Tan[c + d*x])/(12*a^4*(a^2 - b^2)
^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3931

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{7b}{2}+a\sec(c+dx)+\frac{5}{2}b\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx}{2a}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\frac{1}{4}(-4a^2-35b^2)-\frac{5}{2}ab}{(a+b\mathrm{s}}}{}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2(27a^2-35b}{12a^3(a^2-b^2)d(a}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2(27a^2-35b}{12a^3(a^2-b^2)d(a}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2(27a^2-35b}{12a^3(a^2-b^2)d(a}$$

$$= -\frac{(33a^4-170a^2b^2+105b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{12a^4(a-b)(a+b)^{3/2}d}$$

$$= -\frac{(33a^4-170a^2b^2+105b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{12a^4(a-b)(a+b)^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.51, size = 2285, normalized size = 4.07

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b^3*(-13*a^2 + 9*b^2)*Sin[c + d*
x])/(3*a^4*(-a^2 + b^2)^2) - (2*b^5*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*(b + a
*Cos[c + d*x])^2) - (4*(-7*a^2*b^4*Sin[c + d*x] + 5*b^6*Sin[c + d*x]))/(3*a
^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + Sin[2*(c + d*x)]/(4*a^3)))/(d*(a +
 b*Sec[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)
/2]^2)]*(33*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 33*a^4*b^2*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 170*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2] - 170*a^2*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 105*
a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 105*b^6*Sqrt[(-a + b)/(a +
```

```
b)]*Tan[(c + d*x)/2] - 66*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 +
 340*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 210*a*b^5*Sqrt[(-a
 + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 33*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]^5 - 33*a^4*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 170*a^
3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 170*a^2*b^4*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]^5 + 105*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]^5 - 105*b^6*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + (24*I)*a^6*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (162*I)*a^4*b^2*EllipticPi[-
((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
 b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (396*I)*a^2*b^4*EllipticPi[-((a + b)/(
a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + (210*I)*b^6*EllipticPi[-((a + b)/(a - b)), I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + (24*I)*a^6*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] + (162*I)*a^4*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^
2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)] - (396*I)*a^2*b^4*EllipticPi[-((a + b)/(a - b)), I*
ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + (210*I)*b^6*EllipticPi[-((a + b)/(a - b)
), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*b*(-33*a^5 + 33*a^4*b + 170*a^3*b
^2 - 170*a^2*b^3 - 105*a*b^4 + 105*b^5)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - (2*I)*(6*a^6 - 3*a^5*b + 57*a^4*b^2 + 54*a^3*b^3 - 184*a^
2*b^4 - 35*a*b^5 + 105*b^6)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)]))/(12*a^4*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(
5/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c +
d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5637 vs. $2(513) = 1026$.

time = 0.34, size = 5638, normalized size = 10.03

method	result	size
default	Expression too large to display	5638

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(5/2), x)
```

3.578 $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=535

$$\frac{2(58a^4-41a^2b^2+15b^4)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{15a^3(a-b)^2(a+b)^{5/2}d}$$

```
[Out] 2/15*(58*a^4-41*a^2*b^2+15*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/a^3/(a-b)^2/(a+b)^(5/2)/d-2/15*(45*a^4-13*a^3*b-36*a^
2*b^2+5*a*b^3+15*b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/a^3/(a-b)^2/(a+b)^(5/2)/d-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+
c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*
x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/5*b^2*tan(d*x+c)
/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(5/2)+2/15*b^2*(13*a^2-5*b^2)*tan(d*x+c)/a^
2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(3/2)+2/15*b^2*(58*a^4-41*a^2*b^2+15*b^4)*
tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3870, 4145, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^4d}+\frac{2b^2(13a^2-5b^2)\tan(c+dx)}{15a^2d(a^2-b^2)^2(a+b\sec(c+dx))^{3/2}}+\frac{2b^2\tan(c+dx)}{5ad(a^2-b^2)(a+b\sec(c+dx))^{5/2}}+\frac{2(58a^4-41a^2b^2+15b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15a^3d(a-b)^2(a+b)^{5/2}}-\frac{2(45a^4-13a^3b-36a^2b^2+5ab^3+15b^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15a^3d(a-b)^2(a+b)^{5/2}}+\frac{2b^2(58a^4-41a^2b^2+15b^4)\tan(c+dx)}{15a^3d(a^2-b^2)^3\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] (2*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*a^3*(a - b)^2*(a + b)^(
5/2)*d) - (2*(45*a^4 - 13*a^3*b - 36*a^2*b^2 + 5*a*b^3 + 15*b^4)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(15*a^3*(a - b)^2*(a + b)^(5/2)*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a^4*d) + (2*b^2*Tan[c + d*x])/(5*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^(5/2)) + (2*b^2*(13*a^2 - 5*b^2)*Tan[c + d*x])/(15*a^2*(a^2 - b^2)^2*d*(
a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Tan[c +
d*x])/(15*a^3*(a^2 - b^2)^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \frac{2b^2\tan(c+dx)}{5a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}(a^2-b^2)+\frac{5}{2}ab\sec(c+dx)-\frac{3}{2}b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx}{5a\,(a^2-b^2)}$$

$$= \frac{2b^2\tan(c+dx)}{5a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{5/2}} + \frac{2b^2(13a^2-5b^2)\tan(c+dx)}{15a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^{3/2}} +$$

$$= \frac{2b^2\tan(c+dx)}{5a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{5/2}} + \frac{2b^2(13a^2-5b^2)\tan(c+dx)}{15a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^{3/2}} +$$

$$= \frac{2b^2\tan(c+dx)}{5a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{5/2}} + \frac{2b^2(13a^2-5b^2)\tan(c+dx)}{15a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^{3/2}} +$$

$$= \frac{2(58a^4-41a^2b^2+15b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{}}{15a^3(a-b)^2(a+b)^{5/2}d}$$

$$= \frac{2(58a^4-41a^2b^2+15b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{}}{15a^3(a-b)^2(a+b)^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.56, size = 2346, normalized size = 4.39

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^4*Sec[c + d*x]^4*((2*b*(58*a^4 - 41*a^2*b^2 + 15*b^4)
*Sin[c + d*x])/(15*a^3*(-a^2 + b^2)^3) + (2*b^4*Sin[c + d*x])/(5*a^3*(a^2 -
 b^2)*(b + a*Cos[c + d*x])^3) + (2*(-19*a^2*b^3*Sin[c + d*x] + 11*b^5*Sin[c
 + d*x]))/(15*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (2*(74*a^4*b^2*Si
n[c + d*x] - 65*a^2*b^4*Sin[c + d*x] + 23*b^6*Sin[c + d*x]))/(15*a^3*(a^2 -
 b^2)^3*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(7/2)) + (2*(b + a*
Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(58*a^5*b*Sqrt[(-a + b)/(a
 + b)]*Tan[(c + d*x)/2] + 58*a^4*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
] - 41*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 41*a^2*b^4*Sqrt[(-
a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c +
 d*x)/2] + 15*b^6*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 116*a^5*b*Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 82*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^3 - 30*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 58*
a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 58*a^4*b^2*Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]^5 - 41*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]^5 + 41*a^2*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 15*a*b^5*S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*b^6*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^5 - (30*I)*a^6*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + (90*I)*a^4*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (90*I)*
a^2*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (30*I)*b^6*Ellipti
cPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*a^6*EllipticPi[-((a + b)/(
a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b
)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (90*I)*a^4*b^2*EllipticPi[-((
a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (90*I)*a^2*b^4*Ellipt
icPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]]
, (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (30*I)*b^6*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*b*(-58*
a^5 + 58*a^4*b + 41*a^3*b^2 - 41*a^2*b^3 - 15*a*b^4 + 15*b^5)*EllipticE[I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/
```

```
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(15*a^6 + 45*a^5*b - 103*a^4*b^2
- 23*a^3*b^3 + 86*a^2*b^4 + 10*a*b^5 - 30*b^6)*EllipticF[I*ArcSinh[Sqrt[(-a
 + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)]))/(15*a^3*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^3*d*(a
+ b*Sec[c + d*x])^(7/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2
]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c
 + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7837 vs. $2(492) = 984$.
time = 0.29, size = 7838, normalized size = 14.65

method	result	size
default	Expression too large to display	7838

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(7/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(7/2), x)
```

3.579 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$-\frac{6b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+6b\sqrt{}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/5
*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 3853, 3856, 2720, 2719}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6b\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{6b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec^{\frac{5}{2}}(c+dx)\,dx + b\int \sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2b\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int$$
$$= \frac{6b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2b\,\mathrm{se}}{}$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6b\sqrt{\sec(c+dx}}{5}$$
$$= -\frac{6b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+}}{}$$

Mathematica [A]
time = 0.21, size = 97, normalized size = 0.64

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-36b\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20a\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+21b\sin(c+dx)+10a\sin(2(c+dx))+9b\sin(3(c+dx))\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-36*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 2
0*a*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*b*Sin[c + d*x] + 10*a
*Sin[2*(c + d*x)] + 9*b*Sin[3*(c + d*x)]))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(179) = 358$.
time = 0.22, size = 502, normalized size = 3.32

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\right.}}{6\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+
1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-2
4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1
/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 188, normalized size = 1.25

$$\frac{-5i\sqrt{2}\,a\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,a\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-9i\sqrt{2}\,b\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+9i\sqrt{2}\,b\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(9b\cos(dx+c)^2+5a\cos(dx+c)+3b\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0
```

```
, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*b*cos(d*x + c)^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*
I*sqrt(2)*b*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*b*cos(d*x + c)^2 + 5*a*cos(d*x + c
) + 3*b)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
[Out] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

3.580 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=123

$$-\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a\sqrt{}}{}$$

```
[Out] 2/3*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3853, 3856, 2719, 2720}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*b*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec^{\frac{3}{2}}(c+dx)\,dx + b\int \sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - a\int$$
$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \Big(a\sqrt{}$$
$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\cos(c+}$$

Mathematica [A]
time = 0.16, size = 85, normalized size = 0.69

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(-6a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2b\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(b+3a\cos(c+dx))\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-6*a*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
b*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(b + 3*a*Cos[c + d*x])*S
in[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(159) = 318$.
time = 0.19, size = 396, normalized size = 3.22

method	result

default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)
^2*b-6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a-6*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*b+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 167, normalized size = 1.36

$$\frac{-i\sqrt{2}\,b\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+i\sqrt{2}\,b\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3i\sqrt{2}\,a\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3i\sqrt{2}\,a\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+\frac{2\left(3a\cos\left(dx+c\right)+b\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + I*sqrt(2)*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a
*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + b)*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec (c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

3.581 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2b\sqrt{}}{}$$

```
[Out] 2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3856, 2720, 3853, 2719}

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\,dx = a\int \sqrt{\sec(c+dx)}\,dx + b\int \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \Big(a\sqrt{\cos \ldots$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\sec(c+ \ldots}}{}$$
$$= -\frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c \ldots}}{}$$

Mathematica [A]
time = 0.08, size = 71, normalized size = 0.73

$$\frac{2\sqrt{\sec(c+dx)}\left(-b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-(b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*x]))/d
```

Maple [A]
time = 0.12, size = 150, normalized size = 1.55

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ a-2\sqrt{2\ \ldots}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b-(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 124, normalized size = 1.28

$$\frac{-i\sqrt{2}\,a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\,a\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-i\sqrt{2}\,b\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+i\sqrt{2}\,b\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\,b\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sq
rt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + I*sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*b*sin(d*x + c)/sqrt(cos(d*x + c)))/
d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

3.582 $\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*b*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 3856, 2719, 2720}

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + b\int \sqrt{\sec(c+dx)}\,dx$$
$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx + \left(b\sqrt{\cos(c+dx)}\sqrt{\sec(\right.$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{d}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.69

$$\frac{2\sqrt{\cos(c+dx)}\left(aE\left(\frac{1}{2}(c+dx)\middle|2\right)+bF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
[Out] (2*Sqrt[Cos[c + d*x]]*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)/
2, 2])*Sqrt[Sec[c + d*x]])/d
```

Maple [A]
time = 0.11, size = 152, normalized size = 2.03

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(b\,\mathrm{Elliptic}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}\right.\right.}}$
risch	$-\frac{ia\sqrt{2}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\left(\frac{ib\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}\right.\right.}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(b*EllipticF(cos(1/2*d*x+1/2
```

```
*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.04, size = 107, normalized size = 1.43

$$\frac{-i\sqrt{2}\,b\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,b\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sq
rt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec{\left(c + dx \right)}}{\sqrt{\sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

[Out] integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)

3.583 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a\sin}{3d\sqrt{\text{se}}}$$

```
[Out] 2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3854, 3856, 2720, 2719}

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx+\Big(b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\Big)\\ &= \frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\Big(a\sqrt{\mathrm{co}}\\ &= \frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{3d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 76, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\,\left(6b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + a*(
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d)
```

Maple [A]
time = 0.10, size = 228, normalized size = 2.26

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+\sqrt{}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a+
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*a-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 125, normalized size = 1.24

$$\frac{2\,a\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+3i\sqrt{2}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3i\sqrt{2}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*b*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*b*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec\left(c + dx\right)}{\sec^{\frac{3}{2}}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{\cos(c+d\,x)}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

3.584 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a\sin}{5d\sec^{\frac{3}{2}}}$$

```
[Out] 2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*b*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5
*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*b*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3854, 3856, 2719, 2720}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + b\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}b\int \sqrt{\sec$$
$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{5}\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{}$$
$$= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{3d}$$

Mathematica [A]

time = 0.24, size = 88, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(18a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5b+3a\cos(c+dx))\sin(2(c+dx))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(18*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10
*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*b + 3*a*Cos[c + d*x])*
Sin[2*(c + d*x)]))/(15*d)
```

Maple [A]

time = 0.11, size = 262, normalized size = 2.06

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(24a+20b)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{d}{2}\right.\right.}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*b-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 145, normalized size = 1.14

$$\frac{-5i\sqrt{2}\,b\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,b\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+9i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-9i\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3a\cos(dx+c)^2+5b\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + 5*I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
)) + 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c)^2 +
5*b*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{\cos(c+d\,x)}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

3.585 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{6b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

```
[Out] 2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10/
21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 3854, 3856, 2720, 2719}

$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + b\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{7}(5a)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{5}(3b)\int \frac{}{\sqrt{\sec(}}\\ &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(5a)\int \sqrt{\sec(c+a}\\ &= \frac{6b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sin(c+}{5d\sec^{\frac{3}{2}}(c+}\\ &= \frac{6b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+a\right.}{21d}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 99, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(252b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+100a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(65a+42b\cos(c+dx)+15a\cos(2(c+dx)))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1
00*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a + 42*b*Cos[c + d*
x] + 15*a*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A]
time = 0.10, size = 290, normalized size = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(-360a-168b)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*a
-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
a-63*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 156, normalized size = 1.03

$$\frac{-25i\sqrt{2}\,a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}\,a\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+63i\sqrt{2}\,b\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-63i\sqrt{2}\,b\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(15a\cos(dx+c)^{3}+21b\cos(dx+c)^{2}+25a\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/105*(-25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)) + 63*I*sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*b*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a*cos(d*x + c
)^3 + 21*b*cos(d*x + c)^2 + 25*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(7/2),x)
[Out] Integral((a + b*sec(c + d*x))/sec(c + d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + \frac{b}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(7/2), x)
```

3.586 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=200

$$-\frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}$$

```
[Out] 2/21*(7*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a*b*sec(d*x+c)^(5/2)*s
in(d*x+c)/d+2/7*b^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d+12/5*a*b*sin(d*x+c)*sec(d
*x+c)^(1/2)/d-12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*
(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.11, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3853, 3856, 2719, 4131, 2720}

$$\frac{2(7a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{12ab\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{12ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (12*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (
2*(7*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (4*a*b*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*b^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7
*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \sec^{\frac{7}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\, \\
&= \frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(6 \\
&= \frac{12ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(7a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c \\
&= \frac{12ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(7a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c \\
&= -\frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(7a^2+5
\end{aligned}$$

Mathematica [A]
time = 0.58, size = 139, normalized size = 0.70

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(-504ab\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(7a^2+5b^2)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(35a^2+55b^2+273ab\cos(c+dx)+5(7a^2+5b^2)\cos(2(c+dx))+63ab\cos(3(c+dx)))\sin(c+dx)\right)}{210d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]

[Out] (Sec[c + d*x]^(7/2)*(-504*a*b*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2] + 20*(7*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(35*a^2 + 55*b^2 + 273*a*b*Cos[c + d*x] + 5*(7*a^2 + 5*b^2)*Cos[2*(c + d*x)] + 63*a*b*Cos[3*(c + d*x)])*Sin[c + d*x]))/(210*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(224) = 448$.
time = 0.30, size = 689, normalized size = 3.44

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}{6\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4/5*b*a/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.90, size = 235, normalized size = 1.18

$$\frac{-126i\sqrt{2}\,ab\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+126i\sqrt{2}\,ab\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-5\sqrt{2}\left(7i\,a^{2}+5i\,b^{2}\right)\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5\sqrt{2}\left(-7i\,a^{2}-5i\,b^{2}\right)\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+\frac{2\left(126\,ab\cos(dx+c)^{3}+42\,ab\cos(dx+c)+5\left(7\,a^{2}+5\,b^{2}\right)\cos(dx+c)^{2}+15\,b^{2}\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{105\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(-126*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 126*I*sqrt(2)*a*b*cos(d*x
 + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c))) - 5*sqrt(2)*(7*I*a^2 + 5*I*b^2)*cos(d*x + c)^3*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*a^2 - 5*I*b^
2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 2*(126*a*b*cos(d*x + c)^3 + 42*a*b*cos(d*x + c) + 5*(7*a^2 + 5*b^2)*cos(
d*x + c)^2 + 15*b^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2),x)
[Out] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2), x)
```

3.587 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=175

$$-\frac{2(5a^2+3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c-}}{3d}$$

```
[Out] 4/3*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+2/5*(5*a^2+3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*(5*a^2+3*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3853, 3856, 2720, 4131, 2719}

$$\frac{2(5a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2(5a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (2*(5*a^2 + 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d) + (4*a*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*b^2*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \sec^{\frac{5}{2}}(c+dx)\,dx+\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx \\ &= \frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{1}{3}(2a \\ &= \frac{2(5a^2+3b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+ \\ &= \frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2(5a^2+3b^2)\,\sqrt{} \\ &= -\frac{2(5a^2+3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4ab}{}\end{aligned}$$

Mathematica [A]
time = 0.84, size = 126, normalized size = 0.72

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12(5a^2+3b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40ab\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(15(a^2+b^2)+20ab\cos(c+dx)+3(5a^2+3b^2)\cos(2(c+dx)))\sin(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

[Out] (Sec[c + d*x]^(5/2)*(-12*(5*a^2 + 3*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 40*a*b*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a^2 + b^2) + 20*a*b*Cos[c + d*x] + 3*(5*a^2 + 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(30*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(203) = 406$.
time = 0.25, size = 633, normalized size = 3.62

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4ba\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}{6\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 223, normalized size = 1.27

$$\frac{-10i\sqrt{2}\,ab\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}\,ab\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\sqrt{2}\,(5i\,a^2+3i\,b^2)\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\sqrt{2}\,(-5i\,a^2-3i\,b^2)\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(10\,ab\cos(dx+c)+3\,(5\,a^2+3\,b^2)\cos(dx+c)^2+3\,b^2\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(-10*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(5*I*a^2 + 3*I*b^2)*cos(
d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*cos(d*x + c)^2*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
2*(10*a*b*cos(d*x + c) + 3*(5*a^2 + 3*b^2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

3.588 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=135

$$-\frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}$$

```
[Out] 2/3*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(3*a^2+b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3853, 3856, 2719, 4131, 2720}

$$\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*d) + (4*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*b^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\sec(c+dx)}\,\left(a^2+b^2\sec^2(c+dx)\right.$$

$$= \frac{4ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - ($$

$$= \frac{4ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \Big($$

$$= -\frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(3a^2+b^2}{}$$

Mathematica [A]
time = 0.22, size = 93, normalized size = 0.69

$$\frac{2\sec^{\frac{3}{2}}(c+dx)\left(-6ab\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(3a^2+b^2)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+b(b+6a\cos(c+dx))\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^(3/2)*(-6*a*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
+ (3*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + b*(b + 6*a*C
os[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(171) = 342$.
time = 0.20, size = 513, normalized size = 3.80

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-6\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{Ell}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(24*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*
c)^2*a^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^2-12*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a*b-12*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2*a*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 190, normalized size = 1.41

$$\frac{-6i\sqrt{2}\,ab\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+6i\sqrt{2}\,ab\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\sqrt{2}\,(-3i\,a^2-i\,b^2)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}\,(3i\,a^2+i\,b^2)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+\frac{2(6ab\cos(dx+c)+b^2)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + sqrt(2)*(-3*I*a^2 - I*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*a^2 + I*b^2)*cos(d*x + c)*weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*a*b*cos(d*x +
c) + b^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \sqrt{\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{\cos(c + d\,x)}\right)^2 \sqrt{\frac{1}{\cos(c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

3.589 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{2(a^2-b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{d}$$

```
[Out] 2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3873, 3856, 2720, 4131, 2719}

$$\frac{2(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = (2ab)\int \sqrt{\sec(c+dx)}\,dx + \int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \left(a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \Big(2ab\sqrt{\cos(c$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+}{d}$$
$$= \frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4ab\sqrt{\cos(c+dx)}\,}{}$$

Mathematica [A]
time = 0.13, size = 82, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
[Out] (2*Sqrt[Sec[c + d*x]]*((a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + b*(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*x]
)))/d
```

Maple [A]
time = 0.14, size = 202, normalized size = 1.87

method	result

default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2-4\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,ab+2\,\mathrm{Elli}}{\mathrm{s}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima **[F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)

Fricas **[C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 146, normalized size = 1.35

$$\frac{-2i\sqrt{2}\,ab\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+2i\sqrt{2}\,ab\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+\frac{2b^2\sin(dx+c)}{\sqrt{\cos(dx+c)}}+\sqrt{2}\,(i\,a^2-i\,b^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(-i\,a^2+i\,b^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] (-2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*b^2*sin(d*x + c)/sqrt(cos(d*x + c)) + sqrt(2)*(I*a^2 - I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(-I*a^2 + I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**2/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

3.590 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$\frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{3d}$$

```
[Out] 2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3873, 3856, 2719, 4130, 2720}

$$\frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= (2ab)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{1}{3}\left(-a^2-3b^2\right)\int \sqrt{\sec(c+dx)}\,dx+\Big(2ab\sqrt{\cos(c+dx)}\,\sqrt{}\\ &= \frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{1}{3}\Big(\big(-\\ &= \frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}}{}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 87, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a^2*Sin[2*(c
 + d*x)]))/(3*d)
```

Maple [A]
time = 0.11, size = 283, normalized size = 2.53

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2-6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 147, normalized size = 1.31

$$\frac{2\,a^2\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+6i\,\sqrt{2}\,ab\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-6i\,\sqrt{2}\,ab\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+\sqrt{2}\,(-i\,a^2-3i\,b^2)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(i\,a^2+3i\,b^2)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*sqrt(cos(d*x + c))*sin(d*x + c) + 6*I*sqrt(2)*a*b*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6*I*s
qrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sec\left(c+dx\right)\right)^2}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

3.591 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{2(3a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+\ldots}}{3d}$$

```
[Out] 2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a*b*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3854, 3856, 2720, 4130, 2719}

$$\frac{2(3a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= (2ab)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}(2ab)\int \sqrt{\sec(c+dx)}\,dx-\frac{1}{5}\left(-3a^2\right.\\ &= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\\ &= \frac{2(3a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4ab\sqrt{\cos(c+dx}}{}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 100, normalized size = 0.71

$$\frac{\sqrt{\sec(c+dx)}\left(6(3a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a(10b+3a\cos(c+dx))\sin(2(c+dx))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(10*b +
3*a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(173) = 346$.
time = 0.11, size = 357, normalized size = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a^2+24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2+40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2-20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b+10*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-15*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 170, normalized size = 1.21

$$\frac{-10i\sqrt{2}\,ab\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}\,ab\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\sqrt{2}\,(-3i\,a^2-5i\,b^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\sqrt{2}\,(3i\,a^2+5i\,b^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3a^2\cos(dx+c)^2+10ab\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] 1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-3*I*a^2 - 5*I*b^2)*weierstrassZeta(-4, 0, weierstras

```
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*a^2 + 5*I
*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))) + 2*(3*a^2*cos(d*x + c)^2 + 10*a*b*cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
[Out] Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
[Out] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

3.592 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{21d}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+2/21*(5*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a*b*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(5*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3873, 3854, 3856, 2719, 4130, 2720}

$$\frac{2(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{12ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(21*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(5*a^2 + 7*b^2)*Sin[c + d*x
])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= (2ab)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx\\ &= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{5}(6ab)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx-\frac{1}{7}\big(-5a^2-\\ &= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}-\frac{1}{21}\big(-5a^2-7b\\ &= \frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin}{5d\sec}\\ &= \frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(5a^2+7b^2)\sqrt{\cos(c+d}}{}\end{aligned}$$

Mathematica [A]
time = 0.48, size = 120, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(504ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(5a^2+7b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(65a^2+70b^2+84ab\cos(c+dx)+15a^2\cos(2(c+dx)))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(504*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 20*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a^2
+ 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])
)/(210*d)
```

Maple [A]
time = 0.13, size = 362, normalized size = 2.07

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+\left(-360a^2-336ba\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a^2+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2+35*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2-126*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 191, normalized size = 1.09

$$\frac{126i\sqrt{2}\,ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-126i\sqrt{2}\,ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-5\sqrt{2}\,(5i\,a^2+7i\,b^2)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5\sqrt{2}\,(-5i\,a^2-7i\,b^2)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+\frac{2\left(15a^2\cos(dx+c)^3+42ab\cos(dx+c)^2+5(5a^2+7b^2)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/105*(126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 5*sqrt(2)*(5*I
*a^2 + 7*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) -
 5*sqrt(2)*(-5*I*a^2 - 7*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) + 2*(15*a^2*cos(d*x + c)^3 + 42*a*b*cos(d*x + c)^2 + 5*(5*a^
2 + 7*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(c + dx)})^2}{\sec^{\frac{7}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2), x)
```

3.593 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=234

$$-\frac{2a(5a^2+9b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{21d}$$

```
[Out] 2/21*b*(21*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+32/35*a*b^2*sec(d*x+c)^
(5/2)*sin(d*x+c)/d+2/7*b^2*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d+2
/5*a*(5*a^2+9*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*a*(5*a^2+9*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*b*(21*a^2+5*b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3927, 4132, 3853, 3856, 2720, 4131, 2719}

$$\frac{2b(21a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a(5a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}+\frac{32ab^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(5*a^2 + 9*b^2)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (32*a*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*b^2
*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx \\ &= \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx \\ &= \frac{2b(21a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{32ab^2\sec^{\frac{5}{2}}(c+dx)\sin(}{35d} \\ &= \frac{2a(5a^2+9b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2b(21a^2+5b^2)\sec^{\frac{3}{2}}(c}{21d} \\ &= \frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2}{} \\ &= -\frac{2a(5a^2+9b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} +\end{aligned}$$

Mathematica [A]
time = 2.24, size = 177, normalized size = 0.76

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(-168a(5a^2+9b^2)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40b(21a^2+5b^2)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(210a^2b+110b^3+63a(5a^2+13b^2)\cos(c+dx)+10(21a^2b+5b^3)\cos(2(c+dx))+105a^3\cos(3(c+dx))+189ab^2\cos(3(c+dx)))\sin(c+dx)\right)}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-168*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(7/2)*EllipticE[(c
 + d*x)/2, 2] + 40*b*(21*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x
)/2, 2] + 2*(210*a^2*b + 110*b^3 + 63*a*(5*a^2 + 13*b^2)*Cos[c + d*x] + 10*
(21*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 105*a^3*Cos[3*(c + d*x)] + 189*a*b^2*
Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 819 vs. 2(258) = 516.
time = 0.33, size = 820, normalized size = 3.50

method	result	size
default	Expression too large to display	820

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(6*b*a^2*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
```

```
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6/5*b^2*a/(8*sin(1/2*d*x+1/2*c)^6-12*sin(
1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b
^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)+2*a^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.19, size = 270, normalized size = 1.15

$$-\frac{5\sqrt{2}\left(21i a^2b+5i b^3\right)\cos\left(dx+c\right)^3 \text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-21i a^2b-5i b^3\right)\cos\left(dx+c\right)^3 \text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+21\sqrt{2}\left(5i a^3+9i ab^2\right)\cos\left(dx+c\right)^3 \text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+21\sqrt{2}\left(-5i a^3-9i ab^2\right)\cos\left(dx+c\right)^3 \text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(63ab^2\cos(dx+c)+21\left(5a^3+9ab^2\right)\cos(dx+c)^3+15b^3+5\left(21a^2b+5b^3\right)\cos(dx+c)^2\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{105\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*cos(d*x + c)^3*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^2*b - 5*I*b^3)*
cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) +
21*sqrt(2)*(5*I*a^3 + 9*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-5*I*a
^3 - 9*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-
```

```
4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a*b^2*cos(d*x + c) + 21*(5*a^
3 + 9*a*b^2)*cos(d*x + c)^3 + 15*b^3 + 5*(21*a^2*b + 5*b^3)*cos(d*x + c)^2)
*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^3\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

3.594 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=189

$$-\frac{6b(5a^2+b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(a^2+b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{d}$$

```
[Out] 8/5*a*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*sec(d*x+c)^(3/2)*(a+b*sec(d
*x+c))*sin(d*x+c)/d+6/5*b*(5*a^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*b*(
5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*(a^2+b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3927, 4132, 3853, 3856, 2719, 4131, 2720}

$$\frac{6b(5a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8ab^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d}+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (6*b*(5*a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(5*d) + (8*a*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b^2*Sec[
c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3\,dx = \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+$$

$$= \frac{6b(5a^2+b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{8ab^2\sec^{\frac{3}{2}}(c+dx)\sin($$

$$= \frac{6b(5a^2+b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{8ab^2\sec^{\frac{3}{2}}(c+dx)\sin($$

$$= -\frac{6b(5a^2+b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} +$$

Mathematica [A]
time = 0.98, size = 134, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(-3b(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5a(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b(5(3a^2+b^2)+10ab\cos(c+dx)+3(5a^2+b^2)\cos(2(c+dx)))\sin(c+dx)}{2\cos^{\frac{5}{2}}(c+dx)}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*b*(5*a^2 + b^2)*EllipticE[(c +
 d*x)/2, 2] + 5*a*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + (b*(5*(3*a^2 + b^
2) + 10*a*b*Cos[c + d*x] + 3*(5*a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/
(2*Cos[c + d*x]^(5/2))))/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(219) = 438.
time = 0.26, size = 711, normalized size = 3.76

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{Ell}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*b^2
*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b^3/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24
*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+6*b*a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*
```

```
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.15, size = 244, normalized size = 1.29

$$-\frac{5\sqrt{2}\left(i a^{3}+i a b^{2}\right)\cos\left(dx+c\right)^{2} \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-i a^{3}-i a b^{2}\right)\cos\left(dx+c\right)^{2} \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\sqrt{2}\left(5i a^{2} b+i b^{3}\right)\cos\left(dx+c\right)^{2} \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(-5i a^{2} b-i b^{3}\right)\cos\left(dx+c\right)^{2} \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(5ab^{2}\cos\left(dx+c\right)+b^{3}+3\left(5a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{5d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(5*sqrt(2)*(I*a^3 + I*a*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^3 - I*a*b^2)*cos(d*x + c)
^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(5
*I*a^2*b + I*b^3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*a^2*b - I*b^3)*co
s(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) - 2*(5*a*b^2*cos(d*x + c) + b^3 + 3*(5*a^2*b + b^3)*cos
(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^3\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

3.595 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=158

$$\frac{2a(a^2-3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\vphantom{x}}}{3d}$$

```
[Out] 16/3*a*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*b^2*(a+b*sec(d*x+c))*sin(d*x+c
)*sec(d*x+c)^(1/2)/d+2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3927, 4132, 3856, 2720, 4131, 2719}

$$\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}{3d}+\frac{16ab^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (16*a*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}a(3a^2-b^2)+\frac{1}{2}b(9a^2
\\
&= \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}a(3a^2-b^2)+4ab^2\sec}{\sqrt{\sec(c+dx)}}
\\
&= \frac{16ab^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\sin(c+}{3d}
\\
&= \frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{16ab^2\sqrt{\sec(c+d}{3d}
\\
&= \frac{2a(a^2-3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b(9a^2+b^2)\sqrt{\mathrm{co}}
\end{aligned}$$

Mathematica [A]

time = 0.32, size = 106, normalized size = 0.67

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(6a(a^2-3b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2(9a^2+b^2)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2b(b+9a\cos(c+dx))\sin(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(6*a*(a^2 - 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*
x)/2, 2] + b*(2*(9*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]
+ 2*b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(192) = 384$.
time = 0.22, size = 630, normalized size = 3.99

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-18\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(36*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1
/2*c)^2*a^2*b-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^3+6*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3-18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(
1/2*d*x+1/2*c)^2*a*b^2-18*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2-2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b
+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3+9*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*a*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.18, size = 214, normalized size = 1.35

$$\frac{\sqrt{2}\left(-9i\,a^2b-i\,b^3\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(9i\,a^2b+i\,b^3\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\sqrt{2}\left(-i\,a^3+3i\,ab^2\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(i\,a^3-3i\,ab^2\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+\frac{2\left(9\,ab^2\cos(dx+c)+b^3\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-9*I*a^2*b - I*b^3)*cos(d*x + c)*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(9*I*a^2*b + I*b^3)*cos(d*x + c)*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-I*a^3
 + 3*I*a*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*a^3 - 3*I*a*b^2)*cos(d*x +
 c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(9*a*b^2*cos(d*x + c) + b^3)*sin(d*x + c)/sqrt(cos(d*x + c))
)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\sec\left(c + dx\right)\right)^{3}}{\sqrt{\sec\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

3.596 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=166

$$\frac{2b(3a^2-b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}$$

```
[Out] 2/3*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)-2/3*b*(a^2-3*b^2)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d+2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3926, 4132, 3856, 2720, 4131, 2719}

$$-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b*(a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(3*d) + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\int \frac{4a^2b+\frac{1}{2}a(a^2+9b^2)\sec(c+dx)-\frac{1}{2}}{\sqrt{\sec(c+dx)}}\\ &= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\int \frac{4a^2b-\frac{1}{2}b(a^2-3b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx\\ &= -\frac{2b(a^2-3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\\ &= \frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}-\frac{2b(a^2-3b^2)\sqrt{}}{}\\ &= \frac{2b(3a^2-b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2+9b^2)\sqrt{}}{}\end{aligned}$$

Mathematica [A]

time = 0.39, size = 108, normalized size = 0.65

$$\frac{\sqrt{\sec(c+dx)}\left(-6b(-3a^2+b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2a(a^2+9b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3b^3+a^3\cos(c+dx))\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-6*b*(-3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2] + 2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
 + 2*(3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [A]
time = 0.14, size = 303, normalized size = 1.83

method	result
default	$-\frac{2\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3-2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2*a^3-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+9*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*b*a^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 182, normalized size = 1.10

$$\frac{\sqrt{2}\,(-i\,a^3-9i\,ab^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(i\,a^3+9i\,ab^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\sqrt{2}\,(-3i\,a^2b+i\,b^3)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\sqrt{2}\,(3i\,a^2b-i\,b^3)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\,(a^3\cos(dx+c)+3\,b^3)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^3 - 9*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + sqrt(2)*(I*a^3 + 9*I*a*b^2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-3*I*a^2*b + I*b^3)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*s
qrt(2)*(3*I*a^2*b - I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(a^3*cos(d*x + c) + 3*b^3)*sin(d*x +
c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(c + dx)})^3}{\sec^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

3.597 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2b(a^2+b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{se}}}{d}$$

```
[Out] 2/5*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/5*a^2*b*sin(d*x+c)
/d/sec(d*x+c)^(1/2)+6/5*a*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2*b*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3926, 4132, 3856, 2719, 4130, 2720}

$$\frac{2b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^2b\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/d + (8*a^2*b*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
 + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{5}\int \frac{6a^2b+\frac{3}{2}a(a^2+5b^2)\sec(c+dx)+\frac{1}{2}}{\sec^{\frac{3}{2}}(c+dx)}\\ &= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{5}\int \frac{6a^2b+\frac{1}{2}b(a^2+5b^2)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{8a^2b\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}+\frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\left(b\left(a^2+b^2\right)\right)\int \sqrt{\sec}\\ &= \frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{8a^2b\sin(c+dx}{5d\sqrt{\sec(c+dx}}\\ &= \frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2b(a^2+b^2)\sqrt{c}}{}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 106, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(6a(a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b(a^2+b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2(5b+a\cos(c+dx))\sin(2(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]

[Out] (Sqrt[Sec[c + d*x]]*(6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a^2*(5*b + a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(5*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(190) = 380$.
time = 0.13, size = 412, normalized size = 2.64

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3+8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a^3+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3+20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3-10*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 193, normalized size = 1.24

$$-\frac{5\sqrt{2}\left(ia^2b+ib^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-ia^2b-ib^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\sqrt{2}\left(-ia^3-5i\,ab^2\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(ia^3+5i\,ab^2\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(a^3\cos\left(dx+c\right)^2+5a^2b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(5*sqrt(2)*(I*a^2*b + I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^2*b - I*b^3)*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-I*a^3 - 5*I*a*b^2)*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*
sqrt(2)*(I*a^3 + 5*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - 2*(a^3*cos(d*x + c)^2 + 5*a^2*b*cos(d*
x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(c + dx)})^3}{\sec^{\frac{5}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2), x)
```

3.598 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{2b(9a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

```
[Out] 32/35*a^2*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*a^2*(a+b*sec(d*x+c))*sin(d*x+
c)/d/sec(d*x+c)^(5/2)+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2
/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(5
*a^2+21*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3926, 4132, 3854, 3856, 2720, 4130, 2719}

$$\frac{2a(5a^2+21b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{32a^2b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (32*a^2*b*Sin[c + d*x])/(35*d*Sec[c
 + d*x]^(3/2)) + (2*a*(5*a^2 + 21*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x
]]) + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8a^2b+\frac{1}{2}a(5a^2+21b^2)\sec(c+dx)+}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8a^2b+\frac{1}{2}b(3a^2+7b^2)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{32a^2b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(5a^2+21b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{32a^2b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(5a^2+21b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b(9a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(5a^2+21b^2)}{}$$

Mathematica [A]
time = 0.62, size = 132, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(84b(9a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20a(5a^2+21b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a(65a^2+210b^2+126ab\cos(c+dx)+15a^2\cos(2(c+dx)))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 20*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2] + a*(65*a^2 + 210*b^2 + 126*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)
])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A]
time = 0.19, size = 421, normalized size = 2.12

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3+(-360a^3-504b\,a^2)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a^3+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^4*c
```

```
os(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b^2*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-189*b*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 216, normalized size = 1.09

$$-\frac{5\sqrt{2}\,(5i\,a^3+21i\,ab^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-5i\,a^3-21i\,ab^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+21\sqrt{2}\,(-9i\,a^2b-5i\,b^3)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+21\sqrt{2}\,(9i\,a^2b+5i\,b^3)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(15\,a^3\cos(dx+c)^3+63\,a^2b\cos(dx+c)^2+5\,(5\,a^3+21\,ab^2)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^3 - 21*I*a*b^2)*weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-9*I*a^2*b - 5*I*b
^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) + 21*sqrt(2)*(9*I*a^2*b + 5*I*b^3)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*a^3*cos(d*x +
c)^3 + 63*a^2*b*cos(d*x + c)^2 + 5*(5*a^3 + 21*a*b^2)*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2), x)
```

3.599 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{2a(7a^2+27b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2b(15a^2+7b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{21d}$$

```
[Out] 40/63*a^2*b*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*a*(7*a^2+27*b^2)*sin(d*x+c)/
d/sec(d*x+c)^(3/2)+2/9*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2
/21*b*(15*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*a*(7*a^2+27*b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*b*(15*a^2+7*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3926, 4132, 3854, 3856, 2719, 4130, 2720}

$$\frac{2a(7a^2+27b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b(15a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b(15a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(7a^2+27b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a*(7*a^2 + 27*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (2*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (40*a^2*b*Sin[c + d*x])/(63*d*Sec
[c + d*x]^(5/2)) + (2*a*(7*a^2 + 27*b^2)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(
3/2)) + (2*b*(15*a^2 + 7*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*
a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{10a^2b+\frac{1}{2}a(7a^2+27b^2)\sec(c+dx)\,\cdot}{\sec^{\frac{7}{2}}(c+d}$$

$$= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{10a^2b+\frac{1}{2}b(5a^2+9b^2)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,d$$

$$= \frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(7a^2+27b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))\sin(c}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(7a^2+27b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b(15a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\,-$$

$$= \frac{2a(7a^2+27b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{40a^2b\sin(c+}{63d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2a(7a^2+27b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2b(15a^2+7b}{}$$

Mathematica [A]
time = 0.85, size = 159, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(168a(7a^2+27b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+120b(15a^2+7b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(7a(43a^2+108b^2)\cos(c+dx)+5(234a^2b+84b^3+54a^2b\cos(2(c+dx))+7a^3\cos(3(c+dx))))\sin(2(c+dx))\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*a*(7*a^2 + 27*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 120*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] + (7*a*(43*a^2 + 108*b^2)*Cos[c + d*x] + 5*(234*a^2*b + 84*b^3 + 5
4*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(126
0*d)
```

Maple [A]
time = 0.14, size = 470, normalized size = 2.01

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3+(2240a^3+2160b\,a^2)\left(\sin^8\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*a^3+(2240*a^3+2160*a^2*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b+105*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*b^3-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-567*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 238, normalized size = 1.02

$$-\frac{15\sqrt{2}\,(15i\,a^2b+7i\,b^3)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}\,(-15i\,a^2b-7i\,b^3)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+21\sqrt{2}\,(-7i\,a^3-27i\,ab^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+21\sqrt{2}\,(7i\,a^3+27i\,ab^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(35\,a^3\cos(dx+c)^4+135\,a^2b\cos(dx+c)^3+7\,(7\,a^3+27\,ab^2)\cos(dx+c)^2+15\,(15\,a^2b+7\,b^3)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/315*(15*sqrt(2)*(15*I*a^2*b + 7*I*b^3)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-15*I*a^2*b - 7*I*b^3)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-7*I*a^3 - 27*I*
a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c))) + 21*sqrt(2)*(7*I*a^3 + 27*I*a*b^2)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(35*a^3*cos(d*
x + c)^4 + 135*a^2*b*cos(d*x + c)^3 + 7*(7*a^3 + 27*a*b^2)*cos(d*x + c)^2 +
 15*(15*a^2*b + 7*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2),x)
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2), x)
```

3.600 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=287

$$-\frac{2(15a^4+54a^2b^2+7b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{8ab(7a^2+5b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{21d}$$

```
[Out] 8/21*a*b*(7*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+14/45*b^2*(7*a^2+b^2)*
sec(d*x+c)^(5/2)*sin(d*x+c)/d+44/63*a*b^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/9
*b^2*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+2/15*(15*a^4+54*a^2*b
^2+7*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/15*(15*a^4+54*a^2*b^2+7*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/21*a*b*(7*a^2+5*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.28, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3927, 4161, 4132, 3853, 3856, 2720, 4131, 2719}

$$\frac{14b^2(7a^2+b^2)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{8ab(7a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{8ab(7a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(15a^4+54a^2b^2+7b^4)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-\frac{2(15a^4+54a^2b^2+7b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{44ab^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d}+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-2*(15*a^4 + 54*a^2*b^2 + 7*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(15*a^4 + 54*a^2*
b^2 + 7*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (8*a*b*(7*a^2 + 5*b^
2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (14*b^2*(7*a^2 + b^2)*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(45*d) + (44*a*b^3*Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(63*d) + (2*b^2*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(
9*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{9d} + \frac{2}{9}\int \sec^{\frac{3}{2}}(c+dx \\
&= \frac{44ab^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx)}{9d} \\
&= \frac{44ab^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx)}{9d} \\
&= \frac{8ab(7a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{14b^2(7a^2+b^2)\sec^{\frac{5}{2}}(c+}{45d} \\
&= \frac{2(15a^4+54a^2b^2+7b^4)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{8ab(7a^2+5b^2)}{} \\
&= \frac{8ab(7a^2+5b^2)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2(}{} \\
&= -\frac{2(15a^4+54a^2b^2+7b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{15d}
\end{aligned}
$$

Mathematica [A]
time = 1.43, size = 256, normalized size = 0.89

$$
-\frac{2(a+b\sec(c+dx))^4\left(21(15a^4+54a^2b^2+7b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)-60ab(7a^2+5b^2)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-315a^4\sin(c+dx)-1134a^2b^2\sin(c+dx)-147b^4\sin(c+dx)-420a^3b\tan(c+dx)-300ab^3\tan(c+dx)-378a^2b^2\sec(c+dx)\tan(c+dx)-49b^4\sec(c+dx)\tan(c+dx)-180ab^3\sec^2(c+dx)\tan(c+dx)-35b^4\sec^3(c+dx)\tan(c+dx)\right)}{315d(b+a\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-2*(a + b*Sec[c + d*x])^4*(21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2] - 60*a*b*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2] - 315*a^4*Sin[c + d*x] - 1134*a^2*b^2*Sin[c + d*x
] - 147*b^4*Sin[c + d*x] - 420*a^3*b*Tan[c + d*x] - 300*a*b^3*Tan[c + d*x]
- 378*a^2*b^2*Sec[c + d*x]*Tan[c + d*x] - 49*b^4*Sec[c + d*x]*Tan[c + d*x]
- 180*a*b^3*Sec[c + d*x]^2*Tan[c + d*x] - 35*b^4*Sec[c + d*x]^3*Tan[c + d*x
]))/(315*d*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(307) = 614$.
time = 0.47, size = 1147, normalized size = 4.00

method	result	size
default	Expression too large to display	1147

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*b*a^3*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+8*b^3*a*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-
5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+12/5*b^2*a^2/(8*sin(1/2*d*x+1/2*c)
^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(
24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a^4/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^4*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/
180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

[Out] Timed out

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 318, normalized size = 1.11

$$-\frac{60\sqrt{2}\left(7i\,a^{3}b+5i\,ab^{3}\right)\cos\left(dx+c\right)^{4}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+60\sqrt{2}\left(-7i\,a^{3}b-5i\,ab^{3}\right)\cos\left(dx+c\right)^{4}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+21\sqrt{2}\left(15i\,a^{4}+54i\,a^{2}b^{2}+7i\,b^{4}\right)\cos\left(dx+c\right)^{4}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+21\sqrt{2}\left(-15i\,a^{4}-54i\,a^{2}b^{2}-7i\,b^{4}\right)\cos\left(dx+c\right)^{4}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(180\,ab^{3}\cos\left(dx+c\right)+21\left(15\,a^{4}+54\,a^{2}b^{2}+7\,b^{4}\right)\cos\left(dx+c\right)^{4}+35\,b^{4}+60\left(7\,a^{3}b+5\,ab^{3}\right)\cos\left(dx+c\right)^{3}+7\left(54\,a^{2}b^{2}+7\,b^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{315\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/315*(60*sqrt(2)*(7*I*a^3*b + 5*I*a*b^3)*cos(d*x + c)^4*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 60*sqrt(2)*(-7*I*a^3*b - 5*I*a*b
^3)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) + 21*sqrt(2)*(15*I*a^4 + 54*I*a^2*b^2 + 7*I*b^4)*cos(d*x + c)^4*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
21*sqrt(2)*(-15*I*a^4 - 54*I*a^2*b^2 - 7*I*b^4)*cos(d*x + c)^4*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(
180*a*b^3*cos(d*x + c) + 21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*cos(d*x + c)^4 +
35*b^4 + 60*(7*a^3*b + 5*a*b^3)*cos(d*x + c)^3 + 7*(54*a^2*b^2 + 7*b^4)*cos
(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^4\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2), x)
```

3.601 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=247

$$-\frac{8ab(5a^2+3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(21a^4+42a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,F(}{21d}$$

```
[Out] 2/21*b^2*(39*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+36/35*a*b^3*sec(d*x+c
)^(5/2)*sin(d*x+c)/d+2/7*b^2*sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*sin(d*x+c)
/d+8/5*a*b*(5*a^2+3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-8/5*a*b*(5*a^2+3*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(21*a^4+42*a^2*b^2+5*
b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.26, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3927, 4161, 4132, 3853, 3856, 2719, 4131, 2720}

$$\frac{2b^2(39a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{8ab(5a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{8ab(5a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(21a^4+42a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{36ab^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d}+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-8*a*b*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*(21*a^4 + 42*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a*b*(5*a^2 + 3*b^2
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b^2*(39*a^2 + 5*b^2)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(21*d) + (36*a*b^3*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(35*d) + (2*b^2*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(
7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^4\,dx &= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{7d}+\frac{2}{7}\int\sqrt{\sec(c\ldots}\\
&= \frac{36ab^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+\ldots}{7d}\\
&= \frac{36ab^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+\ldots}{7d}\\
&= \frac{8ab(5a^2+3b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2b^2(39a^2+5b^2)\sec\ldots}{2\ldots}\\
&= \frac{8ab(5a^2+3b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2b^2(39a^2+5b^2)\sec\ldots}{2\ldots}\\
&= -\frac{8ab(5a^2+3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}\ldots}{5d}
\end{aligned}
$$

Mathematica [A]
time = 1.02, size = 168, normalized size = 0.68

$$\frac{2\sec^{\frac{7}{2}}(c+dx)\left(-84ab(5a^2+3b^2)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(21a^4+42a^2b^2+5b^4)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+b(15b^3+5b(42a^2+5b^2)\cos^2(c+dx)+84a(5a^2+3b^2)\cos^3(c+dx))\sin(c+dx)+42ab^3\sin(2(c+dx))\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (2*Sec[c + d*x]^(7/2)*(-84*a*b*(5*a^2 + 3*b^2)*Cos[c + d*x]^(7/2)*EllipticE
[(c + d*x)/2, 2] + 5*(21*a^4 + 42*a^2*b^2 + 5*b^4)*Cos[c + d*x]^(7/2)*Ellip
ticF[(c + d*x)/2, 2] + b*(15*b^3 + 5*b*(42*a^2 + 5*b^2)*Cos[c + d*x]^2 + 84
*a*(5*a^2 + 3*b^2)*Cos[c + d*x]^3)*Sin[c + d*x] + 42*a*b^3*Sin[2*(c + d*x)]
))/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(271) = 542$.
time = 0.36, size = 898, normalized size = 3.64

method	result	size
default	Expression too large to display	898

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
```

```
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*b^
2*a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^4*(-1/56*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c
)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+8/5*b^3*a/(8*sin(1/2*d*
x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/
2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)+8*b*a^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 289, normalized size = 1.17

$$-\frac{5\sqrt{2}\left(21i\,a^4+42i\,a^2b^2+5i\,b^4\right)\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-21i\,a^4-42i\,a^2b^2-5i\,b^4\right)\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+84\sqrt{2}\left(5i\,a^3b+3i\,ab^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+84\sqrt{2}\left(-5i\,a^3b-3i\,ab^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(84\,ab^3\cos(dx+c)+15\,b^4+84\left(5\,a^3b+3\,ab^3\right)\cos(dx+c)^3+5\left(42\,a^2b^2+5\,b^4\right)\cos(dx+c)^2\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{105\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
[Out] -1/105*(5*sqrt(2)*(21*I*a^4 + 42*I*a^2*b^2 + 5*I*b^4)*cos(d*x + c)^3*weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^4
- 42*I*a^2*b^2 - 5*I*b^4)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 84*sqrt(2)*(5*I*a^3*b + 3*I*a*b^3)*cos(d*x + c)^3
```

```
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) + 84*sqrt(2)*(-5*I*a^3*b - 3*I*a*b^3)*cos(d*x + c)^3*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(84
*a*b^3*cos(d*x + c) + 15*b^4 + 84*(5*a^3*b + 3*a*b^3)*cos(d*x + c)^3 + 5*(4
2*a^2*b^2 + 5*b^4)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^4\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2), x)
```

3.602 $\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=209

$$\frac{2(5a^4-30a^2b^2-3b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab(3a^2+b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3d}$$

```
[Out] 28/15*a*b^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*(29*a^2+3*b^2)*sin(d*x+c)
*sec(d*x+c)^(1/2)/d+2/5*b^2*(a+b*sec(d*x+c))^2*sin(d*x+c)*sec(d*x+c)^(1/2)/
d+2/5*(5*a^4-30*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/d+8/3*a*b*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.25, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3927, 4161, 4132, 3856, 2720, 4131, 2719}

$$\frac{2b^2(29a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5a^4-30a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{28ab^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(5*a^4 - 30*a^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*(29*a^2 + 3*b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (28*a*b^3*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(15*d) + (2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*
x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(\tfrac{1}{2}\right.}{}$$

$$= \frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\sin(c+}{5d}$$

$$= \frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\sin(c+}{5d}$$

$$= \frac{2b^2(29a^2+3b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} +$$

$$= \frac{8ab(3a^2+b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2b^2(29a^2+3b^2)}{}$$

$$= \frac{2(5a^4-30a^2b^2-3b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8ab(3a^2}{}$$

Mathematica [A]
time = 1.43, size = 146, normalized size = 0.70

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(12(5a^4-30a^2b^2-3b^4)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(80a(3a^2+b^2)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2b(15(6a^2+b^2)+40ab\cos(c+dx)+9(10a^2+b^2)\cos(2(c+dx)))\sin(c+dx)\right)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(12*(5*a^4 - 30*a^2*b^2 - 3*b^4)*Cos[c + d*x]^(5/2)*Ell
ipticE[(c + d*x)/2, 2] + b*(80*a*(3*a^2 + b^2)*Cos[c + d*x]^(5/2)*EllipticF
[(c + d*x)/2, 2] + 2*b*(15*(6*a^2 + b^2) + 40*a*b*Cos[c + d*x] + 9*(10*a^2
+ b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(237) = 474.
time = 0.30, size = 880, normalized size = 4.21

method	result	size
default	Expression too large to display	880

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
```

```
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*b*a^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*b^3*a*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b^4/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d
*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+
1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^
4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*si
n(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+12*b^2*a
^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 264, normalized size = 1.26

$$-\frac{20\sqrt{2}\left(3i\,a^3b+i\,ab^3\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+20\sqrt{2}\left(-3i\,a^3b-i\,ab^3\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+3\sqrt{2}\left(-5i\,a^4+30i\,a^2b^2+3i\,b^4\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(5i\,a^4-30i\,a^2b^2-3i\,b^4\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(20\,ab^3\cos\left(dx+c\right)+3\,b^4+9\left(10\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{15\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(20*sqrt(2)*(3*I*a^3*b + I*a*b^3)*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 20*sqrt(2)*(-3*I*a^3*b - I*a*b^3)*c
os(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3
*sqrt(2)*(-5*I*a^4 + 30*I*a^2*b^2 + 3*I*b^4)*cos(d*x + c)^2*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt
```

```
(2)*(5*I*a^4 - 30*I*a^2*b^2 - 3*I*b^4)*cos(d*x + c)^2*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(20*a*b^3*
cos(d*x + c) + 3*b^4 + 9*(10*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c)/sq
rt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

3.603 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=208

$$\frac{8ab(a^2-b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(a^4+18a^2b^2+b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{3d}$$

```
[Out] -2/3*b^2*(a^2-b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a^2*(a+b*sec(d*x+c))^2
*sin(d*x+c)/d/sec(d*x+c)^(1/2)-4/3*a*b*(a^2-6*b^2)*sin(d*x+c)*sec(d*x+c)^(1
/2)/d+8*a*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(a
^4+18*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3926, 4161, 4132, 3856, 2720, 4131, 2719}

$$-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{4ab(a^2-6b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{8ab(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}+\frac{2(a^4+18a^2b^2+b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (8*a*b*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*(a^4 + 18*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a*b*(a^2 - 6*b^2)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(3*d) - (2*b^2*(a^2 - b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*d) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*
x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+b\sec(c+dx))\left(5a^2b+\frac{1}{2}a(a^2+\right.}{\sqrt{}}$$

$$= -\frac{2b^2(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b^2(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{4ab(a^2-6b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3d}$$

$$= \frac{2(a^4+18a^2b^2+b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} - \frac{4ab(a^2-}{}$$

$$= \frac{8ab(a^2-b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(a^4+18a^2b^2+}{}$$

Mathematica [A]
time = 0.73, size = 130, normalized size = 0.62

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(24ab(a^2-b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(a^4+18a^2b^2+b^4)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{(a^4+2b^4+24ab^3\cos(c+dx)+a^4\cos(2(c+dx)))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(24*a*b*(a^2 - b^2)*EllipticE[(c + d
*x)/2, 2] + 2*(a^4 + 18*a^2*b^2 + b^4)*EllipticF[(c + d*x)/2, 2] + ((a^4 +
2*b^4 + 24*a*b^3*Cos[c + d*x] + a^4*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c +
 d*x]^(3/2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(238) = 476$.
time = 0.19, size = 777, normalized size = 3.74

method	result	size
default	Expression too large to display	777

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-8*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^6*a^4+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
```

```
)^2)^(1/2)*a*(a^3+6*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^4+12*a*b^3+b^4)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+18*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a^2*b^2+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^3*b+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)*
sin(1/2*d*x+1/2*c)^2+a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)+18*b^2*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+12*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 242, normalized size = 1.16

$$\frac{\sqrt{2}\,(-i\,a^4-18i\,a^2b^2-i\,b^4)\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(i\,a^4+18i\,a^2b^2+i\,b^4)\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-12\sqrt{2}\,(-i\,a^3b+i\,ab^3)\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-12\sqrt{2}\,(i\,a^3b-i\,ab^3)\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\left(a^4\cos(dx+c)^2+12\,ab^3\cos(dx+c)+b^4\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^4 - 18*I*a^2*b^2 - I*b^4)*cos(d*x + c)*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^4 + 18*I*a^2*b^2 +
I*b^4)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
)) - 12*sqrt(2)*(-I*a^3*b + I*a*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 12*sqrt(2)*(I*a^
3*b - I*a*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
```

```
0, cos(d*x + c) - I*sin(d*x + c))) + 2*(a^4*cos(d*x + c)^2 + 12*a*b^3*cos(d
*x + c) + b^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(c + dx)})^4}{\sec^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^4}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2), x)
```

3.604 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=207

$$\frac{2(3a^4+30a^2b^2-5b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab(a^2+3b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3d}$$

```
[Out] 2/5*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+28/15*a^3*b*sin(d*
x+c)/d/sec(d*x+c)^(1/2)-2/5*b^2*(a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2
/5*(3*a^4+30*a^2*b^2-5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+
8/3*a*b*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3926, 4159, 4132, 3856, 2720, 4131, 2719}

$$\frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2b^2(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(3a^4+30a^2b^2-5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(3*a^4 + 30*a^2*b^2 - 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (28*a^3*b*Sin[c + d*x])/(1
5*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(5*d) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^
(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(7a^2b+\frac{3}{2}a(a^2+5\right.}{\sec}$$

$$= \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}a^2(3a^2+2}{}$$

$$= \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}a^2(3a^2+2}{}$$

$$= \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2b^2(a^2-5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a^2(a+b\sec(c}{5d\,\mathrm{se}}$$

$$= \frac{8ab(a^2+3b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{28a^3b\sin(c+d}{15d\sqrt{\sec(c+d}}$$

$$= \frac{2(3a^4+30a^2b^2-5b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8ab(a^2}{}$$

Mathematica [A]
time = 0.39, size = 138, normalized size = 0.67

$$\frac{\sqrt{\sec(c+dx)}\left(12(3a^4+30a^2b^2-5b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+80ab(a^2+3b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3a^4+30b^4+40a^3b\cos(c+dx)+3a^4\cos(2(c+dx)))\sin(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*(3*a^4 + 30*a^2*b^2 - 5*b^4)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2] + 80*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2] + 2*(3*a^4 + 30*b^4 + 40*a^3*b*Cos[c + d*x] + 3*a^4*Cos[2*(
c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [A]
time = 0.16, size = 432, normalized size = 2.09

method	result
default	$\frac{\frac{16\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{5}-\frac{16a^4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{32a^3\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b}{3}+\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{5}+16}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a^4-24*a^4*sin(1/2*d*x+1/2
*c)^4*cos(1/2*d*x+1/2*c)-80*a^3*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)*b+6
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^4+40*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2*a^3*b+30*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^4-20*b*a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-60*b^3*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*a^4*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+90*b^2*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.99, size = 215, normalized size = 1.04

$$-\frac{20\sqrt{2}\,(i\,a^3b+3i\,ab^3)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+20\sqrt{2}\,(-i\,a^3b-3i\,ab^3)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3\sqrt{2}\,(-3i\,a^4-30i\,a^2b^2+5i\,b^4)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3\sqrt{2}\,(3i\,a^4+30i\,a^2b^2-5i\,b^4)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(3a^4\cos(dx+c)^2+20a^3b\cos(dx+c)+15b^4\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(20*sqrt(2)*(I*a^3*b + 3*I*a*b^3)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 20*sqrt(2)*(-I*a^3*b - 3*I*a*b^3)*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-3*I*a^4 - 30*I*a^2*
b^2 + 5*I*b^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + 3*sqrt(2)*(3*I*a^4 + 30*I*a^2*b^2 - 5*I*b^4)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - 2*(3*a^4*cos(d*x + c)^2 + 20*a^3*b*cos(d*x + c) + 15*b^4)*sin(d*x + c)/s
qrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2), x)
```

3.605 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{8ab(3a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(5a^4+42a^2b^2+21b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{21d}$$

```
[Out] 36/35*a^3*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*a^2*(a+b*sec(d*x+c))^2*sin(d*
x+c)/d/sec(d*x+c)^(5/2)+2/21*a^2*(5*a^2+39*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/
2)+8/5*a*b*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/2
1*(5*a^4+42*a^2*b^2+21*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.25, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3926, 4159, 4132, 3856, 2719, 4130, 2720}

$$\frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(5a^2+39b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{8ab(3a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(5a^4+42a^2b^2+21b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

```
[Out] (8*a*b*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*(5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (36*a^3*b*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2)) + (2*a^2*(5*a^2 + 39*b^2)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c
 + d*x]^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+b\sec(c+dx))\left(9a^2b+\frac{1}{2}a(5a^2\right.}{}$$

$$= \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}a^2(5a^2+3}{}$$

$$= \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}a^2(5a^2+3}{}$$

$$= \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(5a^2+39b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{8ab(3a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{36a^3b\sin(c+}{35d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{8ab(3a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(5a^4+42a^2}{}$$

Mathematica [A]
time = 0.55, size = 142, normalized size = 0.67

$$\frac{\sqrt{\sec(c+dx)}\left(336ab(3a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(5a^4+42a^2b^2+21b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2(65a^2+420b^2+168ab\cos(c+dx)+15a^2\cos(2(c+dx)))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(336*a*b*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 20*(5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2] + a^2*(65*a^2 + 420*b^2 + 168*a*b*Cos[c + d*x] + 15*a^2
*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A]
time = 0.14, size = 476, normalized size = 2.26

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4+\left(-360a^4-672b\,a^3\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a^4+(-360*a^4-672*a^3*b)*sin(1/2*d*x+1/2*c
```

```
)^6*cos(1/2*d*x+1/2*c)+(280*a^4+672*a^3*b+840*a^2*b^2)*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+(-80*a^4-168*a^3*b-420*a^2*b^2)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+25*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+210*b^2*a^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+105*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
3*b-420*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 234, normalized size = 1.11

$$-\frac{5\sqrt{2}\left(5i\,a^4+42i\,a^2b^2+21i\,b^4\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-5i\,a^4-42i\,a^2b^2-21i\,b^4\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+84\sqrt{2}\left(-3i\,a^3b-5i\,ab^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+84\sqrt{2}\left(3i\,a^3b+5i\,ab^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(15\,a^4\cos\left(dx+c\right)^3+84\,a^3b\cos\left(dx+c\right)^2+5\left(5\,a^4+42\,a^2b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*a^4 + 42*I*a^2*b^2 + 21*I*b^4)*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^4 - 42*I*a^2*b^2 -
 21*I*b^4)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 84*s
qrt(2)*(-3*I*a^3*b - 5*I*a*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 84*sqrt(2)*(3*I*a^3*b + 5*I*a*b^3)
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) - 2*(15*a^4*cos(d*x + c)^3 + 84*a^3*b*cos(d*x + c)^2 + 5*(5*a^4 + 4
2*a^2*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\sec\left(c + dx\right)\right)^4}{\sec^{\frac{7}{2}}\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2), x)
```

3.606 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2(7a^4+54a^2b^2+15b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{8ab(5a^2+7b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{21d}$$

```
[Out] 44/63*a^3*b*sin(d*x+c)/d/sec(d*x+c)^(5/2)+14/45*a^2*(a^2+7*b^2)*sin(d*x+c)/
d/sec(d*x+c)^(3/2)+2/9*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)
+8/21*a*b*(5*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*(7*a^4+54*a^2*b^
2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/21*a*b*(5*a^2+7*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.27, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3926, 4159, 4132, 3854, 3856, 2720, 4130, 2719}

$$\frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{14a^2(a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8ab(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{8ab(5a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2(7a^4+54a^2b^2+15b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(7*a^4 + 54*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (44*a^3*b*Sin[c + d*x
])/(63*d*Sec[c + d*x]^(5/2)) + (14*a^2*(a^2 + 7*b^2)*Sin[c + d*x])/(45*d*Se
c[c + d*x]^(3/2)) + (8*a*b*(5*a^2 + 7*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/
2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\sec(c+dx))\left(11a^2b+\frac{1}{2}a(7a^2\right.}{\mathrm{s}}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{4}{63}\int \frac{-\frac{49}{4}a^2(a^2+7b^2}{}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{4}{63}\int \frac{-\frac{49}{4}a^2(a^2+7b^2}{}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{14a^2(a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{8ab(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} +$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{14a^2(a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{8ab(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} +$$

$$= \frac{2(7a^4+54a^2b^2+15b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{8ab(5a}{}$$

Mathematica [A]
time = 0.99, size = 168, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(168(7a^4+54a^2b^2+15b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+480ab(5a^2+7b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a(7a(43a^2+216b^2)\cos(c+dx)+5(312a^2b+336b^3+72a^2b\cos(2(c+dx))+7a^3\cos(3(c+dx))))\sin(2(c+dx))\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(7*a^4 + 54*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2] + 480*a*b*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2] + a*(7*a*(43*a^2 + 216*b^2)*Cos[c + d*x] + 5*(312*a^2*
b + 336*b^3 + 72*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[2*(c
 + d*x)]))/(1260*d)
```

Maple [A]
time = 0.14, size = 529, normalized size = 2.16

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4+\left(2240a^4+2880b\,a^3\right)\left(\sin^8\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*a^4+(2240*a^4+2880*a^3*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^4-4320*a^3*b-3024*a^2*b^2)*sin(1/2*d*x+
1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^4+3360*a^3*b+3024*a^2*b^2+1680*a*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^4-960*a^3*b-756*a^2*b^2-840*a
*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+300*b*a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+420*b^3*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1
134*b^2*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-315*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 257, normalized size = 1.05

$$-\frac{60\sqrt{2}\,(5i\,a^3b+7i\,ab^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+60\sqrt{2}\,(-5i\,a^3b-7i\,ab^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+21\sqrt{2}\,(-7i\,a^4-54i\,a^2b^2-15i\,b^4)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+21\sqrt{2}\,(7i\,a^4+54i\,a^2b^2+15i\,b^4)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(35\,a^4\cos(dx+c)^4+180\,a^3b\cos(dx+c)^3+7\,(7\,a^4+54\,a^2b^2)\cos(dx+c)^2+60\,(5\,a^3b+7\,ab^3)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/315*(60*sqrt(2)*(5*I*a^3*b + 7*I*a*b^3)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 60*sqrt(2)*(-5*I*a^3*b - 7*I*a*b^3)*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-7*I*a^4 - 54*
I*a^2*b^2 - 15*I*b^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(7*I*a^4 + 54*I*a^2*b^2 + 15*I*b^
4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))) - 2*(35*a^4*cos(d*x + c)^4 + 180*a^3*b*cos(d*x + c)^3 + 7*(7*a^4
+ 54*a^2*b^2)*cos(d*x + c)^2 + 60*(5*a^3*b + 7*a*b^3)*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2), x)
```

3.607 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=289

$$\frac{8ab(7a^2+9b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2(45a^4+330a^2b^2+77b^4)\sqrt{\cos(c+dx)}\,F(}{231d}$$

```
[Out] 52/99*a^3*b*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/77*a^2*(9*a^2+59*b^2)*sin(d*x+c
)/d/sec(d*x+c)^(5/2)+8/45*a*b*(7*a^2+9*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2
/11*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(9/2)+2/231*(45*a^4+330*
a^2*b^2+77*b^4)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+8/15*a*b*(7*a^2+9*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/231*(45*a^4+330*a^2*b^2+77*b^4)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.30, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3926, 4159, 4132, 3854, 3856, 2719, 4130, 2720}

$$\frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a^2(9a^2+59b^2)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8ab(7a^2+9b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8ab(7a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{2(45a^4+330a^2b^2+77b^4)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2(45a^4+330a^2b^2+77b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] (8*a*b*(7*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(15*d) + (2*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (52*a^3*b*Sin[c +
d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*a^2*(9*a^2 + 59*b^2)*Sin[c + d*x])/(77
*d*Sec[c + d*x]^(5/2)) + (8*a*b*(7*a^2 + 9*b^2)*Sin[c + d*x])/(45*d*Sec[c +
 d*x]^(3/2)) + (2*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sin[c + d*x])/(231*d*Sqrt
[Sec[c + d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c +
 d*x]^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+b\sec(c+dx))\left(13a^2b+\frac{3}{2}a(3a\right.}{}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{4}{99}\int \frac{-\frac{9}{4}a^2(9a^2+5}{}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{4}{99}\int \frac{-\frac{9}{4}a^2(9a^2+5}{}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2(9a^2+59b^2)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{8ab(7a^2+9b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2(9a^2+59b^2)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{8ab(7a^2+9b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{8ab(7a^2+9b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{52a^3b\sin(c+}{99d\sec^{\frac{7}{2}}(c+}$$

$$= \frac{8ab(7a^2+9b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2(45a^4+330}{}$$

Mathematica [A]
time = 1.23, size = 199, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(14784ab(7a^2+9b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+240(45a^4+330a^2b^2+77b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(616ab(43a^2+36b^2)\cos(c+dx)+5(1593a^4+10296a^2b^2+1848b^4+72(8a^4+33a^2b^2)\cos(2(c+dx))+616a^3b\cos(3(c+dx))+63a^4\cos(4(c+dx))))\sin(2(c+dx))\right)}{27720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(14784*a*b*(7*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2] + 240*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2] + (616*a*b*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*(159
3*a^4 + 10296*a^2*b^2 + 1848*b^4 + 72*(8*a^4 + 33*a^2*b^2)*Cos[2*(c + d*x)]
 + 616*a^3*b*Cos[3*(c + d*x)] + 63*a^4*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)])
)/(27720*d)
```

Maple [A]
time = 0.14, size = 586, normalized size = 2.03

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(20160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4+\left(-50400a^4-49280b\,a^3\right)\left(\sin^{1}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12*a^4+(-50400*a^4-49280*a^3*b)*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*a^4+98560*a^3*b+47520*a^2*b^2)*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-34920*a^4-91168*a^3*b-71280*a^2*b^2-22
176*a*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*a^4+41888*a^3*b+5
5440*a^2*b^2+22176*a*b^3+4620*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-2790*a^4-7392*a^3*b-15840*a^2*b^2-5544*a*b^3-2310*b^4)*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+675*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4950*b^2*a^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1155*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6468*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^3*b-8316*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.90, size = 286, normalized size = 0.99

$$-\frac{15\sqrt{2}\left(45i\,a^4+330i\,a^2b^2+77i\,b^4\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+15\sqrt{2}\left(-45i\,a^4-330i\,a^2b^2-77i\,b^4\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+924\sqrt{2}\left(-7i\,a^3b-9i\,ab^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+924\sqrt{2}\left(7i\,a^3b+9i\,ab^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(315\,a^4\cos\left(dx+c\right)^5+1540\,a^3b\cos\left(dx+c\right)^4+135\left(3\,a^4+22\,a^2b^2\right)\cos\left(dx+c\right)^3+308\left(7\,a^3b+9\,ab^3\right)\cos\left(dx+c\right)^2+15\left(45\,a^4+330\,a^2b^2+77\,b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/3465*(15*sqrt(2)*(45*I*a^4 + 330*I*a^2*b^2 + 77*I*b^4)*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-45*I*a^4 - 330*I*a^
2*b^2 - 77*I*b^4)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 924*sqrt(2)*(-7*I*a^3*b - 9*I*a*b^3)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 924*sqrt(2)*(7*I*a^3*b + 9
*I*a*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(315*a^4*cos(d*x + c)^5 + 1540*a^3*b*cos(d*x + c)^4 +
135*(3*a^4 + 22*a^2*b^2)*cos(d*x + c)^3 + 308*(7*a^3*b + 9*a*b^3)*cos(d*x +
 c)^2 + 15*(45*a^4 + 330*a^2*b^2 + 77*b^4)*cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2), x)
```

3.608 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=188

$$\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3bd}+\frac{2a^2\sqrt{\mathrm{co}}}{}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/b/d-2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/d+2
*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d+2/3*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

Rubi [A]
time = 0.35, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3936, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*
d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*b*d) + (2*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d) - (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(b^2*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}b\sec(c+dx)-\frac{3}{2}a\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{3b}$$

$$= -\frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{4\int \frac{\frac{3a^2}{4}+ab\sec(c+dx)+}{\sqrt{\sec(c+dx}}}{3}$$

$$= -\frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{4\int \frac{\frac{3a^3}{4}+\frac{1}{4}a^2b\sec(c+dx}{\sqrt{\sec(c+dx)}}}{3a^2b^2}$$

$$= -\frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}}{b^2}$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d} - \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+}{b^2d}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{3bd}$$

Mathematica [A]
time = 31.89, size = 165, normalized size = 0.88

$$-\frac{\cot(c+dx)\left(-b^2\sec^{\frac{5}{2}}(c+dx)+b^2\cos(2(c+dx))\sec^{\frac{5}{2}}(c+dx)+6abE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-2(3a^2+3ab+b^2)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+6a^2\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/3*(Cot[c + d*x]*(-(b^2*Sec[c + d*x]^(5/2)) + b^2*Cos[2*(c + d*x)]*Sec[c
+ d*x]^(5/2) + 6*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c
+ d*x]^2] - 2*(3*a^2 + 3*a*b + b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[-Tan[c + d*x]^2] + 6*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(b^3*d)
```

Maple [A]
time = 0.25, size = 423, normalized size = 2.25

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.}}{3\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^3/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*a/b^2/si
n(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x)),x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x)), x)
```

3.609 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd}-\frac{2a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a+b)d}+$$

```
[Out] 2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/d-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a+
b)/d
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3935, 3853, 3856, 2719, 3934, 2884}

$$-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
- (2*a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(b*(a + b)*d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3935

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d/b, Int[(d*Csc[e + f*x])^(3/2), x], x] - Dist[a*(
d/b), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\int \sec^{\frac{3}{2}}(c+dx)\,dx}{b} - \frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b} - \frac{\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a+b)d} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+}{bd}$$

$$= -\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c-\right.}{b(a+b}$$

Mathematica [A]
time = 33.05, size = 83, normalized size = 0.71

$$\frac{2\cot(c+dx)\left(bE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-(a+b)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+a\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)\sqrt{-\tan^2(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]

[Out] (2*Cot[c + d*x]*(b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] - (a + b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b^2*d)

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(159) = 318$.
time = 0.14, size = 353, normalized size = 3.02

method	result
default	$-\frac{2\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\,(\sin}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
[Out] Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x)), x)
```

3.610 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a+b)d}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a+b)/d
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3934, 2884}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/((a + b)*d)
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx\\ &= \frac{2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a+b)d}\end{aligned}$$

Mathematica [A]
time = 10.23, size = 63, normalized size = 1.29

$$\frac{2\cot(c+dx)\left(F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)\sqrt{-\tan^2(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b
/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(71) = 142.
time = 0.10, size = 150, normalized size = 3.06

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{EllipticPi}\left(\cos\right.}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x)), x)
```

3.611 $\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{2b\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a+b)d}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*b*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a+b)/d
```

Rubi [A]
time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3933, 2882, 2720, 2884}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a*(a + b)*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3933

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d), Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx$$
$$= \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\right.}{a(a+b)}$$

Mathematica [A]
time = 10.17, size = 47, normalized size = 0.51

$$\frac{2\cot(c+dx)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Ta
n[c + d*x]^2])/(a*d)
```

Maple [A]
time = 0.12, size = 187, normalized size = 2.01

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(\text{EllipticF}\right.}{a(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c + dx\right)}}{a + b\sec\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x)), x)
```

3.612 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=135

$$\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2b^2\sqrt{\mathrm{co}}}{}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*b*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a^2/(a+b)/d
```

Rubi [A]
time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3937, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3937

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[b^2/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b
*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx = \frac{\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2}$$

$$= \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{b\int \sqrt{\sec(c+dx)}\,dx}{a^2} + \frac{\Big(b^2\sqrt{\cos(c+dx}}{}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\Pi\big(\frac{2a}{a+b};\frac{1}{2}(c+dx)\big|\,2\big)\sqrt{\sec(c+dx)}}{a^2(a+b)d} + \frac{\Big(\sqrt{\cos}}{}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,E\big(\frac{1}{2}(c+dx)\big|\,2\big)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+}}{}$$

Mathematica [A]

time = 24.01, size = 176, normalized size = 1.30

$$\frac{\cot(c+dx)\left(-a\sec^{\frac{3}{2}}(c+dx)-a\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+a\sec^{\frac{7}{2}}(c+dx)+a\cos(2(c+dx))\sec^{\frac{7}{2}}(c+dx)-2aE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+2aF\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-2b\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (Cot[c + d*x]*(-(a*Sec[c + d*x]^(3/2)) - a*Cos[2*(c + d*x)]*Sec[c + d*x]^(3
/2) + a*Sec[c + d*x]^(7/2) + a*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*a*El
lipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b*EllipticPi[-
(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*d)
```

Maple [A]

time = 0.16, size = 226, normalized size = 1.67

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(\text{EllipticF}\left(\cos\right.\right.}{a^2(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec{(c+dx)})\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

3.613 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=172

$$-\frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{3a^3d}$$

```
[Out] 2/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^2/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^3/d-2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(
a+b)/d
```

Rubi [A]
time = 0.24, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3938, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}+\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a^3*d) - (2*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a + b)*d) + (2*Sin[c + d*x])/(3
*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3b}{2}+\frac{1}{2}a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3ab}{2}-\left(-\frac{a^2}{2}-\frac{3b^2}{2}\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^3} - \frac{b^3\int \frac{\sec^{\frac{3}{2}}(c+dx}{a+b\sec(c+}}{a^3}$$

$$= \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{(a^2+3b^2)\int \sqrt{\sec(c-}}{3a^3}$$

$$= -\frac{2b^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a+b)d} + \frac{2\sin(}{3ad\sqrt{\sec}}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2(a^2+3b^2)\sqrt{}}{}$$

Mathematica [A]
time = 34.19, size = 194, normalized size = 1.13

$$-\frac{\cot(c+dx)\left(-a^2\sqrt{\sec(c+dx)}+6ab\sec^{\frac{3}{2}}(c+dx)-6ab\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+a^2\cos(3(c+dx))\sec^{\frac{3}{2}}(c+dx)-12abE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-4a(a-3b)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-12b^2\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -1/6*(Cot[c + d*x]*(-(a^2*Sqrt[Sec[c + d*x]]) + 6*a*b*Sec[c + d*x]^(3/2) -
6*a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + a^2*Cos[3*(c + d*x)]*Sec[c + d*
x]^(3/2) - 12*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d
*x]^2] - 4*a*(a - 3*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
c + d*x]^2] - 12*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[-Tan[c + d*x]^2]))/(a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(232) = 464$.
time = 0.15, size = 552, normalized size = 3.21

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b+3*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3+3*b*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2+3*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec{(c+dx)})\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

3.614 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=342

$$\frac{a(5a^2-4b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a^2-b^2)\,d}+\frac{(5a^2-2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3b^2(a^2-b^2)\,d}$$

```
[Out] 1/3*(5*a^2-2*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)/d-a^2*sec(d*x+c
)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))-a*(5*a^2-4*b^2)*sin(d*x+c
)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+a*(5*a^2-4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+1/3*(5*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d+a^2*(5*a^2-7*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^3/(a+b)^2/d
```

Rubi [A]
time = 0.63, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3930, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}+\frac{(5a^2-2b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d(a^2-b^2)}+\frac{(5a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d(a^2-b^2)}-\frac{a(5a^2-4b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^3d(a^2-b^2)}+\frac{a(5a^2-4b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a^2-b^2)}+\frac{a^2(5a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*(5*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(b^3*(a^2 - b^2)*d) + ((5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d) + (a^2*(5*a^2 -
 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/((a - b)*b^3*(a + b)^2*d) - (a*(5*a^2 - 4*b^2)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((5*a^2 - 2*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
```

```
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a^2}{2}-ab\sec(c+dx)-\frac{1}{2}\left(5a^2-2b^2\right)\sec^2(c+\cdots}{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{2\int \cdots}{\cdots}$$

$$= -\frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+\cdots}{3b^2\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+\cdots}{3b^2\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+\cdots}{3b^2\left(a^2-b^2\right)d}$$

$$= \frac{a^2\left(5a^2-7b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} - \frac{a\left(5a^2-4\cdots\right.}{\cdots}$$

$$= \frac{a\left(5a^2-4b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sqrt{c\cdots}}{\cdots}$$

Mathematica [A]

time = 35.04, size = 294, normalized size = 0.86

$$\frac{\frac{2b\left(-\frac{3a^2\left(5a^2-4b^2\right)\sin(c+dx)}{a^2-b^2}+2b(-5a+b\sec(c+dx))\tan(c+dx)\right)}{(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}+\frac{\cot(c+dx)\left(-6ab\left(5a^2-4b^2\right)E\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+2\left(15a^4+15a^3b-16a^2b^2-12ab^3-2b^4\right)F\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-6a\left(b\left(-5a^2+4b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin^2(c+dx)+a\left(5a^2-7b^2\right)\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)\right)}{(a-b)(a+b)}}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*b*((-3*a^2*(5*a^2 - 4*b^2)*Sin[c + d*x])/(a^2 - b^2) + 2*b*(-5*a + b*Se
c[c + d*x])*Tan[c + d*x]))/((b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot
[c + d*x]*(-6*a*b*(5*a^2 - 4*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[-Tan[c + d*x]^2] + 2*(15*a^4 + 15*a^3*b - 16*a^2*b^2 - 12*a*b^3 - 2*b
^4)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*a*(
b*(-5*a^2 + 4*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 + a*(5*a^2 - 7*b^2)*El
lipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2])))/(
(a - b)*(a + b)))/(6*b^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(398) = 796$.
time = 0.42, size = 975, normalized size = 2.85

method	result	size
default	Expression too large to display	975

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b^2*(1/b*
a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2
*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2
)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))-4*a^3/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-4/b^3*a/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2, x)
```

3.615 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=279

$$-\frac{(3a^2-2b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)d}-\frac{a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)d}$$

```
[Out] -a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))+(3*a^2-2*b^
2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-(3*a^2-2*b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d-a*(3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^2/(a+b)^2/d
```

Rubi [A]
time = 0.44, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3930, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}+\frac{(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d(a^2-b^2)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd(a^2-b^2)}-\frac{(3a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a^2-b^2)}-\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(b^2*(a^2 - b^2)*d)) - (a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) - (a*(3*a^2 - 5*b^2)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a -
 b)*b^2*(a + b)^2*d) + ((3*a^2 - 2*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b
^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*
(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
```

```
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-ab\sec(c+dx)-\frac{1}{2}\left(3a^2-2b^2\right)\sec}{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{2\int}{}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{2\int}{}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{a\int}{}$$

$$= -\frac{a\left(3a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} + \frac{\left(3a^2-2b^2\right.}{}$$

$$= -\frac{\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} - \frac{a\sqrt{\cos(c+dx)}}{}$$

Mathematica [A]
time = 33.78, size = 351, normalized size = 1.26

$$\frac{2b\left(-3a^3+2ab^2+2b\left(-a^2+b^2\right)\sec(c+dx)\right)\sin(c+dx)}{\left(-a^2+b^2\right)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx)\left(-3a^2b\sec^{\frac{3}{2}}(c+dx)+2b^3\sec^{\frac{3}{2}}(c+dx)+3a^2b\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-2b^3\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+2b\left(3a^2-2b^2\right)E\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-2\left(3a^3+3a^2b-4ab^2-2b^3\right)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+6a^3\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-10ab^2\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{(a-b)(a+b)}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*b*(-3*a^3 + 2*a*b^2 + 2*b*(-a^2 + b^2)*Sec[c + d*x])*Sin[c + d*x])/((-a
^2 + b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-3*a^2*
```

```
b*Sec[c + d*x]^(3/2) + 2*b^3*Sec[c + d*x]^(3/2) + 3*a^2*b*Cos[2*(c + d*x)]*
Sec[c + d*x]^(3/2) - 2*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*b*(3*a^2
 - 2*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] -
 2*(3*a^3 + 3*a^2*b - 4*a*b^2 - 2*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^3*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a*b^2*EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b)))/(2*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(341) = 682$.
time = 0.28, size = 841, normalized size = 3.01

method	result	size
default	Expression too large to display	841

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a/b*(1/b*a^2
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/
(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2/
b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2, x)
```

3.616 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=214

$$\frac{a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d}+\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{\left(a^2-b^2\right)d}+\frac{(a^2-3b^2)}{}$$

```
[Out] -a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))+a*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d+(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b/(a+b)^2/d
```

Rubi [A]
time = 0.27, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3930, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2
 - b^2)*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/((a^2 - b^2)*d) + ((a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b*(a + b)^2*d) - (a^2*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{a^2}{2}-ab\sec(c+dx)-\frac{1}{2}(a^2-2b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx}{b\,(a^2-b^2)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{a^3}{2}-\frac{1}{2}a^2b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2b\,(a^2-b^2)} + \frac{(a^2-3b^2)\int \frac{\sec^{\frac{3}{2}}}{a+b\,\mathrm{se}}}{2b\,(a^2-b}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2\,(a^2-b^2)} + \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}}{2b\,(a^2-b^2)}$$

$$= \frac{(a^2-3b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d} - \frac{a^2\sqrt{\sec(c+}}{b\,(a^2-b^2)\,d(}$$

$$= \frac{a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{b\,(a^2-b^2)\,d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx}{(a^2-b^2)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 582 vs. 2(214) = 428.
time = 36.46, size = 582, normalized size = 2.72

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*((a*Sin[c + d*x])/(b*(-a^2 + b^2)) + (a*Sin[c + d*x])/(
(a^2 - b^2)*(b + a*Cos[c + d*x]))))/d + ((2*(3*a^2 - 4*b^2)*Cos[c + d*x]^2*
(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c +
 d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*b*Cos[c + d*x]^2*
EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqr
t[1 - Sec[c + d*x]^2]*Sin[c + d*x])/((b + a*Cos[c + d*x])*(1 - Cos[c + d*x]
^2)) + (Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^
2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a),
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
 - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*
x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Co
s[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*b*(a + b
)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(278) = 556$.
time = 0.16, size = 608, normalized size = 2.84

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.}}{b\left(a^2-b^2\right)\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-a+b\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*a^2/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/b*a/(a^2-b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/b*a/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/(a^2-a*b
)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2, x)
```

3.617 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$-\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a^2-b^2)\,d}-\frac{b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\,(a^2-b^2)\,d}+\frac{(a^2+b^2)}{}$$

```
[Out] a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))-(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a/d/(a^2-b^2)+(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a/(a-b)/(a+b)^2/d
```

Rubi [A]
time = 0.24, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3929, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\,(a^2-b^2)}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\,(a^2-b^2)}+\frac{(a^2+b^2)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a^2 -
 b^2)*d)) - (b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*(a^2 - b^2)*d) + ((a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*(a + b)^2*d) + (a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3929

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{a}{2}-b\sec(c+dx)+\frac{1}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx}{-a^2+b^2}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2\,(a^2-b^2)} + \frac{(a^2+b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}}{2a\,(a^2-b^2)}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2\,(a^2-b^2)} - \frac{b\int \sqrt{\sec(c+dx)}\,dx}{2a\,(a^2-b^2)} +$$

$$= \frac{(a^2+b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} + \frac{a\sqrt{\sec(c+dx)}}{(a^2-b^2)\,d(a+}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{(a^2-b^2)\,d} - \frac{b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{a\,(a^2-b^2)}$$

Mathematica [A]
time = 35.60, size = 289, normalized size = 1.39

$$\frac{\frac{4a\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{2\cot(c+dx)\left(ab\sec^{\frac{3}{2}}(c+dx)+ab\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-ab\sec^{\frac{7}{2}}(c+dx)-ab\cos(2(c+dx))\sec^{\frac{7}{2}}(c+dx)+2abE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+2a(a-b)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-2a^2\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-2b^2\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{a(a-b)b(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*a*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) +
 (2*Cot[c + d*x]*(a*b*Sec[c + d*x]^(3/2) + a*b*Cos[2*(c + d*x)]*Sec[c + d*x
]^(3/2) - a*b*Sec[c + d*x]^(7/2) - a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2)
+ 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2
*a*(a - b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
- 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*
x]^2] - 2*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
c + d*x]^2]))/(a*(a - b)*b*(a + b)))/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(272) = 544.
time = 0.22, size = 707, normalized size = 3.40

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2b\left(\frac{a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}{b\left(a^2-b^2\right)\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-a+b\right)}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/a*(1/b*a^2
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/
(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2, x)
```

3.618 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=227

$$\frac{b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d}+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d}$$

```
[Out] -b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))+b*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d/(a^2-b^2)+(2*a^2-b^2)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)/(a+b)^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3928, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2
 - b^2)*d) + ((2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - (b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)
*(a + b)^2*d) - (b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*S
ec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3928

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)
*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Si
mp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
 && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{b}{2}-a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx}{-a^2+b^2}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{ab}{2}-\left(a^2-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2\,(a^2-b^2)} - \frac{\left(b\left(3-\frac{b^2}{a^2}\right)\right)\int}{2\,(a^2}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a\,(a^2-b^2)} + \frac{(2a^2-b^2)\int \sqrt{\sec(}}{2a^2\,(a^2-}$$

$$= -\frac{b(3a^2-b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} - \frac{b\sqrt{\sec(c}}{(a^2-b^2)\,d}$$

$$= \frac{b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\,(a^2-b^2)\,d} + \frac{(2a^2-b^2)\,\sqrt{\cos(c+dx)}\,F}{a^2\,(a^2}$$

Mathematica [A]
time = 32.84, size = 251, normalized size = 1.11

$$\frac{\cos(2(c+dx))\csc(c+dx)\sqrt{\sec(c+dx)}\left(a(a-b)(b+a\cos(c+dx))F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}-(3a^2-b^2)(b+a\cos(c+dx))\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}+ab\left(-b\tan^2(c+dx)+(b+a\cos(c+dx))E\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}\right)\right)}{a^2(a-b)(a+b)d(b+a\cos(c+dx))(-2+\sec^2(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Sec[c + d*x]]*(a*(a - b)*(b + a*Cos[c +
 d*x])*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-T
an[c + d*x]^2] - (3*a^2 - b^2)*(b + a*Cos[c + d*x])*EllipticPi[-(b/a), ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + a*b*
(-(b*Tan[c + d*x]^2) + (b + a*Cos[c + d*x])*EllipticE[ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/(a^2*(a - b)*(a + b)
*d*(b + a*Cos[c + d*x])*(-2 + Sec[c + d*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. 2(291) = 582.
time = 0.25, size = 788, normalized size = 3.47

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{Ellipt}}{a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a^2
*b^2*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+4*b/a/(a^2-a*b)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec (c+dx)}}{(a+b\sec (c+dx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos (c+d\,x)}}}{\left(a+\frac{b}{\cos (c+d\,x)}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2, x)
```

3.619 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=244

$$\frac{(2a^2-3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a^2-b^2)\,d}-\frac{b(4a^2-3b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{a^3(a^2-b^2)\,d}$$

```
[Out] b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))+(2*a^2-3*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-b*(4*a^2-3*b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+b^2*(5*a^2
-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*
d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)/(
a+b)^2/d
```

Rubi [A]
time = 0.32, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3932, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{(2a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a^2-b^2)}-\frac{b(4a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a^2-b^2)}+\frac{b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^2*(a^2 - b^2)*d) - (b*(4*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + (b^2*(5*a^2 - 3
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{-a^2+\frac{3b^2}{2}+ab\sec(c+dx)-\frac{1}{2}b^2\sec^2(c+\cdots}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}}{a\,(a^2-b^2)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{a\left(-a^2+\frac{3b^2}{2}\right)-\left(-a^2b+b\left(-a^2+\frac{3b^2}{2}\right)\right)\cdots}{\sqrt{\sec(c+dx)}}}{a^3\,(a^2-b^2)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{(2a^2-3b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}\,d\cdots}{2a^2\,(a^2-b^2)}$$

$$= \frac{b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}\cdots}{a^3(a-b)(a+b)^2d}$$

$$= \frac{(2a^2-3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\,(a^2-b^2)\,d} - \frac{b(4\cdots}{\cdots}$$

Mathematica [A]
time = 34.42, size = 319, normalized size = 1.31

$$\frac{\frac{4b^2\sin(c+dx)}{a(a^2-b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}+\frac{2\cot(c+dx)\left(2a^3\sec^{\frac{3}{2}}(c+dx)-3ab^2\sec^{\frac{3}{2}}(c+dx)-2a^3\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+3ab^2\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-2a(2a^2-3b^2)E\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+2a(2a^2+ab-3b^2)F\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-10a^2b\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+6b^3\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{a^3(a-b)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]
]) + (2*Cot[c + d*x]*(2*a^3*Sec[c + d*x]^(3/2) - 3*a*b^2*Sec[c + d*x]^(3/2)
 - 2*a^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 3*a*b^2*Cos[2*(c + d*x)]*Sec
[c + d*x]^(3/2) - 2*a*(2*a^2 - 3*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*(2*a^2 + a*b - 3*b^2)*EllipticF[ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a^2*b*EllipticPi[-(b/a), A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*b^3*EllipticPi[-(b
/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*(a - b)*(
a + b)))/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(308) = 616$.
time = 0.26, size = 809, normalized size = 3.32

method	result	size
default	Expression too large to display	809

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/a^3*b^3*(1/b*a^2/(a^2-b^2)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a
*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2)))-6*b^2/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec{(c+dx)})^2\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos{(c+d\,x)}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)
```

3.620 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=304

$$-\frac{b(4a^2-5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3\left(a^2-b^2\right)d}+\frac{(2a^4+16a^2b^2-15b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{3a^4\left(a^2-b^2\right)d}$$

```
[Out] 1/3*(2*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)+b^2*sin(d*x+c
)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2)-b*(4*a^2-5*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*a^4+16*a^2*b^2-1
5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d-b^3*(7*
a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a-b
)/(a+b)^2/d
```

Rubi [A]
time = 0.45, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3932, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}+\frac{(2a^2-5b^2)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{(2a^4+16a^2b^2-15b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d\left(a^2-b^2\right)}-\frac{b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a-b)(a+b)^2}-\frac{b(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -((b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a^3*(a^2 - b^2)*d)) + ((2*a^4 + 16*a^2*b^2 - 15*b^4)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d
) - (b^3*(7*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a - b)*(a + b)^2*d) + ((2*a^2 - 5*b^2)
*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (b^2*Sin[c + d*x]
)/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))} - \frac{\int \frac{-a^2+\frac{5b^2}{2}+ab\sec(c \cdots}{\sec^{\frac{3}{2}}(c+dx)(\cdots}}{a\,(a \cdots}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b \cdots}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b \cdots}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b \cdots}$$

$$= -\frac{b^3\left(7a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)} \cdots}{a^4(a-b)(a+b)^2d}$$

$$= -\frac{b\left(4a^2-5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3\left(a^2-b^2\right)d} + \cdots$$

Mathematica [**A**]
time = 34.75, size = 279, normalized size = 0.92

$$\frac{\frac{a^2\left(2a^2b-5b^3+2a\left(a^2-b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx)\left(3ab\left(4a^2-5b^2\right)E\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+a\left(2a^3-12a^2b-5ab^2+15b^3\right)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-3b\left(a\left(4a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin^2(c+dx)+b\left(-7a^2+5b^2\right)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)\right)}{(a-b)(a+b)}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((a^2*(2*a^2*b - 5*b^3 + 2*a*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2
- b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(3*a*b*(4*a
^2 - 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
```

```
 + a*(2*a^3 - 12*a^2*b - 5*a*b^2 + 15*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 3*b*(a*(4*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)
*Sin[c + d*x]^2 + b*(-7*a^2 + 5*b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[-Tan[c + d*x]^2])))/((a - b)*(a + b)))/(3*a^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(362) = 724$.
time = 0.28, size = 1064, normalized size = 3.50

method	result	size
default	Expression too large to display	1064

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^2*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4/a^3*(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*(a^2+2*a*b+3*b^2)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a^4*b^4*(1/b*a^2/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2
-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2)))+8*b^3/a^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})^2 \sec^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
[Out] Integral(1/((a + b*sec(c + d*x))**2*sec(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{b}{\cos(c+d\,x)}\right)^2 \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)
```

3.621 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=388

$$-\frac{(15a^4-29a^2b^2+8b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^3(a^2-b^2)^2d}-\frac{a(5a^2-11b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{4b^2(a^2-b^2)^2}$$

```
[Out] -1/2*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/4*a
^2*(5*a^2-11*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*
x+c))+1/4*(15*a^4-29*a^2*b^2+8*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^3/(a^2-b^
2)^2/d-1/4*(15*a^4-29*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/b^3/(a^2-b^2)^2/d-1/4*a*(5*a^2-11*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d-1/4*a*(15*a^4-38*a^2*b^2+35*b^4)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(
a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^3/(a+b)^3/d
```

Rubi [A]
time = 0.65, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3930, 4183, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{a^2(5a^2-11b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4b^2d(a^2-b^2)^2(a+b\sec(c+dx))}-\frac{a(5a^2-11b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a^2-b^2)^2}+\frac{(15a^4-29a^2b^2+8b^4)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^3d(a^2-b^2)^2}-\frac{(15a^4-29a^2b^2+8b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a^2-b^2)^2}-\frac{a(15a^4-38a^2b^2+35b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)^2*d) - (a*(5*a^2 - 11*b^2)*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b
^2)^2*d) - (a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^3*(a + b)
^3*d) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*
b^3*(a^2 - b^2)^2*d) - (a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^
2)*d*(a + b*Sec[c + d*x])^2) - (a^2*(5*a^2 - 11*b^2)*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
```

```
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a^2}{2}-2ab\sec(c+dx)-\frac{1}{2}\left(5a^2-4b^2\right)\sec^2}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(5a^2-11b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+}{2b\left(a^2-b^2\right)d(a+b\sec(c-}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+}{2b\left(a^2-b^2\right)d(a+b\sec(c-}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+}{2b\left(a^2-b^2\right)d(a+b\sec(c-}$$

$$= -\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^3(a+b)^3d}$$

$$= -\frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a(5a}{}$$

Mathematica [A]
time = 36.39, size = 532, normalized size = 1.37

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*b*(15*a^6 - 13*a^4*b^2 - 24*a^2*b^4 + 16*b^6 + (50*a^5*b - 94*a^3*b^3 +
 32*a*b^5)*Cos[c + d*x] + (15*a^6 - 29*a^4*b^2 + 8*a^2*b^4)*Cos[2*(c + d*x)
])*Tan[c + d*x])/(a^2 - b^2)^2 - (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c
 + d*x]*(a + b*Sec[c + d*x])*(-15*a^4*b + 29*a^2*b^3 - 8*b^5 + 15*a^4*b*Sec
[c + d*x]^2 - 29*a^2*b^3*Sec[c + d*x]^2 + 8*b^5*Sec[c + d*x]^2 - b*(15*a^4
- 29*a^2*b^2 + 8*b^4)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] + (15*a^5 + 15*a^4*b - 33*a^3*b^2 - 29*a^2*b^
3 + 24*a*b^4 + 8*b^5)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] - 15*a^5*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c
 + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + 38*a^3*b^2*Ellipt
icPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c
```

```
 + d*x]^2] - 35*a*b^4*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b^4*d*(
b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1986 vs. 2(436) = 872.
time = 0.51, size = 1987, normalized size = 5.12

method	result	size
default	Expression too large to display	1987

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a/b^2*(1/b*a
^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*
b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*a/b*(1/2/b*a^2/(a^2-b^2)*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/
(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a
^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
```

```
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/
(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),
2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2/b
^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2))+2/b^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c
)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^3, x)
```

3.622 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=315

$$\frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2 d}+\frac{(a^2-7b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}$$

```
[Out] -1/2*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-3/4*a
^2*(a^2-3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))+3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-
b^2)^2/d+1/4*(a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^
2-b^2)^2/d+3/4*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/(a-b)^2/b^2/(a+b)^3/d
```

Rubi [A]

time = 0.48, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3930, 4183, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b*(a^2 - b^2)^2*d) + (3*(a^4 - 2
*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^2*(a + b)^3*d) - (a^2*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (3*a^2*(a^2
 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Se
c[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
```

```
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-2ab\sec(c+dx)-\frac{1}{2}(3a^2-4b}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3a^2(a^2-3b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

$$= -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3a^2(a^2-3b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

$$= -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3a^2(a^2-3b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

$$= \frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^2(a+b)^3d} - \frac{a}{2b\,(}$$

$$= \frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2d} + \frac{(a^2-7b^2)\sqrt{\cos}}{}$$

Mathematica [A]
time = 35.13, size = 335, normalized size = 1.06

$$\frac{\frac{4a^2b\left(-5a^2b+11b^3-3a\left(a^2-3b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2}+\frac{4\cos(c+dx)(b+a\cos(c+dx))\cot(c+dx)(a+b\sec(c+dx))\left(3ab\left(a^2-3b^2\right)\tan^2(c+dx)-3ab\left(a^2-3b^2\right)E\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}+\left(3a^4+3a^3b-5a^2b^2-9ab^3+8b^4\right)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}-3\left(a^4-2a^2b^2+5b^4\right)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}\right)}{(a-b)^2(a+b)^2}}{16b^3d(b+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*a^2*b*(-5*a^2*b + 11*b^3 - 3*a*(a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x]
)/(a^2 - b^2)^2 + (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c + d*x]*(a + b*
Sec[c + d*x])*(3*a*b*(a^2 - 3*b^2)*Tan[c + d*x]^2 - 3*a*b*(a^2 - 3*b^2)*Ell
ipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x
]^2] + (3*a^4 + 3*a^3*b - 5*a^2*b^2 - 9*a*b^3 + 8*b^4)*EllipticF[ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*(a^4 - 2
*a^2*b^2 + 5*b^4)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b^3*d*(b +
a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(367) = 734$.
time = 0.25, size = 1203, normalized size = 3.82

method	result	size
default	Expression too large to display	1203

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/b*a^2/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2*a-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b^2)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*a/(a^2-b^2)^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/4/(a-b)/(a+b)/(a^
2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)
*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
```

```
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))-15/4/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+dx)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^3, x)
```

3.623 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=313

$$\frac{(a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b\,(a^2-b^2)^2\,d}+\frac{3(a^2+b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a\,(a^2-b^2)^2\,d}$$

```
[Out] -1/2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*a
*(a^2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))+1
/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d
+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d
+1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a/(a-b)^2/b/(a+b)^3/d
```

Rubi [A]

time = 0.45, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3930, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\,(a^2-b^2)\,(a+b\sec(c+dx))^2}+\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\,(a^2-b^2)^2\,(a+b\sec(c+dx))}+\frac{3(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\,(a^2-b^2)^2}+\frac{(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\,(a^2-b^2)^2}+\frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4abd(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(4*b*(a^2 - b^2)^2*d) + (3*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^
2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*(a - b)^2*b*(a + b)^3*d) - (a^2*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2 - 7*b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
```

```
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{\int \frac{-\frac{a^2}{2}-2ab\sec(c+dx)-\frac{1}{2}(a^2-4b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx}{2b\,(a^2-b^2)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))} +$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))} +$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))} +$$

$$= \frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a(a-b)^2b(a+b)^3d} - \frac{a^2}{2b\,(a}$$

$$= \frac{(a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b\,(a^2-b^2)^2\,d} + 3(a^2+b^2)\sqrt{\cos(c}$$

Mathematica [A]
time = 35.44, size = 428, normalized size = 1.37

$$\frac{-\frac{4ab\left(-a^2b+7b^3+a(a^2+5b^2)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2}+\frac{4\cos(c+dx)(b+a\cos(c+dx))\cot(c+dx)(a+b\sec(c+dx))\left(\left(-a^3b-5ab^3+a^3b\sec^2(c+dx)+5ab^3\sec^2(c+dx)-ab(a^2+5b^2)\right)E\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}+a(a^3+a^2b-7ab^2+5b^3)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}-a^4\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}+10a^2b^2\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}+3b^4\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}\right)}{a(a-b)^2(a+b)^2}}{16b^2d(b+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-4*a*b*(-(a^2*b) + 7*b^3 + a*(a^2 + 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a
^2 - b^2)^2 + (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c + d*x]*(a + b*Sec[
c + d*x])*(-(a^3*b) - 5*a*b^3 + a^3*b*Sec[c + d*x]^2 + 5*a*b^3*Sec[c + d*x]
^2 - a*b*(a^2 + 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
 + d*x]]*Sqrt[-Tan[c + d*x]^2] + a*(a^3 + a^2*b - 7*a*b^2 + 5*b^3)*Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]
- a^4*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[-Tan[c + d*x]^2] + 10*a^2*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + 3*b^4*EllipticPi[-(b
/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^
2]))/(a*(a - b)^2*(a + b)^2))/(16*b^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[c +
 d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(365) = 730$.
time = 0.40, size = 1760, normalized size = 5.62

method	result	size
default	Expression too large to display	1760

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(1/b*a^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^
2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*b/a*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a
-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
```

```
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^
2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-
a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{2}}{\left(c + d x \right)}}{\left(a + b \sec{\left(c + d x \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^3, x)
```

3.624 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=306

$$-\frac{(5a^2+b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a\left(a^2-b^2\right)^2d}-\frac{b(7a^2-b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{4a^2\left(a^2-b^2\right)^2d}$$

```
[Out] 1/2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+3/4*(a^2+b
^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))-1/4*(5*a^2+b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d-1/4*b*(7*a
^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d+1/4*
(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/a^2/(a-b)^2/(a+b)^3/d
```

Rubi [A]
time = 0.41, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3929, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{b(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{(5a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{(3a^4+10a^2b^2-b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*((5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(a*(a^2 - b^2)^2*d) - (b*(7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4 +
10*a^2*b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*(a + b)^3*d) + (a*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (3*(a^2 + b^2)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3929

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
```

```
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{-\frac{a}{2}-2b\sec(c+dx)+\frac{3}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3(a^2+b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \cdots}{\cdots}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3(a^2+b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \cdots}{\cdots}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3(a^2+b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(b(\cdots}{\cdots}$$

$$= \frac{\left(3a^4+10a^2b^2-b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a-b)^2(a+b)^3d} + \frac{a\cdots}{2(a\cdots}$$

$$= -\frac{\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a\left(a^2-b^2\right)^2 d} - \frac{b(7a^2-b^2)\sqrt{\cos\cdots}}{\cdots}$$

Mathematica [**A**]
time = 35.20, size = 429, normalized size = 1.40

$$\frac{\frac{4a^2\left(3b(a^2+b^2)+a\left(5a^2+b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2} + \frac{4\cos(c+dx)(b+a\cos(c+dx))\cot(c+dx)(a+b\sec(c+dx))\left(5a^3b+ab^3-5a^3b\sec^2(c+dx)-ab^3\sec^2(c+dx)+ab\left(5a^2+b^2\right)E\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}+a\left(3a^3-5a^2b+3ab^2-b^3\right)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}-3a^4\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}-10a^2b^2\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}+b^4\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{-\tan^2(c+dx)}\right)}{(a-b)^2b(a+b)^2}}{16a^2d(b+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*a^2*(3*b*(a^2 + b^2) + a*(5*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2
 - b^2)^2 + (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c + d*x]*(a + b*Sec[c
+ d*x])*(5*a^3*b + a*b^3 - 5*a^3*b*Sec[c + d*x]^2 - a*b^3*Sec[c + d*x]^2 +
a*b*(5*a^2 + b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*
x]]*Sqrt[-Tan[c + d*x]^2] + a*(3*a^3 - 5*a^2*b + 3*a*b^2 - b^3)*EllipticF[A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 3
*a^4*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[-Tan[c + d*x]^2] - 10*a^2*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + b^4*EllipticPi[-(b/a)
, ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])
)/((a - b)^2*b*(a + b)^2))/(16*a^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*
x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. $2(358) = 716$.

time = 0.40, size = 1858, normalized size = 6.07

method	result	size
default	Expression too large to display	1858

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b/a^2*(1/b*a
^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*
b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/a^2*b^2*(1/2/b*a^2/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
```

```
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+
7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3
/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/
b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2/a
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^3, x)
```

3.625 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=323

$$\frac{3b(3a^2-b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2 d}+\frac{(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.$$

```
[Out] -1/2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/4*b*(7*
a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))+3/4*b
*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d
+1/4*(8*a^4-5*a^2*b^2+3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a
^3/(a^2-b^2)^2/d-3/4*b*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)^2/(a+b)^3/d
```

Rubi [A]
time = 0.42, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3928, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad(a^2-b^2)^2(a+b\sec(c+dx))}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{3b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a^2-b^2)^2}+\frac{(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d(a^2-b^2)^2}-\frac{3b(5a^4-2a^2b^2+b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((8*a^4 - 5*a^2*b^2 + 3*b^4)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*
d) - (3*b*(5*a^4 - 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*(a + b)^3*d) - (
b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
 - (b*(7*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(
a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3928

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)
*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Si
mp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
 && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
```

```
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-\frac{b}{2}-2a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} +$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} +$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} +$$

$$= -\frac{3b\left(5a^4-2a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3d} - \frac{}{2}$$

$$= \frac{3b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-5a^2b^2+\right.}{}$$

Mathematica [A]
time = 34.43, size = 286, normalized size = 0.89

$$\frac{\frac{4b\left(-7a^2b+b^3+\left(-9a^3+3ab^2\right)\cos(c+dx)\right)\sin(c+dx)}{a\left(a^2-b^2\right)^2(b+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}+\frac{2\cot(c+dx)\left(6ab\left(3a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin^2(c+dx)-6ab\left(3a^2-b^2\right)E\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-2a\left(7a^3-9a^2b-ab^2+3b^3\right)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+6\left(5a^4-2a^2b^2+b^4\right)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{a^3(a-b)^2(a+b)^2}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*b*(-7*a^2*b + b^3 + (-9*a^3 + 3*a*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a*(
a^2 - b^2)^2*(b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (2*Cot[c + d*x]*(
6*a*b*(3*a^2 - b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 - 6*a*b*(3*a^2 - b^2)
*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*a*(7*a
^3 - 9*a^2*b - a*b^2 + 3*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[-Tan[c + d*x]^2] + 6*(5*a^4 - 2*a^2*b^2 + b^4)*EllipticPi[-(b/a), ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*(a - b)^2*(a + b)^2))
/(16*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(375) = 750$.
time = 0.45, size = 1936, normalized size = 5.99

method	result	size
default	Expression too large to display	1936

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6/a^3
*b^2*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2/a^3*b^3*(1/2/b*a^2/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^
2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
```

```
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)
/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-
a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2)))+6*b/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^3, x)
```

3.626 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=342

$$\frac{(8a^4-29a^2b^2+15b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3\left(a^2-b^2\right)^2d}-\frac{3b(8a^4-11a^2b^2+5b^4)\sqrt{\cos(c+dx}}{4a^4\left(a^2-\right.}$$

```
[Out] 1/2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*b^
2*(11*a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x
+c))+1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^3/(a^2-b^2)^2/d-3/4*b*(8*a^4-11*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/4*b^2*(35*a^4-38*a^2*b^2+15*b^4
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a-b)^2/(a+b)
^3/d
```

Rubi [A]
time = 0.52, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3932, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3b(8a^4-11a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b^2(35a^4-38a^2b^2+15b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d(a-b)^2(a+b)^3}+\frac{(8a^4-29a^2b^2+15b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - (3*b*(8*a^4 - 11*a^2*b^2 + 5
*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a
^4*(a^2 - b^2)^2*d) + (b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a -
 b)^2*(a + b)^3*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^2) + (b^2*(11*a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
```

```
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2a^2+\frac{5b^2}{2}+2ab\sec(c+dx)-\frac{3}{2}b^2\,\mathrm{s}}{\sqrt{\sec(c+dx)}\,(a+b\sec(c}}{2a\,(a^2-b^2)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(11a^2-5b^2)\sqrt{\sec(c+d}}{4a^2\,(a^2-b^2)^2\,d(a+b\,\mathrm{se}}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(11a^2-5b^2)\sqrt{\sec(c+d}}{4a^2\,(a^2-b^2)^2\,d(a+b\,\mathrm{se}}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(11a^2-5b^2)\sqrt{\sec(c+d}}{4a^2\,(a^2-b^2)^2\,d(a+b\,\mathrm{se}}$$

$$= \frac{b^2(35a^4-38a^2b^2+15b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{4a^4(a-b)^2(a+b)^3d}$$

$$= \frac{(8a^4-29a^2b^2+15b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{4a^3\,(a^2-b^2)^2\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 707 vs. 2(342) = 684.

time = 36.57, size = 707, normalized size = 2.07

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((2*(8*a^4 - 7*a^2*b^2 + 5*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c
 + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b
*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x
])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*b + 8*a*b^3)*Cos[c + d*x]^2*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Se
c[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) +
 ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a
*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2
*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*
(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d
*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-1/4*(b^2*(-
13*a^2 + 7*b^2)*Sin[c + d*x])/(a^3*(-a^2 + b^2)^2) + (b^4*Sin[c + d*x])/(2*
a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (3*(-5*a^2*b^3*Sin[c + d*x] + 3*b
^5*Sin[c + d*x]))/(4*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1956 vs. $2(394) = 788$.
time = 0.47, size = 1957, normalized size = 5.72

method	result	size
default	Expression too large to display	1957

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-8/a^4*b^3*(1/b*a^2/(a^2-b^2)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
```

```
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a
*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2)))+2/a^4*b^4*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a
-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8
/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^
2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/
(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-12*b^2/a^3/(a^2-a*b)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec\left(c + dx\right)\right)^{3}\sqrt{\sec\left(c + dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**3*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3}\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x)
```

3.627 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=406

$$-\frac{b(24a^4-65a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4(a^2-b^2)^2d}+\frac{(8a^6+128a^4b^2-223a^2b^4+105$$

```
[Out] 1/12*(8*a^4-61*a^2*b^2+35*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/sec(d*x+c)^(1/2
)+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2)+1/4*
b^2*(13*a^2-7*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/sec(d*x+c)
^(1/2)-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/12*(8*a^6+128*a^4*b^2-223*a^2*b^4+105*b^6)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^5/(a^2-b^2)^2/d-1/4*b^3*(63*a^
4-86*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^5/(a-b)^2/(a+b)^3/d
```

Rubi [A]
time = 0.68, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3932, 4185, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b^2(13a^2-7b^2)\sin(c+dx)}{4a^2d(a^2-b^2)^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)}{2ad(a^2-b^2)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}-\frac{b(24a^4-65a^2b^2+35b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d(a^2-b^2)^2}-\frac{b^3(63a^4-86a^2b^2+35b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^5d(a-b)^2(a+b)^3}+\frac{(8a^4-61a^2b^2+35b^4)\sin(c+dx)}{12a^3d(a^2-b^2)^2\sqrt{\sec(c+dx)}}+\frac{(8a^6+128a^4b^2-223a^2b^4+105b^6)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^5d(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -1/4*(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a^2 - b^2)^2*d) + ((8*a^6 + 128*a^4*b^2
- 223*a^2*b^4 + 105*b^6)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(12*a^5*(a^2 - b^2)^2*d) - (b^3*(63*a^4 - 86*a^2*b^2 + 35*b^
4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sin
[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]) + (b^2*Sin[c + d*x])
/(2*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2) + (b^2*(13*a
^2 - 7*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(a + b*
Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
```

```
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{\int \frac{-2a^2+\frac{7b^2}{2}+2ab\,\mathrm{se}}{\sec^{\frac{3}{2}}(c+dx}}{2a}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{b^2(1}{4a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a}$$

$$= -\frac{b^3(63a^4-86a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}}}{4a^5(a-b)^2(a+b)^3d}$$

$$= -\frac{b(24a^4-65a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c-}}{4a^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A]
time = 36.62, size = 731, normalized size = 1.80

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((2*(-56*a^4*b + 73*a^2*b^3 - 35*b^5)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt
[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*
(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c
 + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^5 + 112*a^3*b^2 - 56*a*b^4)*Cos[c
 + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c +
 d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)) + ((-72*a^4*b + 195*a^2*b^3 - 105*b^5)*Cos[2*(c + d*x)]*(a
 + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*
(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
```

```
[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a),
 ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[
c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec
[c + d*x]]*((b^3*(-17*a^2 + 11*b^2)*Sin[c + d*x])/(4*a^4*(-a^2 + b^2)^2) -
(b^5*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (19*a^2*b^4
*Sin[c + d*x] - 13*b^6*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*
x])) + Sin[2*(c + d*x)]/(3*a^3)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. $2(454) = 908$.
time = 0.51, size = 2216, normalized size = 5.46

method	result	size
default	Expression too large to display	2216

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^3*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(2*a+3*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(a^2+3*a*b+6*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+10/a^5*b^4*(1/b*a^2/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)
/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2)))-2/a^5*b^5*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2
```

```
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a
+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(
a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*
b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+20*b^3/a^4/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^5 + 3*a*b^2*sec(d*x + c)^4 +
3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec{(c+dx)})^3\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**3*sec(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)
```

3.628 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c-}}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2
)/d/(a+b*sec(d*x+c))^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)-(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)
^(1/2)+sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.43, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3940, 4194, 3944, 2886, 2884, 3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}+\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (a*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(d*Sqrt[a + b*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]) + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*
Csc[e + f*x])^(n - 1)/(f*(2*n - 1))), x] + Dist[d^2/(2*n - 1), Int[(d*Csc[e
 + f*x])^(n - 2)*(Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e + f
*x]^2, x]/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4194

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}\int \frac{-a}{\sqrt{\sec(c+}}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}a\int \frac{}{\sqrt{\sec(c+}}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}\int \frac{\sqrt{a+b\,\mathrm{se}}}{\sqrt{\sec(c}}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{\left(b\sqrt{b+a\cos(c}\right.}{}$$

$$= \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{\mathrm{se}}}{}$$

$$= \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{a\sqrt{\frac{b+}{}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.63, size = 321, normalized size = 1.35

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} - \frac{2i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}\,b\sqrt{b+a\cos(c+dx)}}+4\tan(c+dx)\right)}{4d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*a*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/
((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - ((2*I)*Sqrt[-((a*(-1 + Cos[c
 + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b
*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]],
(-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b +
a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a -
b)^(-1)]*b*Sqrt[b + a*Cos[c + d*x]]) + 4*Tan[c + d*x]))/(4*d*Sqrt[Sec[c + d
*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 1.16, size = 789, normalized size = 3.33

method	result
default	$-\frac{\left(-\mathrm{EllipticE}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)(\cos^2(dx+c))\sin(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,a+\mathrm{Elli}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+2*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*a-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+2
*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((
a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a-(
(a-b)/(a+b))^(1/2)*cos(d*x+c)*a+((a-b)/(a+b))^(1/2)*cos(d*x+c)*b-((a-b)/(a+
b))^(1/2)*b)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)
```

3.629 $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(}}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1
/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3939, 3943, 2742, 2740, 3944, 2886, 2884}

$$\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3939

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx = a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\left(a\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{}}{}$$

Mathematica [A]
time = 2.19, size = 96, normalized size = 0.70

$$\frac{2\left(aF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+b\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)\sqrt{a+b\sec(c+dx)}}{(a+b)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2, (c + d*x)/2,
(2*a)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/((a + b)*d*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 265, normalized size = 1.92

method	result
default	$-\frac{2\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)a-\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)b+2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos}{}\right.\right.}{d(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b+2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b)*cos(d*x+c)*((b+a*cos(d*x+c))/cos(d
*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec{\left(c + dx \right)}}\,\sqrt{\sec{\left(c + dx \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

3.630 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+
b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3941, 2734, 2732}

$$\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx &= \frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}\,dx}{\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\\
&= \frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\
&= \frac{2E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(90) = 180$.
time = 0.32, size = 925, normalized size = 13.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*b-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
```

```
+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a+(
(a-b)/(a+b))^(1/2)*cos(d*x+c)*a-((a-b)/(a+b))^(1/2)*cos(d*x+c)*b+((a-b)/(a+
b))^(1/2)*b)/(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 355, normalized size = 5.30

$$\frac{-i\sqrt{2}\sqrt{a}\,b\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+i\sqrt{2}\sqrt{a}\,b\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+3i\sqrt{2}\,a^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)-3i\sqrt{2}\,a^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/
a) + I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a
) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3
*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
- 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)
```

3.631 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/d/sec(d*x+c)^(1/2)+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1
/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3942, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(2*d*n), Int[(d*Csc[e + f*x])^(n + 1)*(Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*Cs
c[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{3a} + \frac{(a^2-b^2)\int}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left((a^2-b^2)\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+\right.}{3a\sqrt{a+b\sec(c}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left((a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c-\right.}{3a\sqrt{a+b\,\mathrm{se}}}$$

$$= \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c-\right.}{3ad\sqrt{\frac{b+}{}}}$$

Mathematica [A]
time = 0.41, size = 156, normalized size = 0.81

$$\frac{2\sqrt{a+b\sec(c+dx)}\left(b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+a(b+a\cos(c+dx))\sin(c+dx)\right)}{3ad(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*(b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*S
in[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1020 vs. 2(228) = 456.
time = 0.27, size = 1021, normalized size = 5.32

method	result	size

default	Expression too large to display	1021

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-cos(d
*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b+cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^3*((a-
b)/(a+b))^(1/2)*a^2+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1
/2)*a^2*cos(d*x+c)-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+cos(d*x+c)*((a-b)/(a+
b))^(1/2)*b^2-((a-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1/2)*b^2)*cos(d*x+c)^2
*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 415, normalized size = 2.16

$$\frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+3i\,\sqrt{2}\,a^{\frac{3}{2}}b\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3i\,\sqrt{2}\,a^{\frac{3}{2}}b\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)+\sqrt{2}\,(-3i\,a^2+2i\,b^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\,(3i\,a^2-2i\,b^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)}{9\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)
/a)) - 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
) + sqrt(2)*(-3*I*a^2 + 2*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 - 2*I*b^2)*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sec{\left(c + dx \right)}}}{\sec^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

3.632 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=244

$$-\frac{4b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b}}{15a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c}}$$

```
[Out] -4/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/5*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/d/sec(d*x+c)^(3/2)+2/15*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d/s
ec(d*x+c)^(1/2)+2/15*(9*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+
c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3942, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{4b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (
2*(9*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (
2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(2*d*n), Int[(d*Csc[e + f*x])^(n + 1)*(Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*Cs
c[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\int \frac{b+3a\sec(c+dx)+2b\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,d$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{2\int -}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{(2b(a}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{\left(2b(\right.}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{\left(2b(\right.}{}$$

$$= -\frac{4b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9-\frac{2b}{a}\right.}{15a}$$

Mathematica [A]
time = 0.59, size = 203, normalized size = 0.83

$$\frac{\sqrt{a+b\sec(c+dx)}\left(4(9a^3+9a^2b-2ab^2-2b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+8b(-a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+2a(3a^2+2b^2+8ab\cos(c+dx)+3a^2\cos(2(c+dx)))\sin(c+dx)\right)}{30a^2d(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*b*(-a^2
+ b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)] + 2*a*(3*a^2 + 2*b^2 + 8*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*Si
n[c + d*x]))/(30*a^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(274) = 548$.
time = 0.25, size = 1736, normalized size = 7.11

method	result	size
default	Expression too large to display	1736

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(9*cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+2*
cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*b^3+7*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-9*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin
(d*x+c)-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b^2*sin(d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+2*((a-b)/(a+b))^(1/2)*b^3+4*cos
(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+
5*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b
^2-9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3+7*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*cos(d*x+c)*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b^2-9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
```

```
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-2*cos(d*x+c)*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2
-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*sin(d*x+c)+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-9*((a-b)/(a+b))^(1/2)*a^2*b-((a-b)/(a+b)
)^(1/2)*a*b^2+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6*cos(d*x+c)^2*((a-b)/
(a+b))^(1/2)*a^3-9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-2*cos(d*x+c)*((a-b)/(
a+b))^(1/2)*b^3)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+
c))/((a-b)/(a+b))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.13, size = 461, normalized size = 1.89

$$\frac{\sqrt{2}\,(-3i\,a^2b-4i\,b^3)\sqrt{a}\,\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\,(3i\,a^2b+4i\,b^3)\sqrt{a}\,\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\,(-9i\,a^3+2i\,ab^2)\sqrt{a}\,\text{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\,(9i\,a^3-2i\,ab^2)\sqrt{a}\,\text{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)+\frac{6\left(3a^3\cos(dx+c)^2+a^2b\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{45\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
[Out] 1/45*(sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*s
in(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2*b + 4*I*b^3)*sqrt(a)*weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d
*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9*I*a^3 + 2*I*a*b^2)*s
qrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3
, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(9*I*a^3
 - 2*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(
3*a^3*cos(d*x + c)^2 + a^2*b*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*
x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec{\left(c + dx \right)}}}{\sec^{\frac{5}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2), x)
```

3.633 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=305

$$\frac{2(25a^4-17a^2b^2-8b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2b(19a^2+8b^2)\,E\left(\frac{1}{2}(c+dx)|\right.}{105a^3d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2/105*(25*a^4-17*a^2*b^2-8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/7*sin(d*x+c)
*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/35*b*sin(d*x+c)*(a+b*sec(d*x+c
))^(1/2)/a/d/sec(d*x+c)^(3/2)+2/105*(25*a^2-4*b^2)*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/105*b*(19*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))
^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3942, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\sec(c+dx)}}+\frac{2b(19a^2+8b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(25a^4-17a^2b^2-8b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{35ad\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^3*d*Sqrt[a + b*Se
c[c + d*x]]) + (2*b*(19*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d*Sec[c + d*
x]^(3/2)) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105
*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(2*d*n), Int[(d*Csc[e + f*x])^(n + 1)*(Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*Cs
c[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\int \frac{b+5a\sec(c+dx)+4b\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,d$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{1}{2}}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25a}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25a}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25a}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25a}{}$$

$$= \frac{2(25a^4-17a^2b^2-8b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [**A**]
time = 0.82, size = 237, normalized size = 0.78

$$\frac{\sqrt{a+b\sec(c+dx)}\left(8b(19a^3+19a^2b+8ab^2+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+8(25a^4-17a^2b^2-8b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a(136a^2b-16b^3+a(145a^2-4b^2)\cos(c+dx)+36a^2b\cos(2(c+dx))+15a^3\cos(3(c+dx)))\sin(c+dx)\right)}{420a^3d(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(8*b*(19*a^3 + 19*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(25*
a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)] + 2*a*(136*a^2*b - 16*b^3 + a*(145*a^2 - 4*b^2)*Cos
[c + d*x] + 36*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*Sin[c + d*
x]))/(420*a^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2049 vs. $2(329) = 658$.
time = 0.26, size = 2050, normalized size = 6.72

method	result	size
default	Expression too large to display	2050

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(18*cos(d*x+c)^4*((a-b)/(a+b))
^(1/2)*a^3*b-cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+26*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*a^3*b+4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-19*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a^3*b+20*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-8*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+25*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+19*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(
d*x+c)-19*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^2*b^2*sin(d*x+c)+8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)-19*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)+2*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
^2*sin(d*x+c)-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
```

+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^3*sin(d*x+c)-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)+25*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)-25*((a-
b)/(a+b))^(1/2)*a^3*b-19*((a-b)/(a+b))^(1/2)*a^2*b^2+4*((a-b)/(a+b))^(1/2)*
a*b^3-8*((a-b)/(a+b))^(1/2)*b^4-8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+19*cos(d*x+c)*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^3*b-19*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+8*cos(d*x+c)*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^3-19*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-8*cos(d*
x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^3+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4+15*cos(d*x+c)^5*
((a-b)/(a+b))^(1/2)*a^4+10*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-25*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*a^4)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b
+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 501, normalized size = 1.64

$$\frac{\sqrt{2}\left(-75i\,a^4+32i\,a^2b^2+16i\,b^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+\sqrt{2}\left(75i\,a^4-32i\,a^2b^2-16i\,b^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)-3\sqrt{2}\left(-19i\,a^3b-8i\,ab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)-3\sqrt{2}\left(19i\,a^3b+8i\,ab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)+\frac{6\left(15\,a^4\cos(dx+c)^3+3\,a^3b\cos(dx+c)^2+(25\,a^4-4\,a^2b^2)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{315\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 1/315*(sqrt(2)*(-75*I*a^4 + 32*I*a^2*b^2 + 16*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*a^4 - 32*I*a^2*b^2 - 16*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-19*I*a^3*b - 8*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(19*I*a^3*b + 8*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(15*a^4*cos(d*x + c)^3 + 3*a^3*b*cos(d*x + c)^2 + (25*a^4 - 4*a^2*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)

[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2), x)

3.634 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 7/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c
)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(
1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1
/2)+1/2*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-5/4*a*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2
)/sec(d*x+c)^(1/2)+5/4*a*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)
/d
```

Rubi [A]

time = 0.66, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3951, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}+\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4d}+\frac{7ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}-\frac{5a\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (7*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) - (5*a*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (5*a*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(4*d) + (b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
```

```
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3951

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Dist[d/(m + n - 1), Int
[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (
b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d}+\frac{1}{2}\int \frac{\sqrt{\sec(c+$$

$$= \frac{5a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d}+\frac{b\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d}+\frac{b\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d}+\frac{b\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d}+\frac{b\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+a}}{4d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}+\frac{(3a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.18, size = 411, normalized size = 1.37

$$\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{4ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}+\frac{(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}-\frac{5i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{\sqrt{\frac{1}{a-b}}\,b(b+a\cos(c+dx))^{3/2}}+\frac{2(5a+2b\sec(c+dx))\tan(c+dx)}{b+a\cos(c+dx)}\right)}{8d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*((4*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + ((a^2 + 8*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(b + a*Cos[c + d*x])^2 - ((5*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]
*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[
I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] +
 a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (
-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[
b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sqrt[(a - b)^(-1)]*b*(b + a*Cos
[c + d*x])^(3/2)) + (2*(5*a + 2*b*Sec[c + d*x])*Tan[c + d*x])/(b + a*Cos[c
+ d*x])))/(8*d*Sec[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 1744, normalized size = 5.83

method	result	size
default	Expression too large to display	1744

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(6*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*cos(d*x+c)^3*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*b^2+2*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*cos(d*x+c)^3*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b-4*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-5*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+5*cos(d*x
+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b+6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*cos(d*x+
c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+2*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
```

```
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*cos(d*x+c
)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b-4*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-5*cos(d*x+c)^2*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^2+5*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+5*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+2*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2
+5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
b^2-7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*((a-b)/(a+b))^(1/2)*b^2)*((b+a*c
os(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(b+a*cos(d*x+c))/sin(d*x+
c)/((a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

3.635 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=249

$$\frac{(2a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d
*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((
b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)-b*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^
(1/2)/sec(d*x+c)^(1/2)+b*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)
/d
```

Rubi [A]

time = 0.48, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3951, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(2a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{3ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (3*a*b*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (b*EllipticE[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*Sqrt[Sec[c + d*x]]) + (b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/d
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3951

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Dist[d/(m + n - 1), Int
[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (
b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx = \frac{b\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \int \frac{-\frac{ab}{2}+a^2 \cdots}{\sqrt{\sec(c \cdots}}$$

$$= \frac{b\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}(3ab)\int \frac{\cdots}{\sqrt{\cdots}}$$

$$= \frac{b\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}b\int \frac{\sqrt{a+\cdots}}{\sqrt{\mathrm{se}\cdots}}$$

$$= \frac{b\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{\left((2a^2+b^2)\cdots\right.}{\cdots}$$

$$= \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{b\cdots}{\cdots}$$

$$= \frac{(2a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}\cdots}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.25, size = 394, normalized size = 1.58

$$\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{8a^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}+\frac{10ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}-\frac{2i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}(b+a\cos(c+dx))^{3/2}}+\frac{4b\tan(c+dx)}{b+a\cos(c+dx)}\right)}{4d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*((8*a^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + (10*a*b*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b +
 a*Cos[c + d*x])^2 - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(
a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b
*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)
/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*C
os[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*(b + a*Cos[c + d*
```

```
x])^(3/2)) + (4*b*Tan[c + d*x])/(b + a*Cos[c + d*x])))/(4*d*Sec[c + d*x]^(3
/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 1207, normalized size = 4.85

method	result	size
default	Expression too large to display	1207

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-cos(d*x+c
)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*sin(d*x+c)*cos(d*x+c)^2*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*a*b+2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*cos(d*x+c)*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-cos
(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b+cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*sin(d*x+c)*cos(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*a*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a*b+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/2)*b^2)*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin
(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec{(c+dx)})^{\frac{3}{2}}\sqrt{\sec{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)
```

3.636 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=209

$$\frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\mathrm{s}}}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^
(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*co
s(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2
)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3953, 3941, 2734, 2732, 3939, 3943, 2742, 2740, 3944, 2886, 2884}

$$\frac{2b^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3939

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3953

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Dist[a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e +
 f*x]], x], x] + Dist[b/d, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}dx = a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx + b\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$= (ab)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx + b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx + \frac{\left(a\sqrt{a+b\,\mathrm{se}\right.}{\sqrt{b\,\cdot}}$$

$$= \frac{\left(ab\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(b^2\sqrt{b\,\cdot\right.}{}$$

$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{\left(ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c\,\cdot\right.}{\sqrt{a+b}}$$

$$= \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c}{a+b}}}{}$$

Mathematica [A]
time = 2.33, size = 129, normalized size = 0.62

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a(a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+b\left(aF\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+b\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)\right)(a+b\sec(c+dx))^{3/2}}{d(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a*(a + b)*EllipticE[(c + d*x)/2, (2*
a)/(a + b)] + b*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)]))*(a + b*Sec[c + d*x])^(3/2))/(d*(b + a*Cos[c +
 d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 1367, normalized size = 6.54

method	result	size
default	Expression too large to display	1367

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-cos(d*x+c)*sin(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*cos(
d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(
d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+2*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b
))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-((a-b)/(a+b))
^(1/2)*a^2*cos(d*x+c)+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1/2
)*a*b)/(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^{\frac{3}{2}}}{\sqrt{\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

3.637 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}}+\frac{8bE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/d/sec(d*x+c)^(1/2)+8/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1
/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3949, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3949

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)/S
qrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e
 + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\int \frac{-4ab-(a^2+3b^2)\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(4b)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx - \frac{1}{3}($$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{\left((-a^2+b^2)\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec\right.}{3\sqrt{a+b\sec}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{\left((-a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\mathrm{se}\right.}{3\sqrt{a+b}}$$

$$= \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c\right.}{3d\sqrt{\frac{b}{}}}$$

Mathematica [A]
time = 0.46, size = 156, normalized size = 0.83

$$\frac{(a+b\sec(c+dx))^{3/2}\left(8b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a(b+a\cos(c+dx))\sin(c+dx)\right)}{3d(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(8*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c + d
*x])*Sin[c + d*x]))/(3*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1218 vs. $2(223) = 446$.
time = 0.25, size = 1219, normalized size = 6.52

method	result	size
default	Expression too large to display	1219

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(4*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-4*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/2)*a*b-4*((a-b)/(a+b))^(1/2)*b^2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.94, size = 415, normalized size = 2.22

$$\frac{6\,a^2\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+12i\,\sqrt{2}\,a^{\frac{3}{2}}b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)-12i\,\sqrt{2}\,a^{\frac{3}{2}}b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)+\sqrt{2}\,(-3i\,a^2-i\,b^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+\sqrt{2}\,(3i\,a^2+i\,b^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b
)/a)) - 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/
a)) + sqrt(2)*(-3*I*a^2 - I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + I*b^2)*sqrt(a)*weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
- 3*I*a*sin(d*x + c) + 2*b)/a))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec{(c + dx)})^{\frac{3}{2}}}{\sec^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)
```

3.638 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=240

$$\frac{2b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5ad\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec}}{5ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+}}$$

```
[Out] 2/5*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*s
ec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)/d/sec(d*x+c)^(3/2)+4/5*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*
x+c)^(1/2)+2/5*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2
)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3949, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(5*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(5*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*b*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3949

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)/S
qrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e
 + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{1}{5}\int \frac{-6ab-(3a^2+5b^2)\sec(c+dx)-2}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2\int}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{(b(}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{(b}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{(b}{}$$

$$= \frac{2b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5ad\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^2+b}{5ad}$$

Mathematica [A]

time = 0.74, size = 197, normalized size = 0.82

$$\frac{(a+b\sec(c+dx))^{3/2}\left(4(3a^3+3a^2b+ab^2+b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+4b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a(a^2+4b^2+6ab\cos(c+dx)+a^2\cos(2(c+dx)))\sin(c+dx)\right)}{10ad(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(4*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 4*b*(a^2 - b
^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
] + 2*a*(a^2 + 4*b^2 + 6*a*b*Cos[c + d*x] + a^2*Cos[2*(c + d*x)])*Sin[c + d
*x]))/(10*a*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1706 vs. $2(270) = 540$.
time = 0.22, size = 1707, normalized size = 7.11

method	result	size
default	Expression too large to display	1707

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c)*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-3*c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b+cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-cos(d*x+c)*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*b^3-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+4*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-cos(d*x+c)*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b^2+cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+3*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-3*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b^2*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
```

```
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)+4*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin
(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*b^2*sin(d*x+c)+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+2*cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*a^3+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-3*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d
*x+c)*((a-b)/(a+b))^(1/2)*b^3-3*((a-b)/(a+b))^(1/2)*a^2*b-2*((a-b)/(a+b))^(
1/2)*a*b^2-((a-b)/(a+b))^(1/2)*b^3)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d
*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.33, size = 463, normalized size = 1.93

$$-\frac{2\sqrt{2}\,(3i\,a^2b-i\,b^3)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+2\sqrt{2}\,(-3i\,a^2b+i\,b^3)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)+3\sqrt{2}\,(-3i\,a^3-i\,ab^2)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)+3\sqrt{2}\,(3i\,a^3+i\,ab^2)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)-\frac{6\left(a^3\cos(dx+c)^2+2a^2b\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
[Out] -1/15*(2*sqrt(2)*(3*I*a^2*b - I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*s
in(d*x + c) + 2*b)/a) + 2*sqrt(2)*(-3*I*a^2*b + I*b^3)*sqrt(a)*weierstrassP
Inverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(
d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(-3*I*a^3 - I*a*b^2)*sq
rt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(3*I*a^3
+ I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
- 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*(a^3
*cos(d*x + c)^2 + 2*a^2*b*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2), x)
```

3.639 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=303

$$\frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^2d\sqrt{a+b\sec(c+dx)}}+\frac{4b(41a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)|\right.}{105a^2d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2/105*(25*a^4-31*a^2*b^2+6*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/7*a*sin(d*x+
c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+16/35*b*sin(d*x+c)*(a+b*sec(d*
x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/105*(25*a^2+3*b^2)*sin(d*x+c)*(a+b*sec(d*x
+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+4/105*b*(41*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^
(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+
c)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3949, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105ad\sqrt{\sec(c+dx)}}+\frac{4b(41a^2-3b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{a+b\sec(c+dx)}}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^2*d*Sqrt[a + b*Se
c[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c +
 d*x]^(5/2)) + (16*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d
*x]^(3/2)) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(10
5*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3949

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)/S
qrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e
 + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{1}{7}\int \frac{-8ab-(5a^2+7b^2)\sec(c+dx)-4}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{-}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{-}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{-}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{-}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{-}$$

$$= \frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 1.15, size = 237, normalized size = 0.78

$$\frac{(a+b\sec(c+dx))^{3/2}\left(16b(41a^3+41a^2b-3ab^2-3b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+8(25a^4-31a^2b^2+6b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a(178a^2b+12b^3+a(145a^2+108b^2)\cos(c+dx)+78a^2b\cos(2(c+dx))+15a^3\cos(3(c+dx)))\sin(c+dx)\right)}{420a^2d(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(16*b*(41*a^3 + 41*a^2*b - 3*a*b^2 - 3*b^3)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(
25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + 2*a*(178*a^2*b + 12*b^3 + a*(145*a^2 + 108*b^2
)*Cos[c + d*x] + 78*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*Sin[c
 + d*x]))/(420*a^2*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2049 vs. $2(327) = 654$.
time = 0.24, size = 2050, normalized size = 6.77

method	result	size
default	Expression too large to display	2050

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(39*cos(d*x+c)^4*((a-b)/(a+b))
^(1/2)*a^3*b+27*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+68*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a^3*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-82*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a^3*b+55*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+6*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+25*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+82*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*s
in(d*x+c)-82*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b^2*sin(d*x+c)-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)-82*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)+51*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b^2*sin(d*x+c)+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
```

```
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)+25*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)-25*
((a-b)/(a+b))^(1/2)*a^3*b-82*((a-b)/(a+b))^(1/2)*a^2*b^2-3*((a-b)/(a+b))^(1
/2)*a*b^3+6*((a-b)/(a+b))^(1/2)*b^4+6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+82*cos(d*x+
c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3*b-82*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-6*cos(d*x+c)*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^3-82*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+51*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+6*c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b^3-6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4+15*cos(d*x+
c)^5*((a-b)/(a+b))^(1/2)*a^4+10*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-25*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^4)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+
c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 501, normalized size = 1.65

$$\frac{\sqrt{2}\left(-75 i a^{4}+11 i a^{2} b^{2}-12 i b^{4}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3 a^{2}-4 b^{2}\right)}{3 a^{2}}, \frac{8\left(9 a^{2} b-8 b^{3}\right)}{27 a^{3}}, \frac{3 a\cos(dx+c)+3 i a\sin(dx+c)+2 b}{3 a}\right)+\sqrt{2}\left(75 i a^{4}-11 i a^{2} b^{2}+12 i b^{4}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3 a^{2}-4 b^{2}\right)}{3 a^{2}}, \frac{8\left(9 a^{2} b-8 b^{3}\right)}{27 a^{3}}, \frac{3 a\cos(dx+c)-3 i a\sin(dx+c)+2 b}{3 a}\right)-6\sqrt{2}\left(-41 i a^{3} b+3 i a b^{3}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3 a^{2}-4 b^{2}\right)}{3 a^{2}}, \frac{8\left(9 a^{2} b-8 b^{3}\right)}{27 a^{3}}, \mathrm{weierstrassPInverse}\left(-\frac{4\left(3 a^{2}-4 b^{2}\right)}{3 a^{2}}, \frac{8\left(9 a^{2} b-8 b^{3}\right)}{27 a^{3}}, \frac{3 a\cos(dx+c)+3 i a\sin(dx+c)+2 b}{3 a}\right)\right)-6\sqrt{2}\left(41 i a^{3} b-3 i a b^{3}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3 a^{2}-4 b^{2}\right)}{3 a^{2}}, \frac{8\left(9 a^{2} b-8 b^{3}\right)}{27 a^{3}}, \mathrm{weierstrassPInverse}\left(-\frac{4\left(3 a^{2}-4 b^{2}\right)}{3 a^{2}}, \frac{8\left(9 a^{2} b-8 b^{3}\right)}{27 a^{3}}, \frac{3 a\cos(dx+c)-3 i a\sin(dx+c)+2 b}{3 a}\right)\right)+\frac{6\left(15 a^{4}\cos(dx+c)^{3}+24 a^{3} b\cos(dx+c)^{2}+\left(25 a^{4}+3 a^{2} b^{2}\right)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{315 a^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/315*(sqrt(2)*(-75*I*a^4 + 11*I*a^2*b^2 - 12*I*b^4)*sqrt(a)*weierstrassPIn
verse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*
x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*a^4 - 11*I*a^2*b^2 +
12*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 6*s
qrt(2)*(-41*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
c) + 2*b)/a)) - 6*sqrt(2)*(41*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierstrassZeta(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
- 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(15*a^4*cos(d*x + c)^3 + 24*a^3*b*cos(d
*x + c)^2 + (25*a^4 + 3*a^2*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2), x)
```

3.640 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=369

$$\frac{b(59a^2+16b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{24d\sqrt{a+b\sec(c+dx)}}+\frac{5a(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi(2}{8d\sqrt{a+b\sec(}$$

```
[Out] 1/24*b*(59*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+5/8*a*(a^2+4*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1
/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b
*sec(d*x+c))^(1/2)+13/12*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(
1/2)/d+1/3*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-1/24*(3
3*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/24*(33*a^2+16*b^2)*sin(d*x+c)*sec(
d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.90, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3927, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{24d}+\frac{b(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{a+b\sec(c+dx)}}-\frac{(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{5a(a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8d\sqrt{a+b\sec(c+dx)}}+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}+\frac{13ab\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sqrt[a + b*Sec[c + d*x]]) + (5
*a*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(8*d*Sqrt[a + b*Sec[c + d*x]]) - ((3
3*a^2 + 16*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((33*a^
2 + 16*b^2)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d
) + (13*a*b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d
) + (b^2*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

```
&& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}\int \frac{\sec^{\frac{3}{2}}(c+$$

$$= \frac{13ab\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d}+\frac{b^2\sec^{\frac{5}{2}}(c+$$

$$= \frac{(33a^2+16b^2)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d}+$$

$$= \frac{(33a^2+16b^2)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d}+$$

$$= \frac{(33a^2+16b^2)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d}+$$

$$= \frac{(33a^2+16b^2)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d}+$$

$$= \frac{5a(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+}{8d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{b(59a^2+16b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+}{24d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.41, size = 602, normalized size = 1.63

$$-\frac{a(a+b\sec(c+dx))^{5/2}\left(-\frac{104ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(3a^2-104b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i(33a^2+16b^2)\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a+a\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)\right)\right)\sin(c+dx)}{\sqrt{\frac{1}{a-b}}\,b\sqrt{1-\cos^2(c+dx)}\sqrt{\frac{a^2-a^2\cos^2(c+dx)}{a^2}}\left(-a^2+2b^2-4b(b+a\cos(c+dx))+2(b+a\cos(c+dx))^2\right)}\right)}{96d(b+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{24}\sec(c+dx)\left(33a^2\sin(c+dx)+16b^2\sin(c+dx)\right)+\frac{13}{12}ab\sec(c+dx)\tan(c+dx)+\frac{1}{3}b^2\sec^2(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]

[Out] -1/96*(a*(a + b*Sec[c + d*x])^(5/2)*((-104*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(3*a^2 - 104*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(33*a^2 + 16*b^2)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*Sec[c + d*x])^(5/2)*((Sec[c + d*x]*(33*a^2*Sin[c + d*x] + 16*b^2*Sin[c + d*x]))/24 + (13*a*b*Sec[c + d*x]*Tan[c + d*x])/12 + (b^2*Sec[c + d*x]^2*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))

Maple [C] Result contains complex when optimal does not.
time = 0.52, size = 2295, normalized size = 6.22

method	result	size
default	Expression too large to display	2295

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/24/d*(33*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-16*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3+8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3-16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*b^3-18*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3+33*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3-18*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)

```
*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticPi((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3+8*(
(a-b)/(a+b))^(1/2)*b^3-26*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b-16*cos(d*x
+c)^4*((a-b)/(a+b))^(1/2)*a*b^2-18*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2+5
9*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-33*cos(d*x+c)^3*((a-b)/(a+b))^(1/2
)*a^2*b+34*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+33*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*
a^3+16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^2-26*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+44*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b^2-120*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticPi((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2
-33*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-26*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b+44*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-120*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*
x+c)^3*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*a*b^2-33*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b-33*cos(d*x+
c)^4*((a-b)/(a+b))^(1/2)*a^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*
x+c))^(3/2)/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)
```

3.641 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=314

$$\frac{a(8a^2+11b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b(15a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi}{4d\sqrt{a+b\sec}}$$

```
[Out] 1/4*a*(8*a^2+11*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1
/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*b*(15*a^2+4*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(
1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+
b*sec(d*x+c))^(1/2)+1/2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/d-9/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+9/4*a*b*sin(d*x+c)*sec(d*x+c)^(1/2)*
(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.71, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3927, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{a(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b(15a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}+\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4d}-\frac{9ab\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(
15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) - (9*a
*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (9*a*b*Sqrt[Sec[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (b^2*Sec[c + d*x]^(3/2)
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{\sec(c}{}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{b(15a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c}{4d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{a(8a^2+11b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+}{4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.32, size = 560, normalized size = 1.78

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2(16a^3+4ab^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(21a^2b+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}-\frac{18ia^2\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a+a\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)\right)\right)\sin(c+dx)}{\sqrt{\frac{1}{a-b}}\sqrt{1-\cos^2(c+dx)}\sqrt{\frac{a^2-a^2\cos^2(c+dx)}{a^2}}\left(-a^2+2b^2-4b(b+a\cos(c+dx))+2(b+a\cos(c+dx))^2\right)}\right)}{16d(b+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{9}{4}ab\tan(c+dx)+\frac{1}{2}b^2\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(16*a^3 + 4*a*b^2)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]]
+ (2*(21*a^2*b + 8*b^3)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] - ((18*I)*a^2*Sqrt[(a -
 a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x
)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c
+ d*x])/(Sqrt[(a - b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c
+ d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c +
d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*
Sec[c + d*x])^(5/2)*((9*a*b*Tan[c + d*x])/4 + (b^2*Sec[c + d*x]*Tan[c + d*x
])/2))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 1982, normalized size = 6.31

method	result	size
default	Expression too large to display	1982

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-30*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b-8*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin
(d*x+c)*b^3-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-2
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b^2+4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^3+9*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
```

```
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2
*sin(d*x+c)*a^2*b-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-30*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*a^2*b-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-8*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x
+c)^2*sin(d*x+c)*a^3+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-2*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x
+c)*a*b^2+4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-9*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*
a^2*b-2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2+9*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a^2*b-9*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-2*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*b^3+11*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+2*((a-b)/(a+b))^(
1/2)*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/(b+a*cos
(d*x+c))/cos(d*x+c)/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

3.642 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=263

$$\frac{b(4a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] b*(4*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+5*a*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+
(2*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d
*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+b^2*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*s
ec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.52, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3927, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{b(4a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2a^2-b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (5*a*b^2*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*a^2 - b^2)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[Sec[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \int \frac{\frac{1}{2}a(2a^2-b^2)+3a^2b\,\mathrm{s}}{\sqrt{\sec(c+dx}}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}(5ab^2)\int \frac{\sec^{\frac{3}{2}}(c+}{\sqrt{a+b\sec(c}}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}(2a^2-b^2)\int \frac{\sqrt{a+b\,\mathrm{s}}}{\sqrt{\sec(}}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{\Big(b(4a^2+b^2)\sqrt{b+a\cos}}{}$$

$$= \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{\sec(c+}}{}$$

$$= \frac{b(4a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b}{}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.26, size = 421, normalized size = 1.60

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{24a^2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^3}+\frac{2a(2a^2+9b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^3}+\frac{2i(2a^2-b^2)\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}\,b(b+a\cos(c+dx))^{5/2}}+\frac{4b^2\tan(c+dx)}{(b+a\cos(c+dx))^2}\right)}{4d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((24*a^2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + (2*a*(2*a^2
+ 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a
)/(a + b)])/(b + a*Cos[c + d*x])^3 + ((2*I)*(2*a^2 - b^2)*Sqrt[-((a*(-1 + C
os[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(
-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]
]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[
b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[S
qrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(
```

```
a - b)^(-1)]*b*(b + a*Cos[c + d*x])^(5/2)) + (4*b^2*Tan[c + d*x])/(b + a*Co
s[c + d*x])^2))/(4*d*Sec[c + d*x]^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 1949, normalized size = 7.41

method	result	size
default	Expression too large to display	1949

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*sin(d*x+c)*cos(d*x+c)^2*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-2*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*c
os(d*x+c)^2*sin(d*x+c)*a^2*b-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+sin(d*x+c)*cos(d
*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*b^3-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3+6*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-4
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*cos(d*x+c)^2*sin(d*x+c)*a*b^2+10*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b
^2+2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-cos(d*x+c)*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*b^2+cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*cos(d
*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
```

```
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2*b-4*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+10*sin(d*x+c)*cos(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-2*cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a^3+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b+cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a*b^2-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-((a-b)/(
a+b))^(1/2)*b^3)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a
+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

3.643 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=262

$$\frac{2a(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\sec(c+dx}}$$

```
[Out] 2/3*a*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2
)+2/3*a^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+14/3*a*b*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(
1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3926, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{14ab\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^
3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\frac{7a^2b}{2}+\frac{1}{2}a(a^2+9b^2)\sec(c+dx)+}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\frac{7a^2b}{2}+\frac{1}{2}a(a^2+9b^2)\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(7ab)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}($$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(a(a^2+2b^2)\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec}\right.}{3\sqrt{a+b\sec}}$$

$$= \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2a^2\sqrt{a+b\sec(}}{3d\sqrt{\text{se}}}$$

$$= \frac{2a(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} + \frac{2b^3\sqrt{\frac{b}{-}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.76, size = 409, normalized size = 1.56

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2a(a^2+9b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^3}+\frac{b(7a^2+6b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^3}+\frac{7i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{\sqrt{\frac{1}{a-b}}(b+a\cos(c+dx))^{5/2}}+\frac{2a^2\sin(c+dx)}{(b+a\cos(c+dx))^2}\right)}{3d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*a*(a^2 + 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + (b*
(7*a^2 + 6*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + ((7*I)*Sqrt[-((a*(-1 + Cos[c +
d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a
 + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a
 + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*C
os[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a
- b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sqrt[(a - b)^(-
```

```
1)]*(b + a*Cos[c + d*x])^(5/2)) + (2*a^2*Sin[c + d*x])/(b + a*Cos[c + d*x])
^2))/(3*d*Sec[c + d*x]^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 1661, normalized size = 6.34

method	result	size
default	Expression too large to display	1661

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-7*cos
(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b+9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*sin(d*x+c)*cos(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b^3+6*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+7*cos(d*x+c)*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b-7*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-7*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+9*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
2*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*b^3*sin(d*x+c)+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+7*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*
x+c)-7*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
```

```
(1/2))*a*b^2*sin(d*x+c)+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+8*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-7*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a^2*b+7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-((a-b)/(a+
b))^(1/2)*a^2*b-7*((a-b)/(a+b))^(1/2)*a*b^2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3
/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

3.644 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=239

$$\frac{16b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+23b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b}}{15d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c}}$$

```
[Out] 16/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2/5*a^2*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/d/sec(d*x+c)^(3/2)+22/15*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/
sec(d*x+c)^(1/2)+2/15*(9*a^2+23*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*
x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3926, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{16b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (16*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9
*a^2 + 23*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (22*a*b*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{11a^2b}{2}+\frac{3}{2}a(a^2+5b^2)\sec(c+dx)+}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} -$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} -$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} +$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} +$$

$$= \frac{16b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15d\sqrt{a+b\sec(c+dx)}} + \frac{2(9a^2+}{1}$$

Mathematica [A]
time = 1.00, size = 200, normalized size = 0.84

$$\frac{(a+b\sec(c+dx))^{5/2}\left(4(9a^3+9a^2b+23ab^2+23b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+32b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+2a(3a^2+22b^2+28ab\cos(c+dx)+3a^2\cos(2(c+dx)))\sin(c+dx)\right)}{30d(b+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(4*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 32*b*(
a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)] + 2*a*(3*a^2 + 22*b^2 + 28*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x
)])*Sin[c + d*x]))/(30*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1930 vs. $2(269) = 538$.

time = 0.22, size = 1931, normalized size = 8.08

method	result	size
default	Expression too large to display	1931

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(15*sin(d*x+c)*cos(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+1
5*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*b^3*sin(d*x+c)+9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-23*cos(d*x+c)*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+1
7*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^2*b*sin(d*x+c)-23*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+23*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+
c)-23*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^3*sin(d*x+c)-23*((a-b)/(a+b))^(1/2)*b^3+14*cos(d*x+c)^3*((a-b)/(a+b
))^(1/2)*a^2*b+34*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-5*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a^2*b-23*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-9*cos(d*x+c)*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^3+17*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
```

```
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-23*cos(d*x+c)*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2
-9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2*b+23*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x
+c)+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3*sin(d*x+c)-9*((a-b)/(a+b))^(1/2)*a^2*b-11*((a-b)/(a+b))^(1/2)*a*b
^2+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*a^3-9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+23*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*b^3)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)
/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.94, size = 462, normalized size = 1.93

$$\frac{\sqrt{2}\,(-33i\,a^2b + i\,b^3)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right) + \sqrt{2}\,(33i\,a^2b - i\,b^3)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right) - 3\,\sqrt{2}\,(-9i\,a^3 - 23i\,ab^2)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right) - 3\,\sqrt{2}\,(9i\,a^3 + 23i\,ab^2)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right) + \frac{6\left(3\,a^3\cos(dx+c)^2+11\,a^2b\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{45\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/45*(sqrt(2)*(-33*I*a^2*b + I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*si
n(d*x + c) + 2*b)/a) + sqrt(2)*(33*I*a^2*b - I*b^3)*sqrt(a)*weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9*I*a^3 - 23*I*a*b^2)*sq
rt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(9*I*a^3
+ 23*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
```

```
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(
3*a^3*cos(d*x + c)^2 + 11*a^2*b*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2), x)
```

3.645 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=303

$$\frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{21ad\sqrt{a+b\sec(c+dx)}}+\frac{2b(29a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2/21*(5*a^4-2*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(
a+b))^(1/2)*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/7*a^2*sin(d*x+c)*
(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+6/7*a*b*sin(d*x+c)*(a+b*sec(d*x+c
))^(1/2)/d/sec(d*x+c)^(3/2)+2/21*(5*a^2+9*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^
(1/2)/d/sec(d*x+c)^(1/2)+2/21*b*(29*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(
a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3926, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(5a^2+9b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{21d\sqrt{\sec(c+dx)}}+\frac{2b(29a^2+3b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{a+b\sec(c+dx)}}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(21*a*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b*(29*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(21*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
+ d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^
(5/2)) + (6*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(3
/2)) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt
[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{15a^2b}{2}+\frac{1}{2}a(5a^2+21b^2)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} -$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{21ad\sqrt{a+b\sec(c+dx)}} +$$

Mathematica [A]
time = 1.51, size = 237, normalized size = 0.78

$$\frac{(a+b\sec(c+dx))^{5/2}\left(8b(29a^3+29a^2b+3ab^2+3b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+8(5a^4-2a^2b^2-3b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a(44a^2b+36b^3+a(29a^2+72b^2)\cos(c+dx)+24a^2b\cos(2(c+dx))+3a^3\cos(3(c+dx)))\sin(c+dx)\right)}{84ad(b+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(8*b*(29*a^3 + 29*a^2*b + 3*a*b^2 + 3*b^3)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(5
*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)] + 2*a*(44*a^2*b + 36*b^3 + a*(29*a^2 + 72*b^2)*Cos[
c + d*x] + 24*a^2*b*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)])*Sin[c + d*x]
))/(84*a*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2049 vs. $2(327) = 654$.
time = 0.24, size = 2050, normalized size = 6.77

method	result	size
default	Expression too large to display	2050

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(12*cos(d*x+c)^4*((a-b)/(a+b))^
(1/2)*a^3*b+18*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+22*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a^3*b+12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-29*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a^3*b+11*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-3*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+5*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+29*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*si
n(d*x+c)-29*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*b^2*sin(d*x+c)+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)-29*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)+27*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^2*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
```

```
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)+5*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)-5*((a
-b)/(a+b))^(1/2)*a^3*b-29*((a-b)/(a+b))^(1/2)*a^2*b^2-9*((a-b)/(a+b))^(1/2)
*a*b^3-3*((a-b)/(a+b))^(1/2)*b^4-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+29*cos(d*x+c)*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^3*b-29*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+3*cos(d*x+c)*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b^3-29*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+27*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-3*cos(
d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^3+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4+3*cos(d*x+c)^5
*((a-b)/(a+b))^(1/2)*a^4+2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-5*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^4)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b+
a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 501, normalized size = 1.65

$$\frac{\sqrt{2}\left(-15i\,a^4-23i\,a^2b^2+6i\,b^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(15i\,a^4+23i\,a^2b^2-6i\,b^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-29i\,a^3b-3i\,ab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(29i\,a^3b+3i\,ab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)+\frac{6\left(3a^4\cos(dx+c)^3+9a^3b\cos(dx+c)^2+(5a^4+9a^2b^2)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{63\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/63*(sqrt(2)*(-15*I*a^4 - 23*I*a^2*b^2 + 6*I*b^4)*sqrt(a)*weierstrassPInve
rse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*a^4 + 23*I*a^2*b^2 - 6*
I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
- 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt
(2)*(-29*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a
^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
+ 2*b)/a)) - 3*sqrt(2)*(29*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a)) + 6*(3*a^4*cos(d*x + c)^3 + 9*a^3*b*cos(d*x +
c)^2 + (5*a^4 + 9*a^2*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2), x)
```

3.646 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=363

$$\frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{315a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4+279a^2b^2-10b^4)\,E}{315a^2d\sqrt{\frac{b+a\,c}{a}}}$$

```
[Out] 4/315*b*(57*a^4-62*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/9*a^2*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)+38/63*a*b*sin(d*x+c)*(a+b*
sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/315*(49*a^2+75*b^2)*sin(d*x+c)*(a+b*
sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/315*b*(163*a^2+5*b^2)*sin(d*x+c)*(a+
b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/315*(147*a^4+279*a^2*b^2-10*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/
(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$,
Rules used = {3926, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(49a^2+75b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b(163a^2+5b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315ad\sqrt{\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4+279a^2b^2-10b^4)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{38ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{63d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(315*a^2*d*Sqrt[a + b*
Sec[c + d*x]]) + (2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(9*d*Sec[c + d*x]^(7/2)) + (38*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2 + 75*b^2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*b*(163*a^2 + 5*b^2)*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{19a^2b}{2}+\frac{1}{2}a(7a^2+27b^2)\sec(c+dx}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a-}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx}}{315a^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 1.73, size = 286, normalized size = 0.79

$$\frac{(a+b\sec(c+dx))^{5/2}\left(16(147a^5+147a^4b+279a^3b^2+279a^2b^3-10ab^4-10b^5)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+32b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+2a(301a^4+1984a^2b^2+40b^4+4ab(619a^2+160b^2)\cos(c+dx)+8(42a^4+85a^2b^2)\cos(2(c+dx))+260a^3b\cos(3(c+dx))+35a^4\cos(4(c+dx)))\sin(c+dx)\right)}{2520a^2d(b+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(16*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a^
2*b^3 - 10*a*b^4 - 10*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)] + 32*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*C
```

```
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(301*a^4
+ 1984*a^2*b^2 + 40*b^4 + 4*a*b*(619*a^2 + 160*b^2)*Cos[c + d*x] + 8*(42*a^
4 + 85*a^2*b^2)*Cos[2*(c + d*x)] + 260*a^3*b*Cos[3*(c + d*x)] + 35*a^4*Cos[
4*(c + d*x)])*Sin[c + d*x]))/(2520*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x
]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2787 vs. $2(381) = 762$.
time = 0.32, size = 2788, normalized size = 7.68

method	result	size
default	Expression too large to display	2788

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(82*cos(d*x+c)^3*((a-b)/(a+b))
^(1/2)*a^4*b+80*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3+272*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a^3*b^2-5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4-65*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b-279*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2
+199*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+10*cos(d*x+c)*((a-b)/(a+b))^(1/
2)*a*b^4+130*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b+170*cos(d*x+c)^4*((a-b)
/(a+b))^(1/2)*a^3*b^2+10*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5-147*cos(d*x+c)*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^5-147*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^4*b*sin(d*x+c)+279*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*sin(d*x+c)-279*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*sin(d*x+c)-10
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b^4*sin(d*x+c)+261*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^4*b*sin(d*x+c)-279*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*sin(d*x+c)+155*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*s
in(d*x+c)+10*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
```

```
a-b))^(1/2))*a*b^4*sin(d*x+c)+35*cos(d*x+c)^6*((a-b)/(a+b))^(1/2)*a^5+14*co
s(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5+98*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5-
147*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5-10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^
5+147*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^5*sin(d*x+c)+10*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b^5*sin(d*x+c)-147*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*sin(d*x+c)+10*((a-b)/(a+b))^
(1/2)*b^5-147*((a-b)/(a+b))^(1/2)*a^4*b-163*((a-b)/(a+b))^(1/2)*a^3*b^2-279
*((a-b)/(a+b))^(1/2)*a^2*b^3-5*((a-b)/(a+b))^(1/2)*a*b^4-147*cos(d*x+c)*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^4*b+279*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-279*cos(d*x+c)*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b^3-10*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+261*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-279*c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^3*b^2+155*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+10*cos(d*x+c
)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^4+147*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5)*cos(d*x+c)^5*(1/cos(d*x+c
))^(9/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.09, size = 541, normalized size = 1.49

$$\frac{\sqrt{2}\left(-489i\,a^4b+93i\,a^2b^3-20i\,b^5\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(489i\,a^4b-93i\,a^2b^3+20i\,b^5\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-147i\,a^5-279i\,a^3b^2+10i\,ab^4\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(147i\,a^5+279i\,a^3b^2-10i\,ab^4\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)+\frac{6\left(35a^5\cos(dx+c)^4+95a^4b\cos(dx+c)^3+(49a^5+75a^3b^2)\cos(dx+c)^2+(163a^4b+5a^2b^3)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{945\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/945*(sqrt(2)*(-489*I*a^4*b + 93*I*a^2*b^3 - 20*I*b^5)*sqrt(a)*weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos
(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(489*I*a^4*b - 93*I*a^2*
b^3 + 20*I*b^5)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
 - 3*sqrt(2)*(-147*I*a^5 - 279*I*a^3*b^2 + 10*I*a*b^4)*sqrt(a)*weierstrassZ
eta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x +
 c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(147*I*a^5 + 279*I*a^3*b^2
- 10*I*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(
35*a^5*cos(d*x + c)^4 + 95*a^4*b*cos(d*x + c)^3 + (49*a^5 + 75*a^3*b^2)*cos
(d*x + c)^2 + (163*a^4*b + 5*a^2*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2), x)
```

3.647 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=312

$$-\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{4b^2d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] -1/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)
^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^
(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c
))^(1/2)+1/2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d+3/4*a*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^2/d/((b+a*cos(d*x+c))/(a
+b))^(1/2)/sec(d*x+c)^(1/2)-3/4*a*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+
c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.59, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3945, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b\sec(c+dx)}}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4b^2d}+\frac{3a\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}-\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] -1/4*(a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(4*b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (3*a*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[Sec[c + d*x]]*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d) + (Sec[c + d*x]^(3/2)*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd} + \frac{\int \frac{\sqrt{\sec(c+dx)}\,(a+2b\sec(c+dx)\cdots}{\sqrt{a+b\sec(c+d\cdots}}}{4b}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec\cdots}}{2b\cdots}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec\cdots}}{2b\cdots}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec\cdots}}{2b\cdots}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec\cdots}}{2b\cdots}$$

$$= \frac{\left(4+\frac{3a^2}{b^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\sec\cdots}}{\cdots}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}} + \frac{\left(4+\frac{3a^2}{b^2}\right)\sqrt{\frac{b+\cdots}{\cdots}}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.05, size = 397, normalized size = 1.27

$$\frac{\sqrt{\sec(c+dx)}\left(4ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+b(9a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+\frac{3i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\sqrt{b+a\cos(c+dx)}\,\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{\sqrt{\frac{1}{a-b}}}+2b(2b-3a\cos(c+dx))(b+a\cos(c+dx))\sec(c+dx)\tan(c+dx)\right)}{8b^3d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + b*(9*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)] + ((3*I)*Sqrt[-((a*(-1 + C
os[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos
[c + d*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)
]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*Ellipt
icPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
 b)/(a + b)])))/Sqrt[(a - b)^(-1)] + 2*b*(2*b - 3*a*Cos[c + d*x])*(b + a*Co
s[c + d*x])*Sec[c + d*x]*Tan[c + d*x]))/(8*b^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 1755, normalized size = 5.62

method	result	size
default	Expression too large to display	1755

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(-3*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-6*cos(d
*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*b^2+6*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*cos(d*x
+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*b^2-3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*cos(d*x+c)^2*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b-6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
```

```
sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c)^2*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)
/(a+b))^(1/2))*b^2+6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*cos(d*x+c)^2*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a*b+4*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2-2*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2
+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
b^2-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+2*((a-b)/(a+b))^(1/2)*b^2)*((b+a*cos
(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^2/sin(d*x+c)/(b+
a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.648 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}-\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c}}{bd\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/
d/(a+b*sec(d*x+c))^(1/2)-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)-(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+
c)^(1/2)+sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.44, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3945, 4194, 3944, 2886, 2884, 3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{bd}+\frac{\sqrt{\sec(c+dx)}\,\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\sec(c+dx)}\,\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{a\sqrt{\sec(c+dx)}\,\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (a*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(b*d*Sqrt[a + b*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4194

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{\int \frac{-a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(}}}{2b}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} - \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec}}}{2b}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{\left(\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(}\right.}{2\sqrt{a+}}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\,\sqrt{}}{}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} - \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{bd\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.49, size = 329, normalized size = 1.34

$$\frac{\sqrt{\sec(c+dx)}\left(-6a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-\frac{2i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\sqrt{b+a\cos(c+dx)}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}\,b}+4(b+a\cos(c+dx))\tan(c+dx)\right)}{4bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-6*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)] - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b)
)]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d*x]]*Csc[c + d*
x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sq
```

```
rt[(a - b)^(-1)]*b) + 4*(b + a*Cos[c + d*x])*Tan[c + d*x]))/(4*b*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 995, normalized size = 4.04

method	result	size
default	Expression too large to display	995

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*a-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+2*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*a-2*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a
-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*b+2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-2*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*a-((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a+((a-b)/(a+b))^(1/2)*cos(
d*x+c)*a-((a-b)/(a+b))^(1/2)*cos(d*x+c)*b+((a-b)/(a+b))^(1/2)*b)*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2/sin(d*x+c)/(b+a
*cos(d*x+c))/((a-b)/(a+b))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.649 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(
1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3944, 2886, 2884}

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}}\\
&= \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}}\\
&= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 1.00

$$
\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 232, normalized size = 3.41

method	result
default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\left(\text{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)-2\,\text{EllipticPi}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a}{a}}}\right.\right.}{d(b+a\cos(dx+c))\left(\frac{1}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)^2\sqrt{\frac{a-b}{a+b}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/d*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))-2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^2/((a-b)/(a+b))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)**(3/2)/sqrt(a + b*sec(c + d*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.650 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2
)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3943, 2742, 2740}

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
```

```
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\left(\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 1.00

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A]
time = 0.20, size = 153, normalized size = 2.28

method	result	si
default	$-\frac{2\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}}{d(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{a-b}{a+b}}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(
d*x+c)))^(1/2)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.54, size = 146, normalized size = 2.18

$$\frac{-i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + I
*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec{\left(c + dx \right)}}}{\sqrt{a + b \sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.651 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=142

$$-\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] -2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(
1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c
))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx = \frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a} - \frac{b\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}$$

$$= -\frac{\left(b\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}}{a\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c}{a}}}}{a\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2}{a}$$

Mathematica [A]
time = 1.75, size = 96, normalized size = 0.68

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*a)
/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])*Sqrt[Sec[c + d*x]])/(a
*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(188) = 376$.
time = 0.23, size = 736, normalized size = 5.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+Elliptic
```

```
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*a-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x
+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a-((a-b)/(a+b))^(1/2)*cos(d*x+c)*a+((a
-b)/(a+b))^(1/2)*cos(d*x+c)*b-((a-b)/(a+b))^(1/2)*b)/(1/cos(d*x+c))^(1/2)/s
in(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.40, size = 355, normalized size = 2.50

$$\frac{2i\sqrt{2}\sqrt{a}\,b\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)-2i\sqrt{2}\sqrt{a}\,b\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)+3i\sqrt{2}a^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3i\sqrt{2}a^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)
/a) - 2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b
)/a) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a))
- 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^2
*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}\,\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.652 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=195

$$\frac{2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{4bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*s
ec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)/a/d/sec(d*x+c)^(1/2)-4/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+
c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3948, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (4*
b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3948

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\int \frac{2b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c}}{3a}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(2b)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{3a^2}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(1+\frac{2b^2}{a^2}\right)\sqrt{b+a\cos(c+dx}}{3\sqrt{}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(1+\frac{2b^2}{a^2}\right)\sqrt{\frac{b+a\cos(c+d}{a+b}}}{}$$

$$= \frac{2\left(1+\frac{2b^2}{a^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 0.42, size = 147, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(-4b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+2a(b+a\cos(c+dx))\sin(c+dx)\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-4*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c + d*x])*
Sin[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(231) = 462$.
time = 0.24, size = 1024, normalized size = 5.25

method	result	size

default	Expression too large to display	1024

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*cos(d*x+c)*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*co
s(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*b^2-cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*cos(d*x+c)*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-cos(d*
x+c)^3*((a-b)/(a+b))^(1/2)*a^2+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+((a-b)/
(a+b))^(1/2)*a^2*cos(d*x+c)-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+2*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*b^2+((a-b)/(a+b))^(1/2)*a*b-2*((a-b)/(a+b))^(1/2)*b^
2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))/a^2/((a-b)
/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.34, size = 415, normalized size = 2.13

$$\frac{6\,a^{2}\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-6i\,\sqrt{2}\,a^{\frac{3}{2}}b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)+6i\,\sqrt{2}\,a^{\frac{3}{2}}b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)+\sqrt{2}\,(-3i\,a^{2}-4i\,b^{2})\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+\sqrt{2}\,(3i\,a^{2}+4i\,b^{2})\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)}{9\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) - 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)
/a)) + 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
) + sqrt(2)*(-3*I*a^2 - 4*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + 4*I*b^2)*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.653 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=249

$$-\frac{2b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+8b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a}}{15a^3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{s}}$$

```
[Out] -2/15*b*(7*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(
1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/5*sin(d*x+c)*(a+b*sec(
d*x+c))^(1/2)/a/d/sec(d*x+c)^(3/2)-8/15*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)
/a^2/d/sec(d*x+c)^(1/2)+2/15*(9*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*
sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3948, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^2d\sqrt{\sec(c+dx)}}-\frac{2b(7a^2+8b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+8b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^3*d*Sqrt[a + b*Sec[c + d*x]])
 + (2*(9*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
 + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (
8*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3948

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{\int \frac{4b-3a\sec(c+dx)-2b\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}}{5a}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\,\sin(c}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\,\sin(c}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\,\sin(c}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\,\sin(c}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+}}{15a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 0.52, size = 193, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(4(9a^3+9a^2b+8ab^2+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)-4b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+2a(3a^2-8b^2-2ab\cos(c+dx)+3a^2\cos(2(c+dx)))\sin(c+dx)\right)}{30a^3d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - 4*b*(7*a^2 + 8*b
^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
] + 2*a*(3*a^2 - 8*b^2 - 2*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(30*a^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(279) = 558$.
time = 0.22, size = 1736, normalized size = 6.97

method	result	size
default	Expression too large to display	1736

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(9*cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-8*
cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*b^3+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-8*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-9*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin
(d*x+c)+8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b^2*sin(d*x+c)-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-8*((a-b)/(a+b))^(1/2)*b^3-cos(d
*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+
10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*
b^2-9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^3+2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-8*cos(d*x+c)*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b^2-9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
```

```
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+8*cos(d*x+c)*sin(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
2-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^3*sin(d*x+c)+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-9*((a-b)/(a+b))^(1/2)*a^2*b+4*((a-b)/(a
+b))^(1/2)*a*b^2+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6*cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a^3-9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+8*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*b^3)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d
*x+c))/a^3/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.77, size = 464, normalized size = 1.86

$$-\frac{4\sqrt{2}\left(-3i\,a^2b-4i\,b^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+4\sqrt{2}\left(3i\,a^2b+4i\,b^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+3\sqrt{2}\left(-9i\,a^3-8i\,ab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)+3\sqrt{2}\left(9i\,a^3+8i\,ab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)-\frac{6\left(3\,a^3\cos(dx+c)^2-4\,a^2b\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{45\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -1/45*(4*sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*
a*sin(d*x + c) + 2*b)/a) + 4*sqrt(2)*(3*I*a^2*b + 4*I*b^3)*sqrt(a)*weierstr
assPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(-9*I*a^3 - 8*I*a*b
^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(9*
I*a^3 + 8*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))
- 6*(3*a^3*cos(d*x + c)^2 - 4*a^2*b*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sec{(c + dx)}} \sec^{\frac{5}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.654 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$\frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}}-\frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c}}{b^2d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] -2*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d
/(a+b*sec(d*x+c))^(1/2)-3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))
/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c))^(1/2)-(3*a^2-b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+(3*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)
*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d
```

Rubi [A]

time = 0.69, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3930, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{b^2d(a^2-b^2)}-\frac{(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{3a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (3*a*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((3*a^2 - b^2)*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[S
ec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
+ b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}(3a^2-}{\sqrt{a+b\sec(c+dx)}}}{b\,(a^2-b^2)}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+}}{b^2\,(a^2-b^2)\,d}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+}}{b^2\,(a^2-b^2)\,d}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+}}{b^2\,(a^2-b^2)\,d}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+}}{b^2\,(a^2-b^2)\,d}$$

$$= -\frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec^{\frac{3}{2}}(c+}{b\,(a^2-b^2)\,d\sqrt{}}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\frac{b+a\cos(c+d}{a+b}}}{b^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.83, size = 478, normalized size = 1.39

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{a(b+a\cos(c+dx))^{3/2}\left(\frac{8ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(9a^2-7b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i(3a^2-b^2)\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a^2\sqrt{\frac{1}{a-b}}\,b}\right)}{(a-b)b^2(a+b)}+\frac{4(b+a\cos(c+dx))(-a^2b+b^3+(-3a^3+ab^2)\cos(c+dx))\tan(c+dx)}{-a^2b^2+b^4}\right)}{4d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-((a*(b + a*Cos[c + d*x])^(3/2)*((8*a*b*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[
c + d*x]] + (2*(9*a^2 - 7*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticP
i[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(3*a^2
- b^2)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))
/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[
Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*Ellipti
cPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
b)/(a + b)])))/(a^2*Sqrt[(a - b)^(-1)]*b)))/((a - b)*b^2*(a + b))) + (4*(b
+ a*Cos[c + d*x])*(-(a^2*b) + b^3 + (-3*a^3 + a*b^2)*Cos[c + d*x])*Tan[c +
d*x])/(-(a^2*b^2) + b^4)))/(4*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.45, size = 1501, normalized size = 4.35

method	result	size
default	Expression too large to display	1501

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+4*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*cos(d*x
+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2+sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-6*cos(d*x+c)^2*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/
(a+b))^(1/2))*a^2-6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+6*cos(d*x+c)*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2+4*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+cos(
```

```
d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*b^2-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-6*sin(d*x+c
)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a
-b),I/((a-b)/(a+b))^(1/2))*a*b+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+cos(d
*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-3*((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)+cos(d*
x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1/2)*b^
2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(7/2)/(b
+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/(a+b)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.655 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=206

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}}+\frac{2aE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b\,(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}-$$

```
[Out] -2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c
),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2
)/b/d/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c)
)^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3930, 4193, 3944, 2886, 2884, 21, 3941, 2734, 2732}

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\,(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\,(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(a^2 - b^2)*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
```

```
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,d}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b} - \frac{2\int \frac{}{\sqrt{\sec(c}}}{}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)} + \frac{\left(\sqrt{b+a}\right.}{}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)}{b\sqrt{a+b\sec(c+}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sqrt{\sec(c+dx}}{b\left(a^2-b^2\right)d\sqrt{a-}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{2aE\left(\frac{1}{2}(c+dx\right.}{b\left(a^2-b^2\right)d\sqrt{\frac{b-}{}}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 14.21, size = 434, normalized size = 2.11

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{(b+a\cos(c+dx))^{3/2}\left(\frac{4ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(3a^2-2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{\sqrt{\frac{1}{a-b}}\,b}\right)}{(a-b)(a+b)}+\frac{4a^2(b+a\cos(c+dx))\sin(c+dx)}{-a^2+b^2}\right)}{2bd(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(((b + a*Cos[c + d*x])^(3/2)*((4*a*b*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c +
d*x]] + (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*Sqrt[-((a*(
-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c +
d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
+ d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*Arc
Sinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sq
rt[(a - b)^(-1)]*b)))/((a - b)*(a + b)) + (4*a^2*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/(-a^2 + b^2)))/(2*b*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 1144, normalized size = 5.55

method	result	size
default	Expression too large to display	1144

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*a+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b-EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*a-2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-2*cos(d*x+c)*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^
(1/2))*b+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
```

```
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*sin(d*x
+c)-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/(
(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)*a-((a-b)/(a
+b))^(1/2)*a)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c
))^(5/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/(a+b)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.656 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)/d/((b+a*cos(d*x+c))/(
a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3929, 21, 3941, 2734, 2732}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/((a^2 -
 b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 3929

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{a+b\sec(c+dx)}\,\int \sqrt{b+a\cos(c+dx)}}{(a^2-b^2)\,\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+d}}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{a+b\sec(c+dx)}\,\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c}{a+}}}{(a^2-b^2)\,\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c}}$$

$$= -\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{\sec(c+dx)}\,\sin(c}{(a^2-b^2)\,d\sqrt{a+b\sec(c}}$$

Mathematica [A]
time = 0.19, size = 103, normalized size = 0.82

$$-\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left((a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-a\sin(c+dx)\right)}{(a-b)(a+b)d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((a + b)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - a*Sin[c + d*x]))/((a -
 b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(145) = 290.
time = 0.19, size = 501, normalized size = 3.98

method	result
default	$-\frac{2\left(\cos(dx+c)\sin(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)-\cos(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))-cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))+((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)-((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)+cos
(d*x+c)*((a-b)/(a+b))^(1/2)-((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+
c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/(b+a*cos(d*x+c))/sin(d*x+c)/(a+
b)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 488, normalized size = 3.87

$$\frac{6\,a^{2}\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\sqrt{2}\left(-i\,ab\cos\left(dx+c\right)-i\,b^{2}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)-\sqrt{2}\left(i\,ab\cos\left(dx+c\right)+i\,b^{2}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+3\,\sqrt{2}\left(-i\,a^{2}\cos\left(dx+c\right)-i\,ab\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)+3\,\sqrt{2}\left(i\,a^{2}\cos\left(dx+c\right)+i\,ab\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}},\frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)}{3\left(\left(a^{4}-a^{2}b^{2}\right)d\cos\left(dx+c\right)+\left(a^{3}b-ab^{3}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) - sqrt(2)*(-I*a*b*cos(d*x + c) - I*b^2)*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(2)*(I*a*b*cos(d*x + c) + I*b^2)*sqr
t(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a
^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(-I*a^
2*cos(d*x + c) - I*a*b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)
/a)) + 3*sqrt(2)*(I*a^2*cos(d*x + c) + I*a*b)*sqrt(a)*weierstrassZeta(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I
*a*sin(d*x + c) + 2*b)/a)))/((a^4 - a^2*b^2)*d*cos(d*x + c) + (a^3*b - a*b^
3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.657 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=200

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}-$$

```
[Out] -2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a/d/(
a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)
/a/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3928, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3928

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)
*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Si
mp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
 && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{b}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,a}{a^2-b^2}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} + \frac{b\int \frac{\sqrt{a+b}}{\sqrt{\sec}}}{a(}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{a\sqrt{a+b\sec(c+d}}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)}{a\sqrt{a+b\sec(c-}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c+dx\right.}{a\,(a^2-b^2)\,d\sqrt{\frac{b+}{}}}$$

Mathematica [A]
time = 0.41, size = 156, normalized size = 0.78

$$\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-ab\sin(c+dx)\right)}{a(a-b)(a+b)d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(b*(a + b)*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] - a*b*Sin[c
 + d*x]))/(a*(a - b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(242) = 484.
time = 0.19, size = 506, normalized size = 2.53

method	result

default	$-\frac{2\left(\text{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)\cos(dx+c)\sin(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,a+\text{Elliptic}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*a+EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*
x+c)-((a-b)/(a+b))^(1/2)*cos(d*x+c)*b+((a-b)/(a+b))^(1/2)*b)*cos(d*x+c)*(1/
cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(
d*x+c)/(a+b)/((a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.55, size = 525, normalized size = 2.62

$$-\frac{6\,a^2b\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}\,(3i\,a^2b-2i\,b^3+(3i\,a^3-2i\,ab^2)\cos(dx+c))\sqrt{a}\,\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\,(-3i\,a^2b+2i\,b^3+(-3i\,a^3+2i\,ab^2)\cos(dx+c))\sqrt{a}\,\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\,(i\,a^2b\cos(dx+c)+i\,ab^2)\sqrt{a}\,\text{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\,(-i\,a^2b\cos(dx+c)-i\,ab^2)\sqrt{a}\,\text{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{3\left((a^5-a^3b^2)d\cos(dx+c)+(a^4b-a^2b^3)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^2*b*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + sqrt(2)*(3*I*a^2*b - 2*I*b^3 + (3*I*a^3 - 2*I*a*b^2)*cos(d*x +
 c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*
```

```
(-3*I*a^2*b + 2*I*b^3 + (-3*I*a^3 + 2*I*a*b^2)*cos(d*x + c))*sqrt(a)*weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*
a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(I*a^2*b*cos(d*x
+ c) + I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3
*sqrt(2)*(-I*a^2*b*cos(d*x + c) - I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*
sin(d*x + c) + 2*b)/a)))/((a^5 - a^3*b^2)*d*cos(d*x + c) + (a^4*b - a^2*b^3
)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.658 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a^2(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] 2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-4*b*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/
a^2/d/(a+b*sec(d*x+c))^(1/2)+2*(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*
sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)
^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3932, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-4*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)
*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}+b^2+\frac{1}{2}ab\sec(c\cdots}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\,\mathrm{s}\cdots}}}{a\,(a^2-b^2)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{(2b)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}}{a^2}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2b\sqrt{b+a\cos(c+dx)}\,\sqrt{\mathrm{s}\cdots}\right.}{a^2\sqrt{\cdots}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\cdots}\right.}{a^2\cdots}$$

$$= -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2(\cdots}{a\cdots}$$

Mathematica [A]
time = 0.45, size = 165, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left((a^3+a^2b-2ab^2-2b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+b\left(-2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+ab\sin(c+dx)\right)\right)}{a^2(a-b)(a+b)d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + b*(-2*(a^2 - b^2)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*
b*Sin[c + d*x])))/(a^2*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(256) = 512$.
time = 0.23, size = 999, normalized size = 4.67

method	result	size
default	Expression too large to display	999

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*cos(d*
x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*cos(d*x+c)*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*sin(d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)-cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^2-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+((a-b)/(a+b))
^(1/2)*a^2*cos(d*x+c)-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2+((a-b)/(a+b))^(1
/2)*a*b+2*((a-b)/(a+b))^(1/2)*b^2)/(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/si
n(d*x+c)/((a-b)/(a+b))^(1/2)/(a+b)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.40, size = 565, normalized size = 2.64

$$\frac{6\,a^2b^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - \sqrt{2}\left(-5i\,a^2b^2+4i\,b^4+(-5i\,a^3b+4i\,ab^3)\cos(dx+c)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right) - \sqrt{2}\left(5i\,a^2b^2-4i\,b^4+(5i\,a^3b-4i\,ab^3)\cos(dx+c)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right) + 3\sqrt{2}\left(i\,a^3b-2i\,ab^3+(i\,a^4-2i\,a^2b^2)\cos(dx+c)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right) + 3\sqrt{2}\left(-i\,a^3b+2i\,ab^3+(-i\,a^4+2i\,a^2b^2)\cos(dx+c)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{3\left((a^6-a^4b^2)d\cos(dx+c)+(a^5b-a^3b^3)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^2*b^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) - sqrt(2)*(-5*I*a^2*b^2 + 4*I*b^4 + (-5*I*a^3*b + 4*I*a*b^3)*co
s(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - s
qrt(2)*(5*I*a^2*b^2 - 4*I*b^4 + (5*I*a^3*b - 4*I*a*b^3)*cos(d*x + c))*sqrt(
a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3
, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(I*a^3*b
 - 2*I*a*b^3 + (I*a^4 - 2*I*a^2*b^2)*cos(d*x + c))*sqrt(a)*weierstrassZeta(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
+ 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(-I*a^3*b + 2*I*a*b^3 + (-I*a^4
 + 2*I*a^2*b^2)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a
^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c)
+ 2*b)/a)))/((a^6 - a^4*b^2)*d*cos(d*x + c) + (a^5*b - a^3*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec{(c+dx)})^{\frac{3}{2}}\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.659 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3d\sqrt{a+b\sec(c+dx)}}-\frac{2b(5a^2-8b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^3\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*
(a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d
*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/3*(a^2-4*b^2)*sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)-2/3*b*(5*a^2-8*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((b+a*cos(d*x+c
))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3932, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{2(a^2+8b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\sqrt{a+b\sec(c+dx)}}-\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*(a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*d*Sqrt[a + b*Sec[c + d*x]]) - (2*
b*(5*a^2 - 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}+2b^2+\frac{1}{2}a}{\sec^{\frac{3}{2}}(c+dx)\sqrt{}}}{a}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{}}{3a^2(a^2}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{}}{3a^2(a^2}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{}}{3a^2(a^2}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{}}{3a^2(a^2}$$

$$= \frac{2(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]

time = 0.63, size = 203, normalized size = 0.70

$$\frac{2\sqrt{\sec(c+dx)}\left(b(-5a^3-5a^2b+8ab^2+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a^4+7a^2b^2-8b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+a(b(a^2-4b^2)+a(a^2-b^2)\cos(c+dx))\sin(c+dx)\right)}{3a^3(a-b)(a+b)d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(b*(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^4 + 7*a^2*
b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a
)/(a + b)] + a*(b*(a^2 - 4*b^2) + a*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d*x])
)/(3*a^3*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(321) = 642$.
time = 0.25, size = 1317, normalized size = 4.56

method	result	size
default	Expression too large to display	1317

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(5*cos(d*x+c)*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-8*
cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*b^3-cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-6*cos(d*x+c)*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^2*b-8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+5*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-8*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
3*sin(d*x+c)-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
```

```
/(a-b))^(1/2))*a^2*b*sin(d*x+c)-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-cos(d*x+c)^3*((a-b)/(a+b
))^(1/2)*a^3-cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+4*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*a^2*b+4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a^3-4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+8*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*b^3+((a-b)/(a+b))^(1/2)*a^2*b-4*((a-b)/(a+b))^(1/2)*a*b^2-8
*((a-b)/(a+b))^(1/2)*b^3)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a
*cos(d*x+c))/a^3/(a+b)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.20, size = 632, normalized size = 2.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*(sqrt(2)*(3*I*a^4*b + 16*I*a^2*b^3 - 16*I*b^5 + (3*I*a^5 + 16*I*a^3*b^
2 - 16*I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
+ c) + 2*b)/a) + sqrt(2)*(-3*I*a^4*b - 16*I*a^2*b^3 + 16*I*b^5 + (-3*I*a^5
- 16*I*a^3*b^2 + 16*I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-5*I*a^3*b^2 + 8*I*a*b^4 + (-5*I*a
^4*b + 8*I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
 c) + 2*b)/a)) - 3*sqrt(2)*(5*I*a^3*b^2 - 8*I*a*b^4 + (5*I*a^4*b - 8*I*a^2*
b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))
- 6*((a^5 - a^3*b^2)*cos(d*x + c)^2 + (a^4*b - 4*a^2*b^3)*cos(d*x + c))*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^7
 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec (c+dx))^{\frac{3}{2}} \sec^{\frac{3}{2}} (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.660 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=360

$$-\frac{8b(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5a^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4+8a^2b^2-16b^4)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5a^4\,(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)-8/5*
b*(a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/a^4/d/(a+b*sec(d*x+c))^(1/2)+2/5*(a^2-6*b^2)*sin(d*x+c)*(a+b*
sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sec(d*x+c)^(3/2)-2/5*b*(3*a^2-8*b^2)*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/sec(d*x+c)^(1/2)+2/5*(3*a^4+8
*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2
-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.67, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3932, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b^2\sin(c+dx)}{ad\,(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d\,(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)}-\frac{8b(a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5a^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5a^4d\,(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^3d\,(a^2-b^2)\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-8*b*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(5*a^4*d*Sqrt[a + b*Sec[c + d*x]]) +
(2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^
(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 6*b^2)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) - (2*b*(3*a^2 - 8*b
^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d*Sqrt[Sec[c
+ d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}+3b^2+\frac{1}{2}a\cdots}{\sec^{\frac{5}{2}}(c+dx)\cdots}}{a\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-6b^2)\sqrt{\cdots}}{5a^2\,(a^2\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-6b^2)\sqrt{\cdots}}{5a^2\,(a^2\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-6b^2)\sqrt{\cdots}}{5a^2\,(a^2\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-6b^2)\sqrt{\cdots}}{5a^2\,(a^2\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-6b^2)\sqrt{\cdots}}{5a^2\,(a^2\cdots}$$

$$= -\frac{8b(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+\cdots}}{5a^4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 0.91, size = 250, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(4(3a^5+3a^4b+8a^3b^2+8a^2b^3-16ab^4-16b^5)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-16b(a^4+3a^2b^2-4b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a(a^4-7a^2b^2+16b^4-4ab(a^2-b^2)\cos(c+dx)+(a^4-a^2b^2)\cos(2(c+dx)))\sin(c+dx)\right)}{10a^4(a-b)(a+b)d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*(3*a^5 + 3*a^4*b + 8*a^3*b^2 + 8*a^2*b^3 - 16*a*b^4
- 16*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(
a + b)] - 16*b*(a^4 + 3*a^2*b^2 - 4*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(a^4 - 7*a^2*b^2 + 16*b^4 - 4*
a*b*(a^2 - b^2)*Cos[c + d*x] + (a^4 - a^2*b^2)*Cos[2*(c + d*x)])*Sin[c + d*
x]))/(10*a^4*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1860 vs. $2(386) = 772$.

time = 0.22, size = 1861, normalized size = 5.17

method	result	size
default	Expression too large to display	1861

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)^4*((a-b)/(a+b))^(1/2
)*a^3*b-2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+2*cos(d*x+c)^2*((a-b)/(a
+b))^(1/2)*a^3*b+8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3+2*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^3*b-6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-2*cos(d*x+c)
^3*((a-b)/(a+b))^(1/2)*a^3*b+8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2-3*c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^4+8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)-4*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)-12*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2
*sin(d*x+c)-16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b^3*sin(d*x+c)+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)-16*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)-3*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*si
n(d*x+c)-3*((a-b)/(a+b))^(1/2)*a^3*b-8*((a-b)/(a+b))^(1/2)*a*b^3-16*((a-b)/
(a+b))^(1/2)*b^4+cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4+2*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a^4+3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-16*cos(d*x+c)*sin(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+
8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^2*b^2-4*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
```

```
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-12*cos(d*x+c)
*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*b^2-16*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+16*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*b^4-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4)*cos(d*x+c)^3*(1/cos(d*x
+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))/a^4/(a+b)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.76, size = 692, normalized size = 1.92

$$-\frac{\sqrt{2}\left(-9i a^4b^2-28i a^2b^4+32i b^6+\left(-9i a^5b-28i a^3b^3+32i ab^5\right)\cos(dx+c)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(9ia^4b^2+28ia^2b^4-32ib^6+\left(9ia^5b+28ia^3b^3-32iab^5\right)\cos(dx+c)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(3ia^5b+8ia^3b^3-16iab^5+\left(3ia^6+8ia^4b^2-16ia^2b^4\right)\cos(dx+c)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(-3ia^5b-8ia^3b^3+16iab^5+\left(-3ia^6-8ia^4b^2+16ia^2b^4\right)\cos(dx+c)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)-\frac{6\left(\left(a^6-a^4b^2\right)\cos(dx+c)^3-2\left(a^5b-a^3b^3\right)\cos(dx+c)^2-\left(3a^4b^2-8a^2b^4\right)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\left(\left(a^8-a^6b^2\right)d\cos(dx+c)+\left(a^7b-a^5b^3\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(sqrt(2)*(-9*I*a^4*b^2 - 28*I*a^2*b^4 + 32*I*b^6 + (-9*I*a^5*b - 28*I
*a^3*b^3 + 32*I*a*b^5)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*s
in(d*x + c) + 2*b)/a) + sqrt(2)*(9*I*a^4*b^2 + 28*I*a^2*b^4 - 32*I*b^6 + (9
*I*a^5*b + 28*I*a^3*b^3 - 32*I*a*b^5)*cos(d*x + c))*sqrt(a)*weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(3*I*a^5*b + 8*I*a^3*b^3 -
 16*I*a*b^5 + (3*I*a^6 + 8*I*a^4*b^2 - 16*I*a^2*b^4)*cos(d*x + c))*sqrt(a)*
weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(-3*I*a^5*b - 8
*I*a^3*b^3 + 16*I*a*b^5 + (-3*I*a^6 - 8*I*a^4*b^2 + 16*I*a^2*b^4)*cos(d*x +
 c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*((a^6 - a^
4*b^2)*cos(d*x + c)^3 - 2*(a^5*b - a^3*b^3)*cos(d*x + c)^2 - (3*a^4*b^2 - 8
*a^2*b^4)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/((a^8 - a^6*b^2)*d*cos(d*x + c) + (a^7*b - a^5*b^3)*d
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.661 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=458

$$\frac{(5a^2-3b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}-\frac{5a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{b^3d\sqrt{a+b\sec(c+dx}}$$

```
[Out] -2/3*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-2
/3*a^2*(5*a^2-9*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec
(d*x+c))^(1/2)+1/3*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c
))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-5*a
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1
/2)/b^3/d/(a+b*sec(d*x+c))^(1/2)-1/3*(15*a^4-26*a^2*b^2+3*b^4)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(
a+b))^(1/2)/sec(d*x+c)^(1/2)+1/3*(15*a^4-26*a^2*b^2+3*b^4)*sin(d*x+c)*sec(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.97, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3930, 4183, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2(5a^2-9b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{(5a^2-3b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^2d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{(15a^4-26a^2b^2+3b^4)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{3b^3d(a^2-b^2)^2}-\frac{(15a^4-26a^2b^2+3b^4)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^3d(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{5a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^3d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((5*a^2 - 3*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c +
d*x]]) - (5*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^3*d*Sqrt[a + b*Sec[c + d*x]]) - ((15
*a^4 - 26*a^2*b^2 + 3*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) - (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*b*(a^2 - b^
2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a^2*(5*a^2 - 9*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + ((15*a^4
 - 26*a^2*b^2 + 3*b^4)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(3*b^3*(a^2 - b^2)^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
```

```
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
```

```
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a^2}{2}-\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}(5a^2-3}{(a+b\sec(c+dx))^{3/2}}}{3b\,(a^2-b^2)}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(5a^2-9b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3b^2\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(5a^2-9b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3b^2\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(5a^2-9b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3b^2\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(5a^2-9b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3b^2\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(5a^2-9b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+d}{3b^2\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{5a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^3d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec^{\frac{5}{2}}(}{3b\,(a^2-b^2)\,d}$$

$$= \frac{(5a^2-3b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b^2\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{5a\sqrt{\frac{b-}{}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.82, size = 561, normalized size = 1.22

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{a(b+a\cos(c+dx))^{5/2}\left(\frac{8ab(5a^2-9b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(45a^4-86a^2b^2+33b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i(15a^4-26a^2b^2+3b^4)\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)\right)\right)}{a^2\sqrt{\frac{1}{a-b}}\,b}\right)}{(a-b)^2(a+b)^2}+\frac{2(b+a\cos(c+dx))(15a^6-20a^4b^2-9a^2b^4+6b^6+4ab(10a^4-17a^2b^2+3b^4)\cos(c+dx)+(15a^6-26a^4b^2+3a^2b^4)\cos(2(c+dx)))\tan(c+dx)}{(a^2-b^2)^2}\right)}{12b^3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-((a*(b + a*Cos[c + d*x])^(5/2)*((8*a*b*(5*a^2 - 9*b^2
)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])
/Sqrt[b + a*Cos[c + d*x]] + (2*(45*a^4 - 86*a^2*b^2 + 33*b^4)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a
*Cos[c + d*x]] + ((2*I)*(15*a^4 - 26*a^2*b^2 + 3*b^4)*Sqrt[-((a*(-1 + Cos[c
 + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b
*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]],
(-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b +
a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a^2*Sqrt[(a
- b)^(-1)]*b)))/((a - b)^2*(a + b)^2)) + (2*(b + a*Cos[c + d*x])*(15*a^6 -
20*a^4*b^2 - 9*a^2*b^4 + 6*b^6 + 4*a*b*(10*a^4 - 17*a^2*b^2 + 3*b^4)*Cos[c
+ d*x] + (15*a^6 - 26*a^4*b^2 + 3*a^2*b^4)*Cos[2*(c + d*x)])*Tan[c + d*x])/
(a^2 - b^2)^2))/(12*b^3*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 4591, normalized size = 10.02

method	result	size
default	Expression too large to display	4591

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(-5*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b+3*cos(d*x+c)^3*((a-b)/(a+b
))^(1/2)*a^2*b^3-29*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2+30*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2
))*cos(d*x+c)^2*sin(d*x+c)*a^5-30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5-30*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*cos(d*x+c)^2*sin(d*x+c)*a*b^4-50*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b+16*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d
*x+c)^2*sin(d*x+c)*a^3*b^2+54*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^3+6*cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a*b^4+20*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b+5*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2-29*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3
-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4+3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
```

os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5+15*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(
d*x+c)^3*sin(d*x+c)*a^5+15*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5+3*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*b^5-15*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5-3*((a-b)/
(a+b))^(1/2)*b^5+30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^5-30*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)
^3*sin(d*x+c)*a^5+15*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5+3*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*
b^5+3*((a-b)/(a+b))^(1/2)*a^3*b^2+3*((a-b)/(a+b))^(1/2)*a^2*b^3-3*((a-b)/(a
+b))^(1/2)*a*b^4+21*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^2-15*cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a^4*b+23*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^3+18*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*a*b^4+30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b+
30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^2-30*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*
sin(d*x+c)*a^2*b^3-30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^4-26*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+
c)^3*sin(d*x+c)*a^3*b^2+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^4+30*cos(d*x+c)^3*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*a^4*b-30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b^
2-30*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^3-20*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))

```
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.662 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{se}}}{b^2d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] -2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-2
/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*sec
(d*x+c))^(1/2)-2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1
/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a
/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d/(a+b*s
ec(d*x+c))^(1/2)+2/3*a*(3*a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*
x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/
2)
```

Rubi [A]
time = 0.75, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3930, 4183, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2
)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*b^2*(a
^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a
^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a
^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
```

```
b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-\frac{3}{2}ab\sec(c+dx)-\frac{3}{2}(a\right.}{(a+b\sec(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec^{\frac{3}{2}}(c+d}{3b\left(a^2-b^2\right)d(a+}$$

$$= -\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\frac{b+a\cos(c+}{a+b}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.48, size = 487, normalized size = 1.32

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{4ab^2(a^2-3b^2)\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{5/2}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a-b)^2}+\frac{b(9a^4-19a^2b^2+6b^4)\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{5/2}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a-b)^2}+\frac{i\left(\frac{1}{a-b}\right)^{3/2}(3a^2-7b^2)\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}(b+a\cos(c+dx))^{5/2}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{(a+b)^2}+\frac{2a^2b^2(b+a\cos(c+dx))\sin(c+dx)}{-a^2+b^2}+\frac{2a^2b(-3a^2+7b^2)(b+a\cos(c+dx))^2\sin(c+dx)}{(a^2-b^2)^2}\right)}{3b^3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*((4*a*b^2*(a^2 - 3*b^2)*((b + a*Cos[c + d*x])/(a + b))^
(5/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(a - b)^2 + (b*(9*a^4 - 19*a^2
*b^2 + 6*b^4)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)])/(a - b)^2 + (I*((a - b)^(-1))^(3/2)*(3*a^2 - 7*b^2)*Sqrt
[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*(
b + a*Cos[c + d*x])^(5/2)*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sq
rt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Elli
pticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
+ d*x]]], (-a + b)/(a + b)])))/(a + b)^2 + (2*a^2*b^2*(b + a*Cos[c + d*x])*
Sin[c + d*x])/(-a^2 + b^2) + (2*a^2*b*(-3*a^2 + 7*b^2)*(b + a*Cos[c + d*x])
^2*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^3*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 3854, normalized size = 10.42

method	result	size
default	Expression too large to display	3854

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+3*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a^3*b+7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-7*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a*b^3-6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2+6*cos(d*x+c)*sin(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^4-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^3*b*sin(d*x+c)+7*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+6*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)+4*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
^2*sin(d*x+c)-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^3*sin(d*x+c)-4*((a-b)/(a+b))^(1/2)*a^3*b-((a-b)/(a+b))^
(1/2)*a^2*b^2+7*((a-b)/(a+b))^(1/2)*a*b^3+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*a^4+4*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
```

in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-9*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-3*c
os(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a*b^3-6*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+
6*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+7*cos(d*x+c)^2*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2
*b^2+6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2-12*cos(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*sin(d*x+c)*a^3*b+12*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-3*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*s
in(d*x+c)+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-6*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b
))^(1/2))*a^3*b*sin(d*x+c)-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+6*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2
))*a*b^3*sin(d*x+c)+6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-6*cos(d*x+c)^2*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)
/(a+b))^(1/2))*a^4-3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-3*cos(d*x+c)*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^

```
4-6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.663 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{8bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}-$$

```
[Out] -2/3*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2
/3*a*(a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(
d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+8/3*b*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(
1/2))*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/s
ec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3930, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*(a^2 - b^2)^
2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) +
(2*a*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sq
rt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}-\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}\left(a^2-3b^2\right)\sec^2(c+dx}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}}{3b\,(a^2-b^2)}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2a(a^2-5b^2)\sqrt{\sec(c+dx)}\,\sin(c+ }{3b\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2a(a^2-5b^2)\sqrt{\sec(c+dx)}\,\sin(c+ }{3b\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2a(a^2-5b^2)\sqrt{\sec(c+dx)}\,\sin(c+ }{3b\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2a(a^2-5b^2)\sqrt{\sec(c+dx)}\,\sin(c+ }{3b\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c+dx\right.}{3\,(a^2-b^2)^2\,d\sqrt{\frac{b }{}}}$$

Mathematica [A]
time = 0.71, size = 169, normalized size = 0.61

$$-\frac{2\sec^{\frac{3}{2}}(c+dx)\left(-4b(a+b)^2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-(a-b)(a+b)^2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+a(-a^2+5b^2+4ab\cos(c+dx))\sin(c+dx)\right)}{3(a-b)^2(a+b)^2d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sec[c + d*x]^(3/2)*(-4*b*(a + b)^2*((b + a*Cos[c + d*x])/(a + b))^(3/2)
*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - (a - b)*(a + b)^2*((b + a*Cos[c +
d*x])/(a + b))^(3/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(-a^2 + 5*b^
2 + 4*a*b*Cos[c + d*x])*Sin[c + d*x]))/(3*(a - b)^2*(a + b)^2*d*(a + b*Sec[
c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1342 vs. $2(307) = 614$.
time = 0.24, size = 1343, normalized size = 4.85

method	result	size
default	Expression too large to display	1343

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*cos(d*x
+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b+cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*cos(d*x+c)*sin(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-
3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b^2+4*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*cos(d*x+c)*sin(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*b^2+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
```

```
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+4*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a^2-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+4*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a*b-4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/
2)*a^2-((a-b)/(a+b))^(1/2)*a*b+4*((a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(d*x+c))
/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*
x+c))^2/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 688, normalized size = 2.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(sqrt(2)*(-3*I*a^2*b^2 - I*b^4 + (-3*I*a^4 - I*a^2*b^2)*cos(d*x + c)^2
- 2*(3*I*a^3*b + I*a*b^3)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*
a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2*b^2 + I*b^4 + (3*I*a^4 + I*a^2*
b^2)*cos(d*x + c)^2 - 2*(-3*I*a^3*b - I*a*b^3)*cos(d*x + c))*sqrt(a)*weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*
a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 12*sqrt(2)*(-I*a^3*b*cos(d*
x + c)^2 - 2*I*a^2*b^2*cos(d*x + c) - I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a)) - 12*sqrt(2)*(I*a^3*b*cos(d*x + c)^2 + 2*I*a^2*
b^2*cos(d*x + c) + I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
2*b)/a)) - 6*(4*a^3*b*cos(d*x + c)^2 - (a^4 - 5*a^2*b^2)*cos(d*x + c))*sqrt
((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^7
```

```
- 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d
*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.664 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{2\left(3a^2+b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*(a
^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)-2/
3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1
/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1
/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3929, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(
3*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^(3/2)) + (4*(a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^
2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3929

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}+\frac{2\int\frac{-\frac{a}{2}-\frac{3}{2}b\sec(c+dx)+a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}+\frac{4(a^2+b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}-$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}+\frac{4(a^2+b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}-$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}+\frac{4(a^2+b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}-$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}+\frac{4(a^2+b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}-$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{2(3a^2+b^2)\,E(}{3a\left(a^2-b^2\right)^2d}$$

Mathematica [A]
time = 0.72, size = 178, normalized size = 0.63

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{2(a+b)\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{5/2}\left((3a^2+b^2)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a-b)bF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{a(a-b)^2}+\frac{2(b+a\cos(c+dx))(2b(a^2+b^2)+a(3a^2+b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}\right)}{3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*((-2*(a + b)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*((3*a
^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a - b)*b*EllipticF[(c +
d*x)/2, (2*a)/(a + b)]))/(a*(a - b)^2) + (2*(b + a*Cos[c + d*x])*(2*b*(a^2
+ b^2) + a*(3*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*d*(
a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1821 vs. $2(311) = 622$.
time = 0.20, size = 1822, normalized size = 6.48

method	result	size
default	Expression too large to display	1822

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-3*sin(d
*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^3-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+3*cos(d*x+c)*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^3+2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*cos(d*x+c
)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
```

```
a-b))^(1/2))*a^3-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-
cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*b^3+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-3*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d
*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^3*sin(d*x+c)+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a^2*b-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+3*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((
a-b)/(a+b))^(1/2)*b^3-2*((a-b)/(a+b))^(1/2)*a^2*b+((a-b)/(a+b))^(1/2)*a*b^2
-((a-b)/(a+b))^(1/2)*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c)
)^(3/2)*cos(d*x+c)^2/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)/(a+b)^2/a/((a-b)/(
a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.14, size = 759, normalized size = 2.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/9*(2*sqrt(2)*(-3*I*a^2*b^3 + I*b^5 + (-3*I*a^4*b + I*a^2*b^3)*cos(d*x +
c)^2 + 2*(-3*I*a^3*b^2 + I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a) + 2*sqrt(2)*(3*I*a^2*b^3 - I*b^5 + (3*I*a^
```

```
4*b - I*a^2*b^3)*cos(d*x + c)^2 + 2*(3*I*a^3*b^2 - I*a*b^4)*cos(d*x + c))*s
qrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(3*I
*a^3*b^2 + I*a*b^4 + (3*I*a^5 + I*a^3*b^2)*cos(d*x + c)^2 + 2*(3*I*a^4*b +
I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b
)/a)) + 3*sqrt(2)*(-3*I*a^3*b^2 - I*a*b^4 + (-3*I*a^5 - I*a^3*b^2)*cos(d*x
+ c)^2 + 2*(-3*I*a^4*b - I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4
/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) -
 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*((3*a^5 + a^3*b^2)*cos(d*x + c)^2 + 2*(a
^4*b + a^2*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c)^2 +
 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c) + (a^6*b^2 - 2*a^4*b^4 + a^
2*b^6)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.665 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=302

$$\frac{2(3a^2-2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}+\frac{4b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b}}{3a^2(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\;\sqrt{}}$$

```
[Out] -2/3*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-2/3*b
*(5*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(
1/2)+2/3*(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+4/3*b*(3*a^2-
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/((
b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3928, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\,(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\,(a^2-b^2)\,(a+b\sec(c+dx))^{3/2}}+\frac{2(3a^2-2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\,(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{4b(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\,(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]]) + (4*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3928

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)
*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Si
mp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
 && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{b}{2}-\frac{3}{2}a\sec(c+dx)+b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2(3a^2-2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{4b(3a^2}{3a^2\,(a}$$

Mathematica [A]
time = 0.84, size = 196, normalized size = 0.65

$$\frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((6a^2b-2b^3)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(3a^3-3a^2b-2ab^2+2b^3)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{(a-b)^2}+\frac{ab(-5a^2b+b^3+(-6a^3+2ab^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}\right)}{3a^2d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*((6*a^2*b - 2*b^3)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (3*a^3 -
3*a^2*b - 2*a*b^2 + 2*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^
2 + (a*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c + d*x])*Sin[c + d*x])/(
a^2 - b^2)^2))/(3*a^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2069 vs. 2(332) = 664.
time = 0.20, size = 2070, normalized size = 6.85

method	result	size
default	Expression too large to display	2070

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(-6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+3*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a*b^3+6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-6*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a^2*b^2-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a^2*b^2+3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+6*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)+
3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^3*b*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+6*cos(d*x+c)^2*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^3*b-2*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-2*((b+a*cos(d*x+c))/(1+cos(
```

```
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)+5*((a-b)/(
a+b))^(1/2)*a^2*b^2-((a-b)/(a+b))^(1/2)*a*b^3-2*((a-b)/(a+b))^(1/2)*b^4-3*c
os(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^3*b-2*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+3*cos(d*x+c)
^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^4-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+6*cos(d*x+c)*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b
+6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-5*cos(d*x+c)
*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*b^2-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+2*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*b^4)*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.86, size = 811, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(sqrt(2)*(-9*I*a^4*b^2 + 9*I*a^2*b^4 - 4*I*b^6 + (-9*I*a^6 + 9*I*a^4*b^
2 - 4*I*a^2*b^4)*cos(d*x + c)^2 - 2*(9*I*a^5*b - 9*I*a^3*b^3 + 4*I*a*b^5)*c
os(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) +
sqrt(2)*(9*I*a^4*b^2 - 9*I*a^2*b^4 + 4*I*b^6 + (9*I*a^6 - 9*I*a^4*b^2 + 4*I
*a^2*b^4)*cos(d*x + c)^2 - 2*(-9*I*a^5*b + 9*I*a^3*b^3 - 4*I*a*b^5)*cos(d*x
 + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
- 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 6*sqrt
(2)*(-3*I*a^3*b^3 + I*a*b^5 + (-3*I*a^5*b + I*a^3*b^3)*cos(d*x + c)^2 + 2*(
-3*I*a^4*b^2 + I*a^2*b^4)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin
(d*x + c) + 2*b)/a)) - 6*sqrt(2)*(3*I*a^3*b^3 - I*a*b^5 + (3*I*a^5*b - I*a^
3*b^3)*cos(d*x + c)^2 + 2*(3*I*a^4*b^2 - I*a^2*b^4)*cos(d*x + c))*sqrt(a)*w
eierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*
a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*(2*(3*a^5*b - a^3*b^3)*c
os(d*x + c)^2 + (5*a^4*b^2 - a^2*b^4)*cos(d*x + c))*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^9 - 2*a^7*b^2 + a^5*b
^4)*d*cos(d*x + c)^2 + 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c) + (a^
7*b^2 - 2*a^5*b^4 + a^3*b^6)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos \left(c+d x\right)}}}{\left(a+\frac{b}{\cos \left(c+d x\right)}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.666 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{2b(9a^2-8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4-15a^2b^2+8b^4)\,E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{3a^3\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2/3*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+8/
3*b^2*(2*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*
x+c))^(1/2)-2/3*b*(9*a^2-8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*
(3*a^4-15*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/
a^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3932, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{8b^2(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2b(9a^2-8b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4-15a^2b^2+8b^4)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*b*(9*a^2 - 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c +
d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+2b^2+\frac{3}{2}ab\sec(c+dx)-}{\sqrt{\sec(c+dx)}\,(a+b\sec}}{3a\,(a^2-b^2)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2)\,\sqrt{\sec(c+}}{3a^2\,(a^2-b^2)^2\,d\sqrt{a+b}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2)\,\sqrt{\sec(c+}}{3a^2\,(a^2-b^2)^2\,d\sqrt{a+b}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2)\,\sqrt{\sec(c+}}{3a^2\,(a^2-b^2)^2\,d\sqrt{a+b}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2)\,\sqrt{\sec(c+}}{3a^2\,(a^2-b^2)^2\,d\sqrt{a+b}}$$

$$= -\frac{2b(9a^2-8b^2)\,\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\,\sqrt{\sec(c}}{3a^3\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [**A**]
time = 0.88, size = 208, normalized size = 0.66

$$\frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((3a^4-15a^2b^2+8b^4)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+b(-9a^3+9a^2b+8ab^2-8b^3)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{(a-b)^2}+\frac{ab^2(8a^2b-4b^3+a(9a^2-5b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}\right)}{3a^3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*((3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
+ b*(-9*a^3 + 9*a^2*b + 8*a*b^2 - 8*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]))/(a - b)^2 + (a*b^2*(8*a^2*b - 4*b^3 + a*(9*a^2 - 5*b^2)*Cos[c + d*x])
*Sin[c + d*x])/(a^2 - b^2)^2))/(3*a^3*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3102 vs. 2(347) = 694.
time = 0.23, size = 3103, normalized size = 9.79

method	result	size
default	Expression too large to display	3103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-3*cos(d*x+c)^3*((a-b)/(a+b))^(1
/2)*a^4*b+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3-18*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a^3*b^2+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5+9*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+
c)*a^4*b-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2-8*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^
3+12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4+6*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a^4*b+12*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2-18*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a^2*b^3-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4-8*cos(d*x+c)*sin(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*b^5+3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
```

b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*sin(d*x+c)+15*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*sin(d*x+c)+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*sin(d*x+c)+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*sin(d*x+c)-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*sin(d*x+c)-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*sin(d*x+c)+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*sin(d*x+c)-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5-8*((a-b)/(a+b))^(1/2)*b^5-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5+3*((a-b)/(a+b))^(1/2)*a^3*b^2+11*((a-b)/(a+b))^(1/2)*a^2*b^3-4*((a-b)/(a+b))^(1/2)*a*b^4+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^2-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b+4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^3+15*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^4-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+15*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+15*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+12*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-14*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(

```
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.04, size = 863, normalized size = 2.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -1/9*(4*sqrt(2)*(-6*I*a^4*b^3 + 9*I*a^2*b^5 - 4*I*b^7 + (-6*I*a^6*b + 9*I*a
^4*b^3 - 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*(-6*I*a^5*b^2 + 9*I*a^3*b^4 - 4*I*
a*b^6)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a) + 4*sqrt(2)*(6*I*a^4*b^3 - 9*I*a^2*b^5 + 4*I*b^7 + (6*I*a^6*b - 9*I*a
^4*b^3 + 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*(6*I*a^5*b^2 - 9*I*a^3*b^4 + 4*I*a
*b^6)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b
)/a) + 3*sqrt(2)*(-3*I*a^5*b^2 + 15*I*a^3*b^4 - 8*I*a*b^6 + (-3*I*a^7 + 15*
I*a^5*b^2 - 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*(-3*I*a^6*b + 15*I*a^4*b^3 - 8*
I*a^2*b^5)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b
)/a)) + 3*sqrt(2)*(3*I*a^5*b^2 - 15*I*a^3*b^4 + 8*I*a*b^6 + (3*I*a^7 - 15*I
*a^5*b^2 + 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*(3*I*a^6*b - 15*I*a^4*b^3 + 8*I*
a^2*b^5)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/
a)) - 6*((9*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c)^2 + 4*(2*a^4*b^3 - a^2*b^5)*c
os(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^2 + 2*(a^9*b - 2*a^
7*b^3 + a^5*b^5)*d*cos(d*x + c) + (a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sec{\left(c + dx \right)}\right)^{\frac{5}{2}} \sqrt{\sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**(5/2)*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.667 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^4\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{8b(2a^4-7a^2b^2+4b^4)\,E\left(\frac{1}{2}(c\right.}{3a^4\left(a^2-b^2\right)^2d\sqrt{\frac{b+a}{}}}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+4/
3*b^2*(5*a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)/(a+b*sec(
d*x+c))^(1/2)+2/3*(a^4+16*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*
cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d/(a+b*sec(d*x+c))^
(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2
-b^2)^2/d/sec(d*x+c)^(1/2)-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^
(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3932, 4185, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{2(a^4+16a^2b^2-16b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{8b(2a^4-7a^2b^2+4b^4)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(a^4-13a^2b^2+8b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(a^4 + 16*a^2*b^2 - 16*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]]) - (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (4*b^2*(5*a
^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+3b^2\cdots}{\sec^{\frac{3}{2}}(\cdots}}{\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{\cdots}{3a^2\left(a^2-b^2\right)\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{\cdots}{3a^2\left(a^2-b^2\right)\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{\cdots}{3a^2\left(a^2-b^2\right)\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{\cdots}{3a^2\left(a^2-b^2\right)\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{\cdots}{3a^2\left(a^2-b^2\right)\cdots}$$

$$= \frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{se}\cdots}}{3a^4\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 1.13, size = 257, normalized size = 0.66

$$\frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left(-4(2a^4b-7a^2b^3+4b^5)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a^5-a^4b+16a^3b^2-16a^2b^3-16ab^4+16b^5)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{(a-b)^2}+\frac{a\left(a^6-25a^2b^4+16b^6+4ab(a^4-8a^2b^2+5b^4)\cos(c+dx)+(a^3-ab^2)^2\cos(2(c+dx))\right)\sin(c+dx)}{2(a^2-b^2)^2}\right)}{3a^4d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*(-4*(2*a^4*b - 7*a^2*b^3 + 4*b^5)*EllipticE[(c + d*x)/2, (2*a)/(a +
b)] + (a^5 - a^4*b + 16*a^3*b^2 - 16*a^2*b^3 - 16*a*b^4 + 16*b^5)*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + (a*(a^6 - 25*a^2*b^4 + 16*b^6 +
4*a*b*(a^4 - 8*a^2*b^2 + 5*b^4)*Cos[c + d*x] + (a^3 - a*b^2)^2*Cos[2*(c + d
```

```
*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)))/(3*a^4*d*(a + b*Sec[c + d*x])^(5/2)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3613 vs. $2(415) = 830$.

time = 0.24, size = 3614, normalized size = 9.24

method	result	size
default	Expression too large to display	3614

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-16*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^6*sin(d*x+c)+16*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*b^6+cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^6-cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^6+9*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+16*cos(d*x+c)^2*si
n(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^4*b^2-12*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-16*cos(d*x+c)^2*sin(d*x
+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b^4-8*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*b+28*cos(d*x+c)^2*sin(d*x+c)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*
b^3-16*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^5+10*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+25*cos(
d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^4*b^2+4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-28*cos(d*x+c)*sin
(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b^4-16*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos
```

```
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5-8*cos(d*x+c)*sin(d*x+c)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*
b-8*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2+28*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^3+28*cos(d
*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*a^2*b^4-16*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^5-16*((a-b)/(a+b))^(1
/2)*b^6+cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5*b-cos(d*x+c)^4*((a-b)/(a+b))^(
1/2)*a^4*b^2-cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^3-6*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a^5*b-6*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b^2+6*cos(d*x+c)
^3*((a-b)/(a+b))^(1/2)*a^3*b^3+6*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^4+7
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*b-6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
a^4*b^2-34*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^3+8*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*a^2*b^4+24*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^5-2*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a^5*b+14*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^2+22*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^3-34*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^
4-16*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^5+cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6+cos(d
*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^6-16*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^6+(1/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*b*sin(d*x+c)+9*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 949, normalized size = 2.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(sqrt(2)*(-3*I*a^6*b^2 - 37*I*a^4*b^4 + 68*I*a^2*b^6 - 32*I*b^8 + (-3*I
*a^8 - 37*I*a^6*b^2 + 68*I*a^4*b^4 - 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*(3*I*
a^7*b + 37*I*a^5*b^3 - 68*I*a^3*b^5 + 32*I*a*b^7)*cos(d*x + c))*sqrt(a)*wei
erstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*
(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^6*b^2 + 3
7*I*a^4*b^4 - 68*I*a^2*b^6 + 32*I*b^8 + (3*I*a^8 + 37*I*a^6*b^2 - 68*I*a^4*
b^4 + 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*(-3*I*a^7*b - 37*I*a^5*b^3 + 68*I*a^
3*b^5 - 32*I*a*b^7)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 -
 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(
d*x + c) + 2*b)/a) - 12*sqrt(2)*(2*I*a^5*b^3 - 7*I*a^3*b^5 + 4*I*a*b^7 + (2
*I*a^7*b - 7*I*a^5*b^3 + 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*(2*I*a^6*b^2 - 7*I
*a^4*b^4 + 4*I*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 -
 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a)) - 12*sqrt(2)*(-2*I*a^5*b^3 + 7*I*a^3*b^5 - 4*I*a*b^7 + (
-2*I*a^7*b + 7*I*a^5*b^3 - 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*(-2*I*a^6*b^2 +
7*I*a^4*b^4 - 4*I*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*si
n(d*x + c) + 2*b)/a)) + 6*((a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^3 + 2*(
a^7*b - 8*a^5*b^3 + 5*a^3*b^5)*cos(d*x + c)^2 + (a^6*b^2 - 13*a^4*b^4 + 8*a
^2*b^6)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/((a^11 - 2*a^9*b^2 + a^7*b^4)*d*cos(d*x + c)^2 + 2*(a^1
0*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c) + (a^9*b^2 - 2*a^7*b^4 + a^5*b^6)
*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.668 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=474

$$-\frac{2b(17a^4+116a^2b^2-128b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^5(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^6+55a^4b^2-212}{15a^5(a^2-}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)+8/
3*b^2*(3*a^2-2*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/sec(d*x+c)^(3/2)/(a+b*sec(
d*x+c))^(1/2)-2/15*b*(17*a^4+116*a^2*b^2-128*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^5/(a^2-b^2)/d/(a+b*sec(
d*x+c))^(1/2)+2/15*(3*a^4-71*a^2*b^2+48*b^4)*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/a^3/(a^2-b^2)^2/d/sec(d*x+c)^(3/2)-4/15*b*(7*a^4-49*a^2*b^2+32*b^4)*sin
(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)+2/15*(9*a
^6+55*a^4*b^2-212*a^2*b^4+128*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+
c))^(1/2)/a^5/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3932, 4185, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{8b^2(3a^2-2b^2)\sin(c+dx)}{3a^2d(a^2-b^2)^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{4b(7a^4-49a^2b^2+32b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^4d(a^2-b^2)^2\sqrt{\sec(c+dx)}}-\frac{2b(17a^4+116a^2b^2-128b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^5d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4-71a^2b^2+48b^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^3d(a^2-b^2)^2\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9a^6+55a^4b^2-212a^2b^4+128b^6)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^5d(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*b*(17*a^4 + 116*a^2*b^2 - 128*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^5*(a^2 - b^2
)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*
b^6)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^
5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) +
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d
*x])^(3/2)) + (8*b^2*(3*a^2 - 2*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*S
ec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4 - 71*a^2*b^2 + 48*b
^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)^2*d*Sec[c +
d*x]^(3/2)) - (4*b*(7*a^4 - 49*a^2*b^2 + 32*b^4)*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(15*a^4*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+4b^2+\frac{3}{2}\cdots}{\sec^{\frac{5}{2}}(c\cdots}}{\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\cdots}{3a^2\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\cdots}{3a^2\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\cdots}{3a^2\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\cdots}{3a^2\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\cdots}{3a^2\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\cdots}{3a^2\left(a^2-b^2\right)^2\cdots}$$

$$= -\frac{2b(17a^4+116a^2b^2-128b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{\cdots}{a\cdots}\right)\cdots}{15a^5\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}\cdots}$$

Mathematica [A]
time = 1.36, size = 292, normalized size = 0.62

$$\frac{(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((9a^6+55a^4b^2-212a^2b^4+128b^6)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+b(-17a^5+17a^4b-116a^3b^2+116a^2b^3+128ab^4-128b^5)F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)}{(a-b)^2}+a\left(\frac{10b^5\sin(c+dx)}{-a^2+b^2}-\frac{10b^4(-15a^2+11b^2)(b+a\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}-28b(b+a\cos(c+dx))^2\sin(c+dx)+3a(b+a\cos(c+dx))^2\sin(2(c+dx))\right)\right)}{15a^5d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((2*((b + a*Cos[c + d*x])/(a + b))
^(3/2)*((9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*b^6)*EllipticE[(c + d*x)/2,
```

```
(2*a)/(a + b)] + b*(-17*a^5 + 17*a^4*b - 116*a^3*b^2 + 116*a^2*b^3 + 128*a
*b^4 - 128*b^5)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + a*((10*
b^5*Sin[c + d*x])/(-a^2 + b^2) - (10*b^4*(-15*a^2 + 11*b^2)*(b + a*Cos[c +
d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28*b*(b + a*Cos[c + d*x])^2*Sin[c + d*x
] + 3*a*(b + a*Cos[c + d*x])^2*Sin[2*(c + d*x)])))/(15*a^5*d*(a + b*Sec[c +
 d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4585 vs. $2(492) = 984$.
time = 0.31, size = 4586, normalized size = 9.68

method	result	size
default	Expression too large to display	4586

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-72*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^
5*b^2-116*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b^3+96*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^
4+128*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^5+55*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5*b^2-21
2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*a^3*b^4+128*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^6-26*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^6*b-89*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b^2-188*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin
(d*x+c)*a^4*b^3-20*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^4+224*((b+a*cos(d*x+c))/(1+c
```

```
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a^2*b^5+128*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^6+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^6*b+55*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*co
s(d*x+c)*sin(d*x+c)*a^5*b^2+55*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b^3-212*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*a^3*b^4-212*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^5+128*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a*b^6-17*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^6*b-48*((a-b)/(a+b))^(1/2)*cos(d*x+
c)^3*a^2*b^5+128*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*b^7*sin(d*x+c)-9*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^7-128*
((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^7+3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^5*a^7+6
*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^7-9*((a-b)/(a+b))^(1/2)*a^5*b^2+5*((a-b
)/(a+b))^(1/2)*a^4*b^3-50*((a-b)/(a+b))^(1/2)*a^3*b^4-148*((a-b)/(a+b))^(1/
2)*a^2*b^5+64*((a-b)/(a+b))^(1/2)*a*b^6+128*((a-b)/(a+b))^(1/2)*b^7-9*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6*b
*sin(d*x+c)-17*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^5*b^2*sin(d*x+c)-72*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^3*sin(d*x+c)-116*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^4*sin(d*x
+c)+96*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 1036, normalized size = 2.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/45*(2*sqrt(2)*(-21*I*a^6*b^3 - 121*I*a^4*b^5 + 260*I*a^2*b^7 - 128*I*b^9
 + (-21*I*a^8*b - 121*I*a^6*b^3 + 260*I*a^4*b^5 - 128*I*a^2*b^7)*cos(d*x +
c)^2 + 2*(-21*I*a^7*b^2 - 121*I*a^5*b^4 + 260*I*a^3*b^6 - 128*I*a*b^8)*cos(
d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + 2*s
qrt(2)*(21*I*a^6*b^3 + 121*I*a^4*b^5 - 260*I*a^2*b^7 + 128*I*b^9 + (21*I*a^
8*b + 121*I*a^6*b^3 - 260*I*a^4*b^5 + 128*I*a^2*b^7)*cos(d*x + c)^2 + 2*(21
*I*a^7*b^2 + 121*I*a^5*b^4 - 260*I*a^3*b^6 + 128*I*a*b^8)*cos(d*x + c))*sqr
t(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a
^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(-9*I*
a^7*b^2 - 55*I*a^5*b^4 + 212*I*a^3*b^6 - 128*I*a*b^8 + (-9*I*a^9 - 55*I*a^7
*b^2 + 212*I*a^5*b^4 - 128*I*a^3*b^6)*cos(d*x + c)^2 + 2*(-9*I*a^8*b - 55*I
*a^6*b^3 + 212*I*a^4*b^5 - 128*I*a^2*b^7)*cos(d*x + c))*sqrt(a)*weierstrass
Zeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInve
rse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(9*I*a^7*b^2 + 55*I*a^5*b^
4 - 212*I*a^3*b^6 + 128*I*a*b^8 + (9*I*a^9 + 55*I*a^7*b^2 - 212*I*a^5*b^4 +
 128*I*a^3*b^6)*cos(d*x + c)^2 + 2*(9*I*a^8*b + 55*I*a^6*b^3 - 212*I*a^4*b^
5 + 128*I*a^2*b^7)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x +
c) + 2*b)/a)) - 6*(3*(a^9 - 2*a^7*b^2 + a^5*b^4)*cos(d*x + c)^4 - 8*(a^8*b
- 2*a^6*b^3 + a^4*b^5)*cos(d*x + c)^3 - 5*(5*a^7*b^2 - 25*a^5*b^4 + 16*a^3*
b^6)*cos(d*x + c)^2 - 2*(7*a^6*b^3 - 49*a^4*b^5 + 32*a^2*b^7)*cos(d*x + c))
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(
(a^12 - 2*a^10*b^2 + a^8*b^4)*d*cos(d*x + c)^2 + 2*(a^11*b - 2*a^9*b^3 + a^
7*b^5)*d*cos(d*x + c) + (a^10*b^2 - 2*a^8*b^4 + a^6*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.669 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=122

$$-\frac{3\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{2+3\sec(c+dx)}}+\frac{\sqrt{5}\,E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{2+3\sec(c+dx)}}{d\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] -3/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(2+3*
sec(d*x+c))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2/5*5^(1/2))*5^(1/2)*(2+3*sec(d*x+c))^(1/2)/d/(3+2*cos(
d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3947, 3941, 2732, 3943, 2740}

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}\,E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

```
[Out] (-3*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]]) + (Sqrt[5]*EllipticE[(c + d*x)/2, 4/
5]*Sqrt[2 + 3*Sec[c + d*x]])/(d*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \frac{1}{2}\int \frac{\sqrt{2+3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx - \frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx$$
$$= -\frac{\left(3\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3+2\cos(c+dx)}}}{2\sqrt{2+3\sec(c+dx)}}$$
$$= -\frac{3\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{2+3\sec(c+dx)}} + \frac{\sqrt{5}\,E}{d\sqrt{}}$$

Mathematica [A]
time = 0.10, size = 78, normalized size = 0.64

$$\frac{\sqrt{3+2\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{2+3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c +
 d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 1.63, size = 405, normalized size = 3.32

method	result
default	$-\frac{\left(2i\cos(dx+c)\sin(dx+c)\sqrt{5}\ \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)},\sqrt{5}\right)\sqrt{10}\ \sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{2}\ \sqrt{\frac{1}{1+\cos(dx+c)}}+i c\cdots\right.}{\cdots}$
risch	$-\frac{i\left(e^{2i(dx+c)}+3\,e^{i(dx+c)}+1\right)}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ \left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e^{2i(dx+c)}+3\,e^{i(dx+c)}+1}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}+3\,e^{i(dx+c)}+1\right)}{\sqrt{\left(e^{2i(dx+c)}+3\,e^{i(dx+c)}+1\right)e^{i(dx+c)}}}+\frac{2\left(\frac{3}{2}-\cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/d*(2*I*cos(d*x+c)*sin(d*x+c)*5^(1/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*
5^(1/2)/sin(d*x+c),5^(1/2))*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)+I*cos(d*x+c)*sin(d*x+c)*5^(1/2)*Elliptic
E(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*10^(1/2)*((3+2*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)+2*I*5^(1/2)*Elli
pticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*10^(1/2)*((3+2*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+I
*5^(1/2)*EllipticE(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*10^(1/
2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)+20*cos(d*x+c)^2+10*cos(d*x+c)-30)*((3+2*cos(d*x+c))/cos(d*x+c))
^(1/2)/(1/cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 95, normalized size = 0.78

$$\frac{i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\sin(dx+c)+1)-i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c)-i\sin(dx+c)+1)+i\,\text{weierstrassZeta}(8,-4,\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\sin(dx+c)+1))-i\,\text{weierstrassZeta}(8,-4,\text{weierstrassPInverse}(8,-4,\cos(dx+c)-i\sin(dx+c)+1))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (I*weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) - I*weiers
trassPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1) + I*weierstrassZeta
(8, -4, weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1)) - I*
weierstrassZeta(8, -4, weierstrassPInverse(8, -4, cos(d*x + c) - I*sin(d*x
+ c) + 1)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\sec{(c+dx)}+2}\,\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*sec(c + d*x) + 2)*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{3}{\cos\left(c+d\,x\right)}+2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3/cos(c + d*x) + 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((3/cos(c + d*x) + 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.670 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{3\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}-\frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{-2+3\sec(c+dx)}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] 3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-2+3*sec(d*x+c))^(1/2)-
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2*I)*(-2+3*sec(d*x+c))^(1/2)/d/(3-2*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3947, 3941, 2732, 3943, 2740}

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}\,E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
```

```
[Out] (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, -4]*Sqrt[-2 + 3*Sec
[c + d*x]])/(d*Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{-2+3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right)+\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+}}$$

$$= \frac{\left(3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3-2\cos(c+dx)}}}{2\sqrt{-2+3\sec(c+dx)}}$$

$$= \frac{3\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}} - \frac{E(}{}$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 0.62

$$-\frac{\sqrt{3-2\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3F\left(\frac{1}{2}(c+dx)\middle|-4\right)\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c +
d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[-2 + 3*Sec[c + d*x]]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(147) = 294.
time = 1.14, size = 374, normalized size = 3.43

method	result

default	$-\frac{\left(2i\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\sqrt{5}\right)\cos(dx+c)\sin(dx+c)\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{-\frac{2(2\cos(dx+c)-3)}{1+\cos(dx+c)}}+i\,\mathrm{EllipticE}\left(\frac{i(-1}{\mathrm{s}}\right.\right.}{}$
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)\sqrt{2}}{d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{-\frac{2\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}-2}{\sqrt{\mathrm{e}^{i(dx+c)}\left(-2\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}-2\right)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(2*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x+c)*sin(
d*x+c)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c))
)^(1/2)+I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*cos(d*x+c)*sin(d*
x+c)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^
(1/2)+2*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+I*Ell
ipticE(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-4*cos(d*x+c)^2+10*
cos(d*x+c)-6)*(-(2*cos(d*x+c)-3)/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin
(d*x+c)/(2*cos(d*x+c)-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 92, normalized size = 0.84

$$-\frac{\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)+i\sin(dx+c)-1)+\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)-i\sin(dx+c)-1)-\mathrm{weierstrassZeta}(8,4,\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)+i\sin(dx+c)-1))-\mathrm{weierstrassZeta}(8,4,\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)-i\sin(dx+c)-1))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + weierstras
sPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1) - weierstrassZeta(8, 4,
weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1)) - weierstrass
Zeta(8, 4, weierstrassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\sec{(c+dx)}-2}\,\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{3}{\cos{(c+d\,x)}}-2}\,\sqrt{\frac{1}{\cos{(c+d\,x)}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.671 $\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{2-3\sec(c+dx)}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{3\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2*I)*(2-3*sec(d*x+c))^(1/2)/d/(3-2*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+3*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(2-3*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (EllipticE[(c + d*x)/2, -4]*Sqrt[2 - 3*Sec[c + d*x]])/(d*Sqrt[3 - 2*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x
)/2, -4]*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\,\sqrt{\sec(c+dx)}}\,dx = \frac{1}{2}\int \frac{\sqrt{2-3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{-3+2\cos(c+dx)}\,dx}{2\sqrt{-3+2\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{\left(3\sqrt{-3+2\right.}{}$$

$$= \frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{3-2\cos(c+dx)}\,dx}{2\sqrt{3-2\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{\left(3\sqrt{3-2\cos\right.}{}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{2-3\sec(c+dx)}}{d\sqrt{3-2\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{3\sqrt{3-2\cos(c+dx)}}{d\sqrt{}}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 0.63

$$-\frac{\sqrt{3-2\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3F\left(\frac{1}{2}(c+dx)\middle|-4\right)\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c +
d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(146) = 292.
time = 0.62, size = 405, normalized size = 3.75

method	result
default	$-\frac{\left(2i\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{-\frac{2(2\cos(dx+c)-3)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{5}-5i\,\mathrm{co}\right.}{}$
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)}{d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)}{\sqrt{\mathrm{e}^{i(dx+c)}\left(\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+1\right)}}+\frac{2\left(\sqrt{\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/d*(2*I*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*
5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)*5^(1/2)-5*I*cos(d*x+c)*EllipticE(I*(-1+cos(d*x+c))*5^(
1/2)/sin(d*x+c),1/5*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(-2*(2*cos(d*
x+c)-3)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*5^(1/2)+2*I*EllipticF(I*(-1+cos(d*
x+c))*5^(1/2)/sin(d*x+c),1/5*5^(1/2))*5^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-5*I*EllipticE(I*(
-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*5^(1/2))*5^(1/2)*2^(1/2)*(1/(1+cos(d*
```

```
x+c)))^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+20*cos(d
*x+c)^2-50*cos(d*x+c)+30)*((2*cos(d*x+c)-3)/cos(d*x+c))^(1/2)/(1/cos(d*x+c)
)^(1/2)/sin(d*x+c)/(2*cos(d*x+c)-3)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 95, normalized size = 0.88

$$\frac{-i\,\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)+i\sin(dx+c)-1)+i\,\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)-i\sin(dx+c)-1)+i\,\mathrm{weierstrassZeta}(8,4,\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)+i\sin(dx+c)-1))-i\,\mathrm{weierstrassZeta}(8,4,\mathrm{weierstrassPInverse}(8,4,\cos(dx+c)-i\sin(dx+c)-1))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] (-I*weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + I*weiers
trassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1) + I*weierstrassZeta(
8, 4, weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1)) - I*wei
erstrassZeta(8, 4, weierstrassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c)
- 1)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\,\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 3*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2-\frac{3}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((2 - 3/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.672 $\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{5}\,E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{-2-3\sec(c+dx)}}{d\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-2-3\sec(c+dx)}}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2/5*5^(1/2))*5^(1/2)*(-2-3*sec(d*x+c))^(1/2)/d/(3+2*cos(d*x+c))^(1/2)/se
c(d*x+c)^(1/2)-3/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d
*5^(1/2)/(-2-3*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}\,E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[5]*EllipticE[(c + d*x)/2, 4/5]*Sqrt[-2 - 3*Sec[c + d*x]])/(d*Sqrt[3
 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])) - (3*Sqrt[3 + 2*Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d
*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{-2-3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right) - \frac{3}{2}\int \frac{\sqrt{\sec(c+dx}}{\sqrt{-2-3\sec(c-}}$$

$$= -\frac{\sqrt{-2-3\sec(c+dx)}\int\sqrt{-3-2\cos(c+dx)}\,dx}{2\sqrt{-3-2\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(3\sqrt{\right.}}{}$$

$$= -\frac{\left(\sqrt{5}\sqrt{-2-3\sec(c+dx)}\right)\int\sqrt{\frac{3}{5}+\frac{2}{5}\cos(c+dx)}\,dx}{2\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} -$$

$$= -\frac{\sqrt{5}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{4}{5}\right)\sqrt{-2-3\sec(c+dx)}}{d\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{3\sqrt{3+2\cos}}{}$$

Mathematica [A]
time = 0.07, size = 78, normalized size = 0.63

$$\frac{\sqrt{3+2\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c +
 d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.43, size = 390, normalized size = 3.17

method	result
default	$-\frac{\left(2i\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\sqrt{2}\text{ EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)-5i\sin(dx+c)\sqrt{\right.}}{}$
risch	$-\frac{i\left(e^{2i(dx+c)}+3e^{i(dx+c)}+1\right)\sqrt{2}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{2\left(e^{2i(dx+c)}+3e^{i(dx+c)}+1\right)}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2e^{2i(dx+c)}-6e^{i(dx+c)}-2}{\sqrt{e^{i(dx+c)}\left(-2e^{2i(dx+c)}-6e^{i(dx+c)}-2\right)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/d*(2*I*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d
*x+c),1/5*5^(1/2))-5*I*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(1/2)*EllipticE(I*(-1+cos(d*x
+c))/sin(d*x+c),1/5*5^(1/2))+2*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1/5
*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)-5*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),1/5*
5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos
```

```
(d*x+c)))^(1/2)*sin(d*x+c)-20*cos(d*x+c)^2-10*cos(d*x+c)+30)*(-(3+2*cos(d*x
+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(3+2*cos(d*x+c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 87, normalized size = 0.71

$$\frac{\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\,\sin(dx+c)+1)+\text{weierstrassPInverse}(8,-4,\cos(dx+c)-i\,\sin(dx+c)+1)+\text{weierstrassZeta}(8,-4,\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\,\sin(dx+c)+1))+\text{weierstrassZeta}(8,-4,\text{weierstrassPInverse}(8,-4,\cos(dx+c)-i\,\sin(dx+c)+1))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) + weierstras
sPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1) + weierstrassZeta(8, -4
, weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1)) + weierstr
assZeta(8, -4, weierstrassPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1
)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\sec(c+dx)-2}\,\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-\frac{3}{\cos\left(c+d\,x\right)}-2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((- 3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.673 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=127

$$-\frac{4\sqrt{2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}\,d\sqrt{3+2\sec(c+dx)}}+\frac{2\sqrt{5}\,E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)\sqrt{3+2\sec(c+dx)}}{3d\sqrt{2+3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] -4/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),1/5*30^(1/2))*(2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(3+
2*sec(d*x+c))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),1/5*30^(1/2))*5^(1/2)*(3+2*sec(d*x+c))^(1/2)/d/(2
+3*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3947, 3941, 2732, 3943, 2740}

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}\,E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{4\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3\sqrt{5}\,d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]]) + (2*Sqrt[5]*EllipticE[(c + d*x)/2
, 6/5]*Sqrt[3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \frac{1}{3}\int \frac{\sqrt{3+2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx - \frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx$$

$$= -\frac{\left(2\sqrt{2+3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2+3\cos(c+dx)}}}{3\sqrt{3+2\sec(c+dx)}}$$

$$= -\frac{4\sqrt{2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}\,d\sqrt{3+2\sec(c+dx)}} + \frac{2\sqrt{5}}{3d}$$

Mathematica [A]
time = 0.10, size = 81, normalized size = 0.64

$$\frac{2\sqrt{2+3\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)-2F\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}\,d\sqrt{3+2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(c
 + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 1.10, size = 409, normalized size = 3.22

method	result
default	$\frac{\left(3\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)},i\sqrt{5}\right)\cos(dx+c)\sin(dx+c)\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{5}-\mathrm{Ellipt}\right.}{}$
risch	$-\frac{i(3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3)\sqrt{2}}{3d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(\mathrm{e}^{2i(dx+c)}+1)\sqrt{\frac{3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{\left(i\left(-\frac{2\left(3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3\right)}{3\sqrt{\mathrm{e}^{i(dx+c)}\left(3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3\right)}}+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/d*(3*EllipticF(1/5*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*cos(d
*x+c)*sin(d*x+c)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*5^(1/2)-EllipticE(1/5*(-1+cos(d*x+c))*5^(1/2)/sin(d
*x+c),I*5^(1/2))*cos(d*x+c)*sin(d*x+c)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*5^(1/2)+3*EllipticF(1/5*(-1+c
os(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*5^(1/2)*10^(1/2)*((2+3*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-Elliptic
E(1/5*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*5^(1/2)*10^(1/2)*((2+3*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)-30*cos(d*x+c)^2+10*cos(d*x+c)+20)*((2+3*cos(d*x+c))/cos(d*x+c))^(1/2)/(1
/cos(d*x+c))^(1/2)/sin(d*x+c)/(2+3*cos(d*x+c))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 108, normalized size = 0.85

$$\frac{4i\sqrt{6}\,\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)+i\sin\left(dx+c\right)+\frac{4}{9}\right)-4i\sqrt{6}\,\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)-i\sin\left(dx+c\right)+\frac{4}{9}\right)+9i\sqrt{6}\,\mathrm{weierstrassZeta}\left(-\frac{44}{27},\frac{784}{729},\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)+i\sin\left(dx+c\right)+\frac{4}{9}\right)\right)-9i\sqrt{6}\,\mathrm{weierstrassZeta}\left(-\frac{44}{27},\frac{784}{729},\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)-i\sin\left(dx+c\right)+\frac{4}{9}\right)\right)}{27\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(4*I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin
(d*x + c) + 4/9) - 4*I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x
 + c) - I*sin(d*x + c) + 4/9) + 9*I*sqrt(6)*weierstrassZeta(-44/27, 784/729
, weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d*x + c) + 4/9)
) - 9*I*sqrt(6)*weierstrassZeta(-44/27, 784/729, weierstrassPInverse(-44/27
, 784/729, cos(d*x + c) - I*sin(d*x + c) + 4/9)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\sec\left(c+dx\right)+3}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*sec(c + d*x) + 3)*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{2}{\cos\left(c+d\,x\right)}+3}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2/cos(c + d*x) + 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((2/cos(c + d*x) + 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.674 $\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{3-2\sec(c+dx)}}{3d\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{4\sqrt{-2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{\sec(c+dx)}}{3d\sqrt{3-2\sec(c+dx)}}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),6^(1/2))*(3-2*sec(d*x+c))^(1/2)/d/(-2+3*cos(d*x+c))^(1/2)/sec(d*x+c)^
(1/2)+4/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),6^(1/2))*(-2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(3-2*sec(d*
x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3947, 3941, 2732, 3943, 2740}

$$\frac{4\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 6]*Sqrt[3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2 + 3*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{1}{3}\int \frac{\sqrt{3-2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{3-2\sec(c+dx)}\int \sqrt{-2+3\cos(c+dx)}\,dx}{3\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{-2+\cdots\right.}{\cdots}$$

$$= \frac{2E\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{3-2\sec(c+dx)}}{3d\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{4\sqrt{-2+3\cos(c+\cdots}}{3d\cdots}$$

Mathematica [A]
time = 0.08, size = 72, normalized size = 0.64

$$\frac{\sqrt{-2+3\cos(c+dx)}\left(2E\left(\frac{1}{2}(c+dx)\middle|6\right)+4F\left(\frac{1}{2}(c+dx)\middle|6\right)\right)\sqrt{\sec(c+dx)}}{3d\sqrt{3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 1.12, size = 381, normalized size = 3.37

method	result
default	$\frac{2\left(3\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{5}\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\sin(dx+c)\cos(dx+c)-5\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$
risch	$-\frac{i\left(3\,e^{2i(dx+c)}-4\,e^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{3\,e^{2i(dx+c)}-4\,e^{i(dx+c)}+3}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(3\,e^{2i(dx+c)}-4\,e^{i(dx+c)}+3\right)}{3\sqrt{e^{i(dx+c)}\left(3\,e^{2i(dx+c)}-4\,e^{i(dx+c)}+3\right)}}+\cdots\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(3*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*5^(1/2)*EllipticF((-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))*sin(d*
x+c)*cos(d*x+c)-5*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*5^(1/2)*EllipticE((-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2)
)*sin(d*x+c)*cos(d*x+c)+3*EllipticF((-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*
I*5^(1/2))*5^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)-5*EllipticE((-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5
^(1/2))*5^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)-15*cos(d*x+c)^2+25*cos(d*x+c)-10)*((-2+3*cos(d*x+c))/cos(
d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2+3*cos(d*x+c))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 108, normalized size = 0.96

$$\frac{-4i\sqrt{6}\,\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos(dx+c)+i\sin(dx+c)-\frac{4}{9}\right)+4i\sqrt{6}\,\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos(dx+c)-i\sin(dx+c)-\frac{4}{9}\right)+9i\sqrt{6}\,\text{weierstrassZeta}\left(-\frac{44}{27},-\frac{784}{729},\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos(dx+c)+i\sin(dx+c)-\frac{4}{9}\right)\right)-9i\sqrt{6}\,\text{weierstrassZeta}\left(-\frac{44}{27},-\frac{784}{729},\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos(dx+c)-i\sin(dx+c)-\frac{4}{9}\right)\right)}{27\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(-4*I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*s
in(d*x + c) - 4/9) + 4*I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(
d*x + c) - I*sin(d*x + c) - 4/9) + 9*I*sqrt(6)*weierstrassZeta(-44/27, -784
/729, weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(d*x + c) -
 4/9)) - 9*I*sqrt(6)*weierstrassZeta(-44/27, -784/729, weierstrassPInverse(
-44/27, -784/729, cos(d*x + c) - I*sin(d*x + c) - 4/9)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\,\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3 - 2*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3-\frac{2}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((3 - 2/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.675 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=129

$$\frac{4\sqrt{2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}\,d\sqrt{-3+2\sec(c+dx)}}-\frac{2\sqrt{5}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{6}{5}\right)\sqrt{-3+2\sec(c+dx)}}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] -4/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x
+1/2*c),1/5*30^(1/2))*(2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(-3
+2*sec(d*x+c))^(1/2)+2/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*El
lipticE(cos(1/2*d*x+1/2*c),1/5*30^(1/2))*5^(1/2)*(-3+2*sec(d*x+c))^(1/2)/d/
(2-3*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3947, 3941, 2733, 3943, 2741}

$$\frac{4\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3\sqrt{5}\,d\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
```

```
[Out] (4*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c + d
*x]])/(3*Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]]) - (2*Sqrt[5]*EllipticE[(c + P
i + d*x)/2, 6/5]*Sqrt[-3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 - 3*Cos[c + d*x]]*S
qrt[Sec[c + d*x]])
```

Rule 2733

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2741

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{-3+2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right)+\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+}}$$

$$= \frac{\left(2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2-3\cos(c+dx)}}}{3\sqrt{-3+2\sec(c+dx)}}$$

$$= \frac{4\sqrt{2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}\,d\sqrt{-3+2\sec(c+dx)}} - \frac{2}{}$$

Mathematica [A]
time = 0.06, size = 72, normalized size = 0.56

$$\frac{\sqrt{-2+3\cos(c+dx)}\left(2E\left(\frac{1}{2}(c+dx)\middle|6\right)+4F\left(\frac{1}{2}(c+dx)\middle|6\right)\right)\sqrt{\sec(c+dx)}}{3d\sqrt{-3+2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 370, normalized size = 2.87

method	result
default	$-\frac{2\left(3i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\sqrt{5}\right)-i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\cdots}\right.}{\cdots}$
risch	$-\frac{i\left(3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{-\frac{3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{\left(i\,\frac{-2\,\mathrm{e}^{2i(dx+c)}+\frac{8\,\mathrm{e}^{i(dx+c)}}{3}-2}{\sqrt{\left(-3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}-3\right)\mathrm{e}^{i(dx+c)}}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(3*I*(1/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))-
I*(1/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)*cos(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))+3*I*Ellipt
icF(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((-2+3
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-I*EllipticE(I*(-1+cos(d*x+c))
/sin(d*x+c),I*5^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-3*cos(d*x+c)^2+5*cos(d*x+c)-2)*(-(-2+3*cos(d*x+c))
/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2+3*cos(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.93, size = 108, normalized size = 0.84

$$-\frac{4\sqrt{6}\,\mathrm{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\frac{4}{9}\right)+4\sqrt{6}\,\mathrm{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)-\frac{4}{9}\right)-9\sqrt{6}\,\mathrm{weierstrassZeta}\left(-\frac{44}{27},-\frac{784}{729},\mathrm{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\frac{4}{9}\right)\right)-9\sqrt{6}\,\mathrm{weierstrassZeta}\left(-\frac{44}{27},-\frac{784}{729},\mathrm{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)-\frac{4}{9}\right)\right)}{27\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(4*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin
(d*x + c) - 4/9) + 4*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x
+ c) - I*sin(d*x + c) - 4/9) - 9*sqrt(6)*weierstrassZeta(-44/27, -784/729,
weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(d*x + c) - 4/9))
 - 9*sqrt(6)*weierstrassZeta(-44/27, -784/729, weierstrassPInverse(-44/27,
-784/729, cos(d*x + c) - I*sin(d*x + c) - 4/9)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\sec\left(c+dx\right)-3}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{2}{\cos\left(c+d\,x\right)}-3}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.676 $\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{2E\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{-3-2\sec(c+dx)}}{3d\sqrt{-2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}-\frac{4\sqrt{-2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{\sec(c+dx)}}{3d\sqrt{-3-2\sec(c+dx)}}$$

```
[Out] 2/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1
/2*c),6^(1/2))*(-3-2*sec(d*x+c))^(1/2)/d/(-2-3*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2)+4/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/
2*d*x+1/2*c),6^(1/2))*(-2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-3-2*sec(
d*x+c))^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3947, 3941, 2733, 3943, 2741}

$$-\frac{4\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}\,E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*EllipticE[(c + Pi + d*x)/2, 6]*Sqrt[-3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2
- 3*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*Sqrt[-2 - 3*Cos[c + d*x]]*Ellipt
icF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 - 2*Sec[c + d*x]]
)
```

Rule 2733

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2741

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\,\sqrt{\sec(c+dx)}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{-3-2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right) - \frac{2}{3}\int \frac{\sqrt{\sec(c+dx}}{\sqrt{-3-2\sec(c-}}$$

$$= -\frac{\sqrt{-3-2\sec(c+dx)}\int \sqrt{-2-3\cos(c+dx)}\,dx}{3\sqrt{-2-3\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\right.}{}$$

$$= -\frac{2E\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{-3-2\sec(c+dx)}}{3d\sqrt{-2-3\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - \frac{4\sqrt{-2-3}}{}$$

Mathematica [A]
time = 0.07, size = 81, normalized size = 0.70

$$\frac{2\sqrt{2+3\cos(c+dx)}\,\left(5E\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)-2F\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}\,d\sqrt{-3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(c
 + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x]]
)
```

Maple [C] Result contains complex when optimal does not.
time = 0.31, size = 394, normalized size = 3.43

method	result
default	$-\frac{\left(3i\sin(dx+c)\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}-5i\sin(dx+c)\,\text{E}\right.}{}$
risch	$-\frac{i\left(3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+3}{e^{2i(dx+c)}+1}}}-\frac{\left(i\left(\frac{-2\,e^{2i(dx+c)}-\frac{8\,e^{i(dx+c)}}{3}-2}{\sqrt{\left(-3\,e^{2i(dx+c)}-4\,e^{i(dx+c)}-3\right)e^{i(dx+c)}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/d*(3*I*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2
))*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*2^(1/2)-5*I*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),
1/5*I*5^(1/2))*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*2^(1/2)+3*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,1/5*I*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-5*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c
),1/5*I*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-30*cos(d*x+c)^2+10*cos(d*x+c)+20)*(-(2+3
*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(2+3*cos(d*x
+c))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 108, normalized size = 0.94

$$\frac{4\sqrt{6}\,\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)+i\sin\left(dx+c\right)+\frac{4}{9}\right)+4\sqrt{6}\,\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)-i\sin\left(dx+c\right)+\frac{4}{9}\right)+9\sqrt{6}\,\mathrm{weierstrassZeta}\left(-\frac{44}{27},\frac{784}{729},\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)+i\sin\left(dx+c\right)+\frac{4}{9}\right)\right)+9\sqrt{6}\,\mathrm{weierstrassZeta}\left(-\frac{44}{27},\frac{784}{729},\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)-i\sin\left(dx+c\right)+\frac{4}{9}\right)\right)}{27\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 1/27*(4*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d
*x + c) + 4/9) + 4*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c
) - I*sin(d*x + c) + 4/9) + 9*sqrt(6)*weierstrassZeta(-44/27, 784/729, weie
rstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d*x + c) + 4/9)) + 9*
sqrt(6)*weierstrassZeta(-44/27, 784/729, weierstrassPInverse(-44/27, 784/72
9, cos(d*x + c) - I*sin(d*x + c) + 4/9)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\sec\left(c+dx\right)-3}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(1/(sqrt(-2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-\frac{2}{\cos\left(c+d\,x\right)}-3}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((- 2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.677 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{2+3\sec(c+dx)}}$$

```
[Out] 2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(2+3*s
ec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3943, 2740}

$$\frac{2\sqrt{2\cos(c+dx)+3}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \frac{\left(\sqrt{3+2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3+2\cos(c+dx)}}\,dx}{\sqrt{2+3\sec(c+dx)}}$$
$$= \frac{2\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{2+3\sec(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 1.00

$$\frac{2\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{2+3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 142, normalized size = 2.33

method	result
default	$-\frac{i\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\cos(dx+c)\left(\sin^2(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{3+2\cos(dx+c)}{\cos(dx+c)}}\,\text{EllipticF}\left(\frac{i(-1+}{}\right.}{5d\left(2\left(\cos^2(dx+c)\right)+\cos(dx+c)-3\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*I/d*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(1/2)*((3+2*cos(d*x+
c))/cos(d*x+c))^(1/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^
(1/2))/(2*cos(d*x+c)^2+cos(d*x+c)-3)*5^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 47, normalized size = 0.77

$$\frac{-i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\,\sin(dx+c)+1)+i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c)-i\,\sin(dx+c)+1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) + I*weier
strassPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{3}{\cos(c+d\,x)}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) + 2)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) + 2)^(1/2), x)
```

3.678 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-2+3*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3943, 2740}

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3-2\cos(c+dx)}}\,dx}{\sqrt{-2+3\sec(c+dx)}}\\ &= \frac{2\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.00

$$\frac{2\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

Maple [A]
time = 0.26, size = 137, normalized size = 2.54

method	result
default	$\frac{i\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{-\frac{2(2\cos(dx+c)-3)}{1+\cos(dx+c)}}\,(\sin^2(dx+c))\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{-}}{d(2(\cos^2(dx+c))-5\cos(dx+c)+3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] I/d*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*cos(d*x+c
)*(1/cos(d*x+c))^(1/2)*(-(2*cos(d*x+c)-3)/cos(d*x+c))^(1/2)/(2*cos(d*x+c)^2
-5*cos(d*x+c)+3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 44, normalized size = 0.81

$$-\frac{\text{weierstrassPInverse}(8,4,\cos(dx+c)+i\,\sin(dx+c)-1)+\text{weierstrassPInverse}(8,4,\cos(dx+c)-i\,\sin(dx+c)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + weierstras
sPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec{(c+dx)}}}{\sqrt{3\sec{(c+dx)}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos{(c+d\,x)}}}}{\sqrt{\frac{3}{\cos{(c+d\,x)}}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) - 2)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) - 2)^(1/2), x)
```

3.679 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(2-3*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3943, 2742, 2740}

$$\frac{2\sqrt{3-2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \frac{\left(\sqrt{-3+2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{2-3\sec(c+dx)}}$$
$$= \frac{\left(\sqrt{3-2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3-2\cos(c+dx)}}\,dx}{\sqrt{2-3\sec(c+dx)}}$$
$$= \frac{2\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 1.00

$$\frac{2\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Maple [A]
time = 0.28, size = 144, normalized size = 2.67

method	result
default	$-\frac{i\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{2\cos(dx+c)-3}{\cos(dx+c)}}\,\cos(dx+c)\left(\sin^2(dx+c)\right)\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)\sqrt{-\frac{2(2\cos(dx+c)-3)}{1+\cos(dx+c)}}\,\sqrt{}}{5d\left(2\left(\cos^2(dx+c)\right)-5\cos(dx+c)+3\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/5*I/d*(1/cos(d*x+c))^(1/2)*((2*cos(d*x+c)-3)/cos(d*x+c))^(1/2)*cos(d*x+c
)*sin(d*x+c)^2*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*5^(1/2))*
(-2*(2*cos(d*x+c)-3)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
/(2*cos(d*x+c)^2-5*cos(d*x+c)+3)*5^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 47, normalized size = 0.87

$$\frac{-i\,\text{weierstrassPInverse}(8,4,\cos(dx+c)+i\,\sin(dx+c)-1)+i\,\text{weierstrassPInverse}(8,4,\cos(dx+c)-i\,\sin(dx+c)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] (-I*weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + I*weiers
trassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(2-3*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(2 - 3*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{2-\frac{3}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(2 - 3/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(2 - 3/cos(c + d*x))^(1/2), x)
```

3.680 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-2-3\sec(c+dx)}}$$

```
[Out] 2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(-2-3*
sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3943, 2742, 2740}

$$\frac{2\sqrt{2\cos(c+dx)+3}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{-3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \frac{\left(\sqrt{-3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-3-2\cos(c+dx)}}\,dx}{\sqrt{-2-3\sec(c+dx)}}$$

$$= \frac{\left(\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{3}{5}+\frac{2}{5}\cos(c+dx)}}\,dx}{\sqrt{5}\sqrt{-2-3\sec(c+dx)}}$$

$$= \frac{2\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-2-3\sec(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 1.00

$$\frac{2\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 139, normalized size = 2.28

method	result
default	$\frac{i\cos(dx+c)\left(\sin^2(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{-\frac{3+2\cos(dx+c)}{\cos(dx+c)}}\sqrt{2}\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\frac{i(-}{}\right.}{5d\left(2\left(\cos^2(dx+c)\right)+\cos(dx+c)-3\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*I/d*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(1/2)*(-(3+2*cos(d*x+c))/cos
(d*x+c))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))/(
2*cos(d*x+c)^2+cos(d*x+c)-3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 44, normalized size = 0.72

$$-\frac{\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\sin(dx+c)+1)+\text{weierstrassPInverse}(8,-4,\cos(dx+c)-i\sin(dx+c)+1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -(weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) + weierstra
ssPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3\sec(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-2-3*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(-3*sec(c + d*x) - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{-\frac{3}{\cos(c+d\,x)}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(- 3/cos(c + d*x) - 2)^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(- 3/cos(c + d*x) - 2)^(1/2), x)
```

3.681 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{3+2\sec(c+dx)}}$$

```
[Out] 2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),1/5*30^(1/2))*(2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(3+2*
sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3943, 2740}

$$\frac{2\sqrt{3\cos(c+dx)+2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{\sqrt{5}\,d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \frac{\left(\sqrt{2+3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2+3\cos(c+dx)}}\,dx}{\sqrt{3+2\sec(c+dx)}}$$
$$= \frac{2\sqrt{2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{3+2\sec(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 1.00

$$\frac{2\sqrt{2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{3+2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 145, normalized size = 2.38

method	result
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{2+3\cos(dx+c)}{\cos(dx+c)}}\,\cos(dx+c)\left(\sin^2(dx+c)\right)\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)},i\sqrt{5}\right)\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{}}{5d\left(3(\cos^2(dx+c))-\cos(dx+c)-2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/d*(1/cos(d*x+c))^(1/2)*((2+3*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*s
in(d*x+c)^2*EllipticF(1/5*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*10^
(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)/(3*cos(d*x+c)^2-cos(d*x+c)-2)*5^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.30, size = 54, normalized size = 0.89

$$\frac{-i\sqrt{6}\,\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)+\frac{4}{9}\right)+i\sqrt{6}\,\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)+\frac{4}{9}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d
*x + c) + 4/9) + I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c
) - I*sin(d*x + c) + 4/9))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{2\sec\left(c+dx\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\sqrt{\frac{2}{\cos\left(c+d\,x\right)}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) + 3)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) + 3)^(1/2), x)
```

3.682 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{-2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{\sec(c+dx)}}{d\sqrt{3-2\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),6^(1/2))*(-2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(3-2*sec(d*x+c))^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3943, 2740}

$$\frac{2\sqrt{3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)}{d\sqrt{3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \frac{\left(\sqrt{-2+3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-2+3\cos(c+dx)}}\,dx}{\sqrt{3-2\sec(c+dx)}}$$
$$= \frac{2\sqrt{-2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,6\right)\sqrt{\sec(c+dx)}}{d\sqrt{3-2\sec(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.00

$$\frac{2\sqrt{-2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,6\right)\sqrt{\sec(c+dx)}}{d\sqrt{3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[3 - 2*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 138, normalized size = 2.56

method	result
default	$\frac{2\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\cos(dx+c)\left(\sin^2(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{-2+3\cos(dx+c)}{\cos(dx+c)}}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{}{1+}}}{5d\left(3\left(\cos^2(dx+c)\right)-5\cos(dx+c)+2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*EllipticF((-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))*cos(d*x+c
)*sin(d*x+c)^2*(1/cos(d*x+c))^(1/2)*((-2+3*cos(d*x+c))/cos(d*x+c))^(1/2)*((
-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/(3*cos(d*x+
c)^2-5*cos(d*x+c)+2)*5^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 54, normalized size = 1.00

$$\frac{-i\sqrt{6}\,\mathrm{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\frac{4}{9}\right)+i\sqrt{6}\,\mathrm{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)-\frac{4}{9}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 1/3*(-I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(
d*x + c) - 4/9) + I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x +
 c) - I*sin(d*x + c) - 4/9))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{3-2\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(3-2*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(3 - 2*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\sqrt{3-\frac{2}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(3 - 2/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(3 - 2/cos(c + d*x))^(1/2), x)
```

3.683 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-3+2\sec(c+dx)}}$$

```
[Out] -2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+
1/2*c),1/5*30^(1/2))*(2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(-3+
2*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3943, 2741}

$$\frac{2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{\sqrt{5}\,d\sqrt{2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c + d
*x]])/(Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Rule 2741

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{\left(\sqrt{2-3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2-3\cos(c+dx)}}\,dx}{\sqrt{-3+2\sec(c+dx)}}$$
$$= \frac{2\sqrt{2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-3+2\sec(c+dx)}}$$

Mathematica [**A**]
time = 0.03, size = 54, normalized size = 0.87

$$\frac{2\sqrt{-2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{\sec(c+dx)}}{d\sqrt{-3+2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.19, size = 136, normalized size = 2.19

method	result
default	$\frac{2i\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{-\frac{-2+3\cos(dx+c)}{\cos(dx+c)}}\left(\sin^2(dx+c)\right)\cos(dx+c)\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\sqrt{5}\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{-2+3}{1+c}}}{d\left(3\left(\cos^2(dx+c)\right)-5\cos(dx+c)+2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d*(1/cos(d*x+c))^(1/2)*(-(-2+3*cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)
^2*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*(1/(1+cos(d
*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(3*cos(d*x+c)^2-5*co
s(d*x+c)+2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.21, size = 52, normalized size = 0.84

$$-\frac{\sqrt{6}\,\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\frac{4}{9}\right)+\sqrt{6}\,\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)-\frac{4}{9}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -1/3*(sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(d*
x + c) - 4/9) + sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c)
- I*sin(d*x + c) - 4/9))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{2\sec\left(c+dx\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\sqrt{\frac{2}{\cos\left(c+d\,x\right)}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) - 3)^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) - 3)^(1/2), x)
```

3.684 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx$

Optimal. Leaf size=55

$$\frac{2\sqrt{-2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{\sec(c+dx)}}{d\sqrt{-3-2\sec(c+dx)}}$$

```
[Out] -2*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/
2*c),6^(1/2))*(-2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-3-2*sec(d*x+c))^
(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3943, 2741}

$$\frac{2\sqrt{-3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{d\sqrt{-2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*
x]])/(d*Sqrt[-3 - 2*Sec[c + d*x]])
```

Rule 2741

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \frac{\left(\sqrt{-2-3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-2-3\cos(c+dx)}}\,dx}{\sqrt{-3-2\sec(c+dx)}}$$
$$= \frac{2\sqrt{-2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)\middle|\,6\right)\sqrt{\sec(c+dx)}}{d\sqrt{-3-2\sec(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 1.11

$$\frac{2\sqrt{2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 142, normalized size = 2.58

method	result
default	$\frac{i\cos(dx+c)\left(\sin^2(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{-\frac{2+3\cos(dx+c)}{\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+}{\mathrm{si}}\right.}{5d\left(3\left(\cos^2(dx+c)\right)-\cos(dx+c)-2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*I/d*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(1/2)*(-(2+3*cos(d*x+c))/cos
(d*x+c))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))
/(3*cos(d*x+c)^2-cos(d*x+c)-2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 52, normalized size = 0.95

$$-\frac{\sqrt{6}\,\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)+\frac{4}{9}\right)+\sqrt{6}\,\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)+\frac{4}{9}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d*x
 + c) + 4/9) + sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) -
I*sin(d*x + c) + 4/9))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{-2\sec\left(c+dx\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-3-2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(-2*sec(c + d*x) - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\sqrt{-\frac{2}{\cos\left(c+d\,x\right)}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(- 2/cos(c + d*x) - 3)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(- 2/cos(c + d*x) - 3)^(1/2), x)
```

3.685 $\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*s
ec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)/(1+sec
(d*x+c))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3919, 144, 143}

$$\frac{\sqrt{2}\,\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Sec[
c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx = -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a\ldots}}}{\sqrt{1-x}\sqrt{1+x}\ldots}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{-\frac{a+b\sec\ldots}{-a\ldots}}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c\ldots}}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7160 vs. 2(105) = 210.
time = 42.73, size = 7160, normalized size = 68.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \sec(dx+c)(a+b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a + b \sec{\left(c + dx \right)}} \sec{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(1/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/3)/cos(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^(1/3)/cos(c + d*x), x)
```

3.686 $\int \sqrt[3]{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\sqrt[3]{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/3),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(1/3), x]
```

Rubi steps

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Mathematica [A]
time = 2.79, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(1/3), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/3),x)
[Out] int((a+b*sec(d*x+c))^(1/3),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a + b\sec{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/3),x)
[Out] Integral((a + b*sec(c + d*x))**(1/3), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.06

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(1/3), x)
```

3.687 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=362

$$\frac{3(9a^2+32b^2)\,(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d}-\frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{44b^2d}+\frac{3\sec(c+dx)(a+b$$

```
[Out] 3/220*(9*a^2+32*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d-9/44*a*(a+b*se
c(d*x+c))^(5/3)*tan(d*x+c)/b^2/d+3/11*sec(d*x+c)*(a+b*sec(d*x+c))^(5/3)*tan
(d*x+c)/b/d+1/220*a*(18*a^2+49*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+
c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^3/d/((a+b
*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/110*(9*a^4+23*a^2*
b^2-32*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x
+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^3/d/(a+b*sec(d*x+c))^(1/3)
*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3950, 4167, 4087, 4092, 3919, 144, 143}

$$\frac{a(18a^2+49b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{110\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\,\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3(9a^2+32b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{220b^2d}-\frac{(9a^4+23a^2b^2-32b^4)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{55\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{44b^2d}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*(9*a^2 + 32*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(220*b^2*d) -
(9*a*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(44*b^2*d) + (3*Sec[c + d*x]*
(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(11*b*d) + (a*(18*a^2 + 49*b^2)*Ap
pellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a
 + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(110*Sqrt[2]*b^3*d*Sqrt[1 +
 Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - ((9*a^4 + 23*a^2*b^2
 - 32*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(55*S
qrt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
```

```
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3950

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Dist[d^3/(b*(m + n -
1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b
*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
IntegersQ[2*m, 2*n]) && !IGtQ[m, 2]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3\sec(c+dx)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{11bd} + \frac{3\int \sec(c+dx)}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{44b^2d} + \frac{3\sec(c+dx)(a+b\sec(}{11}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c}{}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c}{}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c}{}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c}{}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 8052 vs. $2(362) = 724$.
time = 38.01, size = 8052, normalized size = 22.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]
[Out] Result too large to show
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{4}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^{\frac{2}{3}}\sec^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^4,x)
```

```
[Out] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^4, x)
```

3.688 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=305

$$-\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd}+\frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd}-\frac{(6a^2-25b^2)\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1\right.}{20\sqrt{2}}$$

[Out] -9/40*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d+3/8*(a+b*sec(d*x+c))^(5/3)*ta
n(d*x+c)/b/d-1/40*(6*a^2-25*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))
/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d/((a+b*se
c(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)+3/20*a*(a^2-b^2)*Appell
F1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x
+c))/(a+b))^(1/3)*tan(d*x+c)/b^2/d/(a+b*sec(d*x+c))^(1/3)*2^(1/2)/(1+sec(d*
x+c))^(1/2)

Rubi [A]
time = 0.34, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3925, 4087, 4092, 3919, 144, 143}

$$-\frac{(6a^2-25b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{20\sqrt{2}\,b^2d\sqrt{\sec(c+dx)+1}\,\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3a(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{10\sqrt{2}\,b^2d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{40bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-9*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(40*b*d) + (3*(a + b*Sec[c +
 d*x])^(5/3)*Tan[c + d*x])/(8*b*d) - ((6*a^2 - 25*b^2)*AppellF1[1/2, 1/2, -
2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[
c + d*x])^(2/3)*Tan[c + d*x])/(20*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x]]*((a
+ b*Sec[c + d*x])/(a + b))^(2/3)) + (3*a*(a^2 - b^2)*AppellF1[1/2, 1/2, 1/3
, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c
+ d*x])/(a + b))^(1/3)*Tan[c + d*x])/(10*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x
]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && !LtQ[m, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} + \frac{3\int \sec(c+dx)\left(\frac{5b}{3} - a\sec(c\right.}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8bd}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 7783 vs. $2(305) = 610$.
time = 37.79, size = 7783, normalized size = 25.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^3(dx+c)\right)(a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{2}{3}}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
[Out] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

3.689 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=260

$$\frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}+\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+d}{5bd\sqrt{1+\sec(c+dx)}\,\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] 3/5*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/d+2/5*a*AppellF1(1/2,-2/3,1/2,3/2,b*(
1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*2^(1/2)*tan(
d*x+c)/b/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+sec(d*x+c))^(1/2)-2/5*(a^2-b^2
)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b
*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/b/d/(a+b*sec(d*x+c))^(1/3)/(1+
sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3920, 4092, 3919, 144, 143}

$$-\frac{2\sqrt{2}\,(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5bd\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}+\frac{2\sqrt{2}\,a\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5bd\sqrt{\sec(c+dx)+1}\,\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (2*Sqrt[2]*a*AppellF1[1
/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(
a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d*Sqrt[1 + Sec[c + d*x]]*((a +
 b*Sec[c + d*x])/(a + b))^(2/3)) - (2*Sqrt[2]*(a^2 - b^2)*AppellF1[1/2, 1/2
, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*S
ec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(5*b*d*Sqrt[1 + Sec[c + d*x]]*(a
+ b*Sec[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3920

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[
e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{2}{5}\int \frac{\sec(c+dx)(b+a\sec(c}{\sqrt[3]{a+b\sec(c+dx}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{(2a)\int \sec(c+dx)(a+b\sec(}{5b}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{(2a\tan(c+dx))\mathrm{Subst}\Big(\int \frac{}{\sqrt{}}}{5bd\sqrt{1-\sec(c+}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{(2a(a+b\sec(c+dx))^{2/3}\tan}{5bd\sqrt{1-\sec(c}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\right.}{5bd}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2505 vs. $2(260) = 520$.

time = 48.32, size = 2505, normalized size = 9.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(2/3)*((3*a*Sin[c + d*x])/(5*b) + (3*Tan[c + d*x])/5)
)/d - ((-2*b + 3*a*Cos[c + d*x])*(a + b*Sec[c + d*x])^(2/3)*(3*a*(b + a*Cos
[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(2/3) - (3*(b + a*Co
s[c + d*x])^(2/3)*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)
])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 -
 b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-
2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2,
1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x
])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x])))/(5*b*Sqrt[1 - Cos[c + d*x]^
2]*Sec[c + d*x]^(1/3))))/(5*b*d*((3*a*(b + a*Cos[c + d*x])^(2/3)*Sin[c + d*
x])/(Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(1/3)) - (2*a^2*Sqrt[1 - Cos[c +
 d*x]^2]*Sec[c + d*x]^(2/3)*Sin[c + d*x])/(b + a*Cos[c + d*x])^(1/3) + 2*a*
(b + a*Cos[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(5/3)*Sin[
```

c + d*x] - (3*Sqrt[b^(-2)]*(b + a*Cos[c + d*x])^(2/3)*Sec[c + d*x]^(5/3)*Sq
rt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*
AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])),
 (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8
/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a
 + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(10*(1 - a*Sqrt[b^(-
2)])*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a
*Sqrt[b^(-2)])]) + (3*Sqrt[b^(-2)]*(b + a*Cos[c + d*x])^(2/3)*Sec[c + d*x]^
(5/3)*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2
 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^
(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2
, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d
x])/(a + 1/Sqrt[b^(-2)])](a + b*Sec[c + d*x]))*Sin[c + d*x])/(10*(1 + a*S
qrt[b^(-2)])*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x]
)/(1 + a*Sqrt[b^(-2)])]) + (3*(b + a*Cos[c + d*x])^(2/3)*Sqrt[(1 - Sqrt[b^(
-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c +
 d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -
((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/S
qrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a
 + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[
c + d*x]))*Sin[c + d*x])/(5*b*(1 - Cos[c + d*x]^2)^(3/2)*Sec[c + d*x]^(4/3)
) + (2*a*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[
(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*App
ellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a
 + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8/3,
 -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1
/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*x
])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(1/3)) + ((b + a*Cos[c + d*x
])^(2/3)*Sec[c + d*x]^(2/3)*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*S
qrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*
(-5*(a^2 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1
/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[
5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*S
ec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b
*Sqrt[1 - Cos[c + d*x]^2]) - (3*(b + a*Cos[c + d*x])^(2/3)*Sqrt[(1 - Sqrt[b
^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c
 + d*x])/(1 - a*Sqrt[b^(-2)])]*(2*a*b*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b
*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(
-2)])]*Sec[c + d*x]*Tan[c + d*x] - 5*(a^2 - b^2)*((b*AppellF1[5/3, 1/2, 3/2
, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/
(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(5*(a + 1/Sqrt[b^(-2)])) -
 (b*AppellF1[5/3, 3/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)
])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/
(5*(-a + 1/Sqrt[b^(-2)]))) + 2*a*(a + b*Sec[c + d*x])*((5*b*AppellF1[8/3, 1
/2, 3/2, 11/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c

```
+ d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(16*(a + 1/Sqrt[b^
(-2)])) - (5*b*AppellF1[8/3, 3/2, 1/2, 11/3, -((a + b*Sec[c + d*x])/(-a + 1
/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Ta
n[c + d*x])/(16*(-a + 1/Sqrt[b^(-2)])))))/(5*b*Sqrt[1 - Cos[c + d*x]^2]*Sec
[c + d*x]^(1/3))))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\sec^{2}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^{\frac{2}{3}}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
[Out] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

3.690 $\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*s
ec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+sec
(d*x+c))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3919, 144, 143}

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[1 + Sec[
c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left((a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(-\frac{a+b\sec(c+}{-a-b}\right.}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7142 vs. $2(105) = 210$.
time = 36.40, size = 7142, normalized size = 68.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
[Out] Result too large to show
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sec(dx+c)(a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{2}{3}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

3.691 $\int (a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left((a+b\sec(c+dx))^{2/3},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(2/3),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^{2/3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(2/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(2/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int (a+b\sec(c+dx))^{2/3}\,dx$$

Mathematica [A]
time = 3.97, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{2/3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(2/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(2/3), x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(2/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(2/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(2/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \left(a + \frac{b}{\cos(c + d\,x)}\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(2/3), x)
```

3.692 $\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] (a+b)*AppellF1(1/2,-4/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
(a+b*sec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)/
(1+sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3919, 144, 143}

$$\frac{\sqrt{2}\,(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
- Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = -\frac{\tan(c+dx)\mathrm{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= \frac{\left((-a-b)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right.}{\sqrt{1-x}\sqrt{1}}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{-\frac{a+b\sec}{-a}}}$$

$$= \frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+}}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7313 vs. 2(108) = 216.
time = 42.93, size = 7313, normalized size = 67.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\left(a+b\sec(dx+c)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))^{\frac{4}{3}} \sec (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(4/3),x)
[Out] Integral((a + b*sec(c + d*x))**(4/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(4/3)/cos(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^(4/3)/cos(c + d*x), x)
```

3.693 $\int (a+b\sec (c+d x))^{4/3} \, dx$

Optimal. Leaf size=17

$$\text{Int}\left((a+b\sec(c+dx))^{4/3},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(4/3),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^{4/3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(4/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(4/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{4/3}\,dx = \int (a+b\sec(c+dx))^{4/3}\,dx$$

Mathematica [A]
time = 37.65, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{4/3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(4/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(4/3), x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(4/3),x)
```

[Out] int((a+b*sec(d*x+c))^(4/3),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^(4/3), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")

[Out] Timed out

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(4/3),x)

[Out] Integral((a + b*sec(c + d*x))**(4/3), x)

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^(4/3), x)

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int {\left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))^(4/3),x)

[Out] int((a + b/cos(c + d*x))^(4/3), x)

3.694 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=412

$$\frac{3a(18a^2+97b^2)\,(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d}+\frac{3(18a^2+121b^2)\,(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{1232b^2d}-\frac{9a}{}$$

```
[Out] 3/1232*a*(18*a^2+97*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d+3/1232*(18
*a^2+121*b^2)*(a+b*sec(d*x+c))^(5/3)*tan(d*x+c)/b^2/d-9/77*a*(a+b*sec(d*x+c
))^(8/3)*tan(d*x+c)/b^2/d+3/14*sec(d*x+c)*(a+b*sec(d*x+c))^(8/3)*tan(d*x+c)
/b/d+1/1232*(36*a^4+164*a^2*b^2+605*b^4)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec
(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^3/d/
((a+b*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/616*a*(18*a^4
+79*a^2*b^2-97*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2
*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^3/d/(a+b*sec(d*x+c
))^(1/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3950, 4167, 4087, 4092, 3919, 144, 143}

$$\frac{3(18a^2+121b^2)\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{1232b^2d}+\frac{3a(18a^2+97b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{1232b^2d}+\frac{(36a^4+164a^2b^2+605b^4)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{616\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{a(18a^4+79a^2b^2-97b^4)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{308\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{77b^2d}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*(18*a^2 + 97*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(1232*b^2*d
) + (3*(18*a^2 + 121*b^2)*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(1232*b^
2*d) - (9*a*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(77*b^2*d) + (3*Sec[c
+ d*x]*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(14*b*d) + ((36*a^4 + 164*a
^2*b^2 + 605*b^4)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(616*Sq
rt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) -
(a*(18*a^4 + 79*a^2*b^2 - 97*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c +
 d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1
/3)*Tan[c + d*x])/(308*Sqrt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c +
d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
```

```
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3950

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Dist[d^3/(b*(m + n -
1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b
*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
IntegersQ[2*m, 2*n]) && !IGtQ[m, 2]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3\sec(c+dx)(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{14bd} + \frac{3\int \sec(c+dx)}{\cdots}$$

$$= -\frac{9a(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{77b^2d} + \frac{3\sec(c+dx)(a+b\sec(}{14\cdots}$$

$$= \frac{3(18a^2+121b^2)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{1232b^2d} - \frac{9a(a+b\sec}{\cdots}$$

$$= \frac{3a(18a^2+97b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3(18a^2+12}{\cdots}$$

$$= \frac{3a(18a^2+97b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3(18a^2+12}{\cdots}$$

$$= \frac{3a(18a^2+97b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3(18a^2+12}{\cdots}$$

$$= \frac{3a(18a^2+97b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3(18a^2+12}{\cdots}$$

$$= \frac{3a(18a^2+97b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3(18a^2+12}{\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 28057 vs. $2(412) = 824$.

time = 38.67, size = 28057, normalized size = 68.10

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{4}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*(b*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^4,x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^4, x)
```

3.695 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=356

$$-\frac{3(15a^2-64b^2)\,(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd}-\frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{88bd}+\frac{3(a+b\sec(c+dx)}{11}$$

```
[Out] -3/440*(15*a^2-64*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d-9/88*a*(a+b*se
c(d*x+c))^(5/3)*tan(d*x+c)/b/d+3/11*(a+b*sec(d*x+c))^(8/3)*tan(d*x+c)/b/d-1
/440*a*(30*a^2-373*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/
2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d/((a+b*sec(d*x+c))
/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/220*(15*a^4-79*a^2*b^2+64*b^4)
*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*
sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^2/d/(a+b*sec(d*x+c))^(1/3)*2^(1/2)/(1
+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3925, 4087, 4092, 3919, 144, 143}

$$-\frac{a(30a^2-373b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{220\sqrt{2}\,b^2d\sqrt{\sec(c+dx)+1}\,\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{3(15a^2-64b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{440bd}+\frac{(15a^4-79a^2b^2+64b^4)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{110\sqrt{2}\,b^2d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{88bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*(15*a^2 - 64*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(440*b*d) -
(9*a*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(88*b*d) + (3*(a + b*Sec[c +
d*x])^(8/3)*Tan[c + d*x])/(11*b*d) - (a*(30*a^2 - 373*b^2)*AppellF1[1/2, 1/
2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*
Sec[c + d*x])^(2/3)*Tan[c + d*x])/(220*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x]]
*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + ((15*a^4 - 79*a^2*b^2 + 64*b^4)*Ap
pellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a
+ b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(110*Sqrt[2]*b^2*d
*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
```

```
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] &&  !LtQ[m, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{11bd} + \frac{3\int \sec(c+dx)\left(\frac{8b}{3}-a\sec(c\right.}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{88bd} + \frac{3(a+b\sec(c+dx))^{8/3}\text{ta}}{11bd}$$

$$= -\frac{3(15a^2-64b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec}{}$$

$$= -\frac{3(15a^2-64b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec}{}$$

$$= -\frac{3(15a^2-64b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec}{}$$

$$= -\frac{3(15a^2-64b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec}{}$$

$$= -\frac{3(15a^2-64b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 8065 vs. $2(356) = 712$.
time = 38.04, size = 8065, normalized size = 22.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^3(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(b*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^3,x)
[Out] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^3, x)
```

3.696 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=299

$$\frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d}+\frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d}+\frac{(2a^2+5b^2)\,F_1\left(\tfrac{1}{2};\tfrac{1}{2},-\tfrac{2}{3};\tfrac{3}{2};\tfrac{1}{2}(1-\right.}{4\sqrt{2}\,b}$$

```
[Out] 3/8*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/d+3/8*(a+b*sec(d*x+c))^(5/3)*tan(d*
x+c)/d+1/8*(2*a^2+5*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1
/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d/((a+b*sec(d*x+c))/
(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/4*a*(a^2-b^2)*AppellF1(1/2,1/3,
1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))
^(1/3)*tan(d*x+c)/b/d/(a+b*sec(d*x+c))^(1/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3920, 4087, 4092, 3919, 144, 143}

$$\frac{(2a^2+5b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{4\sqrt{2}\,bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{a(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{2\sqrt{2}\,bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}+\frac{3a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*d) + (3*(a + b*Sec[c + d*x
])^(5/3)*Tan[c + d*x])/(8*d) + ((2*a^2 + 5*b^2)*AppellF1[1/2, 1/2, -2/3, 3/
2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(4*Sqrt[2]*b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
+ d*x])/(a + b))^(2/3)) - (a*(a^2 - b^2)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 -
Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a +
 b))^(1/3)*Tan[c + d*x])/(2*Sqrt[2]*b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c
 + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3920

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[
e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{5}{8}\int \sec(c+dx)(b+a\sec($$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8d}$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8d}$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8d}$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8d}$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\text{ta}}{8d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 7809 vs. $2(299) = 598$.
time = 37.78, size = 7809, normalized size = 26.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(b*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^{\frac{5}{3}} \sec^2 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/3),x)
[Out] Integral((a + b*sec(c + d*x))**(5/3)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/3}}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^2,x)
[Out] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^2, x)
```

3.697 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] (a+b)*AppellF1(1/2,-5/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
(a+b*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/
(1+sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3919, 144, 143}

$$\frac{\sqrt{2}\,(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
- Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{(a+bx)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= \frac{((-a-b)(a+b\sec(c+dx))^{2/3}\tan(c+dx))\,\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{b}{-a}\right.}{\sqrt{1-x}\sqrt{}}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(-\frac{a+b\sec(}{-a}\right.}$$

$$= \frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7321 vs. $2(108) = 216$.
time = 36.83, size = 7321, normalized size = 67.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
[Out] Result too large to show
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{\frac{5}{3}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec (c+dx))^{\frac{5}{3}}\sec (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/3),x)
[Out] Integral((a + b*sec(c + d*x))**(5/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x), x)
```

3.698 $\int (a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left((a+b\sec(c+dx))^{5/3},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/3),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^{5/3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \int (a+b\sec(c+dx))^{5/3}\,dx$$

Mathematica [A]
time = 35.63, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{5/3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(5/3), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/3),x)
[Out] Integral((a + b*sec(c + d*x))**(5/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(5/3), x)
```

3.699 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=313

$$-\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d}+\frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd}+\frac{(18a^2+25b^2)\,F_1\Big(}{}$$

```
[Out] -9/20*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d+3/8*sec(d*x+c)*(a+b*sec(d*x
+c))^(2/3)*tan(d*x+c)/b/d+1/40*(18*a^2+25*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*
(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/
b^3/d/((a+b*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/20*a*(9
*a^2+11*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*
x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^3/d/(a+b*sec(d*x+c))^(1/3
)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3950, 4167, 4092, 3919, 144, 143}

$$-\frac{a(9a^2+11b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{10\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}+\frac{(18a^2+25b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{20\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\,\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{20b^2d}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-9*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(20*b^2*d) + (3*Sec[c + d*x]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*b*d) + ((18*a^2 + 25*b^2)*Appe
llF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(20*Sqrt[2]*b^3*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (a*(9*a^2 + 11*b^2)*App
ellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(10*Sqrt[2]*b^3*d*S
qrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3950

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Dist[d^3/(b*(m + n -
1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b
*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
IntegersQ[2*m, 2*n]) && !IGtQ[m, 2]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} + \frac{3\int \frac{\sec(c+dx)\left(a+\frac{5}{3}b\sec(c+dx)-2a\,\text{s}\right.}{\sqrt[3]{a+b\sec(c+d}}}{8b}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\,\text{ta}}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\,\text{ta}}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\,\text{ta}}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\,\text{ta}}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\,\text{ta}}{8bd}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 7796 vs. 2(313) = 626.
time = 37.86, size = 7796, normalized size = 24.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(dx+c)}{(a+b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/3)), x)
```

3.700 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=265

$$\frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd}-\frac{3\sqrt{2}\,aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+d}{5b^2d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] 3/5*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d-3/5*a*AppellF1(1/2,-2/3,1/2,3/2,b
*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*2^(1/2)*ta
n(d*x+c)/b^2/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+sec(d*x+c))^(1/2)+1/5*(3*a
^2+2*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c
))*((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/b^2/d/(a+b*sec(d*x+c))
^(1/3)/(1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3925, 4092, 3919, 144, 143}

$$\frac{\sqrt{2}\,(3a^2+2b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5b^2d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}-\frac{3\sqrt{2}\,a\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5b^2d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d) - (3*Sqrt[2]*a*AppellF1
[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b^2*d*Sqrt[1 + Sec[c + d*x]]*(
(a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2 + 2*b^2)*AppellF1[1/
2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
 + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(5*b^2*d*Sqrt[1 + Sec[c + d
*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] &&  !LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{3\int \frac{\sec(c+dx)\left(\frac{2b}{3}-a\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{5b}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{1}{5}\left(2+\frac{3a^2}{b^2}\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx \cdots$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{\left(\left(-2-\frac{3a^2}{b^2}\right)\tan(c+dx)\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt{1\cdots}}\right.}{5d\sqrt{1-\sec(c+dx\cdots}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{\left(3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Su}\cdots}{5b^2d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\cdots}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} - \frac{3\sqrt{2}\,aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\cdots\right.}{5b^2d\sqrt{1+\sec(c\cdots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7195 vs. $2(265) = 530$.
time = 36.43, size = 7195, normalized size = 27.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
[Out] Result too large to show
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(a+b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/3)), x)
```

3.701 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=219

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}\,aF_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\right.}{}$$

```
[Out] AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*s
ec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/b/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+s
ec(d*x+c))^(1/2)-a*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*
sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/b/d/(a+b*sec(
d*x+c))^(1/3)/(1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3923, 3919, 144, 143}

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}\,a\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*AppellF1[1/2
, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
+ b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*
(a + b*Sec[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3923

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Dist[-a/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + D
ist[1/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
 b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx}{b} - \frac{a\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{b}$$

$$= -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} + \frac{(a\tan(c+dx)}{}$$

$$= -\frac{\left((a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c-\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2759 vs. $2(219) = 438$.

time = 44.46, size = 2759, normalized size = 12.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*(b + a*Cos[c + d*x])*Tan[c + d*x])/(2*b*d*(a + b*Sec[c + d*x])^(1/3)) -
((b + 3*a*Cos[c + d*x])*(3*(b + a*Cos[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]
^2]*Sec[c + d*x]^(2/3) - (3*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b*S
ec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(
1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*
x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*
AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])),
 (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x])))/(5*b*(b
+ a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(4/3))))/(2*b
*d*(a + b*Sec[c + d*x])^(1/3)*((3*(b + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/
(Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(1/3)) - (2*a*Sqrt[1 - Cos[c + d*x]^
2]*Sec[c + d*x]^(2/3)*Sin[c + d*x])/(b + a*Cos[c + d*x])^(1/3) + 2*(b + a*C
os[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(5/3)*Sin[c + d*x]
 - (3*Sqrt[b^(-2)]*Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b
^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2,
5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a
 + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])
/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*
Sec[c + d*x]))*Sin[c + d*x])/(10*(1 - a*Sqrt[b^(-2)])*(b + a*Cos[c + d*x])^
(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 -
a*Sqrt[b^(-2)])]) + (3*Sqrt[b^(-2)]*Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])
*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF
1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b
*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a
+ b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[
b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(10*(1 + a*Sqrt[b^(-2)])*(b +
 a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 - Sqrt[b^(-2)]*b*Se
c[c + d*x])/(1 + a*Sqrt[b^(-2)])]) - (3*Sec[c + d*x]^(2/3)*Sqrt[(1 - Sqrt[b
^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c
 + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*
Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-
2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[
b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))
*Sin[c + d*x])/(5*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]) + (3
*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^
(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*
AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])),
 (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3
```

```
, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a +
1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*
x])^(1/3)*(1 - Cos[c + d*x]^2)^(3/2)*Sec[c + d*x]^(7/3)) - (a*(a + b*Sec[c
+ d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(
1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3,
1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c
+ d*x])/(a + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec
[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)]
)]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*x])^(4/3)*Sqrt
[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(4/3)) + (4*(a + b*Sec[c + d*x])*Sqrt[(1
- Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]
*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2, 5/3, -
((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/S
qrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a +
 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c
+ d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x
]^2]*Sec[c + d*x]^(1/3)) - (3*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b
*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])
/(1 - a*Sqrt[b^(-2)])]*(2*b*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d
*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec
[c + d*x]*Tan[c + d*x] - 5*a*((b*AppellF1[5/3, 1/2, 3/2, 8/3, -((a + b*Sec[
c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])
]*Sec[c + d*x]*Tan[c + d*x])/(5*(a + 1/Sqrt[b^(-2)])) - (b*AppellF1[5/3, 3/
2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c +
d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(5*(-a + 1/Sqrt[b^(-
2)]))) + 2*(a + b*Sec[c + d*x])*((5*b*AppellF1[8/3, 1/2, 3/2, 11/3, -((a +
b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^
(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(16*(a + 1/Sqrt[b^(-2)])) - (5*b*AppellF
1[8/3, 3/2, 1/2, 11/3, -((a + b*Sec[c + d*x])/(...
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(a+b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)

[Out] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2 (c + dx)}{\sqrt[3]{a + b \sec (c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos (c + d\,x)^2 \left(a + \frac{b}{\cos (c+d\,x)}\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/3)), x)
```

3.702 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

```
[Out] AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*s
ec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(1/3)/(1+sec(
d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3919, 144, 143}

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sqrt
[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{-\frac{a}{-a-b}} \cdots}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c \cdots}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c \cdots}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 310 vs. 2(105) = 210.
time = 13.02, size = 310, normalized size = 2.95

$$\frac{15(a-b)^2(a+b)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)\cos(c+dx)\cot^3(c+dx)(1+\sec(c+dx))(b-b\sec(c+dx))(a+b\sec(c+dx))^{2/3}}{b^2(-a+b)d\left(3(a-b)F_1\left(\frac{5}{3};\frac{1}{2},\frac{3}{2};\frac{8}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)(b+a\cos(c+dx))+(a+b)\left(10(a-b)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)\cos(c+dx)+3F_1\left(\frac{5}{3};\frac{3}{2},\frac{1}{2};\frac{8}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)(b+a\cos(c+dx))\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (15*(a - b)^2*(a + b)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a
- b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[c
+ d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(2/3))/(b^2*(-a + b)*d*(3
*(a - b)*AppellF1[5/3, 1/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a - b), (a + b*
Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(10*(a - b)*AppellF1[
2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])/(a +
 b)]*Cos[c + d*x] + 3*AppellF1[5/3, 3/2, 1/2, 8/3, (a + b*Sec[c + d*x])/(a
- b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx + c)}{(a + b\sec (dx + c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (c + dx)}{\sqrt[3]{a + b\sec (c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/3)), x)
```

3.703 $\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\sqrt[3]{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/(a+b*sec(d*x+c))^(1/3),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-1/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-1/3), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-1/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(-1/3), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(1/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + b \sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(1/3),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(1/3), x)
```

3.704 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}$$

```
[Out] AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*s
ec(d*x+c))/(a+b))^(2/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(2/3)/(1+sec(
d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3919, 144, 143}

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(d*Sqrt
[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\,(a+bx)^{2/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\,\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)}\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx)}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+}{d\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 310 vs. 2(105) = 210.
time = 13.85, size = 310, normalized size = 2.95

$$\frac{24(a-b)^2(a+b)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)\cos(c+dx)\cot^3(c+dx)(1+\sec(c+dx))(b-b\sec(c+dx))\sqrt[3]{a+b\sec(c+dx)}}{b^2(-a+b)d\left(3(a-b)F_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)(b+a\cos(c+dx))+(a+b)\left(8(a-b)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)\cos(c+dx)+3F_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)(b+a\cos(c+dx))\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (24*(a - b)^2*(a + b)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a
- b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[c
+ d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(1/3))/(b^2*(-a + b)*d*(3
*(a - b)*AppellF1[4/3, 1/2, 3/2, 7/3, (a + b*Sec[c + d*x])/(a - b), (a + b*
Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(8*(a - b)*AppellF1[1
/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])/(a +
b)]*Cos[c + d*x] + 3*AppellF1[4/3, 3/2, 1/2, 7/3, (a + b*Sec[c + d*x])/(a -
 b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)}{\left(a+b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+dx)\left(a+\frac{b}{\cos(c+dx)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(2/3)),x)
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(2/3)), x)
```

3.705 $\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{2/3}},x\right)$$

```
[Out] Unintegrable(1/(a+b*sec(d*x+c))^(2/3),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-2/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-2/3), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-2/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(-2/3), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(2/3),x)
[Out] int(1/(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec(c + dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(-2/3), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(2/3),x)
[Out] int(1/(a + b/cos(c + d*x))^(2/3), x)
```

3.706 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{(a+b)d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

```
[Out] AppellF1(1/2,4/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*s
ec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/(a+b)/d/(a+b*sec(d*x+c))^(1/3)/(
1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3919, 144, 143}

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/((a + b
)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{4/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{b}{-a}\right.}\right.}{(a+b)d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c}{(a+b)d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 10343 vs. $2(110) = 220$.
time = 38.22, size = 10343, normalized size = 94.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos \left(c+d\,x\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(4/3)),x)
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(4/3)), x)
```

3.707 $\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{4/3}},x\right)$$

```
[Out] Unintegrable(1/(a+b*sec(d*x+c))^(4/3),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-4/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-4/3), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

Mathematica [A]
time = 40.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-4/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(-4/3), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(4/3),x)
[Out] int(1/(a+b*sec(d*x+c))^(4/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(4/3),x)
[Out] Integral((a + b*sec(c + d*x))**(-4/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(4/3),x)
[Out] int(1/(a + b/cos(c + d*x))^(4/3), x)
```

3.708 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=378

$$-\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}}+\frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx)}{4b^2\left(a^2-b^2\right)d}-\frac{a\left(9a^2-7b^2\right)F_1\left(\frac{1}{2};\frac{1}{2},-\right.}{2\sqrt{2}}$$

```
[Out] -3/2*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)+3/4*(3*
a^2-b^2)*(a+b*sec(d*x+c))^(1/3)*tan(d*x+c)/b^2/(a^2-b^2)/d-1/4*a*(9*a^2-7*b
^2)*AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a
+b*sec(d*x+c))^(1/3)*tan(d*x+c)/b^3/(a^2-b^2)/d/((a+b*sec(d*x+c))/(a+b))^(1
/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/4*(9*a^4-10*a^2*b^2-b^4)*AppellF1(1/2,2/
3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b
))^(2/3)*tan(d*x+c)/b^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)*2^(1/2)/(1+sec(d
*x+c))^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3930, 4167, 4092, 3919, 144, 143}

$$-\frac{a\left(9a^2-7b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}+\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)}+\frac{\left(9a^4-10a^2b^2-b^4\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
(2/3)) + (3*(3*a^2 - b^2)*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*b^2*(
a^2 - b^2)*d) - (a*(9*a^2 - 7*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c
 + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[
c + d*x])/(2*Sqrt[2]*b^3*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
 + d*x])/(a + b))^(1/3)) + ((9*a^4 - 10*a^2*b^2 - b^4)*AppellF1[1/2, 1/2, 2
/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[
c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(2*Sqrt[2]*b^3*(a^2 - b^2)*d*Sqrt[1
+ Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
```

```
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
```

```
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} - \frac{3\int \frac{\sec(c+dx)\left(a^2-\frac{2}{3}ab\sec(c+dx)-\frac{2}{3}(3a^2-b^2)\sec^{}\right.}{(a+b\sec(c+dx))^{2/3}}}{2b(a^2-b^2)}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{3(3a^2-b^2)\sqrt[3]{a+b\sec(c+dx)}\tan(c}{4b^2(a^2-b^2)\,d}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{3(3a^2-b^2)\sqrt[3]{a+b\sec(c+dx)}\tan(c}{4b^2(a^2-b^2)\,d}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{3(3a^2-b^2)\sqrt[3]{a+b\sec(c+dx)}\tan(c}{4b^2(a^2-b^2)\,d}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{3(3a^2-b^2)\sqrt[3]{a+b\sec(c+dx)}\tan(c}{4b^2(a^2-b^2)\,d}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{3(3a^2-b^2)\sqrt[3]{a+b\sec(c+dx)}\tan(c}{4b^2(a^2-b^2)\,d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 8160 vs. $2(378) = 756$.
time = 41.69, size = 8160, normalized size = 21.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*
a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/3)),x)
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/3)), x)
```

3.709 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=307

$$-\frac{3a^2\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}}+\frac{(3a^2-2b^2)\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b}}{\sqrt{2}\,b^2\,(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{\frac{a+b\sec(c}{a+b}}}$$

```
[Out] -3/2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)+1/2*(3*a^2-2*b^2)*
AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*s
ec(d*x+c))^(1/3)*tan(d*x+c)/b^2/(a^2-b^2)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)*
2^(1/2)/(1+sec(d*x+c))^(1/2)-1/2*a*(3*a^2-4*b^2)*AppellF1(1/2,2/3,1/2,3/2,b
*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(2/3)*ta
n(d*x+c)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2
)
```

Rubi [A]
time = 0.29, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3924, 4092, 3919, 144, 143}

$$-\frac{a(3a^2-4b^2)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}\,b^2d\,(a^2-b^2)\,\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}+\frac{(3a^2-2b^2)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}\,b^2d\,(a^2-b^2)\,\sqrt{\sec(c+dx)+1}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}-\frac{3a^2\tan(c+dx)}{2bd\,(a^2-b^2)\,(a+b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) + ((3*
a^2 - 2*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Se
c[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(Sqrt[2]*b^2
*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
 - (a*(3*a^2 - 4*b^2)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b
*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c +
d*x])/(Sqrt[2]*b^2*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x]
)^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3924

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{3a^2\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} - \frac{3\int \frac{\sec(c+dx)\left(-\frac{2ab}{3}-\frac{1}{3}(3a^2-2b^2)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}}{2b\,(a^2-b^2)}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} - \frac{(a(3a^2-4b^2))\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx}{2b^2\,(a^2-b^2)} +$$

$$= -\frac{3a^2\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{(a(3a^2-4b^2)\tan(c+dx))\,\mathrm{Subst}\left(\int\right.}{2b^2\,(a^2-b^2)\,d\sqrt{1-\mathrm{se}}}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} - \frac{\left((3a^2-2b^2)\sqrt[3]{a+b\sec(c+dx)}\,\tan\right.}{2b^2\,(a^2-b^2)\,d\sqrt{1-\sec}}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{(3a^2-2b^2)\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec\right.}{\sqrt{2}\,b^2\,(a^2-b^2)\,d\sqrt{}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7918 vs. $2(307) = 614$.
time = 41.39, size = 7918, normalized size = 25.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
[Out] Result too large to show
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2*
a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/3),x)
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/3)),x)
[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/3)), x)
```

3.710 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=289

$$\frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}}-\frac{aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}}{\sqrt{2}\,b\left(a^2-b^2\right)d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] 3/2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)-1/2*a*AppellF1(1/2,-1/3
,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(1/3)*
tan(d*x+c)/b/(a^2-b^2)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)/(1+sec(d*x+
c))^(1/2)+1/2*(a^2-2*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1
/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(2/3)*tan(d*x+c)/b/(a^2-b^2)/d/
(a+b*sec(d*x+c))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3921, 4092, 3919, 144, 143}

$$\frac{\left(a^2-2b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}\,bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}-\frac{a\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}\,bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}+\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) - (a*Appell
F1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b
)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(Sqrt[2]*b*(a^2 - b^2)*d*Sqrt[1
 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + ((a^2 - 2*b^2)*App
ellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(Sqrt[2]*b*(a^2 - b
^2)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \frac{3a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{3\int \frac{\sec(c+dx)\left(-\frac{2b}{3}-\frac{1}{3}a\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}\,dx}{2(a^2-b^2)}$$

$$= \frac{3a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} - \frac{a\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx}{2b(a^2-b^2)}$$

$$= \frac{3a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{(a\tan(c+dx))\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+}}\right.}{2b(a^2-b^2)\,d\sqrt{1-\sec(c+dx)}\,\sqrt{}}$$

$$= \frac{3a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{\left(a\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx)\right)\text{Su}}{2b(a^2-b^2)\,d\sqrt{1-\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{2(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} - \frac{aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-}{}\right.}{\sqrt{2}\,b(a^2-b^2)\,d\sqrt{1+\sec}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7325 vs. 2(289) = 578.
time = 42.69, size = 7325, normalized size = 25.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*
a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/3),x)
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/3)),x)
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/3)), x)
```

3.711 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{(a+b)d\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}$$

```
[Out] AppellF1(1/2,5/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*s
ec(d*x+c))/(a+b))^(2/3)*2^(1/2)*tan(d*x+c)/(a+b)/d/(a+b*sec(d*x+c))^(2/3)/(
1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3919, 144, 143}

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/((a + b
)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{\tan(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\,(a+bx)^{5/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{5/}}\right.}{(a+b)d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+a}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+d}{(a+b)d\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 10363 vs. $2(110) = 220$.
time = 44.39, size = 10363, normalized size = 94.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
[Out] Result too large to show
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/3),x)
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/3)),x)
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/3)), x)
```

3.712 $\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{5/3}},x\right)$$

```
[Out] Unintegrable(1/(a+b*sec(d*x+c))^(5/3),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-5/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-5/3), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

Mathematica [A]
time = 49.96, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-5/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(-5/3), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(1/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sec{\left(c + dx \right)}\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-5/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(5/3),x)
[Out] int(1/(a + b/cos(c + d*x))^(5/3), x)
```

3.713 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{aF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}}-\frac{bF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c+d}}{(a^2-b^2)\,d}$$

```
[Out] a*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x+
c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)/sec(d*x+c)^(1/3)-b*AppellF1(1/2,1/3,1,3
/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sec(d*x+c)
^(2/3)*sin(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3954, 2902, 3268, 440}

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3
)) - (b*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3)*Sin[c + d*x])/((a^2 - b
^2)*d)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \left(\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\sqrt[3]{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) + \left(b\cos^{\frac{2}{3}}(c+dx)\right.$$

$$= -\frac{a\text{Subst}\left(\int \frac{\sqrt[6]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}} + \frac{\left(b\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)\right)\,\mathrm{S}}{}$$

$$= \frac{aF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}} - \frac{bF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+d\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 4543 vs. $2(174) = 348$.
time = 39.54, size = 4543, normalized size = 26.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (9*(a^2 - b^2)*Sec[c + d*x]^(5/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/6, 1, 3/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])
/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -T
```

an[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*Appe
llF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/
(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2
, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(d*
(Sec[c + d*x]^2)^(2/3)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c + d*x]^2)*((9
(a^2 - b^2)(Sec[c + d*x]^2)^(1/3)*((b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 -
b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d
*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]
^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 2
/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 -
b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c +
d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*Se
c[c + d*x]^2) - (18*b^2*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*Tan[c + d*x]^2*(
(b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[
c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2
, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*Ap
pellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)*Tan[c + d*x]^2) + (a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan
[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[
c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3,
 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2
)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b
^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*Sec[c + d*x]^2)^2 - (12*(a^2 - b^2)*Tan
[c + d*x]^2*((b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6,
1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF
1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-
a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)
/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x
]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3,
1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*Appel
lF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] +
2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x
]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^(2/3)*(-a^2 + b^2*Se
c[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((b*AppellF1[1/2, 1/6, 1, 3/2,
 -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2]*Ta
n[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2
*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c +

```
d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6,
 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2
) + (b*Sqrt[Sec[c + d*x]^2]*((2*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x
]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2
 - b^2)) - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2
)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2
, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*
AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2
)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*((2*b^2*AppellF1[3/2, 2/3, 2, 5/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c
+ d*x])/(3*(a^2 - b^2)) - (4*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b
^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 -
 b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan...
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}\left(dx+c\right)}{a+b\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x)), x)
```

3.714 $\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{aF_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)}-\frac{bF_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c+}{(a^2-b^2)\,d}$$

```
[Out] a*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x+
c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)/sec(d*x+c)^(2/3)-b*AppellF1(1/2,1/6,1,3
/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sec(d*x+c)
^(1/3)*sin(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3954, 2902, 3268, 440}

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\,(a^2-b^2)\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3
)) - (b*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3)*Sin[c + d*x])/((a^2 - b
^2)*d)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \left(\sqrt[3]{\cos(c+dx)}\,\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{2}{3}}(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\sqrt[3]{\cos(c+dx)}\,\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) + \left(b\sqrt[3]{\cos(c+dx)}\right.$$

$$= -\frac{a\mathrm{Subst}\left(\int \frac{\sqrt[3]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)} + \frac{\left(b\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}\right)\,\mathrm{S}}{}$$

$$= \frac{aF_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)} - \frac{bF_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4544 vs. $2(174) = 348$.
time = 39.56, size = 4544, normalized size = 26.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (9*(a^2 - b^2)*Sec[c + d*x]^(4/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/3, 1, 3/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])
/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2,
```

-Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*Ap
pellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2,
(b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(d
*(Sec[c + d*x]^2)^(5/6)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c + d*x]^2)*((
9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 -
b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d
*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2
, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2
- b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a
^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*
Sec[c + d*x]^2) - (18*b^2*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*Tan[c + d*x]^2
*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Ta
n[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/
3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2
)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b
^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2
*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -
Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5
/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b
^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*Sec[c + d*x]^2)^2 - (15*(a^2 - b^2)*
Tan[c + d*x]^2*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/
3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*Ap
pellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
+ (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2,
5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*A
ppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)
] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^(5/6)*(-a^2 + b^
2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((b*AppellF1[1/2, 1/3, 1,
3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2
]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2,
(b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Ta

```
n[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2
, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c +
d*x]^2) + (b*Sqrt[Sec[c + d*x]^2]*((2*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c
 + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(
3*(a^2 - b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(9*(a^2 - b^2)*Appe
llF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] +
 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/
(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*((2*b^2*AppellF1[3/2,
5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x
]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (5*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))
/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d
*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2
, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - ...
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{1}{3}}\left(dx+c\right)}{a+b\sec\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec{(c+dx)}}}{a+b\sec{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
[Out] Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{1/3}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x)),x)
[Out] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x)), x)
```

3.715 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=174

$$-\frac{bF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}}+\frac{aF_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c}{(a^2-b^2)\,d}$$

```
[Out] -b*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x
+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)/sec(d*x+c)^(1/3)+a*AppellF1(1/2,-2/3,1
,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sec(d*x+
c)^(2/3)*sin(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3954, 2902, 3268, 440}

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\,(a^2-b^2)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\,(a^2-b^2)\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -((b*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1
/3))) + (a*AppellF1[1/2, -2/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3)*Sin[c + d*x])/((a^2
 - b^2)*d)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx = \left(\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\cos^{\frac{7}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) +$$

$$= \frac{b\text{Subst}\left(\int \frac{\sqrt[6]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}} - \frac{\left(a\sqrt[3]{\cos^2(c+dx)}\ \text{s}\right.}{}$$

$$= -\frac{bF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}} + \frac{aF_1\left(\frac{1}{2}\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 7542 vs. $2(174) = 348$.
time = 126.71, size = 7542, normalized size = 43.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
[Out] Result too large to show
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}\,(a+b\sec(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec(c + dx)) \sqrt[3]{\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/3)),x)
[Out] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/3)), x)
```

3.716 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=174

$$-\frac{bF_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)}+\frac{aF_1\left(\frac{1}{2};-\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c}{(a^2-b^2)\,d}$$

```
[Out] -b*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x
+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)/sec(d*x+c)^(2/3)+a*AppellF1(1/2,-5/6,1
,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sec(d*x+
c)^(1/3)*sin(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3954, 2902, 3268, 440}

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}\,F_1\left(\frac{1}{2};-\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -((b*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
 - b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2
/3))) + (a*AppellF1[1/2, -5/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3)*Sin[c + d*x])/((a^2
 - b^2)*d)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \left(\sqrt[3]{\cos(c+dx)}\ \sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\sqrt[3]{\cos(c+dx)}\ \sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{8}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right)$$

$$= \frac{b\text{Subst}\left(\int \frac{\sqrt[3]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos^2(c+dx)}\ \sec^{\frac{2}{3}}(c+dx)} - \frac{\left(a\sqrt[6]{\cos^2(c+dx)}\ \sqrt[3]{s\right.}{}$$

$$= -\frac{bF_1\left(\frac{1}{2}; -\frac{1}{3}, 1; \frac{3}{2}; \sin^2(c+dx), \frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos^2(c+dx)}\ \sec^{\frac{2}{3}}(c+dx)} + \frac{aF_1\left(\frac{1}{2};\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7588 vs. 2(174) = 348.
time = 126.54, size = 7588, normalized size = 43.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{2}{3}}\left(a+b\sec\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec{(c+dx)})\sec^{\frac{2}{3}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(2/3)),x)
[Out] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(2/3)), x)
```

3.717 $\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,,x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A]
time = 112.04, size = 0, normalized size = 0.00

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{7}{3}}\left(dx+c\right)\right)\sqrt{a+b\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3), x)
```

3.718 $\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A]
time = 113.00, size = 0, normalized size = 0.00

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{5}{3}}(dx+c)\right)\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3), x)
```

3.719 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A]
time = 63.22, size = 0, normalized size = 0.00

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}\left(dx+c\right)\right)\sqrt{a+b\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3), x)
```

3.720 $\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A]
time = 71.38, size = 0, normalized size = 0.00

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{2}{3}}(dx+c)\right)\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{\left(c + dx \right)}} \sec^{\frac{2}{3}}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3), x)
```

3.721 $\int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,,x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A]
time = 7.16, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{1}{3}}(dx+c)\right)\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{\left(c + dx \right)}} \sqrt[3]{\sec{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3), x)
```

3.722 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 22.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec{\left(c + dx \right)}}}{\sqrt[3]{\sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/3),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/3), x)
```

3.723 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 26.77, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec{\left(c + dx \right)}}}{\sec^{\frac{2}{3}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(2/3),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(2/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a + \frac{b}{\cos\left(c + d\,x\right)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(2/3), x)
```

3.724 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 34.56, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sec{(c + dx)}}}{\sec^{\frac{4}{3}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(4/3),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(4/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(4/3), x)
```

3.725 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 79.84, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec{\left(c + dx \right)}}}{\sec^{\frac{5}{3}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/3),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/3), x)
```

3.726 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]
```

```
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 92.97, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/3), x)
```

3.727 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A]
time = 104.99, size = 0, normalized size = 0.00

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{7}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*sec
(d*x + c)^(1/3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos \left(c+d\,x\right)}\right)}^{7/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3), x)
```

3.728 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A]
time = 99.66, size = 0, normalized size = 0.00

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{5}{3}}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(d
*x + c)^(2/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3), x)
```

3.729 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A]
time = 58.41, size = 0, normalized size = 0.00

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(d
*x + c)^(1/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3), x)
```

3.730 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A]
time = 62.13, size = 0, normalized size = 0.00

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{2}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3), x)
```

3.731 $\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx=\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A]
time = 41.29, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{1}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3), x)
```

3.732 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 92.04, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^{\frac{3}{2}}}{\sqrt[3]{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/3), x)
```

3.733 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 51.35, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\sec^{\frac{2}{3}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(2/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(2/3), x)
```

3.734 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 56.22, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^{\frac{3}{2}}}{\sec^{\frac{4}{3}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(4/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(4/3), x)
```

3.735 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 90.73, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(c + dx)})^{\frac{3}{2}}}{\sec^{\frac{5}{3}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(5/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/3), x)
```

3.736 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 123.76, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/3), x)
```

3.737 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A]
time = 112.89, size = 0, normalized size = 0.00

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{7}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3), x)
```

3.738 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A]
time = 107.05, size = 0, normalized size = 0.00

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{5}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3), x)
```

3.739 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A]
time = 110.16, size = 0, normalized size = 0.00

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3), x)
```

3.740 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A]
time = 104.06, size = 0, normalized size = 0.00

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{2}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)*sec(d*x + c)^(2/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3), x)
```

3.741 $\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx=\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A]
time = 90.22, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{1}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3), x)
```

3.742 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 115.19, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(1/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/3), x)
```

3.743 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 103.75, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(2/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(2/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(2/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(2/3), x)
```

3.744 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 116.79, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)

[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(4/3), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")

[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(4/3), x)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(4/3),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(4/3), x)
```

3.745 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 114.25, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(5/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/3), x)
```

3.746 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Mathematica [A]
time = 135.10, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(7/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/3), x)
```

3.747 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 45.40, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(dx+c)}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.748 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 61.02, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(dx+c)}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.749 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 3.62, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(dx+c)}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}\left(c + dx\right)}{\sqrt{a + b\sec\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(4/3)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.750 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 3.65, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(dx+c)}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(2/3)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.751 $\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{1}{3}}(dx+c)}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec{\left(c + dx \right)}}}{\sqrt{a + b\sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(1/3)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.752 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^2 + a*
sec(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sec{\left(c + dx \right)}} \sqrt[3]{\sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3)), x)
```

3.753 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 65.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{2}{3}}\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^2 + a*
sec(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sec{\left(c + dx \right)}}\sec^{\frac{2}{3}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3)), x)
```

3.754 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 78.24, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{4}{3}}\sqrt{a+b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^3 + a*
sec(d*x + c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sec{\left(c + dx \right)}} \sec^{\frac{4}{3}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(4/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos (c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3)), x)
```

3.755 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 70.54, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{5}{3}}\sqrt{a+b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
[Out] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^3 + a*
sec(d*x + c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sec{\left(c + dx \right)}}\sec^{\frac{5}{3}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(5/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+dx)}}\left(\frac{1}{\cos(c+dx)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3)), x)
```

3.756 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 89.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{7}{3}}\sqrt{a+b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^4 + a*
sec(d*x + c)^3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3)), x)
```

3.757 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

[Out] Unintegrable(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]

[Out] Defer[Int][Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2), x]

Rubi steps

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 69.43, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]

[Out] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2), x]

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.758 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 81.02, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)

[Out] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(3/2), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: AttributeError >> type

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.759 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 77.27, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(4/3)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.760 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

[Out] Unintegrable(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]

[Out] Defer[Int][Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2), x]

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 91.43, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]

[Out] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2), x]

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.761 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 82.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{1}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec (c + dx)}}{(a + b \sec (c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.762 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 95.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{1}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^3 +
2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(1/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3)), x)
```

3.763 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 89.80, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{2}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^3 +
2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})^{\frac{3}{2}}\sec^{\frac{2}{3}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(2/3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3)), x)
```

3.764 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 100.49, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{4}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
[Out] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^4 +
2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})^{\frac{3}{2}} \sec^{\frac{4}{3}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(4/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+dx)}\right)^{3/2}\left(\frac{1}{\cos(c+dx)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3)), x)
```

3.765 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 75.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{5}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^4 +
2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3)), x)
```

3.766 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 107.54, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{7}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^5 +
2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3)), x)
```

3.767 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 91.75, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.768 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 104.58, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.769 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 94.02, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.770 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 103.42, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.771 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 100.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{1}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec{\left(c + dx \right)}}}{\left(a + b \sec{\left(c + dx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.772 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 107.81, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}\,(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^4 +
3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sec{\left(c + d x \right)}\right)^{\frac{5}{2}} \sqrt[3]{\sec{\left(c + d x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**(1/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{5/2} \left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3)), x)
```

3.773 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 107.52, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{2}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^4 +
3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3)), x)
```

3.774 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 117.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{4}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^5 +
3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3)), x)
```

3.775 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 78.55, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{5}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^5 +
3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3)), x)
```

3.776 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

[Out] Unintegrable(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] Defer[Int][1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)), x]

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 123.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)), x]

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{7}{3}}(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^6 +
3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3)), x)
```

3.777 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx$

Optimal. Leaf size=251

$$-\frac{ad(3b^2n+a^2(1+n))\ {}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n^2)\sqrt{\sin^2(e+fx)}}+\frac{b(b^2(1+n)+3a^2(2}{}$$

```
[Out] -a*d*(3*b^2*n+a^2*(1+n))*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^
2)*(d*sec(f*x+e))^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+b*(b^2*
(1+n)+3*a^2*(2+n))*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*(d*sec(f
*x+e))^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(1/2)+a*b^2*(5+2*n)*(d*sec(f*x
+e))^n*tan(f*x+e)/f/(1+n)/(2+n)+b^2*(d*sec(f*x+e))^n*(a+b*sec(f*x+e))*tan(f
*x+e)/f/(2+n)
```

Rubi [A]
time = 0.27, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3927, 4132, 3857, 2722, 4131}

$$-\frac{ad(a^2(n+1)+3b^2n)\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n^2)\sqrt{\sin^2(e+fx)}}+\frac{b(3a^2(n+2)+b^2(n+1))\sin(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn(n+2)\sqrt{\sin^2(e+fx)}}+\frac{ab^2(2n+5)\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)(n+2)}+\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((a*d*(3*b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2,
 Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[
Sin[e + f*x]^2])) + (b*(b^2*(1 + n) + 3*a^2*(2 + n))*Hypergeometric2F1[1/2,
 -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*(
2 + n)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*(5 + 2*n)*(d*Sec[e + f*x])^n*Tan[e +
f*x])/(f*(1 + n)*(2 + n)) + (b^2*(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])*Ta
n[e + f*x])/(f*(2 + n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \frac{b^2(d\sec(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+n)} + \frac{\int (d\sec(e$$

$$= \frac{b^2(d\sec(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+n)} + \frac{\int (d\sec(e$$

$$= \frac{ab^2(5+2n)(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)(2+n)} + \frac{b^2(d\sec(e+fx))^n($$

$$= \frac{b(b^2(1+n)+3a^2(2+n))\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)(d\sec}{fn(2+n)\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{a\left(a^2+\frac{3b^2n}{1+n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\,se}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]

time = 0.58, size = 231, normalized size = 0.92

$$-\frac{\csc^3(e+fx)\left(a^3(6+11n+6n^2+n^3)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\sec^2(e+fx)\right)+bn\left(3a^2(6+5n+n^2)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sec^2(e+fx)\right)+b(1+n)\left(3a(3+n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sec^2(e+fx)\right)+b(2+n)\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\sec^2(e+fx)\right)\right)\right)\right)(d\sec(e+fx))^n\left(-\tan^2(e+fx)\right)^{3/2}}{fn(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((Csc[e + f*x]^3*(a^3*(6 + 11*n + 6*n^2 + n^3)*Cos[e + f*x]^3*Hypergeometr
ic2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(3*a^2*(6 + 5*n + n^2)*Cos
[e + f*x]^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2] +
b*(1 + n)*(3*a*(3 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sec[e + f*x]^2] + b*(2 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)
/2, Sec[e + f*x]^2])))*(d*Sec[e + f*x])^n*(-Tan[e + f*x]^2)^(3/2))/(f*n*(1
+ n)*(2 + n)*(3 + n)))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n (a+b\sec(e+fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos(e+f\,x)}\right)^3 \left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^3*(d/cos(e + f*x))^n,x)
[Out] int((a + b/cos(e + f*x))^3*(d/cos(e + f*x))^n, x)
```

3.778 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$

Optimal. Leaf size=181

$$-\frac{d(b^2n+a^2(1+n))\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+n}\sin(e+fx)}{f\,(1-n^2)\sqrt{\sin^2(e+fx)}}+\frac{2ab\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos\right.}{f}$$

```
[Out] -d*(b^2*n+a^2*(1+n))*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(
d*sec(f*x+e))^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+2*a*b*hyper
geom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^n*sin(f*x+e)/f/n/
(sin(f*x+e)^2)^(1/2)+b^2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+n)
```

Rubi [A]

time = 0.11, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3873, 3857, 2722, 4131}

$$-\frac{d(a^2(n+1)+b^2n)\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\,(1-n^2)\sqrt{\sin^2(e+fx)}}+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] -((d*(b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos
[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[
e + f*x]^2])) + (2*a*b*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*
x]^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) + (b^2*(d
*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx = \frac{(2ab)\int (d\sec(e+fx))^{1+n}\,dx}{d} + \int (d\sec(e+fx))^n \left(a^2+b^2\sec\right.$$

$$= \frac{b^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)} + \left(a^2+\frac{b^2n}{1+n}\right)\int (d\sec(e+$$

$$= \frac{2ab\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{\left(a^2+\frac{b^2n}{1+n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.22, size = 171, normalized size = 0.94

$$\frac{\csc(e+fx)\left(a^2(2+3n+n^2)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\sec^2(e+fx)\right)+bn\left(2a(2+n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sec^2(e+fx)\right)+b(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sec^2(e+fx)\right)\right)\right)\sec(e+fx)(d\sec(e+fx))^n\sqrt{-\tan^2(e+fx)}}{fn(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Csc[e + f*x]*(a^2*(2 + 3*n + n^2)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, n/
2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(2*a*(2 + n)*Cos[e + f*x]*Hypergeometri
c2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2] + b*(1 + n)*Hypergeometric2
F1[1/2, (2 + n)/2, (4 + n)/2, Sec[e + f*x]^2]))*Sec[e + f*x]*(d*Sec[e + f*x
])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n)*(2 + n))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^n\,(a+b\sec(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*sec(f*x + e))^n
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(e+fx\right)\right)^{n} \left(a+b \sec \left(e+fx\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^2\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^2*(d/cos(e + f*x))^n,x)
[Out] int((a + b/cos(e + f*x))^2*(d/cos(e + f*x))^n, x)
```

3.779 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx$

Optimal. Leaf size=137

$$-\frac{ad\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}+\frac{b\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)(d\sec(e$$

```
[Out] -a*d*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^(-
1+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)+b*hypergeom([1/2, -1/2*n],[1-1
/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3872, 3857, 2722}

$$\frac{b\sin(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] -((a*d*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(d*Sec[
e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (b*Hyp
ergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^n*S
in[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = a\int (d\sec(e+fx))^n\,dx + \frac{b\int (d\sec(e+fx))^{1+n}\,dx}{d}$$

$$= \left(a\left(\frac{\cos(e+fx)}{d}\right)^n (d\sec(e+fx))^n\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{-n} dx$$

$$= -\frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^n \text{s}}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [**A**]
time = 0.10, size = 107, normalized size = 0.78

$$\frac{\csc(e+fx)\left(a(1+n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\sec^2(e+fx)\right)+bn\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sec^2(e+fx)\right)\right)(d\sec(e+fx))^n\sqrt{-\tan^2(e+fx)}}{fn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
[Out] (Csc[e + f*x]*(a*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2
, Sec[e + f*x]^2] + b*n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e
+ f*x]^2])*(d*Sec[e + f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^n\,(a+b\sec(fx+e))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d \sec (e + fx))^n (a + b \sec (e + fx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{\cos (e + f\,x)}\right) \left(\frac{d}{\cos (e + f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))*(d/cos(e + f*x))^n,x)
[Out] int((a + b/cos(e + f*x))*(d/cos(e + f*x))^n, x)
```

3.780 $\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=192

$$\frac{aF_1\left(\frac{1}{2};\frac{1}{2}(-1+n),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+fx))^n\sin(e+}{(a^2-b^2)\,f}$$

```
[Out] a*AppellF1(1/2,-1/2+1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*co
s(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/
f-b*AppellF1(1/2,1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(cos(
f*x+e)^2)^(1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/f
```

Rubi [A]
time = 0.22, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3954, 2902, 3268, 440}

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^nF_1\left(\frac{1}{2};\frac{n-1}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\,(a^2-b^2)}-\frac{b\sin(e+fx)\cos^2(e+fx)^{n/2}(d\sec(e+fx))^nF_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] (a*AppellF1[1/2, (-1 + n)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(
a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n*S
in[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, n/2, 1, 3/2, Sin[e + f*x]^2
, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*(d*Sec[e + f*x])
^n*Sin[e + f*x])/((a^2 - b^2)*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
```

```
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] && !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx = (\cos^n(e+fx)(d\sec(e+fx))^n)\int \frac{\cos^{1-n}(e+fx)}{b+a\cos(e+fx)}\,dx$$
$$= -\left((a\cos^n(e+fx)(d\sec(e+fx))^n)\int \frac{\cos^{2-n}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx\right) + (b\cos^n(e+fx$$
$$= -\frac{\left(a\cos^{2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}(d\sec(e+fx))^n\right)\mathrm{Subst}\left(\int \frac{(1-x^2)^{\frac{1-n}{2}}}{-a^2+b^2+a^2x}\right.}{f}$$
$$= \frac{aF_1\left(\frac{1}{2};\frac{1}{2}(-1+n),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}}{(a^2-b^2)f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5280 vs. $2(192) = 384$.
time = 24.33, size = 5280, normalized size = 27.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
[Out] Result too large to show
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^n}{a+b\sec(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x)),x)
[Out] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x)), x)
```

3.781 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=299

$$\frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(-3+n),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+fx))^n\sin(e+}{\left(a^2-b^2\right)^2f}$$

```
[Out] a^2*AppellF1(1/2,-3/2+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*
cos(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2
)^2/f+b^2*AppellF1(1/2,-1/2+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-
b^2))*cos(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a
^2-b^2)^2/f-2*a*b*AppellF1(1/2,-1+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2
/(a^2-b^2))*(cos(f*x+e)^2)^(1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2)^2/
f
```

Rubi [A]
time = 0.32, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3954, 2903, 3268, 440}

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^nF_1\left(\frac{1}{2};\frac{n-3}{2},2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^nF_1\left(\frac{1}{2};\frac{n-1}{2},2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}-\frac{2ab\sin(e+fx)\cos^2(e+fx)^{n/2}(d\sec(e+fx))^nF_1\left(\frac{1}{2};\frac{n-2}{2},2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*AppellF1[1/2, (-3 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n
*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 + n)/2, 2, 3/2, S
in[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]
^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n*Sin[e + f*x])/((a^2 - b^2)^2*f) - (2*a*
b*AppellF1[1/2, (-2 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*(d*Sec[e + f*x])^n*Sin[e + f*x])/((a^2 -
b^2)^2*f)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
```

```
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = (\cos^n(e+fx)(d\sec(e+fx))^n)\int \frac{\cos^{2-n}(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= (\cos^n(e+fx)(d\sec(e+fx))^n)\int \left(\frac{b^2\cos^{2-n}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2} - \frac{2ab\cos^{3-n}(e+}{(b^2-a^2\cos^2(e+}\right.$$

$$= \left(a^2\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \frac{\cos^{4-n}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\,dx - (2ab\cos^n(e$$

$$= \frac{\left(a^2\cos^{2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}(d\sec(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x^2)^{\frac{3-}{}}}{(a^2-b^2-a^2}\right.}{f}$$

$$= \frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(-3+n),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-}}{\left(a^2-b^2\right)^2 f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 13816 vs. 2(299) = 598.
time = 40.89, size = 13816, normalized size = 46.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec \left(fx+e\right)\right)^{n}}{\left(a+b\sec \left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec \left(e+fx\right)\right)^{n}}{\left(a+b\sec \left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^2, x)
```

3.782 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left((d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2},x\right)$$

```
[Out] Unintegrable((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] Defer[Int][(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2), x]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$$

Mathematica [A]
time = 21.79, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2), x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec (e+fx))^n (a+b\sec (e+fx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(3/2),x)
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+\frac{b}{\cos \left(e+f\,x\right)}\right)}^{3/2}\,{\left(\frac{d}{\cos \left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
[Out] int((a + b/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

3.783 $\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\, dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left((d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)},x\right)$$

```
[Out] Unintegrable((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
[Out] Defer[Int][(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi steps

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\, dx = \int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\, dx$$

Mathematica [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
[Out] Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]], x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^n \sqrt{a+b\sec(fx+e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec{(e+fx)})^n \sqrt{a+b\sec{(e+fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral((d*sec(e + f*x))**n*sqrt(a + b*sec(e + f*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos{(e+f\,x)}}}\,\left(\frac{d}{\cos{(e+f\,x)}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
[Out] int((a + b/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

3.784 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}},x\right)$$

```
[Out] Unintegrable((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
[Out] Defer[Int][(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Mathematica [A]
time = 2.61, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
[Out] Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]], x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^n}{\sqrt{a+b\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec{(e+fx)})^n}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/sqrt(a + b*sec(e + f*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(1/2), x)
```

3.785 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
[Out] Defer[Int][(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

Mathematica [A]
time = 2.62, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
[Out] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2), x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^n}{(a+b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 +
2*a*b*sec(f*x + e) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(3/2),x)
[Out] Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(3/2),x)
[Out] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(3/2), x)
```

3.786 $\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=24

$$\text{Int}(\sec^n(e+fx)(a+b\sec(e+fx))^m,x)$$

```
[Out] Unintegrable(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
[Out] Defer[Int][Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx = \int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

Mathematica [A]
time = 1.47, size = 0, normalized size = 0.00

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
[Out] Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\sec^n\left(fx+e\right)\right)\left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (e + fx))^m \sec^n (e + fx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+b*sec(f*x+e))**m,x)
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**n, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a + \frac{b}{\cos (e + f\,x)}\right)^m \left(\frac{1}{\cos (e + f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + b/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

3.787 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=26

$$\text{Int}((d\sec(e+fx))^n(a+b\sec(e+fx))^m,x)$$

```
[Out] Unintegrable((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
[Out] Defer[Int][(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx = \int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
[Out] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m, x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n (a+b\sec(e+fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos(e+f\,x)}\right)^m \left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
[Out] int((a + b/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

3.788 $\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=273

$$\frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)}-\frac{\sqrt{2}\,a(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a}{b^2f(2+m)\sqrt{1+\sec(e+}}$$

```
[Out] (a+b*sec(f*x+e))^(1+m)*tan(f*x+e)/b/f/(2+m)-a*(a+b)*AppellF1(1/2,-1-m,1/2,3
/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))*(a+b*sec(f*x+e))^m*2^(1/2)*ta
n(f*x+e)/b^2/f/(2+m)/(((a+b*sec(f*x+e))/(a+b))^m)/(1+sec(f*x+e))^(1/2)+(a^2
+b^2*(1+m))*AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+
e))*(a+b*sec(f*x+e))^m*2^(1/2)*tan(f*x+e)/b^2/f/(2+m)/(((a+b*sec(f*x+e))/(a
+b))^m)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3925, 4092, 3919, 144, 143}

$$\frac{\sqrt{2}\,(a^2+b^2(m+1))\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\sec(e+fx)+1}}-\frac{\sqrt{2}\,a(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\sec(e+fx)+1}}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] ((a + b*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*a*(a +
 b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f
*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 +
 Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2 + b^2*(1 +
 m))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]
))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Se
c[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] &&  !LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} + \frac{\int \sec(e+fx)(b(1+m) -}{}$$
$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{a\int \sec(e+fx)(a+b\sec(e}{b^2(2+m)}$$
$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} + \frac{(a\tan(e+fx))\text{Subst}\Big(\int -}{b^2f(2+m)\sqrt{1-\text{se}}}$$
$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{\Big(a(-a-b)(a+b\sec(e+}{}$$
$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{\sqrt{2}\,a(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 8899 vs. $2(273) = 546$.
time = 25.57, size = 8899, normalized size = 32.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^3(fx+e)\right)(a+b\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(e + fx))^m \sec^3(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e))**m,x)
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m/cos(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x))^m/cos(e + f*x)^3, x)
```

3.789 $\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\tan}{bf\sqrt{1+\sec(e+fx)}}$$

```
[Out] (a+b)*AppellF1(1/2,-1-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))*
(a+b*sec(f*x+e))^m*2^(1/2)*tan(f*x+e)/b/f/(((a+b*sec(f*x+e))/(a+b))^m)/(1+s
ec(f*x+e))^(1/2)-a*AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*s
ec(f*x+e))*(a+b*sec(f*x+e))^m*2^(1/2)*tan(f*x+e)/b/f/(((a+b*sec(f*x+e))/(a+
b))^m)/(1+sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3923, 3919, 144, 143}

$$\frac{\sqrt{2}\,(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{bf\sqrt{\sec(e+fx)+1}}-\frac{\sqrt{2}\,a\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{bf\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(
1 - Sec[e + f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[
1 + Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) - (Sqrt[2]*a*AppellF1[1
/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]))/(a + b)]*(a
+ b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec[e + f*x]]*((a + b*Sec[e
 + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3923

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Dist[-a/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + D
ist[1/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
 b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{\int \sec(e+fx)(a+b\sec(e+fx))^{1+m}\,dx}{b} - \frac{a\int \sec(e+fx)(a+b\,\mathrm{se}}{b}$$

$$= -\frac{\tan(e+fx)\mathrm{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sec(e+fx)\right)}{bf\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}} + \frac{(a}{}$$

$$= \frac{\left(a(a+b\sec(e+fx))^m\left(-\frac{a+b\sec(e+fx)}{-a-b}\right)^{-m}\tan(e+fx)\right)\mathrm{Subst}\left(\int\right.}{bf\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+}}$$

$$= \frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(}{bf\sqrt{1+\sec(e+fx}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5564 vs. $2(220) = 440$.
time = 20.44, size = 5564, normalized size = 25.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\sec^2\left(fx+e\right)\right)\left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)^m\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m/cos(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x))^m/cos(e + f*x)^2, x)
```

3.790 $\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

```
[Out] AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))*(a+b*sec
(f*x+e))^m*2^(1/2)*tan(f*x+e)/f/(((a+b*sec(f*x+e))/(a+b))^m)/(1+sec(f*x+e))
^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3919, 144, 143}

$$\frac{\sqrt{2}\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e +
f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*
x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = -\frac{\tan(e+fx)\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= -\frac{\left((a+b\sec(e+fx))^m\left(-\frac{a+b\sec(e+fx)}{-a-b}\right)^{-m}\tan(e+fx)\right)\text{Subst}\left(\int\right.}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+}{f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2828 vs. 2(103) = 206.
time = 13.80, size = 2828, normalized size = 27.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] (-6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[
(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^(1 + m)*(a + b
*Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(-1 + Tan[(e + f*x)/2]^2)*(3*(a + b)*
AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]
^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x
)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1 + m)*AppellF1[3
/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b
)])*Tan[(e + f*x)/2]^2)*((6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e +
f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*x])^m*Sec
[(e + f*x)/2]^2*Sec[e + f*x]^m*Tan[(e + f*x)/2]^2)/((-1 + Tan[(e + f*x)/2]^
2)^2*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*
Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5
/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1
 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f
```

*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) - (3*(a + b)*AppellF1[1/2, 1 + m,
-m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*C
os[e + f*x])^m*Sec[(e + f*x)/2]^2*Sec[e + f*x]^m)/((-1 + Tan[(e + f*x)/2]^2
)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan
[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2,
 Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1 +
m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)
/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) + (6*a*(a + b)*m*AppellF1[1/2, 1 + m,
 -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*
Cos[e + f*x])^(-1 + m)*Sec[e + f*x]^m*Sin[e + f*x]*Tan[(e + f*x)/2])/((-1 +
 Tan[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)
/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2,
 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b
)]) + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a
 - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) - (6*(a + b)*m*App
ellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)
/(a + b)]*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^(1 + m)*Sin[e + f*x]*Tan[(e +
 f*x)/2])/((-1 + Tan[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/
2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)
*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f
x)/2]^2)/(a + b)]) + (a + b)(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e
+ f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) -
(6*(a + b)*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^m*Tan[(e + f*x)/2]*(-1/3*((a
 - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(
e + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(a + b) + ((1
+ m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*
x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/((-1 + Tan[(e +
f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a
 - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1
- m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a +
 b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[
(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) + (6*(a + b)*AppellF1[1/2, 1
 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b
 + a*Cos[e + f*x])^m*Sec[e + f*x]^m*Tan[(e + f*x)/2]*(2*(-((a - b)*m*Appell
F1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)
/(a + b)]) + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]
^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2] + 3*(a + b)*(-1/3*((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e +
f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/(a + b) + ((1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2
]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/3) + 2*Tan[(e + f*x)/2]^2*(-((a - b)*m*((3*(a - b)*(1 - m)*AppellF1[5/
2, 1 + m, 2 - m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a +
 b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(5*(a + b)) + (3*(1 + m)*AppellF1
[5/2, 2 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(

```
a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)) + (a + b)*(1 + m)*((-3*(a
- b)*m*AppellF1[5/2, 2 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e
 + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(5*(a + b)) + (
3*(2 + m)*AppellF1[5/2, 3 + m, -m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e
 + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/((-1 + Ta
n[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]
^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1
+ m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*T...
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)^{m}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m/cos(e + f*x),x)
```

```
[Out] int((a + b/cos(e + f*x))^m/cos(e + f*x), x)
```

3.791 $\int (a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=15

$$\text{Int}((a+b\sec(e+fx))^m,x)$$

```
[Out] Unintegrable((a+b*sec(f*x+e))^m,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^m,x]
[Out] Defer[Int][(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(e+fx))^m\,dx = \int (a+b\sec(e+fx))^m\,dx$$

Mathematica [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int (a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^m,x]
[Out] Integrate[(a + b*Sec[e + f*x])^m, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^m,x)
```

[Out] int((a+b*sec(f*x+e))^m,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^m, x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^m,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e) + a)^m, x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(e + fx))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**m,x)

[Out] Integral((a + b*sec(e + f*x))**m, x)

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^m,x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^m, x)

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int {\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^m,x)

[Out] int((a + b/cos(e + f*x))^m, x)

3.792 $\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=22

$$\text{Int}(\cos(e+fx)(a+b\sec(e+fx))^m,x)$$

```
[Out] Unintegrable(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Defer[Int][Cos[e + f*x]*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

Mathematica [A]
time = 3.64, size = 0, normalized size = 0.00

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
[Out] int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec{(e + fx)})^m \cos{(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))**m,x)
[Out] Integral((a + b*sec(e + f*x))**m*cos(e + f*x), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \cos\left(e+f\,x\right)\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x))^m,x)
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x))^m, x)
```

3.793 $\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\cos^2(e+fx)(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

Mathematica [A]
time = 2.83, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m, x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
[Out] int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec (e+fx))^m \cos ^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
[Out] Integral((a + b*sec(e + f*x))**m*cos(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\cos \left(e+f\,x\right)}^2\,{\left(a+\frac{b}{\cos \left(e+f\,x\right)}\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x))^m,x)
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x))^m, x)
```

3.794 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=135

$$\frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{10b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{14a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}+\frac{2\ldots}{}$$

```
[Out] 14/15*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+10/21*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+14/45*a*cos(d*x+c)^(3/2)*sin(d*x+
c)/d+2/7*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a*cos(d*x+c)^(7/2)*sin(d*x+c)/
d+10/21*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2715, 2720, 2719}

$$\frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{14a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10b\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (14*a*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*b*EllipticF[(c + d*x)/2, 2])/
(21*d) + (10*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (14*a*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(45*d) + (2*b*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) +
 (2*a*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= \int \cos^{\frac{7}{2}}(c+dx)(b+a\cos(c+dx))\,dx\\ &= a\int \cos^{\frac{9}{2}}(c+dx)\,dx + b\int \cos^{\frac{7}{2}}(c+dx)\,dx\\ &= \frac{2b\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}(7a)\\ &= \frac{10b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{14a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2b}{}\\ &= \frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10b\sqrt{\cos(c+dx)}\,\text{si}}{21d}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 90, normalized size = 0.67

$$\frac{1176aE\left(\frac{1}{2}(c+dx)\middle|2\right)+600bF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(690b\sin(c+dx)+266a\sin(2(c+dx))+90b\sin(3(c+dx))+35a\sin(4(c+dx)))}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
[Out] (1176*a*EllipticE[(c + d*x)/2, 2] + 600*b*EllipticF[(c + d*x)/2, 2] + Sqrt[
Cos[c + d*x]]*(690*b*Sin[c + d*x] + 266*a*Sin[2*(c + d*x)] + 90*b*Sin[3*(c
+ d*x)] + 35*a*Sin[4*(c + d*x)]))/(1260*d)
```

Maple [A]
time = 0.17, size = 318, normalized size = 2.36

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(2240a+720b)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*a+(2240*a+720*b)*sin(1/2*d*x+1/2*c)^8*c
os(1/2*d*x+1/2*c)+(-2072*a-1080*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+
(952*a+840*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a-240*b)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 159, normalized size = 1.18

$$\frac{2\left(35\,a\cos\left(dx+c\right)^3+45\,b\cos\left(dx+c\right)^2+49\,a\cos\left(dx+c\right)+75\,b\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-75i\,\sqrt{2}\,b\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+75i\,\sqrt{2}\,b\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+147i\,\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-147i\,\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/315*(2*(35*a*cos(d*x + c)^3 + 45*b*cos(d*x + c)^2 + 49*a*cos(d*x + c) + 7
5*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 75*I*sqrt(2)*b*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + 75*I*sqrt(2)*b*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 147*I*sqrt(
2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

Mupad [**B**]
time = 1.31, size = 87, normalized size = 0.64

$$-\frac{2\,a\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] - (2*a*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c
+ d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*b*cos(c + d*x)^(9/2)*sin(c +
d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/
2))
```

3.795 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{10a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a\,c}{}$$

```
[Out] 6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b*cos(d*x+c)^(3/2)*sin(d*x+c)/d
+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2715, 2719, 2720}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10a\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (6*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= \int \cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))\,dx \\
&= a\int \cos^{\frac{7}{2}}(c+dx)\,dx + b\int \cos^{\frac{5}{2}}(c+dx)\,dx \\
&= \frac{2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}(5a) \\
&= \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2b\cos^{\frac{3}{2}}(c+ \\
&= \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10a\sqrt{\cos(c+dx)}\,\sin}{21d}
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 77, normalized size = 0.69

$$\frac{126bE\left(\frac{1}{2}(c+dx)\middle|2\right)+50aF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(65a+42b\cos(c+dx)+15a\cos(2(c+dx)))\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (126*b*EllipticE[(c + d*x)/2, 2] + 50*a*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(65*a + 42*b*Cos[c + d*x] + 15*a*Cos[2*(c + d*x)])*Sin[c + d*x]
)/(105*d)
```

Maple [A]
time = 0.16, size = 290, normalized size = 2.61

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(-360a-168b)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*a
-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*a*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 148, normalized size = 1.33

$$\frac{2\left(15\,a\cos\left(dx+c\right)^2+21\,b\cos\left(dx+c\right)+25\,a\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-25i\,\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+25i\,\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+63i\,\sqrt{2}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-63i\,\sqrt{2}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(15*a*cos(d*x + c)^2 + 21*b*cos(d*x + c) + 25*a)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) + 63*I*sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*b*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Mupad [B]
time = 1.14, size = 87, normalized size = 0.78

$$-\frac{2\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] - (2*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c +
d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*b*cos(c + d*x)^(7/2)*sin(c + d*x
)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.796 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

```
[Out] 6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2
/3*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2715, 2720, 2719}

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= \int \cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))\,dx\\ &= a\int \cos^{\frac{5}{2}}(c+dx)\,dx + b\int \cos^{\frac{3}{2}}(c+dx)\,dx\\ &= \frac{2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}(3a\\ &= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sqrt{\cos(c+dx)}\,\sin(c}{3d}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 66, normalized size = 0.76

$$\frac{2\left(9aE\left(\frac{1}{2}(c+dx)\middle|2\right)+5bF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5b+3a\cos(c+dx))\sin(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(9*a*EllipticE[(c + d*x)/2, 2] + 5*b*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(5*b + 3*a*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(127) = 254$.
time = 0.16, size = 262, normalized size = 3.01

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(24a+20b)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 137, normalized size = 1.57

$$\frac{2\left(3\,a\cos\left(dx+c\right)+5\,b\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-5i\,\sqrt{2}\,b\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5i\,\sqrt{2}\,b\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+9i\,\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-9i\,\sqrt{2}\,a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(3*a*cos(d*x + c) + 5*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqrt
(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(
2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2
)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.04, size = 80, normalized size = 0.92

$$\frac{2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,a\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*b*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b*cos(c + d*x)^(1/2)*sin(c + d
*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.797 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {4310, 2827, 2719, 2715, 2720}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= \int \sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))\,dx\\ &= a\int \cos^{\frac{3}{2}}(c+dx)\,dx + b\int \sqrt{\cos(c+dx)}\,dx\\ &= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}a\int \frac{}{\sqrt{\cos(}}\\ &= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c}{3d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 53, normalized size = 0.87

$$\frac{2\left(3bE\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(3*b*EllipticE[(c + d*x)/2, 2] + a*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos
[c + d*x]]*Sin[c + d*x])))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(107) = 214$.
time = 0.15, size = 228, normalized size = 3.74

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a+
a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 125, normalized size = 2.05

$$\frac{2\,a\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-i\,\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+3i\,\sqrt{2}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3i\,\sqrt{2}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*b*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*b*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
```

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 0.17, size = 53, normalized size = 0.87

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b*ellipticE(c/2 + (d*x)/2, 2))
/d + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

3.798 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4310, 2827, 2720, 2719}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))\,dx &= \int \frac{b+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= a\int \sqrt{\cos(c+dx)}\,dx + b\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 0.91

$$\frac{2\left(aE\left(\frac{1}{2}(c+dx)\middle|2\right)+bF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)/2, 2]))/d
```

Maple [A]
time = 0.15, size = 152, normalized size = 4.34

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(b\,\mathrm{Elliptic}\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}\right.\right.}}$
risch	$-\frac{ia\sqrt{2}\,\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d}-\frac{i\left(\frac{ib\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\mathrm{Ellipt}}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(b*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 107, normalized size = 3.06

$$\frac{-i\sqrt{2}\,b\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,b\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+i\sqrt{2}\,a\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,a\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sq
rt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.23, size = 33, normalized size = 0.94

$$\frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*b*ellipticF(c/2 + (d*x)/2, 2))/d
```

3.799 $\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4310, 2827, 2716, 2719, 2720}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*b*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + b\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - b\int \sqrt{\cos(c+dx)}\,dx\\ &= -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 0.89

$$\frac{2\left(-bE\left(\frac{1}{2}(c+dx)\middle|2\right)+aF\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
[Out] (2*(-(b*EllipticE[(c + d*x)/2, 2]) + a*EllipticF[(c + d*x)/2, 2] + (b*Sin[c
 + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [A]
time = 0.15, size = 150, normalized size = 2.63

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,a-2\sqrt{2\left(\mathrm{s}\right.}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b-a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 156, normalized size = 2.74

$$\frac{-i\sqrt{2}\,a\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\,a\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-i\sqrt{2}\,b\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+i\sqrt{2}\,b\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,b\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) - I*sqrt(2)*b*cos(d*x + c)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b*cos(d*x
 + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))) + 2*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec(c + dx)}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 1.25, size = 60, normalized size = 1.05

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b*sin(c + d*x)*hypergeom([-1/4, 1/
2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.800 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*
a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2716, 2720, 2719}

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{b+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + b\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}b\int \frac{1}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 65, normalized size = 0.78

$$\frac{-6aE\left(\frac{1}{2}(c+dx)\middle|2\right)+2bF\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{2(b+3a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2] + 2*b*EllipticF[(c + d*x)/2, 2] + (2*(b + 3
*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(127) = 254.
time = 0.25, size = 396, normalized size = 4.77

method	result

default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\ }\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*b-6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*a-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.20, size = 175, normalized size = 2.11

$$\frac{-i\sqrt{2}\,b\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,b\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,(3\,a\cos(dx+c)+b)\sqrt{\cos(dx+c)}\,\sin(dx+c)}{3\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] 1/3*(-I*sqrt(2)*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + b)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec{\left(c + dx \right)}}{\cos^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.54, size = 87, normalized size = 1.05

$$\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d
*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*b*sin(c + d*x)*hypergeom([-3/4, 1/2]
, 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.801 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+
2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4310, 2827, 2716, 2719, 2720}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-6*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*b*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{b+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx\\ &= a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + b\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx\\ &= \frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}(3b)\int \frac{1}{\cos^{\frac{3}{2}}(c+\\ &= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}\\ &= -\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} +\end{aligned}$$

Mathematica [A]
time = 0.21, size = 95, normalized size = 0.86

$$\frac{-18b\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10a\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+10a\sin(c+dx)+9b\sin(2(c+dx))+6b\tan(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-18*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*Cos[c + d*x]^(3/
2)*EllipticF[(c + d*x)/2, 2] + 10*a*Sin[c + d*x] + 9*b*Sin[2*(c + d*x)] + 6
*b*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(147) = 294$.
time = 0.35, size = 502, normalized size = 4.52

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2b\left(24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*b/(8*sin(1/
2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*
x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.13, size = 188, normalized size = 1.69

$$\frac{-5i\sqrt{2}\,a\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,a\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-9i\sqrt{2}\,b\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+9i\sqrt{2}\,b\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(9b\cos(dx+c)^2+5a\cos(dx+c)+3b\right)\sqrt{\cos(dx+c)}\sin(dx+c)}{15d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*b*cos(d*x + c)^3*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*
I*sqrt(2)*b*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*b*cos(d*x + c)^2 + 5*a*cos(d*x + c
) + 3*b)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Mupad [**B**]
time = 1.70, size = 87, normalized size = 0.78

$$\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c +
 d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*b*sin(c + d*x)*hypergeom([-5/4, 1/
2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))
```

3.802 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{2(7a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{20ab\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2(7a^2+9b^2)\cos^{\frac{3}{2}}(c}{45}$$

```
[Out] 2/15*(7*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/21*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(7*a^2+9*b^2)*c
os(d*x+c)^(3/2)*sin(d*x+c)/d+4/7*a*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^2*
cos(d*x+c)^(7/2)*sin(d*x+c)/d+20/21*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = $\{4349, 3873, 3854, 3856, 2720, 4130, 2719\}$

$$\frac{2(7a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(7a^2+9b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{20ab\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(7*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a*b*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (20*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*(7*a^2 + 9*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a*b*Cos[c +
d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9
*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \Big(2ab$$

$$= \frac{4ab\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{7}\Big(1$$

$$= \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(7a^2+9b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+$$

$$= \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(7a^2+9b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+$$

$$= \frac{2(7a^2+9b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{20ab\sqrt{\cos$$

Mathematica [A]
time = 0.51, size = 113, normalized size = 0.71

$$\frac{84(7a^2+9b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)+600abF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(7(43a^2+36b^2)\cos(c+dx)+5a(156b+36b\cos(2(c+dx))+7a\cos(3(c+dx))))\sin(c+dx)}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (84*(7*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2] + 600*a*b*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(7*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*a*(156*b
+ 36*b*Cos[2*(c + d*x)] + 7*a*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(192) = 384.
time = 0.19, size = 398, normalized size = 2.49

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+(2240a^2+1440ba)\left(\sin^8\left(\frac{d}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*a^2+(2240*a^2+1440*a*b)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^2-2160*a*b-504*b^2)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(952*a^2+1680*a*b+504*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-168*a^2-480*a*b-126*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+150*b*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-189*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.01, size = 195, normalized size = 1.22

$$\frac{-150i\sqrt{2}\,ab\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+150i\sqrt{2}\,ab\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(35a^2\cos(dx+c)^3+90ab\cos(dx+c)^2+150ab+7(7a^2+9b^2)\cos(dx+c)\right)\sqrt{\cos(dx+c)}\sin(dx+c)-21\sqrt{2}(-7ia^2-9ib^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21\sqrt{2}(7ia^2+9ib^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/315*(-150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) + 150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 2*(35*a^2*cos(d*x + c)^3 + 90*a*b*cos(d*x + c)^2 + 150*a*b +
7*(7*a^2 + 9*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 21*sqrt(2
)*(-7*I*a^2 - 9*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(7*I*a^2 + 9*I*b^2)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

Mupad [B]
time = 1.31, size = 135, normalized size = 0.84

$$-\frac{2a^2\cos(c+dx)^{11/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos(c+dx)^2\right)}{11d\sqrt{\sin(c+dx)^2}}-\frac{2b^2\cos(c+dx)^{7/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}}-\frac{4ab\cos(c+dx)^{9/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+dx)^2\right)}{9d\sqrt{\sin(c+dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] - (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(
c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*b^2*cos(c + d*x)^(7/2)*sin(
c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)
^(1/2)) - (4*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4
, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.803 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5a^2+7b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5a^2+7b^2)\,\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{4ab\cos^{\frac{3}{2}}(c$$

```
[Out] 12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+2/21*(5*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a*b*cos(d*x+c)^(3
/2)*sin(d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*(5*a^2+7*b^2)*s
in(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3854, 3856, 2719, 4130, 2720}

$$\frac{2(5a^2+7b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5a^2+7b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a^2 + 7*b^2)*EllipticF[(c
+ d*x)/2, 2])/(21*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (4*a*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \Big(2ab$$

$$= \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}\Big(6$$

$$= \frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} +$$

$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5a^2+7b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2(5a^2+7b}{}$$

Mathematica [A]

time = 0.40, size = 98, normalized size = 0.73

$$\frac{252abE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(5a^2+7b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(65a^2+70b^2+84ab\cos(c+dx)+15a^2\cos(2(c+dx)))\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (252*a*b*EllipticE[(c + d*x)/2, 2] + 10*(5*a^2 + 7*b^2)*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(65*a^2 + 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a^2
*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(171) = 342$.
time = 0.17, size = 362, normalized size = 2.68

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+\left(-360a^2-336ba\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a^2+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+25*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+35*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 180, normalized size = 1.33

$$\frac{126i\sqrt{2}\,ab\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 126i\sqrt{2}\,ab\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) + 2\left(15a^2\cos(dx+c)^2 + 42ab\cos(dx+c) + 25a^2 + 35b^2\right)\sqrt{\cos(dx+c)}\sin(dx+c) - 5\sqrt{2}(5ia^2+7ib^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 5\sqrt{2}(-5ia^2-7ib^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a^2*cos(
d*x + c)^2 + 42*a*b*cos(d*x + c) + 25*a^2 + 35*b^2)*sqrt(cos(d*x + c))*sin(
d*x + c) - 5*sqrt(2)*(5*I*a^2 + 7*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-5*I*a^2 - 7*I*b^2)*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Mupad [B]
time = 1.19, size = 128, normalized size = 0.95

$$\frac{2\left(b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}-\frac{2\,a^2\cos\left(c+d\,x\right)^{9/2}\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{4\,a\,b\cos\left(c+d\,x\right)^{7/2}\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*(b^2*ellipticF(c/2 + (d*x)/2, 2) + b^2*cos(c + d*x)^(1/2)*sin(c + d*x)))
/(3*d) - (2*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4,
 cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4*a*b*cos(c + d*x)^(7/2)*
sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x
)^2)^(1/2))
```

3.804 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

```
[Out] 2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c)^(3/2
)*sin(d*x+c)/d+4/3*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3854, 3856, 2720, 4130, 2719}

$$\frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (4*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \Big(2ab \\
&= \frac{4ab\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\Big(\\
&= \frac{4ab\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\Big(\\
&= \frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} + \frac{4ab\sqrt{\cos(c}{}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 79, normalized size = 0.78

$$\frac{6(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+20abF\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2a\sqrt{\cos(c+dx)}\,(10b+3a\cos(c+dx))\sin(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (6*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*EllipticF[(c + d*x)/2
, 2] + 2*a*Sqrt[Cos[c + d*x]]*(10*b + 3*a*Cos[c + d*x])*Sin[c + d*x])/(15*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(141) = 282$.
time = 0.17, size = 357, normalized size = 3.53

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}+24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a^2+24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4*a^2+40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2*a^2-20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b+10
*b*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-15*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.95, size = 162, normalized size = 1.60

$$\frac{-10i\sqrt{2}\,ab\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}\,ab\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(3a^{2}\cos(dx+c)+10ab\right)\sqrt{\cos(dx+c)}\sin(dx+c)-3\sqrt{2}\left(-3ia^{2}-5ib^{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\sqrt{2}\left(3ia^{2}+5ib^{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) + 2*(3*a^2*cos(d*x + c) + 10*a*b)*sqrt(cos(d*x + c))*sin(d*x + c)
- 3*sqrt(2)*(-3*I*a^2 - 5*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*a^2 + 5*I*b^2)*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Mupad [**B**]
time = 1.11, size = 102, normalized size = 1.01

$$\frac{2\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (4*a*b*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d*x
)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin
(c + d*x)^2)^(1/2))
```

3.805 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=72

$$\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)*cos(d*x+
c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3873, 3856, 2719, 4130, 2720}

$$\frac{2(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2 + 3*b^2)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\Big(2ab\\ &= \frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+(2ab)\int\sqrt{\cos(c+dx)}\,dx-\frac{1}{3}\Big(\Big(\\ &= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}-\frac{1}{3}\left(-a^2-3\right.\\ &= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sqrt{\cos(c+}}{}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 64, normalized size = 0.89

$$\frac{2\left(6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(6*a*b*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*EllipticF[(c + d*x)/2,
2] + a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(118) = 236$.
time = 0.17, size = 283, normalized size = 3.93

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
a^2+a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 147, normalized size = 2.04

$$\frac{2a^2\sqrt{\cos(dx+c)}\,\sin(dx+c)+6i\sqrt{2}\,ab\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-6i\sqrt{2}\,ab\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\sqrt{2}\,(-ia^2-3ib^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}\,(ia^2+3ib^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*sqrt(cos(d*x + c))*sin(d*x + c) + 6*I*sqrt(2)*a*b*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6*I*s
qrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.05, size = 76, normalized size = 1.06

$$\frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b^2*ellipticF(c/2 + (d*x)/2,
 2))/d + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (4*a*b*ellipticE(c
/2 + (d*x)/2, 2))/d
```

3.806 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=68

$$\frac{2(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b^2*sin(d*x+c)/d/cos(d*x+c)^(
1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3873, 3856, 2720, 4131, 2719}

$$\frac{2(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (4*a*b*EllipticF[(c + d*x)/2,
 2])/d + (2*b^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \Big(2a$$

$$= \frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + (2ab)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \Big(\left(a^2-b^2\right)\sqrt{\cos}$$

$$= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(a^2-b^2\right)\int \sqrt{\cos(c+}$$

$$= \frac{2(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b^2\sin(c+}{d\sqrt{\cos(c+}}$$

Mathematica [A]
time = 0.21, size = 62, normalized size = 0.91

$$\frac{2\left(\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2aF\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
[Out] (2*((a^2 - b^2)*EllipticE[(c + d*x)/2, 2] + b*(2*a*EllipticF[(c + d*x)/2, 2
] + (b*Sin[c + d*x])/Sqrt[Cos[c + d*x]])))/d
```

Maple [A]
time = 0.17, size = 202, normalized size = 2.97

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{2}-4ba\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+2\sqrt{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-2*b*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 178, normalized size = 2.62

$$\frac{-2i\sqrt{2}\,ab\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+2i\sqrt{2}\,ab\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2b^{2}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}\,(ia^{2}-ib^{2})\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+\sqrt{2}\,(-ia^{2}+ib^{2})\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 2*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(
2)*(I*a^2 - I*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(-I*a^2 + I*b^2)*cos(d*x +
 c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 1.40, size = 81, normalized size = 1.19

$$\frac{2a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^2\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.807 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b^2*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+4*a*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3853, 3856, 2719, 4131, 2720}

$$\frac{2(3a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a^2 + b^2)*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*b^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a*b*Sin[
c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\,dx \\
&= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,\left(a^2+b^2\sec^2(c+dx)\right)\,dx + \Big(\\
&= \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \left(2ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int - \\
&= \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (2ab)\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}(3a^2+b^2) \\
&= -\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(3a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab}{d\sqrt{\cdot}}
\end{aligned}
$$

Mathematica [A]
time = 0.41, size = 73, normalized size = 0.77

$$
\frac{2\left(-6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+(3a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b(b+6a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(-6*a*b*EllipticE[(c + d*x)/2, 2] + (3*a^2 + b^2)*EllipticF[(c + d*x)/2,
 2] + (b*(b + 6*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(139) = 278$.
time = 0.28, size = 513, normalized size = 5.40

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-6\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{Ell}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(24*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*
c)^2*a^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*b^2-12*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a*b-12*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2*a*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+3*a^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 198, normalized size = 2.08

$$\frac{-6i\sqrt{2}\,ab\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+6i\sqrt{2}\,ab\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+\sqrt{2}\left(-3i\,a^{2}-i\,b^{2}\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\sqrt{2}\left(3i\,a^{2}+i\,b^{2}\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+2\left(6ab\cos\left(dx+c\right)+b^{2}\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c)^
2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c))) + sqrt(2)*(-3*I*a^2 - I*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*a^2 + I*b^2)*cos(d*x + c
)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*a*b*co
s(d*x + c) + b^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\sec\left(c + dx\right)\right)^{2}}{\sqrt{\cos\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 1.52, size = 108, normalized size = 1.14

$$\frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{4\,a\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^2*sin(c + d*x)*hypergeom([-3/4
, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2
)) + (4*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.808 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$-\frac{2(5a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(5a^2+3b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -2/5*(5*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b^2*sin(d*x+c)/d/c
os(d*x+c)^(5/2)+4/3*a*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(5*a^2+3*b^2)*sin
(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3853, 3856, 2720, 4131, 2719}

$$-\frac{2(5a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5a^2+3b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*b^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(5*a^2 + 3*b^2)*Sin[c + d*x])
/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx + \Big(2$$

$$= \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\Big(2ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\Big)\int \sqrt{}$$

$$= \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2+3b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{3}(2ab)\int \frac{}{\sqrt{\mathrm{co}}}$$

$$= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} + \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2+3b^2)\sin(c}{5d\sqrt{\cos(c+dx}}$$

$$= -\frac{2(5a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} + \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a}{3d}$$

Mathematica [**A**]
time = 0.25, size = 124, normalized size = 0.92

$$\frac{-6(5a^2+3b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20ab\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+20ab\sin(c+dx)+15a^2\sin(2(c+dx))+9b^2\sin(2(c+dx))+6b^2\tan(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-6*(5*a^2 + 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*C
os[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 20*a*b*Sin[c + d*x] + 15*a^2*
Sin[2*(c + d*x)] + 9*b^2*Sin[2*(c + d*x)] + 6*b^2*Tan[c + d*x])/(15*d*Cos[c
 + d*x]^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(171) = 342$.
time = 0.37, size = 633, normalized size = 4.69

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4ba\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.\right.}}{6\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b*a*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*
d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*s
in(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2/s
in(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 223, normalized size = 1.65

$$\frac{-10i\sqrt{2}\,ab\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}\,ab\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\sqrt{2}\,(5ia^2+3ib^2)\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\sqrt{2}\,(-5ia^2-3ib^2)\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(10\,ab\cos(dx+c)+3\,(5a^2+3b^2)\cos(dx+c)^2+3b^2\right)\sqrt{\cos(dx+c)}\,\sin(dx+c)}{15\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(5*I*a^2 + 3*I*b^2)*cos(
d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*cos(d*x + c)^3*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
2*(10*a*b*cos(d*x + c) + 3*(5*a^2 + 3*b^2)*cos(d*x + c)^2 + 3*b^2)*sqrt(cos
(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.67, size = 113, normalized size = 0.84

$$\frac{6\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)+30\,a^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)+20\,a\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{15\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{1-{\cos\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

```
[Out] (6*b^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*a^2*c
os(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) + 20
*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))
/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.809 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=160

$$-\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7a^2+5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(7a^2+5b^2)\sin(\ldots}{21d\cos^{\frac{3}{2}}(c+\ldots}$$

```
[Out] -12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+2/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*b^2*sin(d*x+c)/d
/cos(d*x+c)^(7/2)+4/5*a*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*(7*a^2+5*b^2)*
sin(d*x+c)/d/cos(d*x+c)^(3/2)+12/5*a*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3873, 3853, 3856, 2719, 4131, 2720}

$$\frac{2(7a^2+5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{12ab\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2 + 5*b^2)*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (2*b^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(7*a^2 + 5*b^2)*Sin[c + d*x
])/(21*d*Cos[c + d*x]^(3/2)) + (12*a*b*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]
])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx \\
&= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx + \Big(\\
&= \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\Big(6ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\Big)\int \\
&= \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(7a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{12ab\sin(c+}{5d\sqrt{\cos(c+}} \\
&= \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(7a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{12ab\sin(c+}{5d\sqrt{\cos(c+}} \\
&= -\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(7a^2+5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} +
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 142, normalized size = 0.89

$$\frac{-252ab\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7a^2+5b^2)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84ab\sin(c+dx)+252ab\cos^2(c+dx)\sin(c+dx)+35a^2\sin(2(c+dx))+25b^2\sin(2(c+dx))+30b^2\tan(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-252*a*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2 + 5*b^2)
*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 84*a*b*Sin[c + d*x] + 252*a
*b*Cos[c + d*x]^2*Sin[c + d*x] + 35*a^2*Sin[2*(c + d*x)] + 25*b^2*Sin[2*(c
+ d*x)] + 30*b^2*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(192) = 384.
time = 0.46, size = 689, normalized size = 4.31

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2b^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\right.}}{56\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^4}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*(-1/56*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4/5*b*a/si
n(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/
2*d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
```

```
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 235, normalized size = 1.47

$$\frac{-126i\sqrt{2}\,ab\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+126i\sqrt{2}\,ab\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-5\sqrt{2}\,(7i\,a^2+5i\,b^2)\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5\sqrt{2}\,(-7i\,a^2-5i\,b^2)\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\,(126\,ab\cos(dx+c)^3+42\,ab\cos(dx+c)+5\,(7\,a^2+5\,b^2)\cos(dx+c)^2+15\,b^2)\sqrt{\cos(dx+c)}\sin(dx+c)}{105\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
[Out] 1/105*(-126*I*sqrt(2)*a*b*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 126*I*sqrt(2)*a*b*cos(d*x
 + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c))) - 5*sqrt(2)*(7*I*a^2 + 5*I*b^2)*cos(d*x + c)^4*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*a^2 - 5*I*b^
2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 2*(126*a*b*cos(d*x + c)^3 + 42*a*b*cos(d*x + c) + 5*(7*a^2 + 5*b^2)*cos(
d*x + c)^2 + 15*b^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.79, size = 113, normalized size = 0.71

$$\frac{30\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)+70\,a^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)+84\,a\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{105\,d\,{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{1-{\cos\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

```
[Out] (30*b^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 70*a^2*
cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 8
4*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2
))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.810 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=194

$$\frac{2a(7a^2+27b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}+\frac{2b(15a^2+7b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d}+\frac{2b(15a^2+7b^2)\,\sqrt{\cos(c+dx)}\,\sin(c+d}{21d}$$

```
[Out] 2/15*a*(7*a^2+27*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(15*a^2+7*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*
a*(7*a^2+27*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+40/63*a^2*b*cos(d*x+c)^(5/2)
*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d+2/21*b
*(15*a^2+7*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3926, 4132, 3854, 3856, 2719, 4130, 2720}

$$\frac{2b(15a^2+7b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d}+\frac{2a(7a^2+27b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d}+\frac{2a(7a^2+27b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2b(15a^2+7b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{40a^2b\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*a*(7*a^2 + 27*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*b*(15*a^2 + 7*
b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*(7*a^2 + 27*b^2)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(45*d) + (40*a^2*b*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*a
^2*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx \\
&= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{9d} + \frac{1}{9}\Big(2\sqrt{\cos(c+ \\
&= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{9d} + \frac{1}{9}\Big(2\sqrt{\cos(c+ \\
&= \frac{2a(7a^2+27b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{40a^2b\cos^{\frac{5}{2}}(c+dx)\sin(}{63d} \\
&= \frac{2b(15a^2+7b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a(7a^2+27b^2)\cos^{\frac{3}{2}}(}{45} \\
&= \frac{2a(7a^2+27b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2b(15a^2+7b^2)\sqrt{\cos(c+dx)}\,\mathrm{s}}{21d} \\
&= \frac{2a(7a^2+27b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2b(15a^2+7b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}
\end{aligned}$$

Mathematica [A]
time = 0.64, size = 137, normalized size = 0.71

$$\frac{84(7a^3+27ab^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)+60(15a^2b+7b^3)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(7a(43a^2+108b^2)\cos(c+dx)+5(234a^2b+84b^3+54a^2b\cos(2(c+dx))+7a^3\cos(3(c+dx))))\sin(c+dx)}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (84*(7*a^3 + 27*a*b^2)*EllipticE[(c + d*x)/2, 2] + 60*(15*a^2*b + 7*b^3)*El
lipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*a*(43*a^2 + 108*b^2)*Cos[c
+ d*x] + 5*(234*a^2*b + 84*b^3 + 54*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c
 + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(226) = 452.
time = 0.20, size = 470, normalized size = 2.42

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3+(2240a^3+2160b\,a^2)\Big(\sin^8\Big(}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*a^3+(2240*a^3+2160*a^2*b)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*b*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.25, size = 227, normalized size = 1.17

$$\frac{2 \left(35 a^3 \cos(dx+c)^3 + 135 a^2 b \cos(dx+c)^2 + 225 a^2 b + 105 b^3 + 7 \left(7 a^3 + 27 ab^2\right) \cos(dx+c)\right) \sqrt{\cos(dx+c)} \sin(dx+c) - 15 \sqrt{2} \left(15 i a^2 b + 7 i b^3\right) \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c)) - 15 \sqrt{2} \left(-15 i a^2 b - 7 i b^3\right) \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)) - 21 \sqrt{2} \left(-7 i a^3 - 27 i ab^2\right) \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) - 21 \sqrt{2} \left(7 i a^3 + 27 i ab^2\right) \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)))}{315 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/315*(2*(35*a^3*cos(d*x + c)^3 + 135*a^2*b*cos(d*x + c)^2 + 225*a^2*b + 105*b^3 + 7*(7*a^3 + 27*a*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*(15*I*a^2*b + 7*I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 15*sqrt(2)*(-15*I*a^2*b - 7*I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-7*I*a^3 - 27*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(7*I*a^3 + 27*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

Mupad [B]
time = 1.34, size = 178, normalized size = 0.92

$$\frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,a^3\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{6\,a\,b^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,a^2\,b\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*b^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b^3*cos(c + d*x)^(1/2)*sin(c
 + d*x))/(3*d) - (2*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11
/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (6*a*b^2*cos(c
+ d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d
*(sin(c + d*x)^2)^(1/2)) - (2*a^2*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hyperge
om([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))
```

3.811 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=159

$$\frac{2b(9a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5a^2+21b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5a^2+21b^2)\,\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}$$

```
[Out] 2/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(5*a^2+21*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/35*a
^2*b*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)
)*sin(d*x+c)/d+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3926, 4132, 3854, 3856, 2720, 4130, 2719}

$$\frac{2a(5a^2+21b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(9a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5a^2+21b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{32a^2b\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*b*(9*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*a^2 + 21*b^
2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(21*d) + (32*a^2*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d) + (2*a^2*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{1}{7}\Big(2\sqrt{\cos(c+dx}$$
$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{1}{7}\Big(2\sqrt{\cos(c+dx}$$
$$= \frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{32a^2b\cos^{\frac{3}{2}}(c+dx)\sin}{35d}$$
$$= \frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{32a^2b\cos^{\frac{3}{2}}(c+dx)\sin}{35d}$$
$$= \frac{2b(9a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5a^2+21b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} +$$

Mathematica [A]
time = 0.49, size = 110, normalized size = 0.69

$$\frac{42(9a^2b+5b^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(5a^3+21ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sqrt{\cos(c+dx)}\,(65a^2+210b^2+126ab\cos(c+dx)+15a^2\cos(2(c+dx)))\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (42*(9*a^2*b + 5*b^3)*EllipticE[(c + d*x)/2, 2] + 10*(5*a^3 + 21*a*b^2)*Ell
ipticF[(c + d*x)/2, 2] + a*Sqrt[Cos[c + d*x]]*(65*a^2 + 210*b^2 + 126*a*b*C
os[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(195) = 390$.
time = 0.19, size = 421, normalized size = 2.65

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3+\left(-360a^3-504b\,a^2\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a^3+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2*c
```

```
)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b^2*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 205, normalized size = 1.29

$$\frac{2\left(15a^{3}\cos\left(dx+c\right)^{2}+63a^{2}b\cos\left(dx+c\right)+25a^{3}+105ab^{2}\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\sqrt{2}\left(5ia^{3}+21iab^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-5\sqrt{2}\left(-5ia^{3}-21iab^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-21\sqrt{2}\left(-9ia^{2}b-5ib^{3}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-21\sqrt{2}\left(9ia^{2}b+5ib^{3}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(15*a^3*cos(d*x + c)^2 + 63*a^2*b*cos(d*x + c) + 25*a^3 + 105*a*b^
2)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-5*I*a^3
- 21*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2
1*sqrt(2)*(-9*I*a^2*b - 5*I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(9*I*a^2*b + 5*I*b^3)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Mupad [B]
time = 1.22, size = 146, normalized size = 0.92

$$\frac{2\left(b^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a\,b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}-\frac{2\,a^3\cos\left(c+d\,x\right)^{9/2}\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{6\,a^2\,b\cos\left(c+d\,x\right)^{7/2}\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*(b^3*ellipticE(c/2 + (d*x)/2, 2) + a*b^2*ellipticF(c/2 + (d*x)/2, 2) + a
*b^2*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (2*a^3*cos(c + d*x)^(9/2)*sin(c
+ d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(
1/2)) - (6*a^2*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4
, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.812 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=116

$$\frac{6a(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d}+\frac{2a^2\cos^{\frac{3}{2}}(c+$$

```
[Out] 6/5*a*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c
)^(3/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d+8/5*a^2*b*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A]

time = 0.18, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3926, 4132, 3856, 2719, 4130, 2720}

$$\frac{2b(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2b\sin(c+dx)\sqrt{\cos(c+dx)}}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (6*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(a^2 + b^2)*Elli
pticF[(c + d*x)/2, 2])/d + (8*a^2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d)
+ (2*a^2*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
```

```
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{1}{5}\Big(2\sqrt{\cos(c+dx}$$

$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{1}{5}\Big(2\sqrt{\cos(c+dx}$$

$$= \frac{8a^2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx}{5d}$$

$$= \frac{6a(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{8a^2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d} +$$

$$= \frac{6a(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2b(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{8a^2}{}$$

Mathematica [A]
time = 0.26, size = 84, normalized size = 0.72

$$\frac{2\left(3(a^3+5ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5b(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2\sqrt{\cos(c+dx)}\,(5b+a\cos(c+dx))\sin(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(3*(a^3 + 5*a*b^2)*EllipticE[(c + d*x)/2, 2] + 5*b*(a^2 + b^2)*EllipticF
[(c + d*x)/2, 2] + a^2*Sqrt[Cos[c + d*x]]*(5*b + a*Cos[c + d*x])*Sin[c + d*
x]))/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(158) = 316.
time = 0.17, size = 412, normalized size = 3.55

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3+8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a^3+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
*a^3+20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b-2*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2*a^3-10*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b+5
*b*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+5*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a^3-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 185, normalized size = 1.59

$$\frac{2 \left(a^{3} \cos \left(dx+c\right)+5 a^{2} b\right) \sqrt{\cos \left(dx+c\right)} \sin \left(dx+c\right)-5 \sqrt{2} \left(i a^{2} b+i b^{3}\right) \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)+i \sin \left(dx+c\right)\right)-5 \sqrt{2} \left(-i a^{2} b-i b^{3}\right) \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)-i \sin \left(dx+c\right)\right)-3 \sqrt{2} \left(-i a^{3}-5 i a b^{2}\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)+i \sin \left(dx+c\right)\right)\right)-3 \sqrt{2} \left(i a^{3}+5 i a b^{2}\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)-i \sin \left(dx+c\right)\right)\right)}{5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(2*(a^3*cos(d*x + c) + 5*a^2*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqr
t(2)*(I*a^2*b + I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*sqrt(2)*(-I*a^2*b - I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) - 3*sqrt(2)*(-I*a^3 - 5*I*a*b^2)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*
a^3 + 5*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.18, size = 125, normalized size = 1.08

$$\frac{2 b^{3} \mathrm{F}\left(\frac{c}{2}+\frac{d x}{2} \middle| 2\right)}{d}+\frac{6 a b^{2} \mathrm{E}\left(\frac{c}{2}+\frac{d x}{2} \middle| 2\right)}{d}+\frac{2 a^{2} b \mathrm{F}\left(\frac{c}{2}+\frac{d x}{2} \middle| 2\right)}{d}+\frac{2 a^{2} b \sqrt{\cos \left(c+d x\right)} \sin \left(c+d x\right)}{d}-\frac{2 a^{3} \cos \left(c+d x\right)^{7/2} \sin \left(c+d x\right) {}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos \left(c+d x\right)^{2}\right)}{7 d \sqrt{\sin \left(c+d x\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a*b^2*ellipticE(c/2 + (d*x)/2, 2
))/d + (2*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^2*b*cos(c + d*x)^(1/2
)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2,
7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.813 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=126

$$\frac{2b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(a^2+9b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b(a^2-3b^2)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2a^2\sqrt{\cos(c+dx)}}{}$$

```
[Out] 2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d-2/3*b*(a^2-3*b^
2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a^2*(a+b*sec(d*x+c))*sin(d*x+c)*cos(d*
x+c)^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3926, 4132, 3856, 2720, 4131, 2719}

$$\frac{2a(a^2+9b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b(a^2-3b^2)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(a^2 + 9*b^2)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) - (2*b*(a^2 - 3*b^2)*Sin[c + d*x])/(3*d*Sqrt[Cos
[c + d*x]]) + (2*a^2*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/
(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\Big(2\sqrt{\cos(c+$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\Big(2\sqrt{\cos(c+$$

$$= -\frac{2b(a^2-3b^2)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}{3d}$$

$$= \frac{2a(a^2+9b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} - \frac{2b(a^2-3b^2)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2\sqrt{}}{}$$

$$= \frac{2b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2a(a^2+9b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} - $$

Mathematica [A]
time = 0.41, size = 87, normalized size = 0.69

$$\frac{2\left((9a^2b-3b^3)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(a^3+9ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\frac{(3b^3+a^3\cos(c+dx))\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*((9*a^2*b - 3*b^3)*EllipticE[(c + d*x)/2, 2] + (a^3 + 9*a*b^2)*EllipticF
[(c + d*x)/2, 2] + ((3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c + d
*x]]))/(3*d)
```

Maple [A]
time = 0.20, size = 303, normalized size = 2.40

method	result
default	$-\frac{2\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

[Out] -2/3*(4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3-2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2*a^3-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+9*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*a^2*b+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.71, size = 214, normalized size = 1.70

$$\frac{\sqrt{2}\left(-i a^{3}-9 i a b^{2}\right) \cos\left(dx+c\right) \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i \sin\left(dx+c\right)\right)+\sqrt{2}\left(i a^{3}+9 i a b^{2}\right) \cos\left(dx+c\right) \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i \sin\left(dx+c\right)\right)-3 \sqrt{2}\left(-3 i a^{2} b+i b^{3}\right) \cos\left(dx+c\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i \sin\left(dx+c\right)\right)\right)-3 \sqrt{2}\left(3 i a^{2} b-i b^{3}\right) \cos\left(dx+c\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i \sin\left(dx+c\right)\right)\right)+2\left(a^{3} \cos\left(dx+c\right)+3 b^{3}\right) \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{3 d \cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/3*(sqrt(2)*(-I*a^3 - 9*I*a*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^3 + 9*I*a*b^2)*cos(d*x + c)*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-3*I*a
^2*b + I*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*a^2*b - I*b^3)*cos(d*x +
 c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(a^3*cos(d*x + c) + 3*b^3)*sqrt(cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c))

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.24, size = 124, normalized size = 0.98

$$\frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{6\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,b^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (6*a^2*b*ellipticE(c/2 + (d*x)/
2, 2))/d + (6*a*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^3*cos(c + d*x)^(1
/2)*sin(c + d*x))/(3*d) + (2*b^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, c
os(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.814 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=118

$$\frac{2a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d}+\frac{2b(9a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d}+\frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2b^2(a+b\sec(c+dx))\sin(}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+16/3*a*b^2*sin(
d*x+c)/d/cos(d*x+c)^(1/2)+2/3*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^
(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3927, 4132, 3856, 2720, 4131, 2719}

$$\frac{2b(9a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d}+\frac{2a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d}+\frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(9*a^2 + b^2)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (16*a*b^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx \\
&= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}\Big(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+ \\
&= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}\Big(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+ \\
&= \frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}\big(b(9a \\
&= \frac{2b(9a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} + \frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec}{3d\sqrt{}} \\
&= \frac{2a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{2b(9a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d} +
\end{aligned}
$$

Mathematica [A]
time = 0.79, size = 84, normalized size = 0.71

$$
\frac{2\left(3(a^3-3ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+b\left((9a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\frac{b(b+9a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)\right)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(3*(a^3 - 3*a*b^2)*EllipticE[(c + d*x)/2, 2] + b*((9*a^2 + b^2)*Elliptic
F[(c + d*x)/2, 2] + (b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3
/2))))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(160) = 320$.
time = 0.31, size = 630, normalized size = 5.34

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-18\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(36*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-18*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1
/2*c)^2*a^2*b-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^3+6*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3-18*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(
1/2*d*x+1/2*c)^2*a*b^2-18*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2-2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*b*a^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+9*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 222, normalized size = 1.88

$$\frac{\sqrt{2}\,(-9i\,a^2b-i\,b^3)\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(9i\,a^2b+i\,b^3)\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\sqrt{2}\,(-i\,a^3+3i\,ab^2)\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\sqrt{2}\,(i\,a^3-3i\,ab^2)\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,(9\,ab^2\cos(dx+c)+b^3)\sqrt{\cos(dx+c)}\,\sin(dx+c)}{3\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-9*I*a^2*b - I*b^3)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(9*I*a^2*b + I*b^3)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-I
*a^3 + 3*I*a*b^2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*a^3 - 3*I*a*b^2)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) + 2*(9*a*b^2*cos(d*x + c) + b^3)*sqrt(cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^3 \sqrt{\cos(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
[Out] Integral((a + b*sec(c + d*x))**3*sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 2.10, size = 128, normalized size = 1.08

$$\frac{2\left(\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)a^3+3\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)a^2\right)}{d}+\frac{2\,b^3\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\sqrt{\sin(c+d\,x)^2}}+\frac{6\,a\,b^2\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\sqrt{\cos(c+d\,x)}\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3,x)
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^2*b*ellipticF(c/2 + (d*x)/2, 2)))
/d + (2*b^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*
cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*a*b^2*sin(c + d*x)*hypergeo
m([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)
^(1/2))
```

3.815 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=149

$$-\frac{6b(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}+\frac{6b(5a^2+b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+$$

```
[Out] -6/5*b*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a*b^2*sin(d*
x+c)/d/cos(d*x+c)^(3/2)+2/5*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(3
/2)+6/5*b*(5*a^2+b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3927, 4132, 3853, 3856, 2719, 4131, 2720}

$$\frac{2a(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{6b(5a^2+b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b^2\sin(c+dx)(a+b\sec(c+dx))}{5d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-6*b*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(a^2 + b^2)*Ell
ipticF[(c + d*x)/2, 2])/d + (8*a*b^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
 + (6*b*(5*a^2 + b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*(a +
b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\text{s}$$
$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\text{s}$$
$$= \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b(5a^2+b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+d}{5d\cos^{\frac{3}{2}}(c+dx)}$$
$$= \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b(5a^2+b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+d}{5d\cos^{\frac{3}{2}}(c+dx)}$$
$$= -\frac{6b(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8ab^2\sin(c+}{5d\cos^{\frac{3}{2}}(c+}$$

Mathematica [A]
time = 0.62, size = 125, normalized size = 0.84

$$\frac{-6b(5a^2+b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10a(a^2+b^2)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+10ab^2\sin(c+dx)+3(5a^2b+b^3)\sin(2(c+dx))+2b^3\tan(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-6*b*(5*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*(a^
2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a*b^2*Sin[c + d*
x] + 3*(5*a^2*b + b^3)*Sin[2*(c + d*x)] + 2*b^3*Tan[c + d*x])/(5*d*Cos[c +
d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(187) = 374.
time = 0.38, size = 711, normalized size = 4.77

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*b^2*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*b*a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 244, normalized size = 1.64

$$-\frac{5\sqrt{2}\left(i a^3+i ab^2\right)\cos\left(dx+c\right)^3 \text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-i a^3-i ab^2\right)\cos\left(dx+c\right)^3 \text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\sqrt{2}\left(5i a^2b+i b^3\right)\cos\left(dx+c\right)^3 \text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(-5i a^2b-i b^3\right)\cos\left(dx+c\right)^3 \text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-2\left(5ab^2\cos\left(dx+c\right)+b^3+3\left(5a^2b+b^3\right)\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{5d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -1/5*(5*sqrt(2)*(I*a^3 + I*a*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^3 - I*a*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(5*I*a^2*b + I*b^3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*a^2*b - I*b^3)*co

```
s(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) - 2*(5*a*b^2*cos(d*x + c) + b^3 + 3*(5*a^2*b + b^3)*cos
(d*x + c)^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^3}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 2.17, size = 156, normalized size = 1.05

$$\frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)}{5\,d\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{6\,a^2\,b\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a\,b^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^3*sin(c + d*x)*hypergeom([-5/4
, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/
2)) + (6*a^2*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d
*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*b^2*sin(c + d*x)*hyperge
om([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2
)^(1/2))
```

3.816 $\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{2a(5a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2b(21a^2+5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d}+\frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b(21a^2+5b^2)\sin(c+}{21d\cos^{\frac{3}{2}}(c+dx}$$

```
[Out] -2/5*a*(5*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(21*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/35*
a*b^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)/d/cos(
d*x+c)^(3/2)+2/7*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/5*a*(
5*a^2+9*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3927, 4132, 3853, 3856, 2720, 4131, 2719}

$$\frac{2b(21a^2+5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d}-\frac{2a(5a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5a^2+9b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b^2\sin(c+dx)(a+b\sec(c+dx))}{7d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(5*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(21*a^2 + 5*b
^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (32*a*b^2*Sin[c + d*x])/(35*d*Cos[c
 + d*x]^(5/2)) + (2*b*(21*a^2 + 5*b^2)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/
2)) + (2*a*(5*a^2 + 9*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*
(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}$$

$$= \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b^2(a+b\sec(c+dx))\sin(c+}{7d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(5a^2+9b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} +$$

$$= \frac{2b(21a^2+5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b(21a^2+5b^2)\sin(c+d}{21d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2a(5a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b(21a^2+5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{32ab^2\sin}{35d\cos^{\frac{5}{2}}(}$$

Mathematica [A]
time = 0.55, size = 177, normalized size = 0.91

$$\frac{-42a(5a^2+9b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b(21a^2+5b^2)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+126ab^2\sin(c+dx)+210a^3\cos^2(c+dx)\sin(c+dx)+378ab^2\cos^2(c+dx)\sin(c+dx)+105a^2b\sin(2(c+dx))+25b^3\sin(2(c+dx))+30b^3\tan(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-42*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*b*
(21*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 126*a*b^2*S
in[c + d*x] + 210*a^3*Cos[c + d*x]^2*Sin[c + d*x] + 378*a*b^2*Cos[c + d*x]^
2*Sin[c + d*x] + 105*a^2*b*Sin[2*(c + d*x)] + 25*b^3*Sin[2*(c + d*x)] + 30*
b^3*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(226) = 452$.
time = 0.48, size = 820, normalized size = 4.23

method	result	size
default	Expression too large to display	820

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(6*b*a^2*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6/5*b^2*a/(8*sin(1/2*d*x+1/2*c)^6-12*sin(
1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b
^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)+2*a^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 270, normalized size = 1.39

$$-\frac{5\sqrt{2}(21i a^2b + 5i b^3)\cos(dx+c)^4 \,\mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5\sqrt{2}(-21i a^2b - 5i b^3)\cos(dx+c)^4 \,\mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\sin(dx+c)) + 21\sqrt{2}(5i a^3 + 9i ab^2)\cos(dx+c)^4 \,\mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))) + 21\sqrt{2}(-5i a^3 - 9i ab^2)\cos(dx+c)^4 \,\mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\sin(dx+c))) - 2(63ab^2\cos(dx+c) + 21(5a^3 + 9ab^2)\cos(dx+c)^3 + 15b^3 + 5(21a^2b + 5b^3)\cos(dx+c)^2)\sqrt{\cos(dx+c)}\sin(dx+c)}{105\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*cos(d*x + c)^4*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^2*b - 5*I*b^3)*
cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) +
21*sqrt(2)*(5*I*a^3 + 9*I*a*b^2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weie
```

```
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-5*I*a
^3 - 9*I*a*b^2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a*b^2*cos(d*x + c) + 21*(5*a^
3 + 9*a*b^2)*cos(d*x + c)^3 + 15*b^3 + 5*(21*a^2*b + 5*b^3)*cos(d*x + c)^2)
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 2.24, size = 147, normalized size = 0.76

$$\frac{\frac{2\,b^3\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)}{7} + 2\,a^3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right) + \frac{6\,a\,b^2\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)}{5} + 2\,a^2\,b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

```
[Out] ((2*b^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + 2*a^
3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) +
 (6*a*b^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*
x)^2))/5 + 2*a^2*b*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4,
cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.817 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^3d}-\frac{2b(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^4d}+\frac{2b^4\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)}{a^4(a+b)d}-\frac{2b\sqrt{\cos(c+dx)}\ \mathrm{s}}{3a^2d}$$

```
[Out] 2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-2/3*b*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d+2*b^4
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2*a/(a+b),2^(1/2))/a^4/(a+b)/d+2/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d-2/3*
b*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A]

time = 0.41, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4349, 3938, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{2b^4\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)}{a^4d(a+b)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{2b(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^4d}+\frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^3d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2*b*(a^2 + 3*b^2
)*EllipticF[(c + d*x)/2, 2])/(3*a^4*d) + (2*b^4*EllipticPi[(2*a)/(a + b), (
c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
a^2*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx \\
&= \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5b}{2}+\frac{3}{2}a\sec(c+dx)+}{\sec^{\frac{3}{2}}(c+dx)(a+b}}{5a} \\
&= -\frac{2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\left(4\sqrt{\cos(c+dx)}\right.}{} \\
&= -\frac{2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\left(4\sqrt{\cos(c+dx)}\right.}{} \\
&= -\frac{2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{b^4\int \frac{}{\sqrt{\cos(c+dx}}}{a} \\
&= \frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{a^4(a+b)d} - \frac{2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5ad} \\
&= \frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^3d} - \frac{2b(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^4d} + \frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\right.}{a^4(a+b)}
\end{aligned}
$$

Mathematica [A]
time = 11.05, size = 226, normalized size = 1.49

$$
\frac{\frac{2(9a^2+5b^2)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b} + 8b\left(2F\left(\frac{1}{2}(c+dx)\middle|\,2\right) - \frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b}\right) + 4\sqrt{\cos(c+dx)}\,(-5b+3a\cos(c+dx))\sin(c+dx) + \frac{6(3a^2+5b^2)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{30a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*(9*a^2 + 5*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*
b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b)) + 4*Sqrt[Cos[c + d*x]]*(-5*b + 3*a*Cos[c + d*x])*Sin[c + d*x
```

```
] + (6*(3*a^2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Ellip
ticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Si
n[c + d*x]^2]))/(30*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(218) = 436$.
time = 0.20, size = 668, normalized size = 4.39

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\left(-24a^4+24b\,a^3\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(24a^4-44b\,a^3+20b^2a^2\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*a^4+24*
a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a^4-44*a^3*b+20*a^2*b^2)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*a^4+16*a^3*b-10*a^2*b^2)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+5*b^2*a
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*b^4*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+9*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^3*b-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b
^3-15*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^4/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x)),x)
[Out] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x)), x)
```

3.818 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}-\frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a+b)d}+\frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad}$$

```
[Out] -2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^2/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-2*b^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2
^(1/2))/a^3/(a+b)/d+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.27, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4349, 3938, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(a^2 + 3*b^2)*EllipticF[(c +
d*x)/2, 2])/(3*a^3*d) - (2*b^3*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(
a^3*(a + b)*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx \\
&= \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3b}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}}{3a} \\
&= \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3ab}{2}-\left(-\frac{a^2}{2}-\frac{3b^2}{2}\right)}{\sqrt{\sec(c+}}}{3a^3} \\
&= \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{b^3\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx}{a^3} - \frac{\left(b\sqrt{\cos(c+\right.}}{} \\
&= -\frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a+b)d} + \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{b\int\sqrt{\cos(c+dx)}\,dx}{a^2} \\
&= -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(a^2+3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d} - \frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a+b)d} +
\end{aligned}
$$

Mathematica [A]
time = 11.09, size = 158, normalized size = 1.41

$$
\frac{4F\left(\frac{1}{2}(c+dx)\middle|2\right) - \frac{6b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + 4\sqrt{\cos(c+dx)}\,\sin(c+dx) - \frac{6\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2\sqrt{\sin^2(c+dx)}}}{6ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (4*EllipticF[(c + d*x)/2, 2] - (6*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/(6*a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(184) = 368$.
time = 0.18, size = 552, normalized size = 4.93

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x)), x)
```

3.819 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^2/(a+b)
/d
```

Rubi [A]
time = 0.17, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4349, 3937, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticF[(c + d*x)/2, 2])/(a^2*
d) + (2*b^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3937

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[b^2/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b
*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx$$
$$= \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{\left(b^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.}{}$$
$$= \frac{b^2\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx}{a^2} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c}}}{a}$$
$$= \frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d} + \frac{\int\sqrt{\cos(c+dx)}\,dx}{a} - \frac{b\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$
$$= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d}$$

Mathematica [A]
time = 10.20, size = 81, normalized size = 1.08

$$-\frac{2\left(aE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+b\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(a*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSin
[Sqrt[Cos[c + d*x]]], -1] + b*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]]
, -1])*Sin[c + d*x])/(a^2*d*Sqrt[Sin[c + d*x]^2])
```

Maple [A]
time = 0.15, size = 226, normalized size = 3.01

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\left(c\right.\right.}{a^2(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
```

```
,2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] integral(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x)),x)
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x)), x)
```

3.820 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=53

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a/(a+b)/d
```

Rubi [A]
time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4349, 3933, 2882, 2720, 2884}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[(2*a)/(a + b), (c + d
*x)/2, 2])/(a*(a + b)*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3933

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d), Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx \\
&= \int \frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx \\
&= \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx}{a} \\
&= \frac{2F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{ad} - \frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{a(a+b)d}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.91

$$
\frac{2F\left(\frac{1}{2}(c+dx)\middle|\,2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}}{ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2])/(a + b))/(a*d)
```

Maple [A]
time = 0.14, size = 187, normalized size = 3.53

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(\mathrm{EllipticF}\left(\right.\right.}{a(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec{(c+dx)})\sqrt{\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))), x)
```

3.821 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=29

$$\frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a+b)/d

Rubi [A]
time = 0.09, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4349, 3934, 2884}

$$\frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]

[Out] (2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx$$
$$= \frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

Mathematica [A]
time = 0.06, size = 29, normalized size = 1.00

$$\frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
[Out] (2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(55) = 110$.
time = 0.13, size = 150, normalized size = 5.17

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\Big(\mathrm{c}}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})\cos^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))), x)
```

3.822 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=77

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}+\frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] -2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/b/(a+b)/d+2*sin(d*x+c)/b/d/cos(
d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4349, 3935, 3853, 3856, 2719, 3934, 2884}

$$-\frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2])/(b*(a + b)*d) + (2*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3935

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d/b, Int[(d*Csc[e + f*x])^(3/2), x], x] - Dist[a*(
d/b), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx\\
&= \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx}{b} - \frac{\left(a\sqrt{\cos(c+\right.}{}\\
&= \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{a\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx}{b} - \frac{\left(\sqrt{\cos(c\right.}{}\\
&= -\frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d} + \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{\int\sqrt{\cos(c+dx)}\,d}{b}\\
&= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d} + \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(77) = 154.

time = 11.96, size = 195, normalized size = 2.53

$$-\frac{\frac{6a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{2b\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}-\frac{4\sin(c+dx)}{\sqrt{\cos(c+dx)}}+\frac{2\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -1/2*((6*a*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*b*(2*Ell
ipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a
 + b)))/a - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(127) = 254$.
time = 0.18, size = 353, normalized size = 4.58

method	result
default	$-\frac{2\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/b/(a-b)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))), x)
```

3.823 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=128

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}+\frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d}+\frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)}-\frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/b^2/(a+
b)/d+2/3*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)-2*a*sin(d*x+c)/b^2/d/cos(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.37, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4349, 3936, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}+\frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*
b*d) + (2*a^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d) +
(2*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(3/2)) - (2*a*Sin[c + d*x])/(b^2*d*Sqr
t[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
```

```
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+}}{}}{3b}$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}\right.}{}$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}\right.}{}$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{a^2\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\,\mathrm{co}}}{b^2}$$

$$= \frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d} + \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd} + \frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{b^2(a+b)d}$$

Mathematica [A]
time = 12.62, size = 210, normalized size = 1.64

$$\frac{\frac{2(9a^2+2b^2)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b} + 8b\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right) - \frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b}\right) + \frac{4(b-3a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} + \frac{6\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{b\sqrt{\sin^2(c+dx)}}}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((2*(9*a^2 + 2*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*
b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b)) + (4*(b - 3*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)
 + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Ellip
ticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), Arc
Sin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))/(6*b^
2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(198) = 396$.
time = 0.33, size = 423, normalized size = 3.30

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^3/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*a/b^2/si
n(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))), x)
```

3.824 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=244

$$-\frac{b(4a^2-5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3\,(a^2-b^2)\,d}+\frac{(2a^4+16a^2b^2-15b^4)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^4\,(a^2-b^2)\,d}-\frac{b^3(7a^2-5b^2)\,\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)}{a^4(a-b)(a+b)^2d}$$

```
[Out] -b*(4*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3*(2*a^4+16*a^2*b^2-15*b^4)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))/a^4/(a^2-b^2)/d-b^3*(7*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^4/(a-b)
/(a+b)^2/d+1/3*(2*a^2-5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b^
2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.51, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4349, 3932, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{(2a^2-5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d\,(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\,(a^2-b^2)\,(a+b\sec(c+dx))}+\frac{(2a^4+16a^2b^2-15b^4)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^4d\,(a^2-b^2)}-\frac{b^3(7a^2-5b^2)\,\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)}{a^4d(a-b)(a+b)^2}-\frac{b(4a^2-5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b*(4*a^2 - 5*b^2)*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d)) + ((2*
a^4 + 16*a^2*b^2 - 15*b^4)*EllipticF[(c + d*x)/2, 2])/(3*a^4*(a^2 - b^2)*d)
 - (b^3*(7*a^2 - 5*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a
- b)*(a + b)^2*d) + ((2*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^
2*(a^2 - b^2)*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(
a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx \\
&= \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-a^2+\frac{5b^2}{2}+ab\cdots}{\sec^{\frac{3}{2}}(c\cdots}}{a\left(a^2-b^2\right)\cdots} \\
&= \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2\sqrt{\cdots}\right.}{\cdots} \\
&= \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2\sqrt{\cdots}\right.}{\cdots} \\
&= \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(b^3\right(\cdots}{\cdots} \\
&= -\frac{b^3\left(7a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4(a-b)(a+b)^2d} + \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} - \cdots \\
&= -\frac{b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^4+16a^2b^2-15b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4\left(a^2-b^2\right)d} - \frac{b^3(\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 11.24, size = 266, normalized size = 1.09

$$
\frac{4\sqrt{\cos(c+dx)}\left(2+\frac{3b^3}{\left(-a^2+b^2\right)(b+a\cos(c+dx))}\right)\sin(c+dx) - \frac{\frac{2\left(-8a^2b+5b^3\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{8\left(a^2+2b^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} - \frac{6\left(4a^2-5b^2\right)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2\sqrt{\sin^2(c+dx)}}}{(-a+b)(a+b)}}{12a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2 + (3*b^3)/((-a^2 + b^2)*(b + a*Cos[c + d*x])))*Sin
[c + d*x] - ((2*(-8*a^2*b + 5*b^3)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a + b) + (8*(a^2 + 2*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*Ellipt
icPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(4*a^2 - 5*b^2)*(-2*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c
 + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)
))/(12*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(314) = 628$.
time = 0.37, size = 1064, normalized size = 4.36

method	result	size
default	Expression too large to display	1064

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^2*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4/a^3*(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*(a^2+2*a*b+3*b^2)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a^4*b^4*(1/b*a^2/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2
-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2)))+8*b^3/a^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
```

```
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^2,x)
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^2, x)
```

3.825 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=184

$$\frac{(2a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\,(a^2-b^2)\,d}-\frac{b(4a^2-3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\,(a^2-b^2)\,d}+\frac{b^2(5a^2-3b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a-b)(a+b)^2d}+\frac{}{a\,(a^2-b^2)}$$

```
[Out] (2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d-b*(4*a^2-3*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^
2-b^2)/d+b^2*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2/d+b^2*si
n(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4349, 3932, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{(2a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\,(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad\,(a^2-b^2)\,\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}-\frac{b(4a^2-3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\,(a^2-b^2)}+\frac{b^2(5a^2-3b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - (b*(4*a^2
 - 3*b^2)*EllipticF[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d) + (b^2*(5*a^2 - 3*
b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) +
 (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]
))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx \\
&= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.}{a} \\
&= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.}{} \\
&= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{\left(b^2\left(5a^2-3b^2\right)\right)\int \frac{}{\sqrt{\cos(c+dx}}}{2a^3\left(a^2-b^2\right)} \\
&= \frac{b^2\left(5a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a-b)(a+b)^2d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+}\\
&= \frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} - \frac{b\left(4a^2-3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{b^2\left(5a^2-3b^2\right)\Pi}{a^3(a-}
\end{aligned}
$$

Mathematica [**A**]
time = 11.18, size = 252, normalized size = 1.37

$$
\frac{\frac{4b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)(b+a\cos(c+dx))}+\frac{\frac{2\left(2a^2-b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+8b\left(-F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)+\frac{2\left(2a^2-3b^2\right)\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x]))
 + ((2*(2*a^2 - b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8
*b*(-EllipticF[(c + d*x)/2, 2] + (b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2])/(a + b)) + (2*(2*a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2
*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^
2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(260) = 520$.
time = 0.33, size = 809, normalized size = 4.40

method	result	size
default	Expression too large to display	809

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/a^3*b^3*(1/b*a^2/(a^2-b^2)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a
*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2)))-6*b^2/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\left(a+b\sec\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^2, x)
```

3.826 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=167

$$\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d}+\frac{\left(2a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d}-\frac{b(3a^2-b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a-b)(a+b)^2d}-\frac{b\sin(c+}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx}}$$

```
[Out] b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d/(a^2-b^2)+(2*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d-b*(3*a^2-
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2*a/(a+b),2^(1/2))/a^2/(a-b)/(a+b)^2/d-b*sin(d*x+c)/(a^2-b^2)/d/(a+
b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4349, 3928, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{\left(2a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{b(3a^2-b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] (b*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((2*a^2 - b^2)*EllipticF[
(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - (b*(3*a^2 - b^2)*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2])/(a^2*(a - b)*(a + b)^2*d) - (b*Sin[c + d*x])/((a^2
- b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

0] && GtQ[c + d, 0]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3928

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)
*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Si
mp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
 && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

$$= -\frac{b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$= -\frac{b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$= -\frac{b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(b\left(3-\frac{b^2}{a^2}\right)\right)}{}$$

$$= -\frac{b(3a^2-b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a-b)(a+b)^2d} - \frac{b\sin(c+}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}}$$

$$= \frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\,(a^2-b^2)\,d} + \frac{(2a^2-b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\,(a^2-b^2)\,d} - \frac{b(3a^2-b^2)\,}{a^2(a}$$

Mathematica [**A**]
time = 12.57, size = 194, normalized size = 1.16

$$\frac{\frac{4b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(-a^2+b^2)(b+a\cos(c+dx))} - \frac{8F\left(\frac{1}{2}(c+dx)\middle|2\right) - \frac{10b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2\sqrt{\sin^2(c+dx)}}}{(-a+b)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]))
- (8*EllipticF[(c + d*x)/2, 2] - (10*b*EllipticPi[(2*a)/(a + b), (c + d*x)/
2, 2])/(a + b) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Elliptic
Pi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c +
 d*x]^2]))/((-a + b)*(a + b)))/(4*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(243) = 486$.
time = 0.32, size = 788, normalized size = 4.72

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{Ellipt}}{a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a^2*b^2*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+4*b/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2), x)
```

3.827 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d}-\frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\,(a^2-b^2)\,d}+\frac{(a^2+b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a-b)(a+b)^2d}+\frac{a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/(a^2-b^2)/d-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/(a^2-b^2)+(a^2+b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2
^(1/2))/a/(a-b)/(a+b)^2/d+a*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))/cos(d*x
+c)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4349, 3929, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\,(a^2-b^2)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\,(a^2-b^2)}+\frac{(a^2+b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}+\frac{a\sin(c+dx)}{d\,(a^2-b^2)\,\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/((a^2 - b^2)*d)) - (b*EllipticF[(c + d*x)/2, 2]
)/(a*(a^2 - b^2)*d) + ((a^2 + b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a*(a - b)*(a + b)^2*d) + (a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d
*x]]*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3929

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= \frac{a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= \frac{a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{(a^2+b^2)\int \frac{}{\sqrt{\cos(c}}}{2a\,(}$$

$$= \frac{(a^2+b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{a(a-b)(a+b)^2d} + \frac{a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\,\mathrm{se}}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{(a^2-b^2)\,d} - \frac{bF\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a\,(a^2-b^2)\,d} + \frac{(a^2+b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+d\right.}{a(a-b)(a+b)^2d}$$

Mathematica [A]
time = 12.07, size = 229, normalized size = 1.55

$$\frac{\frac{4a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))}-\frac{2\left(-\frac{a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+2b\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)+\frac{\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{b\sqrt{\sin^2(c+dx)}}\right)}{a(a-b)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) -
 (2*(-((a^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + 2*b*(2*El
lipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(
a + b)) + ((-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*
EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b)
, ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2])))
/(a*(a - b)*(a + b)))/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(224) = 448$.
time = 0.29, size = 707, normalized size = 4.78

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2b\left(\frac{a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\cdots\right.}}{b\left(a^2-b^2\right)\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-a+b\right)}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/a*(1/b*a^2
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/
(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})^2 \cos^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**2*cos(c + d*x)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2), x)
```

3.828 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d}+\frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b(a+b)^2d}-\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec$$

```
[Out] a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/b/(a^2-b^2)/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/(a^2-b^2)/d+(a^2-3*b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),
2^(1/2))/(a-b)/b/(a+b)^2/d-a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))/co
s(d*x+c)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4349, 3930, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] (a*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + EllipticF[(c + d*x)/2, 2]
/((a^2 - b^2)*d) + ((a^2 - 3*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/((a - b)*b*(a + b)^2*d) - (a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx \\
&= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx}\right.}{} \\
&= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx}\right.}{} \\
&= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{\left(a^2-3b^2\right)\int -}{\sqrt{}} \\
&= \frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b(a+b)^2d} - \frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+} \\
&= \frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\right.}{(a-b)b(a+b)^2d}
\end{aligned}
$$

Mathematica [A]
time = 12.31, size = 239, normalized size = 1.55

$$
\frac{\frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(-a^2+b^2\right)(b+a\cos(c+dx))}+\frac{\frac{2\left(3a^2-4b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+4b\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)+\frac{2\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{b\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x])
) + ((2*(3*a^2 - 4*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)
+ 4*b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*
x)/2, 2])/(a + b)) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Elli
pticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c
 + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(230) = 460$.
time = 0.30, size = 608, normalized size = 3.95

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.}}{b\left(a^2-b^2\right)\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-a+b\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*a^2/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/b*a/(a^2-b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/b*a/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/(a^2-a*b
)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2), x)
```

3.829 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=219

$$-\frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\,(a^2-b^2)\,d}-\frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\,(a^2-b^2)\,d}-\frac{a(3a^2-5b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b^2(a+b)^2d}+\frac{(3a^2-2b^2)\sin(c-}{b^2\,(a^2-b^2)\,d\sqrt{\cos(c}}$$

```
[Out] -(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d-a*(3*a
^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/
2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)/b^2/(a+b)^2/d-a^2*sin(d*x+c)/b/(a^2-b
^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))+(3*a^2-2*b^2)*sin(d*x+c)/b^2/(a^2-b
^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$,
Rules used = {4349, 3930, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\,(a^2-b^2)}-\frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\,(a^2-b^2)}-\frac{a(3a^2-5b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}+\frac{(3a^2-2b^2)\sin(c+dx)}{b^2d\,(a^2-b^2)\,\sqrt{\cos(c+dx)}}-\frac{a^2\sin(c+dx)}{bd\,(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -(((3*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d)) - (a*Ell
ipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) - (a*(3*a^2 - 5*b^2)*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/((a - b)*b^2*(a + b)^2*d) + ((3*a^2 - 2*b^2)
*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) - (a^2*Sin[c + d*x])/
(b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\ \sqrt{\right.}{}$$

$$= \frac{(3a^2-2b^2)\sin(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(}$$

$$= \frac{(3a^2-2b^2)\sin(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(}$$

$$= \frac{(3a^2-2b^2)\sin(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(}$$

$$= -\frac{a(3a^2-5b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{(a-b)b^2(a+b)^2d} + \frac{(3a^2-2b^2)\sin(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}}$$

$$= -\frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2\,(a^2-b^2)\,d} - \frac{aF\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b\,(a^2-b^2)\,d} - \frac{a(3a^2-5b^2)}{(a-}$$

Mathematica [A]

time = 12.03, size = 278, normalized size = 1.27

$$\frac{-\frac{\frac{2(9a^3-10ab^2)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{(8a^2b-4b^3)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{2(3a^2-2b^2)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}+4\sqrt{\cos(c+dx)}\left(\frac{a^3\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))}+2\tan(c+dx)\right)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] (-((((2*(9*a^3 - 10*a*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b
) + ((8*a^2*b - 4*b^3)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)
/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(3*a^2 - 2*b^2)*(-2*a*b*Ellipti
cE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b))) + 4*S
qrt[Cos[c + d*x]]*((a^3*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) +
2*Tan[c + d*x]))/(4*b^2*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(293) = 586$.

time = 0.38, size = 841, normalized size = 3.84

method	result	size
default	Expression too large to display	841

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a/b*(1/b*a^2
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/
(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2/
b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2), x)
```

3.830 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=346

$$-\frac{b(24a^4-65a^2b^2+35b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2d}+\frac{(8a^6+128a^4b^2-223a^2b^4+105b^6)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^5\left(a^2-b^2\right)^2d}-\frac{b^3(63a^4$$

```
[Out] -1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*a^6+
128*a^4*b^2-223*a^2*b^4+105*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^5/(a^2-b^2)^2/d-1/4*b^3*(63*a
^4-86*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^5/(a-b)^2/(a+b)^3/d+1/12*(8*a
^4-61*a^2*b^2+35*b^4)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d+1/2*b^2
*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*b^2*(13*a
^2-7*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.75, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {4349, 3932, 4185, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b^2(13a^2-7b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{b(24a^4-65a^2b^2+35b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}-\frac{b^3(63a^4-86a^2b^2+35b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^5d(a-b)^2(a+b)^3}+\frac{(8a^4-61a^2b^2+35b^4)\sin(c+dx)\sqrt{\cos(c+dx)}}{12a^3d\left(a^2-b^2\right)^2}+\frac{(8a^6+128a^4b^2-223a^2b^4+105b^6)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^5d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*EllipticE[(c + d*x)/2, 2])/(a^4*(a^2
 - b^2)^2*d) + ((8*a^6 + 128*a^4*b^2 - 223*a^2*b^4 + 105*b^6)*EllipticF[(c
+ d*x)/2, 2])/(12*a^5*(a^2 - b^2)^2*d) - (b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4
)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d)
+ ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*a^3*(
a^2 - b^2)^2*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*
(a + b*Sec[c + d*x])^2) + (b^2*(13*a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
```

```
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-2a^2+\frac{7b^2}{2}}{\sec}}{2a\,(a^2-b^2)}$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b^2(13a^2-7b^2)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))} +$$

$$= \frac{(8a^4-61a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{12a^3\,(a^2-b^2)^2\,d} + \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+}{2a\,(a^2-b^2)\,d(a+b\sec(c+}$$

$$= \frac{(8a^4-61a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{12a^3\,(a^2-b^2)^2\,d} + \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+}{2a\,(a^2-b^2)\,d(a+b\sec(c+}$$

$$= \frac{(8a^4-61a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{12a^3\,(a^2-b^2)^2\,d} + \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+}{2a\,(a^2-b^2)\,d(a+b\sec(c+}$$

$$= -\frac{b^3(63a^4-86a^2b^2+35b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{4a^5(a-b)^2(a+b)^3d} + \frac{(8a^4-61a^2b^2+35b^4)\sqrt{\cos}}{12a^3\,(a^2-b}$$

$$= -\frac{b(24a^4-65a^2b^2+35b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a^4\,(a^2-b^2)^2\,d} + \frac{(8a^6+128a^4b^2-223a^2b^4+105b^6}{12a^5\,(a^2-b^2)^2\,d}$$

Mathematica [A]
time = 12.34, size = 353, normalized size = 1.02

$$\frac{\frac{4\sqrt{\cos(c+dx)}\left(4a^6-57a^2b^4+35b^6+ab(16a^4-83a^2b^2+49b^4)\cos(c+dx)+4(a^3-ab^2)^2\cos(2(c+dx))\right)\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{-\frac{2(56a^4b-73a^2b^3+35b^5)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16(2a^4+14a^2b^2-7b^4)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}-\frac{6(24a^4-65a^2b^2+35b^4)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{48a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(4*a^6 - 57*a^2*b^4 + 35*b^6 + a*b*(16*a^4 - 83*a^2*
b^2 + 49*b^4)*Cos[c + d*x] + 4*(a^3 - a*b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*
x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(56*a^4*b - 73*a^2*b^3 +
35*b^5)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^4 + 1
4*a^2*b^2 - 7*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/
(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(24*a^4 - 65*a^2*b^2 + 35*b^4)*(-2*
a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[C
```

```
os[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a
 + b)^2))/(48*a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. $2(406) = 812$.

time = 0.71, size = 2216, normalized size = 6.40

method	result	size
default	Expression too large to display	2216

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^3*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(2*a+3*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(a^2+3*a*b+6*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+10/a^5*b^4*(1/b*a^2/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)
/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2)))-2/a^5*b^5*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a
+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
```

```
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(
a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*
b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+20*b^3/a^4/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
[Out] integral(cos(d*x + c)^(3/2)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^3, x)
```

3.831 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=282

$$\frac{(8a^4-29a^2b^2+15b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d}-\frac{3b(8a^4-11a^2b^2+5b^4)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d}+\frac{b^2(35a^4-38a^2b^2+15b^4)}{4a^4(a-b)^2(}$$

```
[Out] 1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d-3/4*b*(8*a^4-11*
a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/4*b^2*(35*a^4-38*a^2*b^2+15*b
^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*a/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b)^3/d+1/2*b^2*sin(d*x+c)/a/(a^2-b
^2)/d/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2)+1/4*b^2*(11*a^2-5*b^2)*sin(d*x+c)
/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4349, 3932, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b^2(11a^2-5b^2)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}-\frac{3b(8a^4-11a^2b^2+5b^4)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b^2(35a^4-38a^2b^2+15b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d(a-b)^2(a+b)^3}+\frac{(8a^4-29a^2b^2+15b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((8*a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2
)^2*d) - (3*b*(8*a^4 - 11*a^2*b^2 + 5*b^4)*EllipticF[(c + d*x)/2, 2])/(4*a^
4*(a^2 - b^2)^2*d) + (b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + (b^2*Sin[c + d*x])
/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2) + (b^2*(11*a
^2 - 5*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*
Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
```

```
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sin(c+dx)}{2a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d\right.}{}$$

$$= \frac{b^2\sin(c+dx)}{2a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{b^2(11a^2-5b^2)\sin}{4a^2\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx}}$$

$$= \frac{b^2\sin(c+dx)}{2a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{b^2(11a^2-5b^2)\sin}{4a^2\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx}}$$

$$= \frac{b^2\sin(c+dx)}{2a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{b^2(11a^2-5b^2)\sin}{4a^2\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx}}$$

$$= \frac{b^2(35a^4-38a^2b^2+15b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{4a^4(a-b)^2(a+b)^3d} + \frac{b^2\sin(c+dx}{2a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a}$$

$$= \frac{(8a^4-29a^2b^2+15b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a^3\,(a^2-b^2)^2\,d} - \frac{3b(8a^4-11a^2b^2+5b^4)\,F\left(\frac{1}{2}(c+dx)\right|}{4a^4\,(a^2-b^2)^2\,d}$$

<u>Mathematica</u> [A]

time = 11.85, size = 311, normalized size = 1.10

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\,(11a^2b-5b^3+a(13a^2-7b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{\frac{(8a^4-7a^2b^2+5b^4)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}-\frac{8(4a^2b-b^3)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{(8a^4-29a^2b^2+15b^4)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*b^2*Sqrt[Cos[c + d*x]]*(11*a^2*b - 5*b^3 + a*(13*a^2 - 7*b^2)*Cos[c + d
*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (((8*a^4 - 7*a^
2*b^2 + 5*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(4*a
^2*b - b^3)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]))/(a + b) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*(-2*a*b*Ellipt
icE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos
[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x
]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)
)/(8*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1956 vs. $2(346) = 692$.
time = 0.74, size = 1957, normalized size = 6.94

method	result	size
default	Expression too large to display	1957

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-8/a^4*b^3*(1/b*a^2/(a^2-b^2)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a
*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
```

```
),2*a/(a-b),2^(1/2)))+2/a^4*b^4*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a
-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8
/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^
2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/
(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-12*b^2/a^3/(a^2-a*b)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos{(c+dx)}}}{(a+b\sec{(c+dx)})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^3, x)
```

3.832 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=263

$$\frac{3b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d}+\frac{(8a^4-5a^2b^2+3b^4)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d}-\frac{3b(5a^4-2a^2b^2+b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\right.}{4a^3(a-b)^2(a+b)^3 d}$$

```
[Out] 3/4*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d+1/4*(8*a^4-5*a^2*b^2+3*b^4)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))/a^3/(a^2-b^2)^2/d-3/4*b*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/
2))/a^3/(a-b)^2/(a+b)^3/d-1/2*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/c
os(d*x+c)^(1/2)-1/4*b*(7*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4349, 3928, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{3b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{b(7a^2-b^2)\sin(c+dx)}{4ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}+\frac{(8a^4-5a^2b^2+3b^4)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{3b(5a^4-2a^2b^2+b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] (3*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((8
*a^4 - 5*a^2*b^2 + 3*b^4)*EllipticF[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d
) - (3*b*(5*a^4 - 2*a^2*b^2 + b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(4*a^3*(a - b)^2*(a + b)^3*d) - (b*Sin[c + d*x])/(2*(a^2 - b^2)*d*Sqrt[C
os[c + d*x]]*(a + b*Sec[c + d*x])^2) - (b*(7*a^2 - b^2)*Sin[c + d*x])/(4*a*
(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3928

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)
*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Si
mp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
 && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
```

```
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+\right.}{}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{}{4a\left(a^2-b^2\right)^2}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{}{4a\left(a^2-b^2\right)^2}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{}{4a\left(a^2-b^2\right)^2}$$

$$= -\frac{3b(5a^4-2a^2b^2+b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3(a-b)^2(a+b)^3d} - \frac{b}{2\left(a^2-b^2\right)d\sqrt{\cos}}$$

$$= \frac{3b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d} + \frac{(8a^4-5a^2b^2+3b^4)\,F\left(\frac{1}{2}(c+dx\right.}{4a^3\left(a^2-b^2\right)^2d}$$

Mathematica [A]

time = 11.84, size = 286, normalized size = 1.09

$$\frac{\frac{4b\sqrt{\cos(c+dx)}\left(-7a^2b+b^3+(-9a^3+3ab^2)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{-\frac{2(5a^2b+b^3)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16(2a^2+b^2)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{6(3a^2-b^2)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((4*b*Sqrt[Cos[c + d*x]]*(-7*a^2*b + b^3 + (-9*a^3 + 3*a*b^2)*Cos[c + d*x])
*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(5*a^2*b + b^3
)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^2 + b^2)*((
a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]))/(a + b) + (6*(3*a^2 - b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*
b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2
*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(327) = 654$.
time = 0.64, size = 1936, normalized size = 7.36

method	result	size
default	Expression too large to display	1936

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6/a^3
*b^2*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2/a^3*b^3*(1/2/b*a^2/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
```

```
/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^
2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)
/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-
a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2)))+6*b/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^3\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**3*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3), x)
```

3.833 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=246

$$-\frac{(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d}-\frac{b(7a^2-b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d}+\frac{(3a^4+10a^2b^2-b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2(a-b)^2(a+b)^3 d}+\frac{}{2\,(a^2}$$

```
[Out] -1/4*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)^2/d-1/4*b*(7*a^2-b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^
2/(a^2-b^2)^2/d+1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^2/(a-b)^
2/(a+b)^3/d+1/2*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2
)+3/4*(a^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4349, 3929, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{b(7a^2-b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{3(a^2+b^2)\sin(c+dx)}{4d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}+\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}+\frac{(3a^4+10a^2b^2-b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -1/4*((5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)^2*d) - (b*(7*
a^2 - b^2)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4 + 1
0*a^2*b^2 - b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^2*(a - b)^
2*(a + b)^3*d) + (a*Sin[c + d*x])/(2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a +
b*Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*Sqrt[C
os[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3929

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{3(a^2}{4\left(a^2-b^2\right)^2 d\sqrt{c}}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{3(a^2}{4\left(a^2-b^2\right)^2 d\sqrt{c}}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{3(a^2}{4\left(a^2-b^2\right)^2 d\sqrt{c}}$$

$$= \frac{\left(3a^4+10a^2b^2-b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2(a-b)^2(a+b)^3d} + \frac{a\sin(}{2\left(a^2-b^2\right)d\sqrt{\cos(c+}}$$

$$= -\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2d} - \frac{b\left(7a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d} + \frac{(3a}{}$$

Mathematica [A]
time = 11.34, size = 272, normalized size = 1.11

$$\frac{\frac{4\sqrt{\cos(c+dx)}\left(3b\left(a^2+b^2\right)+a\left(5a^2+b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))^2}-\frac{-\frac{2\left(a^2+5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+24b\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)+\frac{2\left(5a^2+b^2\right)\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(3*b*(a^2 + b^2) + a*(5*a^2 + b^2)*Cos[c + d*x])*Sin
[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) - ((-2*(a^2 + 5*b^2)*Elli
pticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 24*b*(2*EllipticF[(c + d*x
)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + (2*(5*
a^2 + b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*
EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b)
, ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2
]))/((a - b)^2*(a + b)^2))/(16*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. $2(310) = 620$.
time = 0.61, size = 1858, normalized size = 7.55

method	result	size
default	Expression too large to display	1858

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b/a^2*(1/b*a
^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*
b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2/a^2*
b^2*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/
b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2
```

```
-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+
b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a
^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3), x)
```

3.834 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=253

$$\frac{(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b\,(a^2-b^2)^2\,d}+\frac{3(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a\,(a^2-b^2)^2\,d}+\frac{(a^4-10a^2b^2-3b^4)\,\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)}{4a(a-b)^2b(a+b)^3d}-\frac{}{2b\,(a^2 -}$$

```
[Out] 1/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)^2/d+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-
b^2)^2/d+1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a/(a-b)^2/b/(a+b)
^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2)+1
/4*a*(a^2-7*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/
2)
```

Rubi [A]
time = 0.51, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4349, 3930, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{3(a^2+b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4ad\,(a^2-b^2)^2}+\frac{(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4bd\,(a^2-b^2)^2}-\frac{a^2\sin(c+dx)}{2bd\,(a^2-b^2)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}+\frac{a(a^2-7b^2)\sin(c+dx)}{4bd\,(a^2-b^2)^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}+\frac{(a^4-10a^2b^2-3b^4)\,\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)}{4abd(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (3*(a^2 +
 b^2)*EllipticF[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^2
 - 3*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b*(a +
b)^3*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*S
ec[c + d*x])^2) + (a*(a^2 - 7*b^2)*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
```

```
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{a}{4b\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{a}{4b\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{a}{4b\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^4-10a^2b^2-3b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{4a(a-b)^2b(a+b)^3d} - \frac{a^2\sin}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+}}$$

$$= \frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4a\left(a^2-b^2\right)^2 d} + \frac{(a^4-}{}$$

Mathematica [A]

time = 11.87, size = 289, normalized size = 1.14

$$\frac{-\frac{4a\sqrt{\cos(c+dx)}\,(-a^2b+7b^3+a(a^2+5b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{\frac{6(a^3-3ab^2)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8b(a^2+2b^2)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{2(a^2+5b^2)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((-4*a*Sqrt[Cos[c + d*x]]*(-(a^2*b) + 7*b^3 + a*(a^2 + 5*b^2)*Cos[c + d*x])
*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((6*(a^3 - 3*a*b^2)
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(a^2 + 2*b^2)*(2
*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a + b)))/a + (2*(a^2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*
b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b
*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(317) = 634$.
time = 0.62, size = 1760, normalized size = 6.96

method	result	size
default	Expression too large to display	1760

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(1/b*a^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^
2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*b/a*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a
```

```
-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^
2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-
a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3), x)
```

3.835 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=255

$$\frac{3a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^2\left(a^2-b^2\right)^2d}+\frac{(a^2-7b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b\left(a^2-b^2\right)^2d}+\frac{3(a^4-2a^2b^2+5b^4)\,\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)}{4(a-b)^2b^2(a+b)^3d}-\frac{}{2b\,(a}$$

```
[Out] 3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+1/4*(a^2-7*b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b
/(a^2-b^2)^2/d+3/4*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)^2/b^2/
(a+b)^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)
)^2-3/4*a^2*(a^2-3*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d
*x+c)^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4349, 3930, 4183, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{(a^2-7b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{3a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}-\frac{3a^2(a^2-3b^2)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}+\frac{3(a^4-2a^2b^2+5b^4)\,\Pi\left(\frac{2a}{a+b};\,\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^2d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] (3*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((a
^2 - 7*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (3*(a^4 - 2*
a^2*b^2 + 5*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^
2*(a + b)^3*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(
a + b*Sec[c + d*x])^2) - (3*a^2*(a^2 - 3*b^2)*Sin[c + d*x])/(4*b^2*(a^2 - b
^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
```

```
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{3a^2}{4b^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{3a^2}{4b^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{3a^2}{4b^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{3(a^4-2a^2b^2+5b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{4(a-b)^2b^2(a+b)^3d} - \frac{a^2\sin(}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{3a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{(a^2-7b^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{3(a^4}{}$$

Mathematica [A]

time = 11.87, size = 297, normalized size = 1.16

$$\frac{-\frac{4a^2\sqrt{\cos(c+dx)}\,(5a^2b-11b^3+3a(a^2-3b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{\frac{2(9a^4-19a^2b^2+16b^4)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16b(a^2-4b^2)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{6(a^2-3b^2)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((-4*a^2*Sqrt[Cos[c + d*x]]*(5*a^2*b - 11*b^3 + 3*a*(a^2 - 3*b^2)*Cos[c + d
*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(9*a^4 - 19
*a^2*b^2 + 16*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16
*b*(a^2 - 4*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2]))/(a + b) + (6*(a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])
*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(319) = 638$.
time = 0.38, size = 1203, normalized size = 4.72

method	result	size
default	Expression too large to display	1203

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/b*a^2/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2*a-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b^2)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*a/(a^2-b^2)^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
```

```
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/4/(a-b)/(a+b)/(a^
2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)
*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))-15/4/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3), x)
```

3.836 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=328

$$-\frac{(15a^4-29a^2b^2+8b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d}-\frac{a(5a^2-11b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d}-\frac{a(15a^4-38a^2b^2+35b^4)\,\Pi\left(\frac{2a}{a+b};\ \right.}{4(a-b)^2b^3(a+b)^3d}$$

```
[Out] -1/4*(15*a^4-29*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d-1/4*a*(5*a^2-11
*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d-1/4*a*(15*a^4-38*a^2*b^2+35*b^4)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/
(a+b),2^(1/2))/(a-b)^2/b^3/(a+b)^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d
*x+c)^(5/2)/(a+b*sec(d*x+c))^2-1/4*a^2*(5*a^2-11*b^2)*sin(d*x+c)/b^2/(a^2-b
^2)^2/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))+1/4*(15*a^4-29*a^2*b^2+8*b^4)*sin
(d*x+c)/b^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.71, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {4349, 3930, 4183, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{a(5a^2-11b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{a^2(5a^2-11b^2)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}-\frac{(15a^4-29a^2b^2+8b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{a(15a^4-38a^2b^2+35b^4)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a-b)^2(a+b)^3}+\frac{(15a^4-29a^2b^2+8b^4)\sin(c+dx)}{4b^3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -1/4*((15*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, 2])/(b^3*(a^2 -
b^2)^2*d) - (a*(5*a^2 - 11*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^
2)^2*d) - (a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*
Sin[c + d*x])/(4*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) - (a^2*Sin[c + d*x
])/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2) - (a^2*(5*
a^2 - 11*b^2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a +
b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
```

```
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx\right.}}{}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{a^2(5}{4b^2\left(a^2-b^2\right)^2}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a}$$

$$= -\frac{a\left(15a^4-38a^2b^2+35b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4(a-b)^2b^3(a+b)^3d} + \frac{\left(15a^4-29a^2b^2\right.}{4b^3\left(a^2-b^2\right)}$$

$$= -\frac{\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d} - \frac{a\left(5a^2-11b^2\right)F\left(\frac{1}{2}(c\right.}{4b^2\left(a^2-b^2\right)^2}$$

Mathematica [**A**]
time = 12.28, size = 334, normalized size = 1.02

$$\frac{-\frac{\frac{2\left(45a^5-95a^3b^2+56ab^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8b\left(5a^4-10a^2b^2+2b^4\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{2\left(15a^4-29a^2b^2+8b^4\right)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}+4\sqrt{\cos(c+dx)}\left(\frac{a^3\left(9a^2b-15b^3+a\left(7a^2-13b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))^2}+8\tan(c+dx)\right)}{16b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] (-(((2*(45*a^5 - 95*a^3*b^2 + 56*a*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(a + b) + (8*b*(5*a^4 - 10*a^2*b^2 + 2*b^4)*(2*EllipticF[(c + d*x)/
2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(1
5*a^4 - 29*a^2*b^2 + 8*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -
1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*
EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt
[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + 4*Sqrt[Cos[c + d*x]]*((a^3*(9*a
```

```
^2*b - 15*b^3 + a*(7*a^2 - 13*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)
^2*(b + a*Cos[c + d*x])^2) + 8*Tan[c + d*x]))/(16*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1986 vs. $2(388) = 776$.

time = 0.76, size = 1987, normalized size = 6.06

method	result	size
default	Expression too large to display	1987

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a/b^2*(1/b*a
^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*
b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2/b^3/(a^2-a*b)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-
2*a/b*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2
)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2
)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
```

```
F(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(
a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/
(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2)))+2/b^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c
)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{9/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3), x)
```

3.837 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=244

$$-\frac{4b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2-2b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+}}{15a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] -4/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*cos(d*x+c)^(3/2)*sin(d*x
+c)*(a+b*sec(d*x+c))^(1/2)/d+2/15*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*
x+c))^(1/2)/a/d+2/15*(9*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1
/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3942, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{4b(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5d}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (
2*(9*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (
2*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2
*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(2*d*n), Int[(d*Csc[e + f*x])^(n + 1)*(Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*Cs
c[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{1}{5}\Big(\sqrt{\cos(c+dx}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx}{}$$

$$= -\frac{4b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(9a^2-2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.71, size = 340, normalized size = 1.39

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\left(a(b+3a\cos(c+dx))\sin(c+dx)-\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i\left(9a^3+9a^2b-2ab^2-2b^3\right)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia\left(9a^2+7ab-2b^2\right)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-\left(9a^2-2b^2\right)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\right)}{15a^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(a*(b + 3*a*Cos[c + d*x])*Si
n[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(9*a^3 + 9*a^2*
b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b
)] + I*a*(9*a^2 + 7*a*b - 2*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2)/(a + b)] - (9*a^2 - 2*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^
(3/2)*Tan[(c + d*x)/2]))/((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2))))/(15*a^
2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. 2(274) = 548.
time = 1.30, size = 1726, normalized size = 7.07

method	result	size
default	Expression too large to display	1726

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(4*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*a*b^2+9*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2
)*a^3+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+2*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^3-9*
cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*sin(d*x+c)*a^3-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+7*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin
```

```
(d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b^2*sin(d*x+c)+2*((a-b)/(a+b))^(1/2)*b^3+5*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a^2*b+2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-9*((a-b)/(a+b))^(1/
2)*a^2*b-((a-b)/(a+b))^(1/2)*a*b^2-9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-2*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)+2*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-9*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*s
in(d*x+c)-9*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-2*cos(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2+7*cos
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*a^2*b+2*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2)*((b+a*cos(d*x+c))/c
os(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))
^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.08, size = 453, normalized size = 1.86

$$\frac{6\left(3a^{3}\cos\left(dx+c\right)+a^{2}b\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}\left(-3i\,a^{2}b-4i\,b^{3}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(3i\,a^{2}b+4i\,b^{3}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-9i\,a^{3}+2i\,ab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(9i\,a^{3}-2i\,ab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{45\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(6*(3*a^3*cos(d*x + c) + a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sqrt(a)*
weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
```

```
/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2*b +
4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sq
rt(2)*(-9*I*a^3 + 2*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a
^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
 2*b)/a)) - 3*sqrt(2)*(9*I*a^3 - 2*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a
*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2), x)
```

3.838 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d
/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)*(a
+b*sec(d*x+c))^(1/2)/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a
+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4349, 3942, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d
*x]])/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3942

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(2*d*n), Int[(d*Csc[e + f*x])^(n + 1)*(Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*Cs
c[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\Big(\sqrt{\cos(c+}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{\Big(b\sqrt{\cos(c+}}{}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} - \frac{\Big((-a^2+b^2)}{3a}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} - \frac{\Bigg((-a^2+b^2)}{}$$

$$= \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.36, size = 273, normalized size = 1.42

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(ib(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}-ia(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}+a^2\sin(c+dx)+ab\tan\left(\frac{1}{2}(c+dx)\right)+b^2\sec(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)+ab\tan(c+dx)\right)}{3ad(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(I*b*(a + b)*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(a +
 b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c
```

```
 + d*x]] + a^2*Sin[c + d*x] + a*b*Tan[(c + d*x)/2] + b^2*Sec[c + d*x]*Tan[(
c + d*x)/2] + a*b*Tan[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(228) = 456$.
time = 0.24, size = 1011, normalized size = 5.27

method	result	size
default	Expression too large to display	1011

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+sin(d*x+c)*cos(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2-sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(
d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a*b*sin(d*x+c)+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+2*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a*b+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/2)*a*
b-((a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(
1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/a/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 415, normalized size = 2.16

$$\frac{6\,a^2\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3i\,\sqrt{2}\,a^{\frac{3}{2}}b\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\,\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\,\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\,\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)-3i\,\sqrt{2}\,a^{\frac{3}{2}}b\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\,\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\,\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\,\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)+\sqrt{2}\,(-3i\,a^2+2i\,b^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\,\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\,\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+\sqrt{2}\,(3i\,a^2-2i\,b^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\,\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\,\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)}{9\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)
/a)) - 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
) + sqrt(2)*(-3*I*a^2 + 2*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 - 2*I*b^2)*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3433 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2), x)
```

3.839 $\int \sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+
a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3941, 2734, 2732}

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx \\
&= \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\right)\int \sqrt{b+a\cos(c+dx)}\,dx}{\sqrt{b+a\cos(c+dx)}} \\
&= \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\right)\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\
&= \frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.28, size = 198, normalized size = 2.96

$$
\frac{\sqrt{\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(iE\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)-iF\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+\sqrt{\frac{1}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\sin(c+dx)\right)}{d\sqrt{\frac{1}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(I*Elliptic
E[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - I*EllipticF[I*ArcSinh[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] + Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sin[c + d*x]))/(d*Sqrt[(1 +
 Cos[c + d*x])^(-1)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(90) = 180$.
time = 0.21, size = 923, normalized size = 13.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*a-sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b-sin(d*x+c)*cos(d*x+c)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
a+sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*b+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+
c)-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a+((a-b)/(a+b))^(1/2)*cos(d*x+c)*a-((a-
b)/(a+b))^(1/2)*cos(d*x+c)*b+((a-b)/(a+b))^(1/2)*b)*cos(d*x+c)^(1/2)*((b+a*
cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 355, normalized size = 5.30

$$\frac{-i\sqrt{2}\sqrt{a}\,b\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+i\sqrt{2}\sqrt{a}\,b\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)+3i\sqrt{2}a^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3i\sqrt{2}a^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\operatorname{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)}{3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/
```

```
a) + I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a
) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3
*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
- 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec(c + dx)}\sqrt{\cos(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c + d\,x)}\,\sqrt{a + \frac{b}{\cos(c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2), x)
```

3.840 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^
(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4349, 3939, 3943, 2742, 2740, 3944, 2886, 2884}

$$\frac{2a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3939

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + \left(b\sqrt{\cos(c+d\right.$$

$$= \frac{\left(a\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{b+a\cos(c+dx)}\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{\frac{b+a\cos(c}{a+b}}\right.}{\sqrt{\cos}}$$

$$= \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c-\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 26.99, size = 14885, normalized size = 107.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 257, normalized size = 1.86

method	result
default	$-\frac{2\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)a-\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)b+2\,\mathrm{EllipticPi}\left(\frac{(-1+c}{}\right.\right.}{d(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
```

```
)/(a-b))^(1/2))*b+2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))
/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)/(b+a*cos(d
*x+c))/(1/(1+cos(d*x+c)))^(1/2)/((a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec{\left(c + dx \right)}}}{\sqrt{\cos{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{\sqrt{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

3.841 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{d\sqrt{\frac{b-}{}}}$$

```
[Out] b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1
/2)/(a+b*sec(d*x+c))^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1
/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {4349, 3940, 4194, 3944, 2886, 2884, 3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (a*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*
Csc[e + f*x])^(n - 1)/(f*(2*n - 1))), x] + Dist[d^2/(2*n - 1), Int[(d*Csc[e
 + f*x])^(n - 2)*(Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e + f
*x]^2, x]/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4194

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\mathrm{se}}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\mathrm{s}}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a}}{\sqrt{}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+}}}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 28.89, size = 23549, normalized size = 99.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 781, normalized size = 3.30

method	result

default	$-\frac{\left(2\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)a-s\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-sin(d*x+c)*cos(d*x+c)^2*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+s
in(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*b+2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-sin(d*x+c)
*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*a+sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-((
a-b)/(a+b))^(1/2)*cos(d*x+c)*a+((a-b)/(a+b))^(1/2)*cos(d*x+c)*b-((a-b)/(a+b
))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)
^(1/2)/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

3.842 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=303

$$\frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{4b(41a^2-3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{}{}\right.}{105a^2d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2/105*(25*a^4-31*a^2*b^2+6*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+16/35*b*cos(d*
x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/7*a*cos(d*x+c)^(5/2)*sin(d
*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/105*(25*a^2+3*b^2)*sin(d*x+c)*cos(d*x+c)^(
1/2)*(a+b*sec(d*x+c))^(1/2)/a/d+4/105*b*(41*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^
(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b
))^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {4349, 3949, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(25a^2+3b^2)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{105ad}+\frac{4b(41a^2-3b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{7d}+\frac{16b\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]) + (2*(25*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(105*a*d) + (16*b*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(35*d) + (2*a*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(7*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3949

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)/S
qrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e
 + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
```

```
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d} - \frac{1}{7}\left(\sqrt{\cos(c+\right.$$

$$= \frac{16b\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d} + \frac{2a\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{2(25a^2+3b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{105ad} +$$

$$= \frac{2(25a^2+3b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{105ad} +$$

$$= \frac{2(25a^2+3b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{105ad} +$$

$$= \frac{2(25a^2+3b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{105ad} +$$

$$= \frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.17, size = 383, normalized size = 1.26

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(a(b+a\cos(c+dx))\left(65a^2+6b^2+48ab\cos(c+dx)+15a^2\cos(2(c+dx))\right)\sin(c+dx)-\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(2ib\left(-41a^3-41a^2b+3ab^2+3b^3\right)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia\left(25a^3+82a^2b+51ab^2-6b^3\right)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+2b\left(-41a^2+3b^2\right)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sec^{\frac{3}{2}}(c+dx)}\right)}{105a^2d(b+a\cos(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(a*(b + a*Cos[c + d*x])*(65*
a^2 + 6*b^2 + 48*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x] -
 (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-41*a^3 - 41*a^2*b +
3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
```

```
ec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] +
 I*a*(25*a^3 + 82*a^2*b + 51*a*b^2 - 6*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + 2*b*(-41*a^2 + 3*b^2)*(b + a*Cos[c + d*x])*(Sec
[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(105*a^2*d*(
b + a*Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2039 vs. $2(327) = 654$.
time = 0.22, size = 2040, normalized size = 6.73

method	result	size
default	Expression too large to display	2040

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/d*(-82*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+55*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a^2*b^2+6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+6*sin(d*x+c)*cos(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^4-82*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^3*b*sin(d*x+c)+51*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)+6*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+82*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3*b*sin(d*x+c)-82*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+25*sin(d*x+c)*cos(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^4+39*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+27*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a^2*b^2+68*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-3*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a*b^3-25*((a-b)/(a+b))^(1/2)*a^3*b-82*((a-b)/(a+b))
^(1/2)*a^2*b^2-3*((a-b)/(a+b))^(1/2)*a*b^3+6*((a-b)/(a+b))^(1/2)*b^4+25*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4
*sin(d*x+c)+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
```

```
(a-b))^(1/2))*b^4*sin(d*x+c)+15*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*cos
(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-25*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+6*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+82*sin(d*x+
c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3*b-82*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-6*sin(d*x+c)*cos(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^3-82*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+51*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2)*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x
+c)/a^2/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.79, size = 491, normalized size = 1.62

$$\frac{6\left(15a^4\cos(dx+c)^2+24a^3b\cos(dx+c)+25a^4+3a^2b^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}\left(-75ia^4+11ia^2b^2-12ib^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(75ia^4-11ia^2b^2+12ib^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)-6\sqrt{2}\left(-41ia^3b+3iab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-6\sqrt{2}\left(41ia^3b-3iab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)}{315a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(6*(15*a^4*cos(d*x + c)^2 + 24*a^3*b*cos(d*x + c) + 25*a^4 + 3*a^2*b^
2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
+ sqrt(2)*(-75*I*a^4 + 11*I*a^2*b^2 - 12*I*b^4)*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*a^4 - 11*I*a^2*b^2 + 12*I*
b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 6*sqrt(2
)*(-41*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
```

```
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) - 6*sqrt(2)*(41*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I
*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2), x)
```

3.843 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=240

$$\frac{2b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{5ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2/5*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c
)*(a+b*sec(d*x+c))^(1/2)/d+4/5*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c
))^(1/2)/d+2/5*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b
*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3949, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5d}+\frac{4b\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(5*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (4*b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d
*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3949

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)/S
qrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e
 + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} - \frac{1}{5}\Big(\sqrt{\cos(c+$$

$$= \frac{4b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+$$

$$= \frac{4b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+$$

$$= \frac{4b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+$$

$$= \frac{4b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+$$

$$= \frac{2b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^2+b^2$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.36, size = 344, normalized size = 1.43

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(2(b+a\cos(c+dx))(2b+a\cos(c+dx))\sin(c+dx)-\frac{2(\cos^2(\frac{1}{2}(c+dx))\sec(c+dx))^{3/2}\left(-i(3a^3+3a^2b+ab^2+b^3)E\left(i\sinh^{-1}(\tan(\frac{1}{2}(c+dx)))|\frac{-a+b}{a+b}\right)\sec^2(\frac{1}{2}(c+dx))\sqrt{\frac{(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))}{a+b}}+ia(3a^2+4ab+b^2)F\left(i\sinh^{-1}(\tan(\frac{1}{2}(c+dx)))|\frac{-a+b}{a+b}\right)\sec^2(\frac{1}{2}(c+dx))\sqrt{\frac{(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))}{a+b}}-(3a^2+b^2)(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))^{3/2}\tan(\frac{1}{2}(c+dx))\right)}{a\sec^{\frac{3}{2}}(c+dx)}\right)}{5d(b+a\cos(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(2*(b + a*Cos[c + d*x])*(2*b
 + a*Cos[c + d*x])*Sin[c + d*x] - (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2
)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b + b^2)*EllipticF[I*ArcSinh[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2 + b^2)*(b + a*Cos[c + d*x])*(Sec[(
c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(3/2))))/(5*d*(b +
a*Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. $2(270) = 540$.
time = 0.18, size = 1697, normalized size = 7.07

method	result	size
default	Expression too large to display	1697

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(3*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3-3*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+cos(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*sin(d*x+c)*a*b^2-cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^3-3*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3+4*
cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*sin(d*x+c)*a^2*b-cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
```

```
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2+cos(d*x+c)^4*((a-b)
/(a+b))^(1/2)*a^3+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*
sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^3*sin(d*x+c)+4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+3*cos(d*x+c
)^3*((a-b)/(a+b))^(1/2)*a^2*b+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+3*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-3*((a-
b)/(a+b))^(1/2)*a^2*b-2*((a-b)/(a+b))^(1/2)*a*b^2-((a-b)/(a+b))^(1/2)*b^3)*
((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d
*x+c)/a/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.95, size = 455, normalized size = 1.90

$$\frac{6\left(a^{3}\cos\left(dx+c\right)+2a^{2}b\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2\sqrt{2}\left(3i\,a^{2}b-i\,b^{3}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)-2\sqrt{2}\left(-3i\,a^{2}b+i\,b^{3}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-3i\,a^{3}-i\,ab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(3i\,a^{3}+i\,ab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{15\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(6*(a^3*cos(d*x + c) + 2*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*sqrt(2)*(3*I*a^2*b - I*b^3)*sqrt(a)*w
eierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/
3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - 2*sqrt(2)*(-3*I*a^2*b
```

```
+ I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sq
rt(2)*(-3*I*a^3 - I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2
*b)/a)) - 3*sqrt(2)*(3*I*a^3 + I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin
(d*x + c) + 2*b)/a)))/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2), x)
```

3.844 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=187

$$\frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/c
os(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)*(a
+b*sec(d*x+c))^(1/2)/d+8/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a
+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4349, 3949, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3d}+\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3949

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)/S
qrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e
 + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} - \frac{1}{3}\Big(\sqrt{\cos(c\cdots}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\Big(4b\sqrt{\cos(\cdots}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} - \frac{\Big((-a^2+b^2\cdots}{\cdots}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} - \frac{\Big((-a^2+b^2\cdots}{\cdots}$$

$$= \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{8b\sqrt{\cos(c\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.05, size = 284, normalized size = 1.52

$$\frac{2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}\left(\frac{1}{2}a(b+a\cos(c+dx))\sin(2(c+dx))+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(4ib(a+b)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-i(a^2+4ab+3b^2)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+4b(b+a\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sec^{\frac{3}{2}}(c+dx)}\right)}{3d(b+a\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*((a*(b + a*Cos[c + d*x])*S
in[2*(c + d*x)])/2 + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*b*(a + b
)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)*EllipticF
[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c + d*x])*
```

```
Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/
2]^2]*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(3*d*(b + a*Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1208 vs. $2(223) = 446$.
time = 0.24, size = 1209, normalized size = 6.47

method	result	size
default	Expression too large to display	1209

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+sin(d*x+c)*
cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2-4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*sin(d*x+c)*cos(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+4*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b*sin(d*x+c)-4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*b^2*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-4*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+3*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x
+c)+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*a*b-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*
b+4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/2)*a*b-4*((a-b)/(a+
b))^(1/2)*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*co
s(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.24, size = 415, normalized size = 2.22

$$\frac{6\,a^2\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+12i\,\sqrt{2}\,a^{\frac{3}{2}}b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)-12i\,\sqrt{2}\,a^{\frac{3}{2}}b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)+\sqrt{2}\,(-3i\,a^2-i\,b^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+\sqrt{2}\,(3i\,a^2+i\,b^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + sqrt(2)*(-3*I*a^2 - I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a*d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2), x)
```

3.845 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=209

$$\frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}$$

```
[Out] 2*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c
)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*co
s(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {4349, 3953, 3941, 2734, 2732, 3939, 3943, 2742, 2740, 3944, 2886, 2884}

$$\frac{2b^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3939

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3953

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Dist[a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e +
 f*x]], x], x] + Dist[b/d, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx +$$

$$= \left(ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx +$$

$$= \frac{\left(ab\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(b^2\sqrt{b-\right.}{}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \left(ab\right.$$

$$= \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c-}{a+b}}}{d\sqrt{\cos(c+dx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 27.90, size = 25369, normalized size = 121.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 1361, normalized size = 6.51

method	result	size
default	Expression too large to display	1361

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+sin(d*x+c)*cos(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^2-sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*sin(d*x+c)*
cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*b^2+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)
-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^2*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-cos(d*x+c)^2*
((a-b)/(a+b))^(1/2)*a^2+((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a*b+((a-b)/(a+b))^(1/2)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(
1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3433 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2), x)
```

3.846 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=249

$$\frac{(2a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{b\sqrt{\cos(c}{}$$

```
[Out] (2*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos
(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((
b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+b*si
n(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-b*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1
/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/
2)
```

Rubi [A]

time = 0.53, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {4349, 3951, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(2a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{3ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (3*a*b*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (b*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
```

```
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3951

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Dist[d/(m + n - 1), Int
[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (
b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{ab}{2}+ \cdots}{\sqrt{s\cdots}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cdots}{\sqrt{\sec\cdots}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{c\cdots}}{\cdots}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left((2a^2+b^2)\sqrt{b+a\cos(c+dx)}\right)\int \frac{\cdots}{\sqrt{b+\cdots}}}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c\cdots}}$$

$$= \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+\cdots}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{(2a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\cdots}{d\sqrt{\cos(c+dx)}\sqrt{a+\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 29.52, size = 24604, normalized size = 98.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 1205, normalized size = 4.84

method	result	size

default	Expression too large to display	1205

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(2*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)/((a-b)/(a+b))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec (c + dx))^{\frac{3}{2}}}{\sqrt{\cos (c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sqrt(cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos (c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

3.847 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=299

$$\frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{5a\sqrt{\cos}}{}$$

```
[Out] 7/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x
+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(
1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1
/2)+1/2*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+5/4*a*sin(d*
x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-5/4*a*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1
/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/
2)
```

Rubi [A]
time = 0.70, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4349, 3951, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(3a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{5a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\cos(c+dx)}}+\frac{7ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{5a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (7*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (5*a*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(2*d*Cos[c + d*x]^(3/2)) + (5*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
4*d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3951

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Dist[d/(m + n - 1), Int
[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (
b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\mathrm{se}}}{}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{}\right.}{}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{}\right.}{}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{1}{8}\left(5\right.$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(7a\right.}{}$$

$$= \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{b\sqrt{a+b\sec(c+dx)}}{2d\cos^{\frac{3}{2}}(c+d}$$

$$= \frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.94, size = 36737, normalized size = 122.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 1742, normalized size = 5.83

method	result	size
default	Expression too large to display	1742

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(2*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*sin(d
*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b^2-5*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+5*sin(d*x+c)*cos(d*x+c
)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b+6*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*sin(d*x+c)*cos(d*x+c)
^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*b^2+2*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*sin(d*x+c)*cos(d*x+c)^
2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b-4*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-5*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
```

```
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+5*sin(d*x
+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b+6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*sin(d*x+
c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+5*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+2*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2
+5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
b^2-7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*((a-b)/(a+b))^(1/2)*b^2)*((b+a*c
os(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)/(
(a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)/cos(c + d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

3.848 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=363

$$\frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4+279a^2b^2-10b^4)\sqrt{\cos(c+dx)}\,E}{315a^2d\sqrt{\frac{b+a\,\mathrm{co}}{a}}}$$

```
[Out] 4/315*b*(57*a^4-62*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/315*(49*a^
2+75*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+38/63*a*b*co
s(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/9*a^2*cos(d*x+c)^(7/2)
*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/315*b*(163*a^2+5*b^2)*sin(d*x+c)*cos
(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d+2/315*(147*a^4+279*a^2*b^2-10*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/(
(b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.92, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3926, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(49a^2+75b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{315d}+\frac{2b(163a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{315ad}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{9d}+\frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4+279a^2b^2-10b^4)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{38ab\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (4*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]) + (2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(163*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(49*a^2 + 75*b^2)*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (38*a*b*
Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(63*d) + (2*a^2*C
os[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d} + \frac{1}{9}\Big(2\sqrt{\cos(c$$

$$= \frac{38ab\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d} + \frac{2a^2\cos^{\frac{7}{2}}(c$$

$$= \frac{2(49a^2+75b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315d} +$$

$$= \frac{2b(163a^2+5b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315ad}$$

$$= \frac{2b(163a^2+5b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315ad}$$

$$= \frac{2b(163a^2+5b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315ad}$$

$$= \frac{2b(163a^2+5b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315ad}$$

$$= \frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.02, size = 477, normalized size = 1.31

$$\frac{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{b(747a^2+20b^2)\sin(c+dx)}{630a}+\frac{1}{630}(133a^2+150b^2)\sin(2(c+dx))+\frac{19}{126}ab\sin(3(c+dx))+\frac{1}{36}a^2\sin(4(c+dx))\right)}{d(b+a\cos(c+dx))^2} - \frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a+b\sec(c+dx))^{5/2}\left(-i(147a^5+147a^4b+279a^3b^2+279a^2b^3-10ab^4-10b^5)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia(147a^4+261a^3b+279a^2b^2+155ab^3-10b^4)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-(147a^4+279a^2b^2-10b^4)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{315a^2d(b+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*((b*(747*a^2 + 20*b^2)*Sin[c
 + d*x])/(630*a) + ((133*a^2 + 150*b^2)*Sin[2*(c + d*x)])/630 + (19*a*b*Sin
```

```
[3*(c + d*x)])/126 + (a^2*Sin[4*(c + d*x)])/36))/(d*(b + a*Cos[c + d*x])^2)
 - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec
[c + d*x])^(5/2)*((-I)*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a^2*b^3 - 1
0*a*b^4 - 10*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
+ I*a*(147*a^4 + 261*a^3*b + 279*a^2*b^2 + 155*a*b^3 - 10*b^4)*EllipticF[I*
ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a^4 + 279*a^2*b^2 - 10*
b^4)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(31
5*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2777 vs. $2(381) = 762$.
time = 0.28, size = 2778, normalized size = 7.65

method	result	size
default	Expression too large to display	2778

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*(130*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b+170*cos(d*x+c)^4*((a-b
)/(a+b))^(1/2)*a^3*b^2+82*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b+80*cos(d*x
+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3+272*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*
b^2-5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4+147*sin(d*x+c)*cos(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-
65*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b-279*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
a^3*b^2+199*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+10*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a*b^4-147*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5-10*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*b^5+10*((a-b)/(a+b))^(1/2)*b^5-147*((a-b)/(a+b))^(1/2)*a^4*b-
163*((a-b)/(a+b))^(1/2)*a^3*b^2-279*((a-b)/(a+b))^(1/2)*a^2*b^3-5*((a-b)/(a
+b))^(1/2)*a*b^4+35*cos(d*x+c)^6*((a-b)/(a+b))^(1/2)*a^5+14*cos(d*x+c)^4*((
a-b)/(a+b))^(1/2)*a^5+98*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5-147*sin(d*x+c
)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^4*b+279*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-279*sin(d*x+c)*cos(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b^3-10*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+261*sin(d*x+c)*cos(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
```

```
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-
279*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+155*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+10*sin(
d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^4+147*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^5*sin(d*x+c)+10*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*sin(d*x+c)-147*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*sin(d*
x+c)+10*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5-147*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-147*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*s
in(d*x+c)+279*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3*b^2*sin(d*x+c)-279*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*sin(d*x+c)-10*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*sin(d*x+c)
+261*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^4*b*sin(d*x+c)-279*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*sin(d*x+c)+155*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*sin(d*x+c)+10*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
4*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos
(d*x+c))/sin(d*x+c)/a^2/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.05, size = 531, normalized size = 1.46

$$\frac{6\left(35a^5\cos(dx+c)^3+95a^4b\cos(dx+c)^2+163a^4b+5a^2b^3+(49a^5+75a^3b^2)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}\left(-489i\,a^4b+93i\,a^2b^3-20i\,b^5\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(489i\,a^4b-93i\,a^2b^3+20i\,b^5\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-147i\,a^5-279i\,a^3b^2+10i\,ab^4\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(147i\,a^5+279i\,a^3b^2-10i\,ab^4\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{945a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/945*(6*(35*a^5*cos(d*x + c)^3 + 95*a^4*b*cos(d*x + c)^2 + 163*a^4*b + 5*a
^2*b^3 + (49*a^5 + 75*a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-489*I*a^4*b + 93*I*a^
2*b^3 - 20*I*b^5)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/
a) + sqrt(2)*(489*I*a^4*b - 93*I*a^2*b^3 + 20*I*b^5)*sqrt(a)*weierstrassPIn
verse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*
x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-147*I*a^5 - 279*I*a^3*b
^2 + 10*I*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) -
3*sqrt(2)*(147*I*a^5 + 279*I*a^3*b^2 - 10*I*a*b^4)*sqrt(a)*weierstrassZeta(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
- 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{9/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.849 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=303

$$\frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b(29a^2+3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{21ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2/21*(5*a^4-2*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(
a+b))^(1/2)/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+6/7*a*b*cos(d*x+c)^
(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+
c)*(a+b*sec(d*x+c))^(1/2)/d+2/21*(5*a^2+9*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)*
(a+b*sec(d*x+c))^(1/2)/d+2/21*b*(29*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*c
os(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {4349, 3926, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(5a^2+9b^2)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{21d}+\frac{2b(29a^2+3b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{7d}+\frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{6ab\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(21*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b*(29*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(21*a*d*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]) + (2*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(21*d) + (6*a*b*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(7*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(7*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d} + \frac{1}{7}\Big(2\sqrt{\cos(c$$

$$= \frac{6ab\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{5}{2}}(c+$$

$$= \frac{2(5a^2+9b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d} + \frac{6$$

$$= \frac{2(5a^2+9b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d} + \frac{6$$

$$= \frac{2(5a^2+9b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d} + \frac{6$$

$$= \frac{2(5a^2+9b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d} + \frac{6$$

$$= \frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{21ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.13, size = 386, normalized size = 1.27

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{1}{2}(b+a\cos(c+dx))\left(13a^2+18b^2+18ab\cos(c+dx)+3a^2\cos(2(c+dx))\right)\sin(2(c+dx))+\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(ib(29a^3+29a^2b+3ab^2+3b^3)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(5a^3+29a^2b+27ab^2+3b^3)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+b(29a^2+3b^2)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a\sec^{\frac{5}{2}}(c+dx)}\right)}{21d(b+a\cos(c+dx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(((b + a*Cos[c + d*x])*(13*a
^2 + 18*b^2 + 18*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)
])/2 + (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*b*(29*a^3 + 29*a^2*b +
 3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
```

```
Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
- I*a*(5*a^3 + 29*a^2*b + 27*a*b^2 + 3*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + b*(29*a^2 + 3*b^2)*(b + a*Cos[c + d*x])*(Sec[(c
 + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(5/2))))/(21*d*(b +
a*Cos[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2039 vs. $2(327) = 654$.
time = 0.22, size = 2040, normalized size = 6.73

method	result	size
default	Expression too large to display	2040

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(-29*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+11*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a^2*b^2-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-3*sin(d*x+c)*cos(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*b^4-29*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^3*b*sin(d*x+c)+27*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+29*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
3*b*sin(d*x+c)-29*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+5*sin(d*x+c)*cos(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^4+12*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+18*cos(d*x+c)^3*((a-b)/(
a+b))^(1/2)*a^2*b^2+22*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+12*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a*b^3-5*((a-b)/(a+b))^(1/2)*a^3*b-29*((a-b)/(a+b))^(
1/2)*a^2*b^2-9*((a-b)/(a+b))^(1/2)*a*b^3-3*((a-b)/(a+b))^(1/2)*b^4+5*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*si
n(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
```

```
b))^(1/2))*b^4*sin(d*x+c)+3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+2*cos(d*x+
c)^3*((a-b)/(a+b))^(1/2)*a^4+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-5*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a^4-3*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+29*sin(d*x+c)*cos
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^3*b-29*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+3*sin(d*x+c)*cos(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
3-29*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+27*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2)*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/a/
((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.98, size = 491, normalized size = 1.62

$$\frac{6\left(3a^4\cos(dx+c)^2+9a^3b\cos(dx+c)+5a^4+9a^2b^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}\left(-15ia^4-23ia^2b^2+6ib^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(15ia^4+23ia^2b^2-6ib^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-29ia^3b-3iab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(29ia^3b+3iab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)}{63a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/63*(6*(3*a^4*cos(d*x + c)^2 + 9*a^3*b*cos(d*x + c) + 5*a^4 + 9*a^2*b^2)*s
qrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sq
rt(2)*(-15*I*a^4 - 23*I*a^2*b^2 + 6*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3
*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*a^4 + 23*I*a^2*b^2 - 6*I*b^4)*s
qrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-29
*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
```

```
7*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a
)) - 3*sqrt(2)*(29*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin
(d*x + c) + 2*b)/a)))/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.850 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=239

$$\frac{16b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+23b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+}}{15d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 16/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*a^2*cos(d*x+c)^(3/2)*sin(d*x
+c)*(a+b*sec(d*x+c))^(1/2)/d+22/15*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec
(d*x+c))^(1/2)/d+2/15*(9*a^2+23*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^
(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3926, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{16b(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+23b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5d}+\frac{22ab\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (16*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9
*a^2 + 23*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (22*a*b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a^2*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{1}{5}\Big(2\sqrt{\cos(c$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(}{}$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(}{}$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(}{}$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(}{}$$

$$= \frac{16b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(9a^2+2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.80, size = 358, normalized size = 1.50

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(a(b+a\cos(c+dx))(11b+3a\cos(c+dx))\sin(2(c+dx))-\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(9a^3+9a^2b+23ab^2+23b^3)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+i(9a^3+17a^2b+23ab^2+15b^3)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-(9a^2+23b^2)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sec^{\frac{5}{2}}(c+dx)}\right)}{15d(b+a\cos(c+dx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(a*(b + a*Cos[c + d*x])*(11*
b + 3*a*Cos[c + d*x])*Sin[2*(c + d*x)] - (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(3/2)*((-I)*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[I*ArcSinh[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*(9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^
3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2
]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^2 + 23*b
^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[
c + d*x]^(5/2)))/(15*d*(b + a*Cos[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1920 vs. 2(269) = 538.
time = 0.20, size = 1921, normalized size = 8.04

method	result	size
default	Expression too large to display	1921

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(-14*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-34*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*a*b^2-9*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3-3*cos(d*x+c)^4*((a-b)/(a+b))^
(1/2)*a^3-6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+23*cos(d*x+c)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b
^3+9*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*sin(d*x+c)*a^3+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-23*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-17*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
```

```
2*b*sin(d*x+c)+23*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+23*((a-b)/(a+b))^(1/2)*b^3+5*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a^2*b+23*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-15*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*si
n(d*x+c)-15*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+9*((a-b)/(a+b))^(1/2)*a^2*b+11*((a-b)
/(a+b))^(1/2)*a*b^2+9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-23*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*b^3-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)+23*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+9*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)+9*c
os(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*sin(d*x+c)*a^2*b-23*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-17*cos(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin
(d*x+c)*a^2*b+23*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.20, size = 454, normalized size = 1.90

$$\frac{6\left(3a^{3}\cos\left(dx+c\right)+11a^{2}b\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}\left(-33i\,a^{2}b+i\,b^{3}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(33i\,a^{2}b-i\,b^{3}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-9i\,a^{3}-23i\,ab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(9i\,a^{3}+23i\,ab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)}{45\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/45*(6*(3*a^3*cos(d*x + c) + 11*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-33*I*a^2*b + I*b^3)*sqrt(a
)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(33*I*a^2*b
 - I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*s
qrt(2)*(-9*I*a^3 - 23*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
 + 2*b)/a)) - 3*sqrt(2)*(9*I*a^3 + 23*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*
I*a*sin(d*x + c) + 2*b)/a)))/(a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.851 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=262

$$\frac{2a(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{14ab\sqrt{}}{}$$

```
[Out] 2/3*a*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2
)+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d+14/3*a*b*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.60, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {4349, 3926, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2a(a^2+2b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b^3\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{14ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^
3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\Big(2\sqrt{\cos(c$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\Big(2\sqrt{\cos(c$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\Big(7ab\sqrt{\cos$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{\Big(a(a^2+2b^2}{}$$

$$= \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}}{}$$

$$= \frac{2a(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2b^3\sqrt{\frac{b+a}{}}}{d\sqrt{\cos}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.93, size = 36372, normalized size = 138.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 1651, normalized size = 6.30

method	result	size

default	Expression too large to display	1651

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3-7*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+9*cos(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*sin(d*x+c)*a*b^2-3*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+6*sin(d*x+c)*cos(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b
))^(1/2))*b^3+7*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-7*cos(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2+(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3*sin(d*x+c)-7*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+6*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b
))^(1/2))*b^3*sin(d*x+c)+7*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-7*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+cos(d*x+c)^3
*((a-b)/(a+b))^(1/2)*a^3+8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^3-7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+7*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*a*b^2-((a-b)/(a+b))^(1/2)*a^2*b-7*((a-b)/(a+b))^(1/2
)*a*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+
c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c)
 + a^2*cos(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.852 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=263

$$\frac{b(4a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(2a^2-$$

```
[Out] b*(4*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/c
os(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+5*a*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+
b^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+(2*a^2-b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.58, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {4349, 3927, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{b(4a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{5ab^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (5*a*b^2*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*a^2 - b^2)*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\left(2a^2-b^2\right)\sqrt{\cos(c+d\right.$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b(4a^2+b^2)\sqrt{b+a\cos(c\right.}{2\sqrt{\cos(c+dx}}$$

$$= \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{a+b\sec(c}{d\sqrt{\mathrm{co}}}$$

$$= \frac{b(4a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b+}{}}}{d\sqrt{\mathrm{co}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.61, size = 20828, normalized size = 79.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 1947, normalized size = 7.40

method	result	size

default	Expression too large to display	1947

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(2*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3-2*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3+10*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2-2*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3+6*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-4*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+2*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3-2*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2+cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^3+10*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2-2*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3+6*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-4*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(

```
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2+2*cos(d*x+c)^
3*((a-b)/(a+b))^(1/2)*a^3-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+2*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a^2*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-2*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b^3-((a-b)/(a+b))^(1/2)*b^3)*((b+a*cos(d*x+c))/c
os(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^(1/2)/((a-b)/(a+b))
^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.853 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=314

$$\frac{a(8a^2+11b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{b(15a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 1/4*a*(8*a^2+11*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1
/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*b*(15*a^2+4*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(
1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+
b*sec(d*x+c))^(1/2)+1/2*b^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^
(3/2)+9/4*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-9/4*a*b*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*
cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.79, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4349, 3927, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{a(8a^2+11b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{b(15a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\cos(c+dx)}}-\frac{9ab\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (b*(
15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (9*a
*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)) + (9*a*b*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

```
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{}}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + $$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + $$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - $$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + $$

$$= \frac{b(15a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{a+b\sec(c+}}{2d\cos^{\frac{3}{2}}(c}$$

$$= \frac{a(8a^2+11b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{b(15a^2+4b^2)\sqrt{\frac{b+}{}}}{4d\sqrt{\cos(c+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.36, size = 52888, normalized size = 168.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 1972, normalized size = 6.28

method	result	size
default	Expression too large to display	1972

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(30*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b+8*sin(d*x+c)*cos(d*x
+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/(
(a-b)/(a+b))^(1/2))*b^3+8*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-6*sin(d*x+c)*cos(d*x+
c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*b+2*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-4*sin(d*x+c)*cos(d*x+c)^3*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-9*
sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^2*b+9*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+30*sin(d*x+c)
*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*a^2*b+8*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b
^3+8*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
```

```
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3-6*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b+2*sin(d*x+
c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*a*b^2-4*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-9*sin(d*x+c)*cos(d*x+c)
^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*a^2*b+9*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+9*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a^2*b+2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2-9*cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*a^2*b+9*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+2*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*b^3-11*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-2*((a-b)/(a+b))^
(1/2)*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^
(3/2)/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

3.854 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=369

$$\frac{b(59a^2+16b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{5a(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 1/24*b*(59*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+5/8*a*(a^2+4*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1
/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b
*sec(d*x+c))^(1/2)+1/3*b^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(
5/2)+13/12*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/24*(3
3*a^2+16*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/24*(33
*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(
1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.97, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4349, 3927, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{b(59a^2+16b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{(33a^2+16b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{5a(a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\cos^{\frac{5}{2}}(c+dx)}+\frac{13ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{12d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (5
*a*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)])/(8*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((3
3*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)) + (13*a*b*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d*Cos[c + d*x]^(3/2)) + ((33*a^2 +
16*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
```

```
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \text{se}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{5a(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{a+b\sec(c+d}}{3d\cos^{\frac{5}{2}}(c}$$

$$= \frac{b(59a^2+16b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{5a(a^2+4b^2)\sqrt{\frac{b+}{}}}{8d\sqrt{\cos(c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.61, size = 61979, normalized size = 167.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

[Out] Result too large to show

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 2285, normalized size = 6.19

method	result	size
default	Expression too large to display	2285

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/24/d*(33*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+26*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b+16*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2+18*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2+26*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-44*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+33*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-16*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+33*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-8*((a-b)/(a+b))^(1/2)*b^3-8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3-59*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-34*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+16*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3-33*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+18*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+30*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3+18*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-33*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+16*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+30*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3-33*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+16*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+

```
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*b^3+120*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+
26*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-44*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+33*sin(d*
x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b-16*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+120*sin(d*x+c)*cos
(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*a*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos
(d*x+c))/sin(d*x+c)/cos(d*x+c)^(5/2)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

3.855 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=249

$$-\frac{2b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+8b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a}}{15a^3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] -2/15*b*(7*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(
1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*cos(d*x+c)^(3/2)*sin
(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d-8/15*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*
sec(d*x+c))^(1/2)/a^2/d+2/15*(9*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3948, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{8b\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{15a^2d}-\frac{2b(7a^2+8b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+8b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)])/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
 + (2*(9*a^2 + 8*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
 - (8*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d
) + (2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3948

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a + b*Csc[e + f*x]]/(a*d*f*n)), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+a\right.}}{}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{s}}}{5a}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{s}}}{5a}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{s}}}{5a}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{s}}}{5a}$$

$$= -\frac{2b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(9a^2+8b^2)\sqrt{\cos(}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.76, size = 340, normalized size = 1.37

$$\frac{2a(b+a\cos(c+dx))(-4b+3a\cos(c+dx))\sin(c+dx)+\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(9a^3+9a^2b+8ab^2+8b^3)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(9a^2+2ab+8b^2)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+(9a^2+8b^2)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sec^{\frac{3}{2}}(c+dx)}}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*a*(b + a*Cos[c + d*x])*(-4*b + 3*a*Cos[c + d*x])*Sin[c + d*x] + (2*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)*E
llipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(9*a^2 + 2*a*
b + 8*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (9*a^
2 + 8*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]
))/Sec[c + d*x]^(3/2))/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. 2(279) = 558.
time = 0.20, size = 1725, normalized size = 6.93

method	result	size
default	Expression too large to display	1725

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-4*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a*b^2-9*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3-3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)
*a^3-6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+8*cos(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^3+9*c
os(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*sin(d*x+c)*a^3+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-8*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-2*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(
```

```
d*x+c)+8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b^2*sin(d*x+c)+8*((a-b)/(a+b))^(1/2)*b^3-10*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a^2*b+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+9*((a-b)/(a+b))^(1/
2)*a^2*b-4*((a-b)/(a+b))^(1/2)*a*b^2+9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-8
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)+8*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+9*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
*sin(d*x+c)+9*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-8*cos(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-2*c
os(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*sin(d*x+c)*a^2*b+8*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2)*((b+a*cos(d*x+c))
/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b
))^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.59, size = 456, normalized size = 1.83

$$\frac{6\,(3\,a^3\cos(dx+c)-4\,a^2b)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-4\,\sqrt{2}\,(-3i\,a^2b-4i\,b^3)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)-4\,\sqrt{2}\,(3i\,a^2b+4i\,b^3)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)-3\,\sqrt{2}\,(-9i\,a^3-8i\,ab^2)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)-3\,\sqrt{2}\,(9i\,a^3+8i\,ab^2)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)}{45\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(6*(3*a^3*cos(d*x + c) - 4*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sqrt
(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^
```

```
3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - 4*sqrt(2)*(3*I*a^
2*b + 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 3*sqrt(2)*(-9*I*a^3 - 8*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*
b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
+ c) + 2*b)/a)) - 3*sqrt(2)*(9*I*a^3 + 8*I*a*b^2)*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4
/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) -
 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^4*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.856 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=195

$$\frac{2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2/3*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a
^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*cos(d*x+c)^(1/2
)*(a+b*sec(d*x+c))^(1/2)/a/d-4/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1
/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4349, 3948, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2+2b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (4*
b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3948

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad} - \frac{\left(2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad} + \frac{\left((a^2+2b^2)\sqrt{b+a\cos(c\cdots\right.}{3a^2\sqrt{\cos(c+d\cdots}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad} + \frac{\left((a^2+2b^2)\sqrt{\frac{b+a\cos(c\cdots}{a+b\cdots}}\right.}{3a^2\sqrt{\cos(c\cdots}}$$

$$= \frac{2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{4b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c\cdots\right.}{3a^2d\sqrt{\frac{b\cdots}{\cdots}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.87, size = 265, normalized size = 1.36

$$\frac{2\sqrt{\cos(c+dx)}\left(-2ib(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}-ia(a-2b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}+a^2\sin(c+dx)-2ab\tan\left(\frac{1}{2}(c+dx)\right)-2b^2\sec(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)+ab\tan(c+dx)\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*((-2*I)*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(a - 2*b)*Sqrt[(b + a*Co
```

```
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] + a^2*Sin
[c + d*x] - 2*a*b*Tan[(c + d*x)/2] - 2*b^2*Sec[c + d*x]*Tan[(c + d*x)/2] +
a*b*Tan[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(231) = 462$.
time = 0.23, size = 1014, normalized size = 5.20

method	result	size
default	Expression too large to display	1014

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*sin(d*x+c)*
cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b+2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+2*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-
2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b*sin(d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2-cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)+2*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(
a+b))^(1/2)*a*b+2*((a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1
/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 415, normalized size = 2.13

$$\frac{6\,a^2\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right) - 6i\,\sqrt{2}\,a^{\frac{3}{2}}b\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2}, \frac{8\left(9a^2b-8b^3\right)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2}, \frac{8\left(9a^2b-8b^3\right)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right) + 6i\,\sqrt{2}\,a^{\frac{3}{2}}b\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2}, \frac{8\left(9a^2b-8b^3\right)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2}, \frac{8\left(9a^2b-8b^3\right)}{27a^3}, \frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right) + \sqrt{2}\left(-3i\,a^2-4i\,b^2\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2}, \frac{8\left(9a^2b-8b^3\right)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) + \sqrt{2}\left(3i\,a^2+4i\,b^2\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2}, \frac{8\left(9a^2b-8b^3\right)}{27a^3}, \frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)}{9\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) - 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)
/a)) + 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
) + sqrt(2)*(-3*I*a^2 - 4*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + 4*I*b^2)*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d x)^{3/2}}{\sqrt{a+\frac{b}{\cos (c+d x)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.857 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=142

$$-\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] -2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d/cos(d*x+
c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/
2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4349, 3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{\left(b\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{a+\right.}{a}$$

$$= -\frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right.}{ad\sqrt{\frac{b+a\cos(c}{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.58, size = 216, normalized size = 1.52

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\left(i(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right) - ia\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right) + \sqrt{\frac{1}{1+\cos(c+dx)}}\,(b+a\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*(I*(a + b)*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - I*a*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + Sqrt[(1 + Cos[c + d*x])^(-1)]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2
]))/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(188) = 376$.
time = 0.22, size = 732, normalized size = 5.15 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*a-sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+sin(d*x+c)*cos(d*x+c)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
b+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
+((a-b)/(a+b))^(1/2)*cos(d*x+c)*a-((a-b)/(a+b))^(1/2)*cos(d*x+c)*b+((a-b)/(
a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*co
s(d*x+c))/sin(d*x+c)/a/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.90, size = 355, normalized size = 2.50

$$\frac{2i\sqrt{2}\sqrt{a}\,b\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) - 2i\sqrt{2}\sqrt{a}\,b\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right) + 3i\sqrt{2}a^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right) - 3i\sqrt{2}a^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)
/a) - 2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b
)/a) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a))
- 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
```

```
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^2
*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.858 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1
/2)/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3943, 2742, 2740}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \frac{\sqrt{b+a\cos(c+dx)}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} \\
&= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.54, size = 102, normalized size = 1.52

$$-\frac{2i\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{\frac{1}{1+\cos(c+dx)}}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] ((-2*I)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[(
1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A]
time = 0.19, size = 145, normalized size = 2.16

method	result	size
default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\ \text{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}}{d(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{a-b}{a+b}}}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*((b
+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/
((a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.73, size = 146, normalized size = 2.18

$$\frac{-i\sqrt{2}\,\sqrt{a}\ \text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+i\sqrt{2}\,\sqrt{a}\ \text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + I
*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}\,\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.859 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)
^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4349, 3944, 2886, 2884}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\\
&= \frac{\sqrt{b+a\cos(c+dx)}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\\
&= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\\
&= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 27.09, size = 5763, normalized size = 84.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 222, normalized size = 3.26

method	result

default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\left(\operatorname{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)-2\operatorname{EllipticPi}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a}{a}}}\right.\right.}{d(b+a\cos(dx+c))\left(\frac{1}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)^{2}\sqrt{\frac{a-b}{a+b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))-2*EllipticPi((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))
)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^(1/2)/(b+a
*cos(d*x+c))/(1/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^2/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.860 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+\ldots\right.}{bd\sqrt{\frac{b+\ldots}{\ldots}}}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2
)/(a+b*sec(d*x+c))^(1/2)-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^
(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {4349, 3945, 4194, 3944, 2886, 2884, 3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (a*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4194

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}\right.}{}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}\right.$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{b+a\cos(c+dx)}\int \frac{}{\sqrt{b+}}}{2\sqrt{\cos(c+dx)}\,\sqrt{a+b}}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+d}}{bd\sqrt{\cos(c}}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{bd\sqrt{\cos(c+dx)}\,\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 28.26, size = 21698, normalized size = 88.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 985, normalized size = 4.00

method	result	size
default	Expression too large to display	985

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-2*sin(d*x+c)*cos(d*x+c)^2*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+
sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a-sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*sin(d*x+c)*cos(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)
/(a+b))^(1/2))*a-2*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+sin(d*x+c)*cos(d*x+c)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-sin(d*x+
c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*b-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a+((a-b)/(a+b))^(1/2)*cos(
d*x+c)*a-((a-b)/(a+b))^(1/2)*cos(d*x+c)*b+((a-b)/(a+b))^(1/2)*b)*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)/((a-
b)/(a+b))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.861 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=312

$$-\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{3a\sqrt{\cos(c}{}$$

```
[Out] -1/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*
x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^
(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c
))^(1/2)+1/2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d/cos(d*x+c)^(3/2)-3/4*a*s
in(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)+3/4*a*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/
(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/d/((b+a*cos(d*x+c
))/(a+b))^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4349, 3945, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(3a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{3a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\cos(c+dx)}}+\frac{3a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd\cos^{\frac{3}{2}}(c+dx)}-\frac{a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] -1/4*(a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b
)])/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (3*a*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4
*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)) - (3*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c
+ d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\,\sin(c-}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\,\sin(c-}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\,\sin(c-}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\,\sin(c-}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\,\mathrm{s}}}{2}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2+4b^2)\sqrt{\frac{b+a}{}}}{4b^2d\sqrt{\cos(c-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.50, size = 51323, normalized size = 164.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 1744, normalized size = 5.59

method	result	size
default	Expression too large to display	1744

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(3*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-6*sin(d
*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2+2*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*sin(d*x+c)*cos(d*x+c
)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b^2+6*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*sin(d*x+c)*cos(d*x+c)
^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*b^2+3*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*sin(d*x+c)*cos(d*x+c)^
2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b-6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
```

```
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*sin(d*x
+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*b^2+6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*sin(d*x+
c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+2*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2
-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
b^2+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*((a-b)/(a+b))^(1/2)*b^2)*((b+a*cos
(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)/((a
-b)/(a+b))^(1/2)/b^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.862 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=360

$$-\frac{8b(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5a^4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\right.}{5a^4(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2*b^2*cos(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-8/5*
b*(a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^4
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*(a^2-6*b^2)*cos(d*x+c)^(3/2)
*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d-2/5*b*(3*a^2-8*b^2)*sin(
d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d+2/5*(3*a^4+8
*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c
))^(1/2)/a^4/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3932, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2-6b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d(a^2-b^2)}-\frac{8b(a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5a^4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5a^4d(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{5a^3d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-8*b*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
(2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) + (2*b^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b
^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*b*(3*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d) + (2*(a^2 - 6*b
^2)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 -
 b^2)*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{a^2}{2}+3\cdots}{\sec^{\frac{5}{2}}(\cdots}}{a\,(a^2-b^2)\cdots}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-6b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d\cdots}}{5a^2\,(a^2-b^2)\,d\cdots}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2b(3a^2-8b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c\cdots}}{5a^3\,(a^2-b^2)\,d\cdots}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2b(3a^2-8b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c\cdots}}{5a^3\,(a^2-b^2)\,d\cdots}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2b(3a^2-8b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c\cdots}}{5a^3\,(a^2-b^2)\,d\cdots}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2b(3a^2-8b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c\cdots}}{5a^3\,(a^2-b^2)\,d\cdots}$$

$$= -\frac{8b(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{5a^4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^4+8a^2b^2-16b^4\cdots}{5\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.97, size = 419, normalized size = 1.16

$$\frac{(b+a\cos(c+dx))\left(a\sec^{\frac{3}{2}}(c+dx)\left(10b^4\sin(c+dx)+6b(-a^2+b^2)(b+a\cos(c+dx))\sin(c+dx)+a(a^2-b^2)(b+a\cos(c+dx))\sin(2(c+dx))\right)+2(a^2+4b^2)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(3a^3+3a^2b-4ab^2-4b^3)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(3a^2-ab-4b^2)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+(3a^2-4b^2)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{5a^4(a^2-b^2)\,d\cos^{\frac{5}{2}}(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(a*Sec[c + d*x]^(3/2)*(10*b^4*Sin[c + d*x] + 6*b*(-a^
2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] + a*(a^2 - b^2)*(b + a*Cos[c + d
*x])*Sin[2*(c + d*x)]) + 2*(a^2 + 4*b^2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^
```

```
(3/2)*(I*(3*a^3 + 3*a^2*b - 4*a*b^2 - 4*b^3)*EllipticE[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)] - I*a*(3*a^2 - a*b - 4*b^2)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (3*a^2 - 4*b^2)*(b + a*Cos[c + d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(5*a^4*(a^2 - b^2)*d*Cos[c
+ d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1850 vs. $2(386) = 772$.
time = 0.18, size = 1851, normalized size = 5.14

method	result	size
default	Expression too large to display	1851

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-6*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^2*b^2-16*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-4*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)-12*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin
(d*x+c)-16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b^3*sin(d*x+c)+8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)+8*cos(d*x+c)^2*((a-b)/(a+b
))^(1/2)*a^2*b^2-2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b-3*sin(d*x+c)*cos(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^4+cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b-2*cos(d*x+c)^3*((a-b)/(a+
b))^(1/2)*a^2*b^2+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+8*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a*b^3-3*((a-b)/(a+b))^(1/2)*a^3*b-8*((a-b)/(a+b))^(1/2)*
a*b^3-16*((a-b)/(a+b))^(1/2)*b^4+cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4+2*cos
(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)-16*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)+3*sin(
d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
```

```
+b)/(a-b))^(1/2))*a^4+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)+16*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^
4-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-16*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+8*sin
(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b^2-4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-12*sin(d*x+c)*cos(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*c
os(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/(a+b)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 702, normalized size = 1.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(6*(3*a^4*b^2 - 8*a^2*b^4 - (a^6 - a^4*b^2)*cos(d*x + c)^2 + 2*(a^5*b
 - a^3*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) + (sqrt(2)*(-9*I*a^5*b - 28*I*a^3*b^3 + 32*I*a*b^5)*
cos(d*x + c) + sqrt(2)*(-9*I*a^4*b^2 - 28*I*a^2*b^4 + 32*I*b^6))*sqrt(a)*we
ierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3
*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + (sqrt(2)*(9*I*a^5*b + 2
8*I*a^3*b^3 - 32*I*a*b^5)*cos(d*x + c) + sqrt(2)*(9*I*a^4*b^2 + 28*I*a^2*b^
4 - 32*I*b^6))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 3*(sqrt(2)*(3*I*a^6 + 8*I*a^4*b^2 - 16*I*a^2*b^4)*cos(d*x + c) + sqrt(2)*
(3*I*a^5*b + 8*I*a^3*b^3 - 16*I*a*b^5))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
```

```
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin
(d*x + c) + 2*b)/a)) - 3*(sqrt(2)*(-3*I*a^6 - 8*I*a^4*b^2 + 16*I*a^2*b^4)*c
os(d*x + c) + sqrt(2)*(-3*I*a^5*b - 8*I*a^3*b^3 + 16*I*a*b^5))*sqrt(a)*weie
rstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^8 - a^6*b^2)*d*cos(d*x + c
) + (a^7*b - a^5*b^3)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.863 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2b(5a^2-8b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\,\mathrm{s}}}{3a^3(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*
(a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^3/d
/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(a^2-4*b^2)*sin(d*x+c)*cos(d*x
+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d-2/3*b*(5*a^2-8*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/
d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3932, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3a^2d(a^2-b^2)}+\frac{2(a^2+8b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2b(5a^2-8b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*
b*(5*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{a^2}{2}}{\sec^{\frac{3}{2}}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{2b(5a^2-8b^2)\sqrt{\cos(c}}{3a^3\,(a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.60, size = 382, normalized size = 1.32

$$-\frac{2(b+a\cos(c+dx))\left(a(3b^3-(a^2-b^2)(b+a\cos(c+dx)))\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(ib(-5a^3-5a^2b+8ab^2+8b^3)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(a^3-5a^2b+2ab^2+8b^3)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+b(-5a^2+8b^2)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{3a^3(a^2-b^2)d\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a*(3*b^3 - (a^2 - b^2)*(b + a*Cos[c + d*x]))*Sec[
c + d*x]^(3/2)*Sin[c + d*x] - (Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*b*
(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2)/(a + b)] - I*a*(a^3 - 5*a^2*b + 2*a*b^2 + 8*b^3)*EllipticF[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Co
```

```
s[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(-5*a^2 + 8*b^2)*(b + a*Cos[c
+ d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(3*a^3*(a^2 - b^2)*d
*Cos[c + d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(321) = 642$.
time = 0.24, size = 1305, normalized size = 4.52

method	result	size
default	Expression too large to display	1305

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3+6*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+8*cos(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*sin(d*x+c)*a*b^2-5*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+8*cos(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^
3+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^3*sin(d*x+c)+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-5*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+8
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*b^3*sin(d*x+c)+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^2*b-4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-4*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a*b^2-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+4*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a^2*b-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-((a-b)/(a+b))^(
1/2)*a^2*b+4*((a-b)/(a+b))^(1/2)*a*b^2+8*((a-b)/(a+b))^(1/2)*b^3)*((b+a*cos
(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a
-b)/(a+b))^(1/2)/(a+b)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.89, size = 644, normalized size = 2.23

$$\frac{6\left(a^{4}b-4a^{2}b^{3}+\left(a^{5}-a^{3}b^{2}\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\sqrt{2}\left(3i\,a^{5}+16i\,a^{3}b^{2}-16i\,ab^{4}\right)\cos\left(dx+c\right)+\sqrt{2}\left(3i\,a^{4}b+16i\,a^{2}b^{3}-16i\,b^{5}\right)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)+3i\,a\sin\left(dx+c\right)+2b}{3a}\right)-\left(\sqrt{2}\left(-3i\,a^{5}-16i\,a^{3}b^{2}+16i\,ab^{4}\right)\cos\left(dx+c\right)+\sqrt{2}\left(-3i\,a^{4}b-16i\,a^{2}b^{3}+16i\,b^{5}\right)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)-3i\,a\sin\left(dx+c\right)+2b}{3a}\right)+3\left(\sqrt{2}\left(-5i\,a^{4}b+8i\,a^{2}b^{3}\right)\cos\left(dx+c\right)+\sqrt{2}\left(-5i\,a^{3}b^{2}+8i\,ab^{4}\right)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)+3i\,a\sin\left(dx+c\right)+2b}{3a}\right)\right)+3\left(\sqrt{2}\left(5i\,a^{4}b-8i\,a^{2}b^{3}\right)\cos\left(dx+c\right)+\sqrt{2}\left(5i\,a^{3}b^{2}-8i\,ab^{4}\right)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)-3i\,a\sin\left(dx+c\right)+2b}{3a}\right)\right)}{9\left(\left(a^{7}-a^{5}b^{2}\right)d\cos\left(dx+c\right)+\left(a^{6}b-a^{4}b^{3}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*(a^4*b - 4*a^2*b^3 + (a^5 - a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(3*I*a^5
+ 16*I*a^3*b^2 - 16*I*a*b^4)*cos(d*x + c) + sqrt(2)*(3*I*a^4*b + 16*I*a^2*b
^3 - 16*I*b^5))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
 - (sqrt(2)*(-3*I*a^5 - 16*I*a^3*b^2 + 16*I*a*b^4)*cos(d*x + c) + sqrt(2)*(
-3*I*a^4*b - 16*I*a^2*b^3 + 16*I*b^5))*sqrt(a)*weierstrassPInverse(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a
*sin(d*x + c) + 2*b)/a) + 3*(sqrt(2)*(-5*I*a^4*b + 8*I*a^2*b^3)*cos(d*x + c
) + sqrt(2)*(-5*I*a^3*b^2 + 8*I*a*b^4))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin
(d*x + c) + 2*b)/a)) + 3*(sqrt(2)*(5*I*a^4*b - 8*I*a^2*b^3)*cos(d*x + c) +
sqrt(2)*(5*I*a^3*b^2 - 8*I*a*b^4))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*
b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x
+ c) + 2*b)/a)))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.864 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+d}}{a^2\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-4*b*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^2/d/cos(d*x+c)^
(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))
^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4349, 3932, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-4*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Si
n[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\left(-\frac{a^2}{2}+b^2\right)\sqrt{\cos(c+\right.}{}$$

$$= \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\left(-\frac{a^2b}{2}+b\left(-\frac{a^2}{2}+b^2\right)\right)\right.}{a^2(a^2-b^2)}$$

$$= \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\left(-\frac{a^2b}{2}+b\left(-\frac{a^2}{2}+b^2\right)\right.\right.}{a^2(a^2-}$$

$$= -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}\,E\left(\right.}{a^2(a^2-b^2)\,d\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.87, size = 330, normalized size = 1.54

$$\frac{2(b+a\cos(c+dx))\left(ab^2\sin(c+dx)+\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(a^3+a^2b-2ab^2-2b^3)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(a^2-ab-2b^2)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+(a^2-2b^2)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sec^{\frac{3}{2}}(c+dx)}\right)}{a^2(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(a*b^2*Sin[c + d*x] + ((Cos[(c + d*x)/2]^2*Sec[c +
d*x])^(3/2)*(I*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^2 - a*b - 2*b^2)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a^2 - 2*b^2)*(b + a*Cos[c + d*x])*(
Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(a^2*(a^2
 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(256) = 512$.

time = 0.20, size = 997, normalized size = 4.66

method	result	size
default	Expression too large to display	997

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-sin(d*x+c)*cos(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^2+2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+2*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*s
in(d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*b^2*sin(d*x+c)-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a*b+((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-2*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*b^2+((a-b)/(a+b))^(1/2)*a*b+2*((a-b)/(a+b))^(1/2)*b^2)*(
(b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*
x+c)/a^2/(a+b)/((a-b)/(a+b))^(1/2)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.38, size = 585, normalized size = 2.73

$$\frac{6\,a^2b^2\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\sqrt{2}\left(-5i\,a^3b+4i\,ab^3\right)\cos\left(dx+c\right)+\sqrt{2}\left(-5i\,a^2b^2+4i\,b^4\right)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)-\left(\sqrt{2}\left(5i\,a^3b-4i\,ab^3\right)\cos\left(dx+c\right)+\sqrt{2}\left(5i\,a^2b^2-4i\,b^4\right)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)+3\left(\sqrt{2}\left(i\,a^4-2i\,a^2b^2\right)\cos\left(dx+c\right)+\sqrt{2}\left(i\,a^3b-2i\,ab^3\right)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)+3\left(\sqrt{2}\left(-i\,a^4+2i\,a^2b^2\right)\cos\left(dx+c\right)+\sqrt{2}\left(-i\,a^3b+2i\,ab^3\right)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{3\left(\left(a^6-a^4b^2\right)d\cos\left(dx+c\right)+\left(a^5b-a^3b^3\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^2*b^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) - (sqrt(2)*(-5*I*a^3*b + 4*I*a*b^3)*cos(d*x + c) + sqrt(2)*(-5*
I*a^2*b^2 + 4*I*b^4))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2
*b)/a) - (sqrt(2)*(5*I*a^3*b - 4*I*a*b^3)*cos(d*x + c) + sqrt(2)*(5*I*a^2*b
^2 - 4*I*b^4))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
+ 3*(sqrt(2)*(I*a^4 - 2*I*a^2*b^2)*cos(d*x + c) + sqrt(2)*(I*a^3*b - 2*I*a*
b^3))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*(sqrt(2)*
(-I*a^4 + 2*I*a^2*b^2)*cos(d*x + c) + sqrt(2)*(-I*a^3*b + 2*I*a*b^3))*sqrt(
a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, we
ierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3
*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^6 - a^4*b^2)*d*cos(
d*x + c) + (a^5*b - a^3*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.865 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=200

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a\,(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}-\frac{}{(a^2-b}$$

```
[Out] -2*b*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d/cos(d*x+c)^(1/2)/(
a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*
sec(d*x+c))^(1/2)/a/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4349, 3928, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2b\sin(c+dx)}{d\,(a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\,(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3928

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)
*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Si
mp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
 && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c\right.}{\cdots}$$

$$= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c-\right.}{\cdots}$$

$$= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{b+a\cos}}{a\sqrt{\cos}}$$

$$= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{\frac{b+a\cos}{a}}}{a\sqrt{}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+dx)}\,E}{a\left(a^2-\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.10, size = 245, normalized size = 1.22

$$\frac{2\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(ib(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{1+\sec(c+dx)} - ia(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{1+\sec(c+dx)} + b(-a+b)\sqrt{\sec(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(I*b*(a + b)*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh
```

[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] - I*a*(a + b)*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + b*(-a + b)*S
qrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(242) = 484$.
time = 0.19, size = 498, normalized size = 2.49

method	result
default	$-\frac{2\left(\operatorname{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{-\frac{a+b}{a-b}}\right)\cos(dx+c)\sin(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\, a+\mathrm{Elliptic}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*a+sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*
x+c)-((a-b)/(a+b))^(1/2)*cos(d*x+c)*b+((a-b)/(a+b))^(1/2)*b)*cos(d*x+c)^(1/
2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/(a+b)/((
a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.05, size = 541, normalized size = 2.70

$$-\frac{6\,a^{2}b\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\left(\sqrt{2}\,(3i\,a^{3}-2i\,ab^{2})\cos(dx+c)+\sqrt{2}\,(3i\,a^{2}b-2i\,b^{3})\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}}, \frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}}, \frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+\left(\sqrt{2}\,(-3i\,a^{3}+2i\,ab^{2})\cos(dx+c)+\sqrt{2}\,(-3i\,a^{2}b+2i\,b^{3})\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}}, \frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}}, \frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)-3\left(i\sqrt{2}\,a^{2}b\cos(dx+c)+i\sqrt{2}\,ab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}}, \frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}}, \mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}}, \frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}}, \frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)-3\left(-i\sqrt{2}\,a^{2}b\cos(dx+c)-i\sqrt{2}\,ab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}}, \frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}}, \mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^{2}-4\,b^{2})}{3\,a^{2}}, \frac{8\,(9\,a^{2}b-8\,b^{3})}{27\,a^{3}}, \frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right)}{3\left((a^{5}-a^{3}b^{2})d\cos(dx+c)+(a^{4}b-a^{2}b^{3})d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^2*b*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + (sqrt(2)*(3*I*a^3 - 2*I*a*b^2)*cos(d*x + c) + sqrt(2)*(3*I*a^2
*b - 2*I*b^3))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
+ (sqrt(2)*(-3*I*a^3 + 2*I*a*b^2)*cos(d*x + c) + sqrt(2)*(-3*I*a^2*b + 2*I*
b^3))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*(I*sq
rt(2)*a^2*b*cos(d*x + c) + I*sqrt(2)*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a)) - 3*(-I*sqrt(2)*a^2*b*cos(d*x + c) - I*sqrt(2)*a*b
^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^5 - a^3*b^
2)*d*cos(d*x + c) + (a^4*b - a^2*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.866 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}+\frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)/d/((
b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4349, 3929, 21, 3941, 2734, 2732}

$$\frac{2a\sin(c+dx)}{d\,(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\,(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/((a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*Sin[c
 + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 3929

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+d\right.}{}$$

$$= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx\right.}{}$$

$$= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx\right.}{}$$

$$= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx\right.}{}$$

$$= -\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{}{(a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.09, size = 260, normalized size = 2.06

$$\frac{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(-i(b+a\cos(c+dx))E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{1+\sec(c+dx)}+i(b+a\cos(c+dx))F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{1+\sec(c+dx)}+(a-b)\sqrt{\sec(c+dx)}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}\sin(c+dx)\right)}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]^(3
/2)*((-I)*(b + a*Cos[c + d*x])*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + I*(b + a*Cos[c + d*x])*EllipticF[I*Ar
cSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + (a - b)
*Sqrt[Sec[c + d*x]]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))]
*Sin[c + d*x]))/((a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(145) = 290.
time = 0.18, size = 491, normalized size = 3.90

method	result
default	$-\frac{2\left(\cos(dx+c)\sin(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)-\cos(dx+c\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))-sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))+((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)-((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)+cos
(d*x+c)*((a-b)/(a+b))^(1/2)-((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+
c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/
(a+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.93, size = 500, normalized size = 3.97

$$\frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-\left(-i\sqrt{2}\,ab\cos(dx+c)-i\sqrt{2}\,b^2\right)\sqrt{a}\,\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)-\left(i\sqrt{2}\,ab\cos(dx+c)+i\sqrt{2}\,b^2\right)\sqrt{a}\,\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)+3\left(-i\sqrt{2}\,a^2\cos(dx+c)-i\sqrt{2}\,ab\right)\sqrt{a}\,\text{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)+3\left(i\sqrt{2}\,a^2\cos(dx+c)+i\sqrt{2}\,ab\right)\sqrt{a}\,\text{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{3\left((a^4-a^2b^2)d\cos(dx+c)+(a^3b-ab^3)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) - (-I*sqrt(2)*a*b*cos(d*x + c) - I*sqrt(2)*b^2)*sqrt(a)*weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos
```

```
(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - (I*sqrt(2)*a*b*cos(d*x + c) + I*
sqrt(2)*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) +
3*(-I*sqrt(2)*a^2*cos(d*x + c) - I*sqrt(2)*a*b)*sqrt(a)*weierstrassZeta(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3
*I*a*sin(d*x + c) + 2*b)/a)) + 3*(I*sqrt(2)*a^2*cos(d*x + c) + I*sqrt(2)*a*
b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^4 - a^2*b^2
)*d*cos(d*x + c) + (a^3*b - a*b^3)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.867 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=206

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}-\frac{\quad}{b\left(a^2-\right.}$$

```
[Out] -2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c
),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^
(1/2)/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2
)*(a+b*sec(d*x+c))^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3930, 4193, 3944, 2886, 2884, 21, 3941, 2734, 2732}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(
a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c + d*x])/(b*
(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
```

```
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(a\sqrt{\cos(c\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(\left(-a^2+b\right.\right.}{b}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{2a}{b\,(a^2-b^2)\,d\sqrt{\cos}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}}{b\,(a^2-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.67, size = 34326, normalized size = 166.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 1134, normalized size = 5.50

method	result	size

default	Expression too large to display	1134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(2*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*a+sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b-sin(d*x+c)*cos(d*x+c
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*a-2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-2*sin(d*x+c)*cos(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^
(1/2))*b+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*sin(d*x
+c)-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/(
(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)*a-((a-b)/(a
+b))^(1/2)*a)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos
(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/(a+b)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.868 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$\frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{(3a^2-b^2)\sqrt{\cos(c+d}}{b^2(a^2-}$$

```
[Out] -2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)+(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^(1/2)
/(a+b*sec(d*x+c))^(1/2)-3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))
/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+(3*a^2-b^2)*sin
(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)-(3*a^2-b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^
2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4349, 3930, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2a^2\sin(c+dx)}{bd(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2-b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{b^2d(a^2-b^2)\sqrt{\cos(c+dx)}}-\frac{(3a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{3a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/
(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (3*a*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b^2*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b
^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c + d*x])/(b*(a^2 -
b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
```

```
b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+d\right.}}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{a}}{b^2\,(a^2-}$$

$$= -\frac{2a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{a}}{b^2\,(a^2-}$$

$$= -\frac{2a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{a}}{b^2\,(a^2-}$$

$$= -\frac{2a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{a}}{b^2\,(a^2-}$$

$$= -\frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{2}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.26, size = 36944, normalized size = 107.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 1492, normalized size = 4.32

method	result	size
default	Expression too large to display	1492

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*sin(d*x+c)
*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*a^2+6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b
-6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2-4*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*sin(d*x+c)*c
os(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^2-sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2
))*a^2+6*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-6*sin(d*x+c)*cos(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2-4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a*b+3*((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-cos(d*x
+c)*((a-b)/(a+b))^(1/2)*b^2+((a-b)/(a+b))^(1/2)*a*b+((a-b)/(a+b))^(1/2)*b^2
)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c
)^(1/2)/((a-b)/(a+b))^(1/2)/(a+b)/b^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.869 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{8b(2a^4-7a^2b^2+4b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{3a^4\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\,c}{}}}$$

```
[Out] 2/3*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/
3*b^2*(5*a^2-3*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(
d*x+c))^(1/2)+2/3*(a^4+16*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*
cos(d*x+c))/(a+b))^(1/2)/a^4/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^
(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c
))^(1/2)/a^3/(a^2-b^2)^2/d-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/((b+a*co
s(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4349, 3932, 4185, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{4b^2(5a^2-3b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{8b(2a^4-7a^2b^2+4b^4)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(a^4-13a^2b^2+8b^4)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^4 + 16*a^2*b^2 - 16*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^4*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) - (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2
 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*b^2*(5*a
^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
+ b*Sec[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3a}{2}}{}}{3a(a^2-b}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\,\sin(c+d}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\,\sin(c+d}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\,\sin(c+d}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\,\sin(c+d}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\,\sin(c+d}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^4(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{8b(2a^4-7a^2b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.34, size = 527, normalized size = 1.35

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*Sin[c + d*x])/(3*a^3) + (2*b^4*Sin[c + d*x])/(3
*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (8*(-3*a^2*b^3*Sin[c + d*x] + 2*
b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c +
```

```
d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c
 + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-4*
I)*b*(2*a^5 + 2*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 4*a*b^4 + 4*b^5)*EllipticE[
I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^5 - 8*a^4*b + 7*a^3
*b^2 + 28*a^2*b^3 - 4*a*b^4 - 16*b^5)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
d*x)/2]^2)/(a + b)] - 4*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*(b + a*Cos[c + d*x])*
(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*(a + b
*Sec[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3603 vs. $2(415) = 830$.
time = 0.21, size = 3604, normalized size = 9.22

method	result	size
default	Expression too large to display	3604

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6-16*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^6+((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b*sin(d*
x+c)+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^4*b^2*sin(d*x+c)+16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3*sin(d*x+c)-12*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4*sin(d*x+c)-16*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^5*sin(d*x+c)-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^4*b^2*sin(d*x+c)+28*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4*sin(d*x+c)+sin(d*x+c)*cos(d*
x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^6-34*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^4-16*cos(d*x+c)*((a-b)/(
```

```
a+b))^(1/2)*a*b^5-16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*b^6*sin(d*x+c)+9*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+16*sin(
d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^4*b^2-12*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-16*sin(d*x+c
)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b^4-8*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+28*sin(d*x+c)*cos(d*
x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3*b^3-16*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5+10*sin(d*x+c)*cos(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
5*b+25*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-28*sin
(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b^4-16*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5-8*sin(d*x+c)*cos(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^5*b-8*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+28*sin(d*x+c)*cos(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b
^3+28*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-16*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c...
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.21, size = 971, normalized size = 2.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*(a^6*b^2 - 13*a^4*b^4 + 8*a^2*b^6 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(
d*x + c)^2 + 2*(a^7*b - 8*a^5*b^3 + 5*a^3*b^5)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-3*I*
a^8 - 37*I*a^6*b^2 + 68*I*a^4*b^4 - 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*sqrt(2
)*(3*I*a^7*b + 37*I*a^5*b^3 - 68*I*a^3*b^5 + 32*I*a*b^7)*cos(d*x + c) + sqr
t(2)*(-3*I*a^6*b^2 - 37*I*a^4*b^4 + 68*I*a^2*b^6 - 32*I*b^8))*sqrt(a)*weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + (sqrt(2)*(3*I*a^8 + 37*I*a
^6*b^2 - 68*I*a^4*b^4 + 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*sqrt(2)*(-3*I*a^7*
b - 37*I*a^5*b^3 + 68*I*a^3*b^5 - 32*I*a*b^7)*cos(d*x + c) + sqrt(2)*(3*I*a
^6*b^2 + 37*I*a^4*b^4 - 68*I*a^2*b^6 + 32*I*b^8))*sqrt(a)*weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x +
 c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 12*(sqrt(2)*(2*I*a^7*b - 7*I*a^5*b^3 +
 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(2*I*a^6*b^2 - 7*I*a^4*b^4 + 4*I*a
^2*b^6)*cos(d*x + c) + sqrt(2)*(2*I*a^5*b^3 - 7*I*a^3*b^5 + 4*I*a*b^7))*sqr
t(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 12*(sqrt(2)*(-2*I*a^
7*b + 7*I*a^5*b^3 - 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(-2*I*a^6*b^2 +
 7*I*a^4*b^4 - 4*I*a^2*b^6)*cos(d*x + c) + sqrt(2)*(-2*I*a^5*b^3 + 7*I*a^3*
b^5 - 4*I*a*b^7))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/
((a^11 - 2*a^9*b^2 + a^7*b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^
6*b^5)*d*cos(d*x + c) + (a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.870 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{2b(9a^2-8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4-15a^2b^2+8b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\right.}{3a^3\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+8/
3*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*
x+c))^(1/2)-2/3*b*(9*a^2-8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)/a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*
(3*a^4-15*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*s
ec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3932, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{8b^2(2a^2-b^2)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}-\frac{2b(9a^2-8b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4-15a^2b^2+8b^4)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*(9*a^2 - 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)])/(3*a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^
2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*
Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3932

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= -\frac{2b(9a^2-8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^4-15a^2b^2+8b}{}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 11.00, size = 427, normalized size = 1.35

$$\frac{2(b+a\cos(c+dx))^2\left(-\frac{b^2(-8a^2b+4b^3+(-9a^3+5ab^2)\cos(c+dx))\sin(c+dx)}{b+a\cos(c+dx)}-\frac{(\cos^2(\frac{1}{2}(c+dx))\sec(c+dx))^{3/2}\left(-i(3a^5+3a^4b-15a^3b^2-15a^2b^3+8ab^4+8b^5)E\left(i\sinh^{-1}(\tan(\frac{1}{2}(c+dx)))\middle|\frac{-a+b}{a+b}\right)\sec^2(\frac{1}{2}(c+dx))\sqrt{\frac{(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))}{a+b}}+ia(3a^4-6a^3b-15a^2b^2+2ab^3+8b^4)F\left(i\sinh^{-1}(\tan(\frac{1}{2}(c+dx)))\middle|\frac{-a+b}{a+b}\right)\sec^2(\frac{1}{2}(c+dx))\sqrt{\frac{(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))}{a+b}}-(3a^4-15a^2b^2+8b^4)(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))^{3/2}\tan(\frac{1}{2}(c+dx))\right)}{a\sec^{\frac{3}{2}}(c+dx)}\right)}{3\left(a^3-ab^2\right)^2 d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])^2*(-((b^2*(-8*a^2*b + 4*b^3 + (-9*a^3 + 5*a*b^2)*Co
s[c + d*x])*Sin[c + d*x])/(b + a*Cos[c + d*x])) - ((Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(3/2)*((-I)*(3*a^5 + 3*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 8*a*b^4
+ 8*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*
```

```
a^4 - 6*a^3*b - 15*a^2*b^2 + 2*a*b^3 + 8*b^4)*EllipticF[I*ArcSinh[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)] - (3*a^4 - 15*a^2*b^2 + 8*b^4)*(b + a*Cos[c + d
*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(3/2))))
/(3*(a^3 - a*b^2)^2*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3100 vs. $2(347) = 694$.

time = 0.20, size = 3101, normalized size = 9.78

method	result	size
default	Expression too large to display	3101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b-3*cos(d*x+c)^3*((a-b)/(a+b
))^(1/2)*a^2*b^3+18*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2-12*cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a*b^4-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^2+3*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b-4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2
*b^3+3*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+3*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-6*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a^4*b-12*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2+18*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4-
8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5+8*((a-b)/(a+b))^(1/2)*b^5-3*((a-b)/(a+
b))^(1/2)*a^3*b^2-11*((a-b)/(a+b))^(1/2)*a^2*b^3+4*((a-b)/(a+b))^(1/2)*a*b^
4+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5-3*sin(d*x+c)*cos(d*x+c)^2*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5-15*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+8*sin
(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b^4-9*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+6*sin(d*x+c)*cos
(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^3*b^2+8*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+3*sin(d*x+c)*cos(d*x+c)
```

```
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^4*b-15*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-15*sin(d*x+c)*cos(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+8*
sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b^4-12*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-3*sin(d*x+c)*cos
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^3*b^2+14*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+8*sin(d*x+c)*cos(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^4+8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^5*sin(d*x+c)+8*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5-3*sin(d*x+c)*cos(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
5+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^4*b*sin(d*x+c)-15*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^2*b^3*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*sin(d*x+c)-9*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*sin
(d*x+c)+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^2*b^3*sin(d*x+c)+8*((b+a*cos(d*x+c)...
```

Maximа [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.72, size = 884, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*(8*a^4*b^3 - 4*a^2*b^5 + (9*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c))*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(sqr
t(2)*(-6*I*a^6*b + 9*I*a^4*b^3 - 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(-
6*I*a^5*b^2 + 9*I*a^3*b^4 - 4*I*a*b^6)*cos(d*x + c) + sqrt(2)*(-6*I*a^4*b^3
 + 9*I*a^2*b^5 - 4*I*b^7))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c
) + 2*b)/a) - 4*(sqrt(2)*(6*I*a^6*b - 9*I*a^4*b^3 + 4*I*a^2*b^5)*cos(d*x +
c)^2 + 2*sqrt(2)*(6*I*a^5*b^2 - 9*I*a^3*b^4 + 4*I*a*b^6)*cos(d*x + c) + sqr
t(2)*(6*I*a^4*b^3 - 9*I*a^2*b^5 + 4*I*b^7))*sqrt(a)*weierstrassPInverse(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) -
3*I*a*sin(d*x + c) + 2*b)/a) - 3*(sqrt(2)*(-3*I*a^7 + 15*I*a^5*b^2 - 8*I*a^
3*b^4)*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*a^6*b + 15*I*a^4*b^3 - 8*I*a^2*b^5)
*cos(d*x + c) + sqrt(2)*(-3*I*a^5*b^2 + 15*I*a^3*b^4 - 8*I*a*b^6))*sqrt(a)*
weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*(sqrt(2)*(3*I*a^7 - 15*
I*a^5*b^2 + 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*sqrt(2)*(3*I*a^6*b - 15*I*a^4*b
^3 + 8*I*a^2*b^5)*cos(d*x + c) + sqrt(2)*(3*I*a^5*b^2 - 15*I*a^3*b^4 + 8*I*
a*b^6))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8
*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*
b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^10 - 2
*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^2 + 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*co
s(d*x + c) + (a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.871 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=302

$$\frac{2(3a^2-2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{4b(3a^2-b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\cdots}}{3a^2(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] -2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-2/3*b
*(5*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(
1/2)+2/3*(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+4/3*b*(3*a^2-
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2
/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3928, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2b(5a^2-b^2)\sin(c+dx)}{3ad(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)}{3d(a^2-b^2)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}+\frac{2(3a^2-2b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d(a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{4b(3a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d(a^2-b^2)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) + (4*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*
x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2)*Sin[c + d*x])/(3*a*(a^
2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3928

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)
*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Si
mp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
 && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2b\sin(c+dx)}{3\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(\right.}{}$$

$$= -\frac{2b\sin(c+dx)}{3\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{}{3a\,(a^2-}$$

$$= -\frac{2b\sin(c+dx)}{3\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{}{3a\,(a^2-}$$

$$= -\frac{2b\sin(c+dx)}{3\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{}{3a\,(a^2-}$$

$$= -\frac{2b\sin(c+dx)}{3\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{}{3a\,(a^2-}$$

$$= \frac{2(3a^2-2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{4b(3a^2-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.68, size = 398, normalized size = 1.32

$$\frac{(b+a\cos(c+dx))^2\left(\frac{2b(-5a^2b+b^3+(-6a^3+2ab^2)\cos(c+dx))\sin(c+dx)}{a(a^2-b^2)^2(b+a\cos(c+dx))}-\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(2ib(-3a^3-3a^2b+ab^2+b^3)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia(3a^3+6a^2b+ab^2-2b^3)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+2b(-3a^2+b^2)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^3-ab^2)^2\sec^{\frac{3}{2}}(c+dx)}\right)}{3d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((2*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c +
d*x])*Sin[c + d*x])/(a*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) - (2*(Cos[(c + d
*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-3*a^3 - 3*a^2*b + a*b^2 + b^3)*Elli
pticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqr
t[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^3 + 6*a^2*b
```

```
 + a*b^2 - 2*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
+ 2*b*(-3*a^2 + b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c
 + d*x)/2]))/((a^3 - a*b^2)^2*Sec[c + d*x]^(3/2))))/(3*d*Cos[c + d*x]^(5/2)
*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2061 vs. $2(332) = 664$.

time = 0.19, size = 2062, normalized size = 6.83

method	result	size
default	Expression too large to display	2062

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-6*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^2*b^2-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-2*sin(d*x+c)*cos(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*b^4+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3*b*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+6*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*s
in(d*x+c)+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2+3*sin(d*x+c)*cos(d*x+c)^
2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^4+3*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+
3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3+5*((a-b)/(a+b))^(1/2)*a^2*b^2-((a-
b)/(a+b))^(1/2)*a*b^3-2*((a-b)/(a+b))^(1/2)*b^4-2*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)+2*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*b^4-3*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-2*sin(d*x+c)*cos
(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^2*b^2+6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)
```

```
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-2*sin(d*x+c)*cos(d*x+c)^2
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b^3-2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+6*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+6*sin(d*
x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b^2-2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-5*sin(d*x+c)*cos(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*b^2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d
*x+c))^2/sin(d*x+c)/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 835, normalized size = 2.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -1/9*(6*(5*a^4*b^2 - a^2*b^4 + 2*(3*a^5*b - a^3*b^3)*cos(d*x + c))*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*
(-9*I*a^6 + 9*I*a^4*b^2 - 4*I*a^2*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(9*I*a^5*
b - 9*I*a^3*b^3 + 4*I*a*b^5)*cos(d*x + c) + sqrt(2)*(-9*I*a^4*b^2 + 9*I*a^2
*b^4 - 4*I*b^6))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a
) - (sqrt(2)*(9*I*a^6 - 9*I*a^4*b^2 + 4*I*a^2*b^4)*cos(d*x + c)^2 - 2*sqrt(
2)*(-9*I*a^5*b + 9*I*a^3*b^3 - 4*I*a*b^5)*cos(d*x + c) + sqrt(2)*(9*I*a^4*b
^2 - 9*I*a^2*b^4 + 4*I*b^6))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^
```

```
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x +
 c) + 2*b)/a) + 6*(sqrt(2)*(-3*I*a^5*b + I*a^3*b^3)*cos(d*x + c)^2 + 2*sqrt
(2)*(-3*I*a^4*b^2 + I*a^2*b^4)*cos(d*x + c) + sqrt(2)*(-3*I*a^3*b^3 + I*a*b
^5))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(sqrt(2)*(
3*I*a^5*b - I*a^3*b^3)*cos(d*x + c)^2 + 2*sqrt(2)*(3*I*a^4*b^2 - I*a^2*b^4)
*cos(d*x + c) + sqrt(2)*(3*I*a^3*b^3 - I*a*b^5))*sqrt(a)*weierstrassZeta(-4
/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) -
3*I*a*sin(d*x + c) + 2*b)/a)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^
2 + 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c) + (a^7*b^2 - 2*a^5*b^4 +
 a^3*b^6)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.872 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(3a^2+b^2)\,\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\,\sqrt{a+b\sec}}{3a\,(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+4/3*(a
^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2/
3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/(a^2-b^2)/d
/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b+a*cos(
d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.49, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3929, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{4(a^2+b^2)\sin(c+dx)}{3d\,(a^2-b^2)^2\,\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)}{3d\,(a^2-b^2)\,\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}-\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\,(a^2-b^2)\,\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(3a^2+b^2)\,\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\,(a^2-b^2)^2\,\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(
3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c +
d*x])^(3/2)) + (4*(a^2 + b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3929

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{}{3\left(a^2-b^2\right)^2}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{}{3\left(a^2-b^2\right)^2}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{}{3\left(a^2-b^2\right)^2}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{}{3\left(a^2-b^2\right)^2}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{2(3a^2+b^{\cdots}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.67, size = 447, normalized size = 1.59

$$\frac{(b+a\cos(c+dx))^3\left(\frac{2b\sin(c+dx)}{3(-a^2+b^2)(b+a\cos(c+dx))^2}+\frac{2(3a^2\sin(c+dx)+b^2\sin(c+dx))}{3(-a^2+b^2)^2(b+a\cos(c+dx))}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} + \frac{2\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(3a^3+3a^2b+ab^2+b^3)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia(3a^2+4ab+b^2)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-(3a^2+b^2)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c +
d*x])^2) + (2*(3*a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b
 + a*Cos[c + d*x]))))/(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (
2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*
x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE
[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
```

```
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b + b^2)
*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^
2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2 + b^2)*(
b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a*(a^2
 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1811 vs. $2(311) = 622$.
time = 0.19, size = 1812, normalized size = 6.45

method	result	size
default	Expression too large to display	1812

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(3*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3-sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-3*sin(d
*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*a^3-sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+3*cos(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c
)*a^3+2*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-3*cos(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*sin(d*x+c)*a^3-3*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-
cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*sin(d*x+c)*b^3+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c
```

```
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-3*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d
*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^3*sin(d*x+c)+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a^2*b-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+3*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((
a-b)/(a+b))^(1/2)*b^3-2*((a-b)/(a+b))^(1/2)*a^2*b+((a-b)/(a+b))^(1/2)*a*b^2
-((a-b)/(a+b))^(1/2)*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1
/2)/(b+a*cos(d*x+c))^2/sin(d*x+c)/(a-b)/(a+b)^2/a/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.02, size = 782, normalized size = 2.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(6*(2*a^4*b + 2*a^2*b^3 + (3*a^5 + a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*(sqrt(2)*(-3
*I*a^4*b + I*a^2*b^3)*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*a^3*b^2 + I*a*b^4)*c
os(d*x + c) + sqrt(2)*(-3*I*a^2*b^3 + I*b^5))*sqrt(a)*weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
+ 3*I*a*sin(d*x + c) + 2*b)/a) - 2*(sqrt(2)*(3*I*a^4*b - I*a^2*b^3)*cos(d*x
 + c)^2 + 2*sqrt(2)*(3*I*a^3*b^2 - I*a*b^4)*cos(d*x + c) + sqrt(2)*(3*I*a^2
*b^3 - I*b^5))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 3*(sqrt(2)*(3*I*a^5 + I*a^3*b^2)*cos(d*x + c)^2 + 2*sqrt(2)*(3*I*a^4*b +
I*a^2*b^3)*cos(d*x + c) + sqrt(2)*(3*I*a^3*b^2 + I*a*b^4))*sqrt(a)*weierstr
assZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d
*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*(sqrt(2)*(-3*I*a^5 - I*a^3*b^2)
*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*a^4*b - I*a^2*b^3)*cos(d*x + c) + sqrt(2)
```

```
*(-3*I*a^3*b^2 - I*a*b^4))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2
*b)/a)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c)^2 + 2*(a^7*b - 2*a^5*b
^3 + a^3*b^5)*d*cos(d*x + c) + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.873 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}-$$

```
[Out] -2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+2
/3*a*(a^2-5*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c
))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/(a^2
-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+8/3*b*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(
1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)^2/d/((b+a*cos(d*x+c
))/(a+b))^(1/2)
```

Rubi [A]

time = 0.51, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4349, 3930, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\left(a^2-b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c +
 d*x])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) +
(2*a*(a^2 - 5*b^2)*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{}{3b\,(a^2-b^2}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{}{3b\,(a^2-b^2}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{}{3b\,(a^2-b^2}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{}{3b\,(a^2-b^2}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{8b\sqrt{\cos(c+d}{3\,(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.93, size = 311, normalized size = 1.12

$$\frac{2(b+a\cos(c+dx))^2\left(\frac{a(a^2-5b^2-4ab\cos(c+dx))\sin(c+dx)}{b+a\cos(c+dx)}+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(4ib(a+b)E\left(i\sinh^{-1}(\tan(\frac{1}{2}(c+dx)))|\frac{-a+b}{a+b}\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))}{a+b}}-i(a^2+4ab+3b^2)F\left(i\sinh^{-1}(\tan(\frac{1}{2}(c+dx)))|\frac{-a+b}{a+b}\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))}{a+b}}+4b(b+a\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\tan(\frac{1}{2}(c+dx))\right)}{\sqrt{\sec(c+dx)}}\right)}{3\,(a^2-b^2)^2\,d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])^2*((a*(a^2 - 5*b^2 - 4*a*b*Cos[c + d*x])*Sin[c + d*
x])/(b + a*Cos[c + d*x]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*b*
(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)*Ell
ipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c +
```

```
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Sec[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]))/Sqrt[Sec[c + d*x]]))/(3*(a^2 - b^2)^2*d*Cos[
c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1332 vs. $2(307) = 614$.
time = 0.23, size = 1333, normalized size = 4.81

method	result	size
default	Expression too large to display	1333

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*sin(d*x
+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*sin(d*x+c)*cos(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-
3*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b^2+4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*sin(d*x+c)*cos(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*b^2+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+4*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a^2-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+4*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a*b-4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/
2)*a^2-((a-b)/(a+b))^(1/2)*a*b+4*((a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(d*x+c))
/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))^2/sin(d*x+c)/(a-b)/(a+
b)^2/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.37, size = 709, normalized size = 2.56

$$-\frac{6\left(4a^{3}b\cos\left(dx+c\right)-a^{4}+5a^{2}b^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\sqrt{2}\left(-3ia^{4}-ia^{2}b^{2}\right)\cos\left(dx+c\right)^{2}-2\sqrt{2}\left(3ia^{3}b+iab^{3}\right)\cos\left(dx+c\right)+\sqrt{2}\left(-3ia^{2}b^{2}-ib^{4}\right)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)+3ia\sin\left(dx+c\right)+2b}{3a}\right)-\left(\sqrt{2}\left(3ia^{4}+ia^{2}b^{2}\right)\cos\left(dx+c\right)^{2}-2\sqrt{2}\left(-3ia^{3}b-iab^{3}\right)\cos\left(dx+c\right)+\sqrt{2}\left(3ia^{2}b^{2}+ib^{4}\right)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)-3ia\sin\left(dx+c\right)+2b}{3a}\right)+12\left(-i\sqrt{2}a^{3}b\cos\left(dx+c\right)^{2}-2i\sqrt{2}a^{2}b^{2}\cos\left(dx+c\right)-i\sqrt{2}ab^{3}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)+3ia\sin\left(dx+c\right)+2b}{3a}\right)\right)+12\left(i\sqrt{2}a^{3}b\cos\left(dx+c\right)^{2}+2i\sqrt{2}a^{2}b^{2}\cos\left(dx+c\right)+i\sqrt{2}ab^{3}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)-3ia\sin\left(dx+c\right)+2b}{3a}\right)\right)}{9\left(\left(a^{7}-2a^{5}b^{2}+a^{3}b^{4}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{6}b-2a^{4}b^{3}+a^{2}b^{5}\right)d\cos\left(dx+c\right)+\left(a^{5}b^{2}-2a^{3}b^{4}+ab^{6}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/9*(6*(4*a^3*b*cos(d*x + c) - a^4 + 5*a^2*b^2)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-3*I*a^4 - I*a^2*
b^2)*cos(d*x + c)^2 - 2*sqrt(2)*(3*I*a^3*b + I*a*b^3)*cos(d*x + c) + sqrt(2
)*(-3*I*a^2*b^2 - I*b^4))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
 + 2*b)/a) - (sqrt(2)*(3*I*a^4 + I*a^2*b^2)*cos(d*x + c)^2 - 2*sqrt(2)*(-3*
I*a^3*b - I*a*b^3)*cos(d*x + c) + sqrt(2)*(3*I*a^2*b^2 + I*b^4))*sqrt(a)*we
ierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3
*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 12*(-I*sqrt(2)*a^3*b*co
s(d*x + c)^2 - 2*I*sqrt(2)*a^2*b^2*cos(d*x + c) - I*sqrt(2)*a*b^3)*sqrt(a)*
weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 12*(I*sqrt(2)*a^3*b*cos(d
*x + c)^2 + 2*I*sqrt(2)*a^2*b^2*cos(d*x + c) + I*sqrt(2)*a*b^3)*sqrt(a)*wei
erstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstr
assPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*
d*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^5*b^
2 - 2*a^3*b^4 + a*b^6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.874 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a(3a^2-7b$$

```
[Out] -2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)-2
/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec
(d*x+c))^(1/2)-2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1
/2)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a
/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*s
ec(d*x+c))^(1/2)+2/3*a*(3*a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^
(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/
2)
```

Rubi [A]

time = 0.84, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {4349, 3930, 4183, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2(3a^2-7b^2)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a(3a^2-7b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
)/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2
)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a
^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])
^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
```

```
b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+\cdots\right.}{\cdots}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{\cdots}{3b^2\,(a^2-b^2\cdots}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{\cdots}{3b^2\,(a^2-b^2\cdots}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{\cdots}{3b^2\,(a^2-b^2\cdots}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{\cdots}{3b^2\,(a^2-b^2\cdots}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\cdots}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c\cdots}$$

$$= -\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\frac{b+a\cos\cdots}{a\cdots}}}{b^2d\sqrt{\cos(\cdots}}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 32.29, size = 92128, normalized size = 248.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 3844, normalized size = 10.39

method	result	size
default	Expression too large to display	3844

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-7*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a^2*b^2+7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-6*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)-4*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b^2*sin(d*x+c)+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)-7*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+
6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2-6*sin(d*x+c)*cos(d*x+c)^2*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-6*
sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^4-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+4*((a-b)/(
a+b))^(1/2)*a^3*b+((a-b)/(a+b))^(1/2)*a^2*b^2-7*((a-b)/(a+b))^(1/2)*a*b^3-3
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4+3*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin
(d*x+c)-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4
-4*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+9*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+3*sin(d*
x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
```

```
+b)/(a-b))^(1/2))*a^4+6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^4+3*sin(d*x
+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*b^4+6*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^4-6*sin(d*x+c)*c
os(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*b^4+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+6*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*a^2*b^2*sin(d*x+c)-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+12*sin(d*x+c)*cos(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*a^3*b-12*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^3-7*sin(
d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b^2+3*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+6*sin(d*x+c)*co
s(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*a^3*b-6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.875 $\int (d\cos(e+fx))^n (a+b\sec(e+fx))^3\,dx$

Optimal. Leaf size=266

$$-\frac{b(b^2(1-n)+3a^2(2-n))\,(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a(a^2(1-n)-3b^2n)\,\mathrm{c}}{}$$

```
[Out] -b*(b^2*(1-n)+3*a^2*(2-n))*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n
],cos(f*x+e)^2)*sin(f*x+e)/f/(2-n)/n/(sin(f*x+e)^2)^(1/2)-a*(a^2*(1-n)-3*b^
2*n)*cos(f*x+e)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos
(f*x+e)^2)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+a*b^2*(5-2*n)*(d*cos(
f*x+e))^n*tan(f*x+e)/f/(n^2-3*n+2)+b^2*(d*cos(f*x+e))^n*(a+b*sec(f*x+e))*ta
n(f*x+e)/f/(2-n)
```

Rubi [A]
time = 0.34, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4349, 3927, 4132, 3857, 2722, 4131}

$$-\frac{b(3a^2(2-n)+b^2(1-n))\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a(a^2(1-n)-3b^2n)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}+\frac{ab^2(5-2n)\tan(e+fx)(d\cos(e+fx))^n}{f(1-n)(2-n)}+\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\cos(e+fx))^n}{f(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((b*(b^2*(1 - n) + 3*a^2*(2 - n))*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2
, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 - n)*n*Sqrt[Sin[e + f
*x]^2])) - (a*(a^2*(1 - n) - 3*b^2*n)*Cos[e + f*x]*(d*Cos[e + f*x])^n*Hyper
geometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1
 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*(5 - 2*n)*(d*Cos[e + f*x])^n*T
an[e + f*x])/(f*(1 - n)*(2 - n)) + (b^2*(d*Cos[e + f*x])^n*(a + b*Sec[e + f
*x])*Tan[e + f*x])/(f*(2 - n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3927

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
 && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx &= ((d\cos(e+fx))^n(d\sec(e+fx))^n)\int (d\sec(e+fx))^{-n}(a+b\sec \\
&= \frac{b^2(d\cos(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2-n)} + \frac{((d\cos(e+}{} \\
&= \frac{b^2(d\cos(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2-n)} + \frac{((d\cos(e+}{} \\
&= \frac{ab^2(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{b^2(d\cos(e+fx))^n(a}{} \\
&= -\frac{b(b^2(1-n)+3a^2(2-n))\,(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(\right.}{f(2-n)n\sqrt{\sin^2(e+fx)}} \\
&= -\frac{b(b^2(1-n)+3a^2(2-n))\,(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(\right.}{f(2-n)n\sqrt{\sin^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.47, size = 222, normalized size = 0.83

$$
-\frac{(d\cos(e+fx))^n\csc(e+fx)\left(b^3n(-1+n^2)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(e+fx)\right)+\frac{1}{2}a(-2+n)\cos(e+fx)\left(6b^2n(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(e+fx)\right)+2a(-1+n)\cos(e+fx)\left(3b(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)+an\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\right)\right)\right)\sec^2(e+fx)\sqrt{\sin^2(e+fx)}}{f(-2+n)(-1+n)n(1+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -(((d*Cos[e + f*x])^n*Csc[e + f*x]*(b^3*n*(-1 + n^2)*Hypergeometric2F1[1/2,
 (-2 + n)/2, n/2, Cos[e + f*x]^2] + (a*(-2 + n)*Cos[e + f*x]*(6*b^2*n*(1 +
n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + 2*a*(-1
+ n)*Cos[e + f*x]*(3*b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e
 + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2])))/2)*Sec[e + f*x]^2*Sqrt[Sin[e + f*x]^2])/(f*(-2 + n)*(-1
+ n)*n*(1 + n)))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$
\int (d\cos(fx+e))^n\,(a+b\sec(fx+e))^3\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*cos(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(e+fx))^n (a+b\sec(e+fx))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
[Out] Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,\cos(e+f\,x))^n\,\left(a+\frac{b}{\cos(e+f\,x)}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^3,x)
[Out] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^3, x)
```

3.876 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx$

Optimal. Leaf size=186

$$-\frac{2ab(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}-\frac{(a^2(1-n)-b^2n)\cos(e+fx)(d\cos(e+fx}{f(1-n)(1+}$$

[Out] -2*a*b*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)-(a^2*(1-n)-b^2*n)*cos(f*x+e)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+b^2*(d*cos(f*x+e))^n*tan(f*x+e)/f/(1-n)

Rubi [A]

time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4349, 3873, 3857, 2722, 4131}

$$-\frac{(a^2(1-n)-b^2n)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}-\frac{2ab\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\cos(e+fx))^n}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a*b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f
*x]^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) - ((a^2*(1 - n) - b^2*n)*Co
s[e + f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (b
^2*(d*Cos[e + f*x])^n*Tan[e + f*x])/(f*(1 - n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int (d\cos(e+fx))^n (a+b\sec(e+fx))^2\,dx = ((d\cos(e+fx))^n (d\sec(e+fx))^n) \int (d\sec(e+fx))^{-n} (a+b\operatorname{sec}$$

$$= ((d\cos(e+fx))^n (d\sec(e+fx))^n) \int (d\sec(e+fx))^{-n} \left(a^2+b^2\right.$$

$$= \frac{b^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)} + \frac{\left(2ab\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\cos(e+\right.}{a}$$

$$= -\frac{2ab(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} +$$

$$= -\frac{2ab(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} -$$

Mathematica [A]
time = 0.26, size = 161, normalized size = 0.87

$$-\frac{d(d\cos(e+fx))^{-1+n}\csc(e+fx)\left(b^2n(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(e+fx)\right)+a(-1+n)\cos(e+fx)\left(2b(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)+an\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\right)\right)\sqrt{\sin^2(e+fx)}}{f(-1+n)n(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] -((d*(d*Cos[e + f*x])^(-1 + n)*Csc[e + f*x]*(b^2*n*(1 + n)*Hypergeometric2F
1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + a*(-1 + n)*Cos[e + f*x]*(2*
```

```
b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2] + a*n*Cos[
e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]))*Sqr
t[Sin[e + f*x]^2])/(f*(-1 + n)*n*(1 + n)))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (d\cos(fx+e))^n (a+b\sec(fx+e))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
[Out] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*cos(f*x + e))^n
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(e+fx))^n (a+b\sec(e+fx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
[Out] Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\cos\left(e+f\,x\right))^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^2,x)
[Out] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^2, x)
```

3.877 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx$

Optimal. Leaf size=132

$$-\frac{b(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a(d\cos(e+fx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+\right.}{df(1+n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x+
e)/f/n/(sin(f*x+e)^2)^(1/2)-a*(d*cos(f*x+e))^(1+n)*hypergeom([1/2, 1/2+1/2*
n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4310, 16, 2827, 2722}

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)\sqrt{\sin^2(e+fx)}}-\frac{b\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] -((b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])
/(d*f*(1 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx &= \int (d\cos(e+fx))^n(b+a\cos(e+fx))\sec(e+fx)\,dx\\ &= d\int (d\cos(e+fx))^{-1+n}(b+a\cos(e+fx))\,dx\\ &= a\int (d\cos(e+fx))^n\,dx+(bd)\int (d\cos(e+fx))^{-1+n}\,dx\\ &= -\frac{b(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a(\end{aligned}$$

Mathematica [A]
time = 0.08, size = 106, normalized size = 0.80

$$-\frac{(d\cos(e+fx))^n\csc(e+fx)\left(b(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)+an\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\right)\sqrt{\sin^2(e+fx)}}{fn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
[Out] -(((d*Cos[e + f*x])^n*Csc[e + f*x]*(b*(1 + n)*Hypergeometric2F1[1/2, n/2, (
2 + n)/2, Cos[e + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)
/2, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
[Out] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(e+fx))^n (a+b\sec(e+fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\cos(e+f\,x))^n \left(a+\frac{b}{\cos(e+f\,x)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x)),x)
```

```
[Out] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x)), x)
```

3.878 $\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=196

$$\frac{aF_1\left(\frac{1}{2};\frac{1}{2}(-1-n),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}(-1-n)}\sin(e+fx}{(a^2-b^2)\,f}$$

```
[Out] a*AppellF1(1/2,-1/2-1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*co
s(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a^2-b^2)/
f-b*AppellF1(1/2,-1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(d*c
os(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/f/((cos(f*x+e)^2)^(1/2*n))
```

Rubi [A]
time = 0.27, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4349, 3954, 2902, 3268, 440}

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^nF_1\left(\frac{1}{2};\frac{1}{2}(-n-1),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin(e+fx)\cos^2(e+fx)^{-n/2}(d\cos(e+fx))^nF_1\left(\frac{1}{2};-\frac{n}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] (a*AppellF1[1/2, (-1 - n)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(
a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)*S
in[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, -1/2*n, 1, 3/2, Sin[e + f*x
]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(d*Cos[e + f*x])^n*Sin[e + f*x])/((a
^2 - b^2)*f*(Cos[e + f*x]^2)^(n/2))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2902

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
```

```
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = ((d\cos(e+fx))^n(d\sec(e+fx))^n)\int \frac{(d\sec(e+fx))^{-n}}{a+b\sec(e+fx)}\,dx$$
$$= \left(\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{1+n}(e+fx)}{b+a\cos(e+fx)}\,dx$$
$$= -\left(\left(a\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{2+n}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx\right) + \left(b\cos^{-n}(e\right.$$
$$= -\frac{\left(a\cos^{2\left(\frac{1}{2}+\frac{n}{2}\right)-n}(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{-\frac{1}{2}-\frac{n}{2}}\right)\text{Subst}\left(\int \frac{(1-x^2)^{\frac{1+}{2}}}{-a^2+b^2+a^2}\right.}{f}$$
$$= \frac{aF_1\left(\frac{1}{2};\frac{1}{2}(-1-n),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\text{co}}{(a^2-b^2)\,f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5216 vs. $2(196) = 392$.
time = 24.51, size = 5216, normalized size = 26.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(d\cos{(fx+e)})^n}{a+b\sec{(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
[Out] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cos{(e+fx)})^n}{a+b\sec{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e)),x)
[Out] Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\cos(e+fx))^n}{a+\frac{b}{\cos(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n/(a + b/cos(e + f*x)),x)
```

```
[Out] int((d*cos(e + f*x))^n/(a + b/cos(e + f*x)), x)
```

3.879 $\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=309

$$\frac{a^2 F_1\left(\frac{1}{2};\frac{1}{2}(-3-n),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}(-1-n)}\sin(e+f}{\left(a^2-b^2\right)^2 f}$$

```
[Out] a^2*AppellF1(1/2,-3/2-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*
cos(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a^2-b^2
)^2/f+b^2*AppellF1(1/2,-1/2-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-
b^2))*cos(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a
^2-b^2)^2/f-2*a*b*AppellF1(1/2,-1-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2
/(a^2-b^2))*(d*cos(f*x+e))^n*sin(f*x+e)/(a^2-b^2)^2/f/((cos(f*x+e)^2)^(1/2*
n))
```

Rubi [A]

time = 0.36, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4349, 3954, 2903, 3268, 440}

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-3),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-1),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}-\frac{2ab\sin(e+fx)\cos^2(e+fx)^{-n/2}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-2),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*AppellF1[1/2, (-3 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)
*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 - n)/2, 2, 3/2, S
in[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*
x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) - (2*a*
b*AppellF1[1/2, (-2 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(d*Cos[e + f*x])^n*Sin[e + f*x])/((a^2 - b^2)^2*f*(Cos[e + f*x]^
2)^(n/2))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2903

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
```

```
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3954

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4349

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = ((d\cos(e+fx))^n(d\sec(e+fx))^n)\int \frac{(d\sec(e+fx))^{-n}}{(a+b\sec(e+fx))^2}\,dx$$

$$= \left(\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{2+n}(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \left(\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \left(\frac{b^2\cos^{2+n}(e+fx)}{(b^2-a^2\cos^2(e+fx))^2} - \frac{2ab\cos^{3+n}(\right.$$

$$= \left(a^2\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{4+n}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\,dx - \left(2ab\cos\right.$$

$$= \frac{\left(a^2\cos^{2\left(\frac{1}{2}+\frac{n}{2}\right)-n}(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{-\frac{1}{2}-\frac{n}{2}}\right)\mathrm{Subst}\left(\int \frac{(1-x^2)}{(a^2-b^2-}\right.}{f}$$

$$= \frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(-3-n),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))}{\left(a^2-b^2\right)^2 f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 13974 vs. $2(309) = 618$.
time = 42.16, size = 13974, normalized size = 45.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(fx+e))^n}{(a+b\sec(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,\cos\left(e+f\,x\right))^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n/(a + b/cos(e + f*x))^2,x)
```

```
[Out] int((d*cos(e + f*x))^n/(a + b/cos(e + f*x))^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational):  #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational):  #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul):  #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func):  #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func):  #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args)))  #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```